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Heavy Flavor Theory

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Heavy Flavor Theory

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This is a limited review and update of the status of Heavy Flavor Physics. After we review the flavor problem we discuss a number of topics: recent puzzles in purely leptonic D and B decays and their possible resolutions, mixing in neutral B and D mesons, the determination of $|V_{cb}|$ and $|V_{ub}|$ from semileptonic decays, and we conclude with radiative B decays.

1. Introduction: The Flavor Problem

The Standard Model (SM) of electroweak interactions¹ correctly accounts for all known particle physics data. Hints of small anomalies exist but none is firmly established. If there is new physics (NP) it must be hiding, and one good way to hide it is by making it active only at shorter distances than we have yet probed. Future high energy particle collision experiments may directly probe such new short distance physics. We can ask, in the mean time, what are the indirect effects of such new physics at the longer distances that are probed in current experiments? A model independent way to address this question is by supplementing the Lagrangian of the SM with local terms, or “operators,” of dimension greater than four. Such terms render the theory non-renormalizable. Hence a momentum cut-off Λ_{NP} must be introduced and the theory is considered only as an Effective Field Theory (EFT) valid only at energies below Λ_{NP} . Colliding particles with center of mass energy in excess of Λ_{NP} surely produces new states that require further specific modification of the theory.

The scale Λ_{NP} also serves to make the terms in the Lagrangian dimensionally correct. An operator of dimension $n > 4$ appears with coefficient $c/\Lambda_{\text{NP}}^{n-4}$, with c a dimensionless constant. The natural expectation is that c is of order unity. A very large number renders the theory ineffective, breaking down at energies below Λ_{NP} . This does not happen: one simply chooses a smaller number for Λ_{NP} . On the other hand, the coefficients of some operators could be very small. But short of explaining why some coefficients unexpectedly small, we must assume that in fact we underestimated Λ_{NP} . Hence, we may proceed by assuming $c \sim 1$ and see what current data implies for Λ_{NP} .

The EFT generically contains $\Delta F = 2$ FCNCs, that is, terms that induce neutral interactions that change flavor by two units. For example one may include

$$\frac{1}{\Lambda_{\text{NP}}^2} [c_1(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu s_L) + c_2(\bar{u}_L\gamma^\mu c_L)(\bar{u}_L\gamma_\mu c_L)]. \quad (1)$$

Then, if one ignores the SM contribution, neutral meson mixing data gives[1]

$$c_1^{(\text{data})} = (8.8 + 0.033i) \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2, \quad (2)$$

$$c_2^{(\text{data})} = (5.9 + 1.0i) \times 10^{-7} \left(\frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right)^2. \quad (3)$$

There being no reason to expect a cancellation between the SM and NP contributions, the NP contributions should be smaller than $c_i^{(\text{data})}$. Therefore Λ_{NP} should be larger than the electroweak scale by some four orders of magnitude!

This in itself is not a problem. But there is one good reason to expect NP at the electroweak scale. In the SM there are quadratically divergent radiative corrections to the higgs mass. In terms of our cut-off EFT, the shift in the higgs mass from L -loops is of order $\Lambda_{\text{NP}}/(4\pi)^L$ so a counterterm must be fine tuned to one part per mil to cancel this at one loop, and further fine tuned to one per cent at two loops, etc. If instead $\Lambda_{\text{NP}} \sim 1 \text{ TeV}$ there is no need for any fine tuning. This is the Flavor Problem, that NP at the EW scale requires extraordinarily small dimensionless couplings c_i .

Much of the work on Heavy flavor physics aims at testing the SM in the flavor sector with high precision. It gives additional restrictions on NP, that can be described as bounds on additional coefficients of higher dimension operators. The purpose of this talk is to present some of the main results in heavy flavor theory. While it is interesting to investigate models of NP that address the Flavor Problem, a prerequisite is to understand the restrictions that heavy flavors place on the models. In the absence of glaring anomalies in the data, this is best done by verifying consistency of the SM to as high precision as possible. I will focus on precision SM determinations, but will here and there indicate implications on models of NP.

¹Supplemented with right handed neutrinos and masses.

2. Purely Leptonic Decays

2.1. The evanescent f_{D_s} puzzle

The theory of purely leptonic decays is simple,

$$\Gamma(D_s \rightarrow \ell \nu_\ell) = \frac{m_{D_s}}{8\pi} f_{D_s}^2 G_F^2 m_\ell^2 |V_{cs}|^2 (1 - m_\ell^2/m_{D_s}^2)^2 \quad (4)$$

with f_{D_s} the D_s decay constant and V the Kobayashi-Maskawa matrix. There are analogous formulas with obvious modifications when replacing B , B_s or D for D_s . Last year a discrepancy became apparent in the value of f_{D_s} obtained from Monet-Carlo simulations of QCD on the lattice and the one from the experimental measurement of the purely leptonic branching fraction. From a recent compilation of lattice results[2]

$$f_D = 206(4) \text{ MeV}, \quad f_{D_s} = 243(3) \text{ MeV} \quad (5)$$

while experimentally[3, 4]

$$f_D = 205.8(8.5)(2.5) \text{ MeV}, \quad f_{D_s} = 275(16)(12) \text{ MeV} \quad (6)$$

While this anomaly was not firmly established, the agreement between lattice and experiment in the value of f_D suggests the discrepancy in f_{D_s} may well remain once the errors are reduced.

