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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 43(43)

ISSN
1069-7977

Authors
Schneider, Rose M.
Feiman, Roman
Mendes, Madeleine A
et al.

Publication Date
2021

Peer reviewed

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Rose M. Schneider
roschnei@ucsd.edu
Department of Psychology
University of California, San Diego

Roman Feiman
roman_feiman@brown.edu
Department of Cognitive, Linguistic, & Psychological Sciences
Brown University

Madeleine Mendes
mendesma@cardiff.ac.uk
Department of Psychology
Cardiff University

David Barner
dbarner@ucsd.edu
Department of Psychology
Department of Linguistics
University of California, San Diego

Abstract

The distinctly human ability to both represent number exactly and develop symbolic number systems has raised the question of whether such number concepts are culturally constructed through symbolic systems. Although previous work with innumerate and semi-numerate groups has provided some evidence that understanding exact equality is related to numeracy, it is possible that previous failures were driven by pragmatic factors, rather than the absence of conceptual knowledge. Here, we test whether such factors affect performance on a test of exact equality in 3- to 5-year-old children by modifying previous methods to draw children's attention to number. We find no effect of highlighting exact equality, either through framing the task as a "Number" game or as a "Sharing" game. Instead, we replicate previous findings showing a link between numeracy and an understanding of exact equality, strengthening the proposal that exact number concepts are facilitated by the acquisition of symbolic number systems.

Keywords: Number; language; cognitive development; conceptual development

Introduction

Human numerical capacities are unique in the animal kingdom in two ways: Only we are capable of thinking about large exact numbers (like 2021), and only we have developed rich and varied symbolic systems with which to express them (like "2021"). The presence of exact number concepts and their symbolic expression in humans raises a question: Are these concepts part of humanity's unique cognitive endowment, with numerical language and notation invented only to express them? Or is the ability to think about exact numbers culturally constructed through our unique, exact symbolic number systems?

There are several hypotheses about the relation between exact number concepts and symbolic number systems. One possibility is that humans who do not have symbolic number language are not only unable to represent precise magnitudes such as 2021, but also cannot recognize that large magnitudes can be exactly equal to one another. Instead, on this view, innumerate humans are limited to the non-symbolic numerical systems they share with non-human animals, such as the Approximate Number System (Feigenson, Dehaene, & Spelke, 2004) and Parallel Individuation (Gordon, 2004), which are incapable of supporting precise representations of large numerosities (Carey & Barner, 2019). An alternative to this account, however, is that large exact number representations are innately available (Gelman & Butterworth, 2005; Gelman & Gallistel, 2000), and are simply expressed by symbolic number (Gelman & Gallistel, 1992). Finally, a third possibility is that humans without symbolic number systems lack representations of individual precise magnitudes, but nevertheless understand that large magnitudes can be exactly equal to one another.

This final possibility entails an understanding that any two sets that have the same number of elements are equinumerous if there is a one-to-one correspondence between elements in the set—sometimes referred to as "Hume's Principle" (Boolos, 1986; Hume, 1739). While equality is easily established by counting, it can also be established through non-symbolic one-to-one procedures. For example, by pairing off items in two sets such that each item in one set corresponds to an item in the other (and vice versa), it is possible to verify that two sets are equal without counting or knowing exactly how many items are in each set. Consequently, it is possible, at least in principle, that an understanding of exact equality may be present even in individuals who have little or no exposure to symbolic number systems.

Previous work has tested whether innumerate individuals can use one-to-one correspondence to establish exact equality, using a measure known as the "set-matching task." In the simplest version of this task, the experimenter presents a number of items in a row, and then asks the participant to create a matching row. The experimenter's row stays visible, such that participants can create their row by matching each element in the experimenter's row one-to-one, without using symbolic number. However, this task has produced mixed results in work done with innumerate adults, such as the Pirahã, an indigenous Amazonian group. While two studies (Everett & Madora, 2012; Gordon, 2004) found that the Pirahã failed to exactly match sets greater than 3 items (see also Pica et al., 2004), another found that Pirahã participants succeeded for all set sizes (Frank et al., 2008). In addition to being in conflict, these findings are also limited in that they tested only individuals who were fully innumerate. Numeracy consists of many different components (such as access to a shared count list, knowledge of number words, and counting) that may be learned individually. While it is possible that knowledge of a culturally shared count list or exposure to number words is alone sufficient to support reasoning about exact equality, another possibility is that additional components of numeracy are required to support this ability. To address these limitations, recent work has focused on another group with limited symbolic number knowledge, and in whom these components are sequentially acquired; young children in the US.