Perhaps for this reason several groups have looked for a viable interpretation of this result in terms of NP[5, 6, 7]. In the SM this is a Cabibbo allowed, tree level decay. Hence for the NP to have a significant effect it must be neither loop nor Cabibbo suppressed. Moreover the mass M of the new particle mediating this interaction cannot be too large: for constructive interference the amplitude should be about 6% of the SM's, so roughly $M \approx M/\sqrt{0.06} = 320 \text{ GeV}$. Dobrescu and Kleinfeld argued that (i) s -channel charged higgs exchange could explain the effect, with $y_s \ll y_c$ and both y_c and y_τ of order unity, but then found this explanation disfavoured by D decay data (ii) t -channel charge $+2/3$ leptoquark exchange could also account for the data but is disfavoured by the bound on $\tau \rightarrow \bar{s}s\mu$ (iii) u -channel charge $-1/3$ leptoquark exchange (d -squark like object, \tilde{d}) is a viable explanation. They introduce the interaction

$$\mathcal{L}_{LQ} = \kappa_\ell (\bar{c}_L \ell_L^c - \bar{s}_L \nu_{\ell L}^c) \tilde{d} + \kappa'_\ell \bar{c}_R \ell_R^c \tilde{d} + \text{h.c.}, \quad (7)$$

which is already present in supersymmetric extensions of the SM without R parity, and show that for $|\kappa'_\ell/\kappa_\ell| \ll m_\ell m_c/m_{D_s}^2$ the resulting interference is automatically constructive. In addition, if $|\kappa_\ell| \approx |\kappa_\tau|$ the deviations in $\mu\nu$ and $\tau\nu$ are approximately equal.

Earlier this year the CLEO collaboration published new results on D_s purely leptonic decays to both $\mu\nu$ and $\tau\nu$ final states[8]. Their value for the decays constant, $f_{D_s} = 259.5(6.6)(3.1) \text{ MeV}$ has significantly reduced errors but also has moved significantly in the direction of eliminating the anomaly.

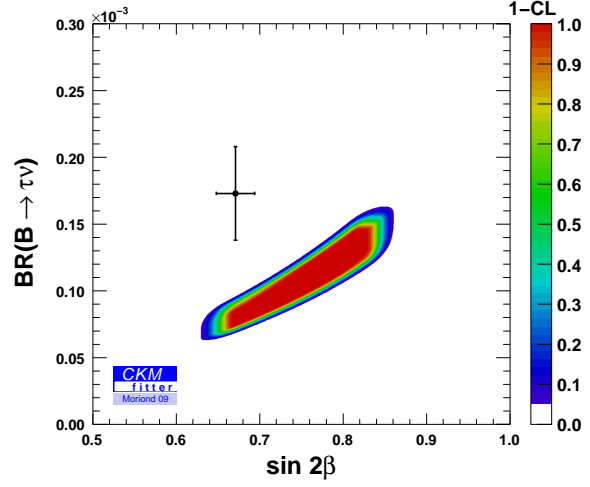


Figure 1: Tension in the CKM global fit from $B \rightarrow \tau\nu$ [9]. The cross corresponds to the experimental values with 1 sigma uncertainties. The shaded region is the result of the global fit performed without using these measurements.

2.2. $B \rightarrow \tau\nu$

The direct determination of the branching fraction for $B \rightarrow \tau\nu$ and $\sin 2\beta$, which measures the CP asymmetry in interference between mixing and decay in $B \rightarrow J/\psi K_s$, is in slight disagreement with a global fit to these quantities from other measurements. This is presented in Fig. 1 which shows both the result of the direct measurements (as a cross) and that of the global fit (as a shaded area).

That there is a correlation between these quantities can be seen from the following formula, obtained from the expression for the $B^0 - \bar{B}^0$ mass difference² Δm_d , by eliminating the decay constant f_B in favor of the purely leptonic branching fraction:

$$\frac{\text{Br}(B \rightarrow \tau\nu)}{\Delta m_d} = \frac{3\pi m_\tau^2 (1 - \frac{m_\tau^2}{m_B^2})^2 \tau_{B^+}}{4m_W^2 S(x_t) |V_{ud}|^2} \frac{1}{B_{B_d}} \left(\frac{\sin \beta}{\sin \gamma} \right)^2. \quad (8)$$

Here $S(x_t)$ is an Inami-Lim function with the top quark mass as argument, and B_{B_d} is the ‘‘bag’’ constant that parametrizes the matrix element of the short distance four-quark operator that gives rise to mixing. The main uncertainties come from B_{B_d} and the angles α and γ .

The deviation is 2.4 sigmas if one compares the indirect fit prediction for the $B \rightarrow \tau\nu$ branching fraction with the direct measurement. Alternatively one can use the fit to determine B_{B_d} and compare with the value determined from Monte-Carlo simulations

²See next section.

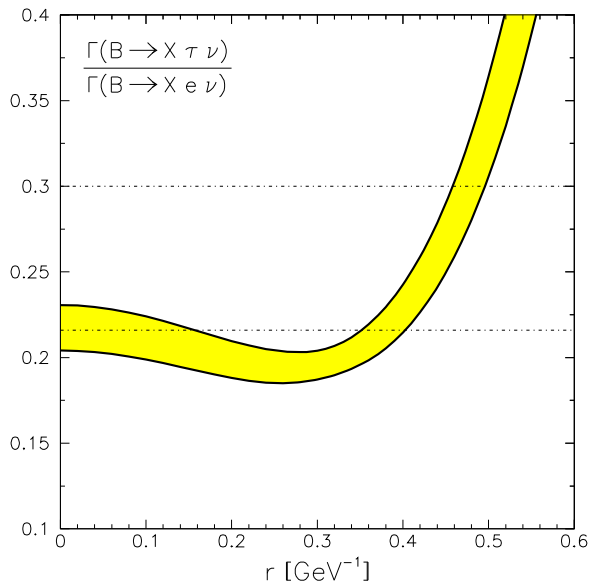


Figure 2: The shaded area gives the predicted ratio of B semileptonic branching fractions into τ vs. electron, as a function of $r = \tan \beta / M_{H^+}$, in a two higgs doublet model[12]. The dashed-dotted lines give experimental 1σ bounds.

of QCD on the lattice. Then the deviation is 2.7 sigmas.

As in the D_s decay case this is a tree level process, but in contrast, this is Cabibbo suppressed, which gives a bit more leeway in giving a NP interpretation. In a two higgs doublet model tree level charged higgs exchange can easily modify the leptonic branching fraction[10, 11]. In this model the Cabibbo suppression is still present but there is an amplification factor proportional to $\tan^2 \beta = (v_2/v_1)^2$, where v_1 and v_2 are the vacuum expectation values of the higgs doublets responsible for up and down masses, respectively. The parameter that controls the NP correction is $r = \tan \beta / M_{H^+}$. The semileptonic branching fraction will also be affected, and a sensitive test of this interpretation is the ratio of semileptonic decay branching fractions into τ versus electrons[12]; see Fig. 2.

3. Neutral Meson Mixing

3.1. Generalities

The recent measurements of mixing of neutral D mesons and their unfamiliar properties suggest we begin our discussion by a taking a general look at neutral meson mixing. Let us briefly review the quantities that enter the description of neutral meson mixing. Parameters p and q are introduced to express the physical states in terms of flavor eigenstates: $|P_{L,H}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle$. These, together with

the mass and width differences, $\Delta m = m_H - m_L$ and $\Delta \Gamma = \Gamma_H - \Gamma_L$, are given in terms of the off-diagonal elements of the 2×2 non-hermitian hamiltonian $M - \frac{i}{2}\Gamma$ by

$$\begin{aligned} (\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2, \\ \Delta m \Delta \Gamma &= 4\text{Re}(M_{12}\Gamma_{12}^*), \\ \frac{q^2}{p^2} &= \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}. \end{aligned} \quad (9)$$

We expect M_{12} rather than Γ_{12} to be more prone to modifications from new physics because Γ is largely given by long distance physics. Finally, the decay amplitudes are

$$A_f = \langle f | \mathcal{H} | P^0 \rangle \quad \text{and} \quad \bar{A}_f = \langle f | \mathcal{H} | \bar{P}^0 \rangle. \quad (10)$$

CP violation (CPV) can be searched through processes that probe different quantities. If $|\bar{A}_f/A_f| \neq 1$ there is CPV in decays, while $|q/p| \neq 1$ gives CPV in mixing. A non-vanishing imaginary part of $\lambda_f = (q/p)(\bar{A}_f/A_f)$ gives CPV in the interference between mixing and decay. The phase $\phi_{12} = \arg(-M_{12}/\Gamma_{12})$ is sensitive to NP that may show up in M_{12} . The parameter that controls the di-lepton asymmetry is $\text{Im}\Gamma_{12}/M_{12} = (1 - |q/p|^4)/(1 + |q/p|^4)$; it is non-perturbative and hence difficult to compute (the OPE is no better that for lifetimes, perhaps worse).

The behavior of the mixing system depends rather sensitively on which of Δm and $\Delta \Gamma$ is largest. Consider first the case $\Delta m \gg \Delta \Gamma$ which is the situation for B and B_s mesons. This condition corresponds to small Γ_{12}/M_{12} and one can find approximate solutions of (9): $\Delta m = 2|M_{12}|(1 + \dots)$ and $\Delta \Gamma = -2\Gamma_{12} \cos \phi_{12}(1 + \dots)$, where the ellipsis indicate corrections of order Γ_{12}/M_{12} . Keeping in mind that ϕ_{12} is suppressed in the SM, we see that the effects of NP can only reduce the magnitude of $\Delta \Gamma$. On the other hand since $q/p = -\arg(M_{12})(1 + \dots)$ time dependent CP asymmetries are sensitive to NP that may show up in M_{12} .

The other extreme case has $\Delta \Gamma \gg \Delta m$ and this condition corresponds to small M_{12}/Γ_{12} . As before, one can find approximate solutions of (9): $\Delta m = 2|M_{12} \cos \phi_{12}|(1 + \dots)$, $\Delta \Gamma = \mp 2\Gamma_{12}(1 + \dots)$ and $q/p = -\arg(\Gamma_{12})(1 + \dots)$ depends weakly on M_{12} (the ellipsis now indicate corrections of order M_{12}/Γ_{12}). For example, if D -mesons satisfy $\Delta \Gamma \gg \Delta m$ and there in negligible CPV in the decay, then $\arg \lambda_{K^+K^-} \propto |M_{12}/\Gamma_{12}|^2 \sin(2\phi_{12})$. Hence there is reduced sensitivity to NP in M_{12} , even for dominant NP.

3.2. $B^0\bar{B}^0$ and $B_s\bar{B}_s$

CPV in B decays and mixing is discussed at length in other talks at this conference. In order to avoid unnecessary duplication we limit ourselves to discussing the mass difference measurements and theory.

In the SM B^0 and B_s mixing have the same underlying dynamics (double W exchange, with virtual top-antitop quark intermediate state). Hence the expressions³ for Δm are virtually identical, except for obvious change of parameters. Taking the ratio not just simplifies the expressions but also cancels some uncertainties. Solving for the ratio of KM elements

$$\frac{|V_{ts}|}{|V_{td}|} = \xi \sqrt{\frac{\Delta m_s m_{B_s}}{\Delta m_d m_{B_d}}}, \quad \text{where} \quad \xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}. \quad (11)$$

Here f_B and B_B are the decay and bag constants (see (8)) introduced in previous sections. The quantity ξ , required to extract a value for the ratio of KM elements, contains all of the hard to estimate hadronic physics and has the property that it is unity in the $SU(2)_v$ symmetry limit ($m_d = m_s$). Monte-Carlo simulations of lattice QCD give

$$\begin{aligned} \xi &= 1.205(52) && \text{FNAL/MILC[14],} \\ \xi &= 1.258(25)(21) && \text{HPQCD[13].} \end{aligned} \quad (12)$$

Using Belle and BaBar accurate measurements of Δm_d and CDF and D0 measurements of Δm_s , Evans reports[15]

$$\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0012(\text{exp})_{-0.0060}^{+0.0081}(\text{theor}). \quad (13)$$

I hasten to point out that Evans uses a value for ξ reported in 2003[16], one of the early unquenched calculations. Luckily, this value interpolates between the two more recent results, $\xi = 1.210^{(+47)}_{(-35)}$, and the errors are also similar. Rather than fussing over the best central value and theoretical error let us stop to think how well we can trust lattice calculations with this extraordinary, $\sim 3\%$ precision. In fact, the precision of the calculation is only about 16% since only the deviation of ξ^2 from unity needs be computed. Some other rather crude methods should therefore work rather well too. For example, a very early computation of the chiral logs gives $\xi = 1.14$ [17]. If we estimate the errors by $\Delta \xi^2 \sim (m_K/\Lambda_\chi)^2 \approx 20\%$, we would write $\xi = 1.14 \pm 0.08$, or a 7% error. Remarkably, this crude determination agrees with the lattice result, within expected errors!

3.3. $D^0\bar{D}^0$

In the two years that followed the first evidence for $D^0\bar{D}^0$ oscillation by BaBar[18] and Belle[23], these two collaborations have reported a number of related and improved measurements[19, 20, 21, 22, 24, 25,

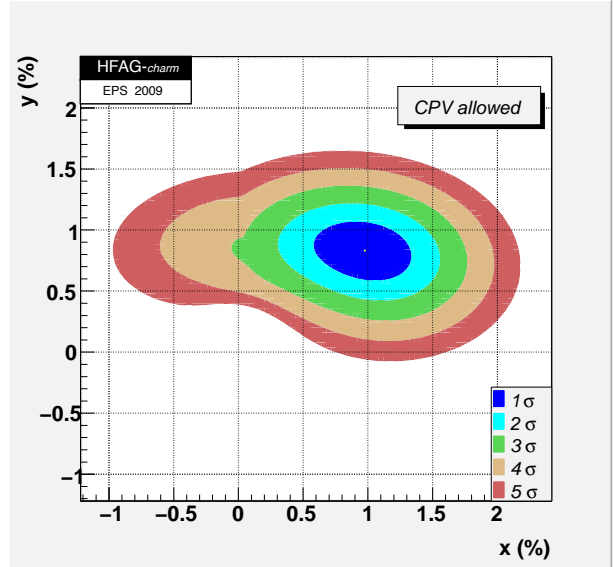


Figure 3: Contours of allowed values of the parameters $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/\Gamma$ from an HFAG analysis of the Belle and BaBar data on D^0 mixing and decays[27].

26]. It is customary to introduce $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/\Gamma$. The experimental situation is summarized in Fig. 3, which shows the allowed region in the x, y plane.

The two- W exchange graph that gives $\Delta F = 2$ processes in the SM has, in the $D\bar{D}$ case, an intermediate $q\bar{q}'$ state with $q, q' = d, s, b$. Since these are light compared to the W , GIM suppression is very effective in x and y . Therefore x, y are small. But they are not perturbatively calculable. They are probably dominated by the same long distance physics so x may well be comparable in size to y . Moreover, y has a Cabibbo suppression factor $\sin^2\theta_C$ and vanishes in the $SU(2)_v$ limit ($m_s = m_d$).

The experimental measurement $x \approx y \approx 1\%$ is very compatible with SM expectations. However, since we can't compute accurately, precision tests of the SM are not possible. Still one can use the measured values of x and y to constrain models of NP. M_{12} is more sensitive to NP because, in the SM, it starts at 1-loop while Γ_{12} starts at tree level. Moreover, NP is short distance dominated. Excluding a possible perverse cancellation between the short and long distance contributions to M_{12} we can then restrict the NP by demanding that its contribution be no larger than what is measured. We have already alluded to this in a model independent way in (3). The implications for many specific models have been studied, including the MSSM and a sequential fourth generation of quarks [28, 29]. The MSSM can evade the bound by the same mechanisms that are available for the other flavor problems of that model: either make the SUSY breaking scale uncomfortably high or find a mechanism, like gauge mediation, that implements Minimal Flavor Violation in

³Which are rather involved, witness Eq. (8).

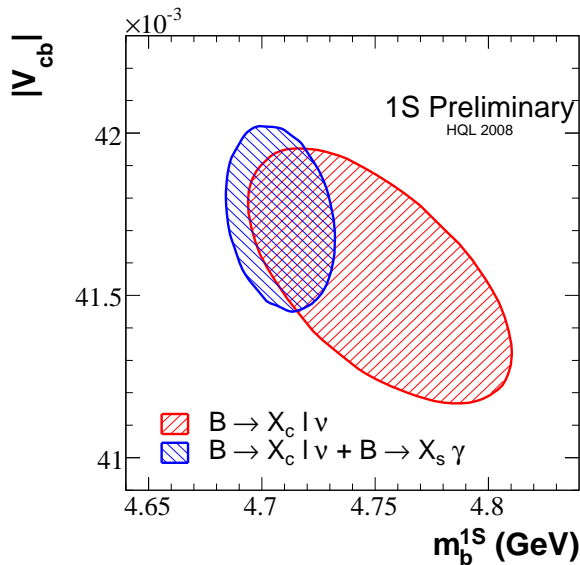


Figure 4: Results of the t combining all available moment data, from Ref. [40]. The $\Delta\chi = 1$ contours are shown for the t with and without $B \rightarrow X_s\gamma$ data. The analysis is performed assuming a 1S (bottom) subtraction scheme.

the MSSM. Because it must be very heavy, a fourth generation quark in the two- W exchange graph does not have an effective GIM mechanism. But its contribution can be suppressed by small KM (4×4) matrix elements, $|V_{ub'}V_{cb'}| \sim 0.001$ for $m_{b'} \sim \text{few } 100\text{'s GeV}$.

4. Determination of $|V_{cb}|$

This section and the next are concerned with the precision determination of the magnitudes of the two KM elements V_{cb} and V_{ub} . It goes without saying that a precise determination is fundamental in testing the KM theory of CP violation in the SM. Traditionally these magnitudes are determined from the semileptonic decay rate of B mesons. Hence these sections are largely about the theory of semileptonic B decay, and we separate each one into inclusive and exclusive sub-sections since the theory is different.

4.1. Inclusive

The theory of inclusive semileptonic B decay is based on a double expansion, combining the heavy mass expansion of Heavy Quark Effective Theory (HQET) with the Operator Product Expansion (OPE)[30, 31, 32]. The decay rate can then be expressed as an expansion, $\Gamma = \Gamma_0 + \Gamma_1 + \dots$, where $\Gamma_n \sim (\Lambda_{\text{QCD}}/m_b)^n$. The main results are that Γ_0 is actually given by the perturbative expression for the semileptonic decay rate for an unconfined b quark and that $\Gamma_1 = 0$.

The method of moments gives a very accurate determination of $|V_{cb}|$ from inclusive semileptonic B decays. In QCD, the rate $d\Gamma(B \rightarrow X_c\ell\nu)/dx dy = |V_{cb}|^2 f(x, y)$, where x and y are the invariant lepton pair mass and energy in units of m_B , is given in terms of four parameters: $|V_{cb}|$, α_s , m_c and m_b . $|V_{cb}|$, which is what we are after, drops out of normalized moments, $\langle z^n \rangle = \int dx dy f(x, y) z^n / \int dx dy f(x, y)$ where $z = x$ or y . Since α_s is well known, the idea is to fix m_c and m_b from normalized moments and then use them to compute the normalization, hence determining $|V_{cb}|$. In reality we cannot solve QCD to give the moments in terms of m_c and m_b , but we can use the combined HQET/OPE to write the moments in terms of m_c , m_b and a few constants that parametrize our ignorance[33, 34, 35, 36, 37, 38]. These constants are in fact matrix elements of operators in the HQET/OPE. If terms of order $1/m_Q^3$ are retained in the expansion one needs to introduce five such constants; and an additional two are determined by meson masses. All five constants and two quark masses can be over-determined from a few normalized moments that are functions of E_{cut} , the lowest limit of the lepton energy integration. The error in the determination of $|V_{cb}|$ is a remarkably small 2% [39]; see Fig. 4. But even most remarkable is that this estimate for the error is truly believable. It is obtained by assigning the last term *retained* in the expansion to the error, as opposed to the less conservative guessing of the next order *not* kept in the expansion. Since there is also a perturbative expansion, the assigned error is the combination of the last term kept in all expansions, of order $\beta_0\alpha_s^2$, $\alpha_s\Lambda_{\text{QCD}}/m_b$ and $(\Lambda_{\text{QCD}}/m_b)^3$.

There is only one assumption in the calculation that is not fully justified from first principles. The moment integrals can be computed perturbatively (in the $1/m_Q$ expansion) only because the integral can be turned into a contour over complex energy E away from the physical region[30]. However, the contour is pinned at the minimal energy, E_{cut} , on the real axis, right on the physical cut. So there is a small region of integration where quark-hadron duality cannot be justified and has to be invoked. Parametrically this region of integration is small, a fraction of order Λ/m_Q of the total. But this is a disaster because this is parametrically much larger than the claimed error of order $(\Lambda/m_Q)^3$. However, this is believed not to be a problem. For one thing, the fits to moments as functions of E_{cut} are extremely good: the system is over-constrained and these internal checks work. And for another, it has been shown[41] that duality works exactly in the Shifman-Voloshin (small velocity) limit, to order $1/m_Q^2$. It seems unlikely that the violation to local quark-hadron duality mainly changes the normalization and has mild dependence on E_{cut} , and that this effect only shows up away from the SV limit.

4.2. Exclusive

The exclusive determination of $|V_{cb}|$ is in pretty good shape theoretically, and only last year has become competitive with the inclusive one. So it provides a sanity check, but not an improvement. The semileptonic rates into either D or D^* are parametrized by functions \mathcal{F} , \mathcal{F}_* , of the rapidity of the charmed meson in the B rest-frame, w . Luke's theorem[42] states $\mathcal{F} = \mathcal{F}_* = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_c)^2$ at $w = 1$. The rate is measured at $w > 1$ and extrapolated to $w = 1$. The extrapolation is made with a first principles calculation to avoid introducing extraneous errors[43]. The resulting determination of $|V_{cb}|$ has a 4% error equally shared by theory and experiment; the theory error is dominated by the uncertainty in the determination of \mathcal{F} , \mathcal{F}_* at $w = 1$. FNAL/MILC combines the 2008 PDG average for $|V_{cb}|\mathcal{F}_*(1)$ with their computed value $\mathcal{F}_*(1) = 0.921(13)(20)$ to obtain, after applying a small electromagnetic correction, $|V_{cb}| = (38.7 \pm 0.9_{\text{exp}} \pm 1.0_{\text{theor}}) \times 10^{-3}$ [44].

There is some tension between theory and experiment in these exclusive decays that needs attention. The ratios of form factors $R_{1,2}$ are at variance from theory by three and two sigma respectively[45]. Also, in the heavy quark limit the slopes ρ^2 of \mathcal{F} and \mathcal{F}_* should be equal. One can estimate symmetry violations and obtains[46] $\rho_{\mathcal{F}}^2 - \rho_{\mathcal{F}_*}^2 \simeq 0.19$, while experimentally this is -0.22 ± 0.20 , a deviation in the opposite direction. This is a good place for the lattice to make post-dictions at the few percent error level that may lend it some credibility in other areas where it is needed to determine a fundamental parameter.

5. Determination of $|V_{ub}|$

The magnitude $|V_{ub}|$ determines the rate for $B \rightarrow X_u \ell \nu$. The well known experimental difficulty is that since $|V_{ub}| \ll |V_{cb}|$ the semileptonic decay rate is dominated by charmed final states. To measure a signal it is necessary to either look at exclusive final states or suppress charm kinematically. The interpretation of the measurement requires, in the exclusive case, knowledge of hadronic matrix elements parametrized in terms of form-factors, and for inclusive decays, understanding of the effect of the kinematic cuts on the perturbative expansion and quark-hadron duality.

5.1. Inclusive

This has been the method of choice until recently, since it was thought that the perturbative calculation was reliable and systematic and hence could be made sufficiently accurate. However it has become increasingly clear of late that the calculation cannot be made arbitrarily precise. The method uses effective field

theories to expand the amplitude systematically in inverse powers of a large energy, either the heavy mass or the energy of the up-quark (or equivalently, of the hadronic final state). One shows that in the restricted kinematic region needed for experiment (to enhance the up-signal to charm-background) the inclusive amplitude is governed by a non-perturbative ‘‘shape function,’’ which is, however, universal: it also determines other processes, like the radiative $B \rightarrow X_s \gamma$. So the strategy has been to eliminate this unknown, non-perturbative function from the rates for semileptonic and radiative decays.

Surprisingly, most analysis do not eliminate the shape function dependence between the two processes. Instead, practitioners commonly use parametrized fits that unavoidably introduce uncontrolled errors. It is not surprising that errors quoted in the determination of $|V_{ub}|$ are smaller if by a parametrized fit than by the elimination method of [47]. The problem is that parametrized fits introduce systematic errors that are unaccounted for.

Parametrized fits aside, there is an intrinsic problem with the method. Universality is violated by sub-leading terms[48] in the large energy expansion (‘‘sub-leading shape functions’’). One can estimate this uncontrolled correction to be of order $\alpha_s \Lambda/m_b$, where Λ is hadronic scale that characterizes the sub-leading effects (in the effective theory language: matrix elements of higher dimension operators). We can try to estimate these effects using models of sub-leading shape functions but then one introduces uncontrolled errors into the determination. At best one should use models to estimate the errors. I think it is fair, albeit unpopular, to say that this method is limited to a precision of about 15%: since there are about 10 sub-leading shape functions, I estimate the precision as $\sqrt{10} \alpha_s \Lambda/m_b$. This is much larger than the error commonly quoted in the determination of $|V_{ub}|$.

This is just as well, since the value of $|V_{ub}|$ from inclusions is in disagreement not only with the value from exclusives but also with the global unitarity triangle fit. You can quantify this if you like, but it is graphically obvious from Fig. 5. The location of the apex of the unitarity triangle differs in the two panels, and the agreement would be much better if the green ring, whose radius is given by $|V_{ub}/V_{cb}|$ and is dominated in the fit by the determination from the inclusive decay, had smaller radius.

5.2. Exclusives

The branching fraction $\text{Br}(B \rightarrow \pi \ell \nu)$ is known[49] to 8%. A comparable determination of $|V_{ub}|$ requires knowledge of the $B \rightarrow \pi$ form factor $f_+(q^2)$ to 4%. There are some things we do know about f_+ : (i) The shape is constrained by dispersion relations[50]. This means that if we know f_+ at a few well spaced points

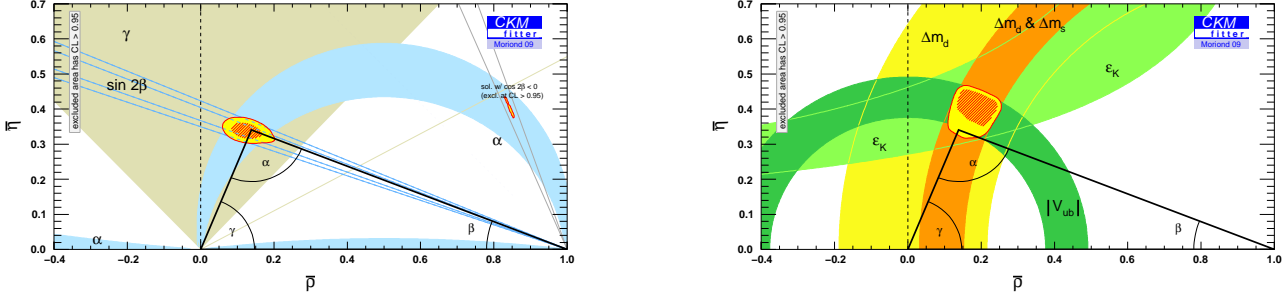


Figure 5: Latest CKMfitter fit of data on the $\rho-\eta$ plane describing the unitarity triangle[9]. The left panel includes only measurements of CPV angles, while the right panel excludes them. The green ring, $|V_{ub}|$, is dominated by the inclusive semileptonic determination.

we can pretty much determine the whole function f_+ (ii) We can get a rough measurement of the form factor at $q^2 = m_\pi^2$ from the rate for $B \rightarrow \pi\pi$ [51]. This requires a sophisticated effective theory (SCET) analysis which both shows that the leading order contains a term with $f_+(m_\pi^2)$ and systematically characterizes the corrections to the lowest order SCET. It is safe to assume that this determination of $f_+(m_\pi^2)$ will not improve beyond the 10% mark.

Lattice QCD can determine the form factor, at least over a limited region of large q^2 . The experimental and lattice measurements can be combined using constraints from dispersion relations and unitarity[53]. Because these constraints follow from fundamentals, they do not introduce additional uncertainties. They improve the determination of $|V_{ub}|$ significantly. The lattice determination is for the q^2 -region where the rate is smallest. This is true even if the form factor is largest there, because in that region the rate is phase space suppressed. But a rough shape of the spectrum is experimentally observed, through a binned measurement[49], and the dispersion relation constraints allows one to combine the full experimental spectrum with the restricted- q^2 lattice measurement. The best lattice calculations are in good agreement; however use the same MILC ensembles[52]. They give 3.55(25)(50) and 3.38(36) for $10^3|V_{ub}|$. The 11% error in $|V_{ub}|$ is dominated by lattice errors.

5.3. Alternatives

Exclusive and inclusive determinations of $|V_{ub}|$ have comparable precisions. Neither is very good and the prospect for significant improvement is limited. Other methods need be explored, if not to improve on existing $|V_{ub}|$ to lend confidence to the result. A lattice-free method would be preferable. A third method, proposed a while ago[54], uses the idea of double ratios[55] to reduce hadronic uncertainties. Two independent approximate symmetries protect double ratios from deviations from unity, which are therefore

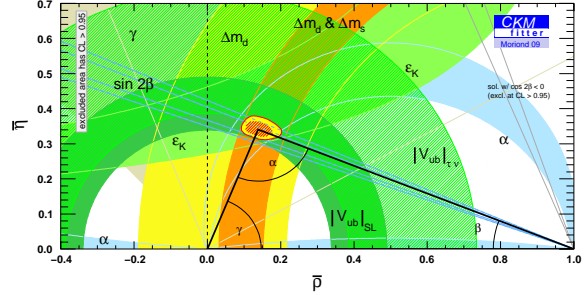


Figure 6: CKMfitter constraints in the $\rho-\eta$ plane including the most recent inputs in the global CKM fit[9]. The $|V_{ub}|$ constraint has been split in two contributions: from inclusive and exclusive semileptonic decays (plain dark green) and from $B^+ \rightarrow \tau\nu$ (hashed green). The red hashed region of the global combination corresponds to 68% CL.

of the order of the product of two small symmetry breaking parameters. For example, the double ratio $(f_{B_s}/f_{B_d})/(f_{D_s}/f_{D_d}) = (f_{B_s}/f_{D_s})/(f_{B_d}/f_{D_d}) = 1 + \mathcal{O}(m_s/m_c)$ because $f_{B_s}/f_{B_d} = f_{D_s}/f_{D_d} = 1$ by $SU(3)$ flavor, while $f_{B_s}/f_{D_s} = f_{B_d}/f_{D_d} = \sqrt{m_c/m_b}$ by heavy flavor symmetry. One can extract $|V_{ub}/V_{ts}V_{tb}|$ by measuring the ratio,

$$\frac{d\Gamma(\bar{B}_d \rightarrow \rho\ell\nu)/dq^2}{d\Gamma(\bar{B}_d \rightarrow K^*\ell^+\ell^-)/dq^2} = \frac{|V_{ub}|^2}{|V_{ts}V_{tb}|^2} \cdot \frac{8\pi^2}{\alpha^2} \cdot \frac{1}{N(q^2)} \cdot R_B, \quad (14)$$

where q^2 is the lepton pair invariant mass, and $N(q^2)$ is a known function[56]. When expressed as functions of the rapidity of the vector meson, $y = E_V/m_V$, the ratios of helicity amplitudes

$$R_B = \frac{\sum_\lambda |H_\lambda^{B \rightarrow \rho}(y)|^2}{\sum_\lambda |H_\lambda^{B \rightarrow K^*}(y)|^2}, \quad R_D = \frac{\sum_\lambda |H_\lambda^{D \rightarrow \rho}(y)|^2}{\sum_\lambda |H_\lambda^{D \rightarrow K^*}(y)|^2}, \quad (15)$$

are related by a double ratio: $R_B(y) = R_D(y)(1 + \mathcal{O}(m_s(m_c^{-1} - m_b^{-1})))$. This measurement could be done today: CLEO has accurately measured the required semileptonic D decays[57, 58].

More methods are available if we are willing to use rarer decays. To extract $|V_{ub}|$ from $\text{Br}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.8 \pm 0.6) \times 10^{-4}$ [59] one needs a lattice determination of f_B . This was discussed in Sec. 2.2: although the determination is still imprecise and relies on lattice hadronic parameters, it gives an even larger value for $|V_{ub}|$; see Fig. 6 which, however, uses as input $\text{Br}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.73 \pm 0.35) \times 10^{-4}$, with substantially lower errors than in the combined BaBar-Belle result of Ref. [59]. Since we want to move away from relying on non-perturbative methods (lattice) to extract $|V_{ub}|$ we have proposed a cleaner but more difficult measurement, the double ratio

$$\frac{\frac{\Gamma(B_u \rightarrow \tau \nu)}{\Gamma(B_s \rightarrow \ell^+ \ell^-)}}{\frac{\Gamma(D_d \rightarrow \ell \nu)}{\Gamma(D_s \rightarrow \ell \nu)}} \sim \frac{|V_{ub}|^2}{|V_{ts} V_{tb}|^2} \cdot \frac{\pi^2}{\alpha^2} \cdot \left(\frac{f_B/f_{B_s}}{f_D/f_{D_s}} \right)^2. \quad (16)$$

In the SM $\text{Br}(B_s \rightarrow \mu^+ \mu^-) \approx 3.5 \times 10^{-9} \times (f_{B_s}/210 \text{ MeV})^2 (|V_{ts}|/0.040)^2$ is the only presently unknown quantity in the double ratio and is expected to be well measured at the LHC [60].

The ratio $\Gamma(B^+ \rightarrow \tau^+ \nu)/\Gamma(B_d \rightarrow \mu^+ \mu^-)$ gives us a fifth method. It has basically no hadronic uncertainty, since the hadronic factor $f_B/f_{B_d} = 1$, by isospin. It involves $|V_{ub}|^2/|V_{td} V_{tb}|^2$, an unusual combination of CKMs. In the $\rho - \eta$ plane it forms a circle centered at $\sim (-0.2, 0)$ of radius ~ 0.5 . Of course, measuring $\Gamma(B_d \rightarrow \mu^+ \mu^-)$ is extremely hard.

In a sixth method one studies wrong charm decays $\bar{B}_{d,s} \rightarrow \bar{D}X$ (really $b\bar{q} \rightarrow u\bar{c}$). This can be done both in semi-inclusive decays [61] (an experimentally challenging measurement) or in exclusive decays [62] (where an interesting connection to $B_{d,s}$ mixing matrix elements is involved).

6. Rare Radiative B decays.

The rare decays $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$ are flavor changing neutral processes that occur first at one loop in the SM. As such they are sensitive probes of NP. They are complimentary to other FCNCs and put stringent bounds on NP models beyond what is obtained from neutral meson mixing measurements. Here I will focus on the radiative decay, $B \rightarrow X_s \gamma$ both because of its higher rate and because of recent progress in theory. Both the total rate and CP violating asymmetries can probe NP. The average of experimental measurement of the rate is rather precise [63],

$$\text{Br}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} = (355 \pm 24_{-10}^{+9} \pm 3) \times 10^{-6}. \quad (17)$$

Here we have indicated that the measured rate is only for energetic photons, $E_\gamma > 1.6 \text{ GeV}$. The combination of data requires some mild extrapolation since measurements have differing photon energy cuts. BaBar recently reported a rather strong constraint in

the CPV asymmetry, $0.033 < A_{\text{CP}}(B \rightarrow K \gamma) < 0.028$ [64].

The theory of $B \rightarrow X_s \gamma$ has two parts. The first one is the computation of the low energy effective Hamiltonian. This is necessary in order to re-sum large logarithms, $\ln(m_t/m_b)$ or $\ln(M_W/m_b)$, in the perturbative expansion. The second step is the computation of the rate from this effective Hamiltonian.

The effective Hamiltonian, to lowest order in an expansion in G_F , is $\mathcal{H} = -(4G_F/\sqrt{2})V_{ts}^* V_{tb} \sum_{i=1}^8 C_i(\mu) Q_i$. The Q_i are dimension 6 $\Delta B = -\Delta S = 1$ operators. Roughly they are the tree level four quark operator and the one it mixes pronouncedly with, $Q_1 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$, $Q_2 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)$, four penguin operators, $Q_{3-6} = (\bar{s} \Gamma_a b) \sum_q (\bar{q} \Gamma'_a q)$ (with Γ matrices in color and spinor space), and two transition magnetic moment operators $Q_7 = (em_b/16\pi^2) \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$ and $Q_8 = (g_s m_b/16\pi^2) \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a$. The problem is to compute reliably the coefficients at a low renormalization scale, $\mu \sim m_b$. This requires computation of the coefficients at a short distance scale, $\mu \sim M_W$, and then using the renormalization group to “run” the coefficients to the physical scale, $\mu \sim m_b$, for which the anomalous dimensions of the operators needs to be computed. The leading logarithms (LO) were first summed 20 years ago [65, 66]. The result is a correction of more than 30% to the un-resummed coefficient $C_7(m_b)$ (and therefore a whopping 60% effect in the rate). To achieve accuracy comparable with present experimental measurement it is important to re-sum the next-to-leading (NLO) and the next-to-NLO (NNLO) logs. This is a challenging enterprise that has taken the better part of two decades. The NNLO calculation requires two loop matching of C_{1-6} [67], three loop matching of C_{7-8} [68], three-loop calculation of the $(1-6) \times (1-6)$ and $(7-8) \times (7-8)$ blocks of the anomalous dimension matrix [69], and four-loop of the $(1-6) \times (7-8)$ block [70]. The magnitude of the NNLO coefficients are, roughly, $|C_{1,2}(m_b)| \sim 1$, $|C_{3,4,5,6}(m_b)| < 0.07$, $C_7(m_b) \simeq -0.3$ and $C_8(m_b) \simeq -0.15$.

The second step, the computation of the rate from this effective Hamiltonian is no smaller a feat. To match the accuracy of the NNLO coefficients one needs a perturbative calculation of the matrix elements of Q_{1-6} to three-loops [71] and of Q_{7-8} to two-loops [72, 73, 74]. The calculation is not complete, some interference terms are missing; see Ref. [75] for a detailed account and for an account of non-perturbative effects.

The SM prediction for the branching fraction restricted to energetic photons $E_\gamma > 1.6 \text{ GeV}$ is [76]

$$\text{Br}(B \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4} \quad (18)$$

in magnificent agreement with the experimental average Eq. (17).

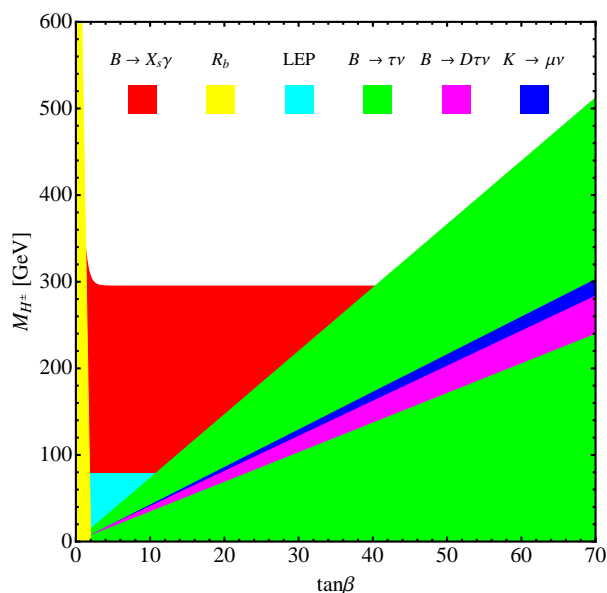


Figure 7: Direct and indirect bounds on M_{H^\pm} in the Two Higgs Doublet Model, type II, as a function of $\tan \beta$, from Ref. [77]. The shaded areas are excluded by the constraints at 95% CL.

The effectiveness of this process in limiting models of NP is nicely illustrated in Fig. 7 which shows the constraints form a variety of measurements on a two higgs doublet model of type II (charge-2/3 quarks get masses from the vacuum expectation value (VEV) of one higgs, charge-1/3 quarks form the VEV of the other higgs) in the $(M_{H^\pm}, \tan \beta)$ plane. Several measurement nicely compliment each other, but it is clear that radiative B decays plays a leading role in excluding parameter space.

We close with a couple of remarks on $B \rightarrow X \ell \ell$. The NNLO calculation in the SM requires one less loop than $B \rightarrow X \gamma$ so it has been complete for quite some time. Much attention has been given to the observation that the forward-backward asymmetry in $B \rightarrow K^* \ell \ell$ has a zero in the SM[78]. The presence and location of the zero suffer little from hadronic uncertainties and from contamination from non-resonant $B \rightarrow K \pi \ell \ell$ decays[79]. Less well known is the fact that at large invariant lepton-pair mass $B \rightarrow K^* \ell \ell$ is well understood and that tests of the SM can be done, largely free of hadronic uncertainties, by a method of double ratios[56] as described in Sec. 5.3. Unfortunately experimentalists have not conducted this test, even if the data is available.

Acknowledgments

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