Schneider and Barner (2020) used a modified version of the set-matching task to test 3- to 5-year-old children with varying levels of symbolic number knowledge and found that their ability to exactly match sets was related to their level of numeracy. Although US children are unlike the Pirahã in that they receive linguistic number input almost from birth, it takes years for them to acquire its meaning, making them effectively semi-numerate for several years. Critically, components of numeracy emerge individually in children over the first few years of life, making them an ideal case-study of how each component might support reasoning about exact equality. Starting around 2.5 years of age, children begin to sequentially learn the meanings of *one, two,* and *three*, but do not yet understand how the meanings of these words are related to the logic of counting (Wynn, 1992). During this phase, children are called "subset-knowers," as they have meanings for only a subset of the number words in their count list. It is not until around 3.5-4 years of age that they become "Cardinal Principle," or CP-knowers, meaning that they know that the last word said while counting indicates the cardinality of a set (Gelman & Gallistel, 1978).

Schneider and Barner (2020) asked whether subset- and CP-knowers could use one-to-one correspondence to generate numerically equal sets for both small (3, 4) and large (6, 8, 10) sets. To do this, they framed the set-matching task as a "matching game" with fish in a pond, where children were instructed to make their pond "look like" the experimenter's, which was presented directly above the child's, as in Gordon (2004). Using this measure, they found that, like some Pirahã participants, subset-knowers—who have limited knowledge of counting and number words—failed to create exact matches for sets >3. In contrast, CP-knowers—who can use counting to generate large sets—were significantly more likely to generate exact matches despite not being allowed to count during the task. On the basis of these results, Schneider and Barner argued that learning how to generate sets through counting, rather than knowledge of number language in general (as tested by work with innumerate populations), is implicated in supporting the ability to reason about exact equality non-symbolically.

Although these findings suggest a link between exact equality and one component of numeracy—specifically, acquiring the cardinal principle—it is possible that Schneider and Barner underestimated children's competence. Previous work has shown that when asked to establish equivalence between two sets, children frequently use perceptual properties other than number (such as color, shape, length, density, or size), leading to failures that appear numerical in nature. Children may have failed in Schneider and Barner's task due to the ambiguously phrased instruction to "make your pond look like mine," which could be compatible with dimensions other than number. Importantly, other work has found that, when instructed to attend to number, children are able to do so, though on different tasks. For example, Negen and Sarnecka (2015) found that subset-knowers who had failed to determine which of two dot arrays "had more" were interpreting the question in terms of surface area rather than numerosity. When children were trained that the word "more" referred to the number of dots, they succeeded. A similar pragmatic failure may also explain failures sometimes observed in innumerate groups (Laurence & Margolis, 2007). Individuals who fail the task may possess an understanding of exact equality, but fail to demonstrate it if the goal of the set-matching task is not made sufficiently clear. On this account, previously reported differences between innumerate/semi-numerate and numerate populations may reflect differences in how participants perceive the relevance of number when instructions are ambiguous.

We address this possibility in the current work. Using the set-matching task from Schneider and Barner (2020), we tested whether motivational and pragmatic factors, rather than numeracy, accounted for children's ability to establish exact set matches. In Experiment 1, we explicitly disambiguate number as the dimension of interest by telling children that this is a *number* matching game, and that they should match the two rows *without counting*. In Experiment 2, we framed the set-matching task as a "sharing" game, in which each row was given to a character, and children had to ensure that the distribution of resources was "fair" and "the same." Historically, human concerns for fairness and equitable distribution of resources motivated some of the earliest creations of symbolic number systems, often including one-to-one tallies to represent goods and trading (Ifrah, 2000; Schmandt-Besserat, 1978). In addition, children develop expectations for equitable resource distribution as early as 19 months (Sloane, Baillargeon, & Premack, 2012), and by 3 years of age believe that equal resource distribution carries normative force (Rakoczy, Kaufman, & Lohse, 2016). Moreover, CP-knowers have been shown to favor 'fair' (equal) distributions more often than subset-knowers in cases where the quantities were small enough that equality could be established by approximation or subitizing, without deploying one-to-one correspondence (Chernyak, Harris, & Cordes, 2019), suggesting that having greater knowledge of symbolic number can support establishing exact equality when fairness is at stake. This may even be true for subset-knowers; because Chernyak et al. did not include a non-sharing condition, it is unknown whether concerns for fairness might motivate subset-knowers to generate numerically equal distributions, despite doing so less often overall than CP-knowers. Given these considerations, we hypothesized that fairness concerns may motivate children to attend to exact numerical equality in this task.

Experiment 1: Number Matching

Methods

This study was not pre-registered. Data collection was halted in March, 2020 due to COVID-19.

Participants Our current sample includes 32 children (M_{age} = 4.11 years, SD_{age} = 0.64 years) out of a planned sample of 80 recruited from local preschools and the surrounding area in San Diego, CA. In this sample, 20 children were identified as CP-knowers (M_{age}=4.41 years, SD_{age}=0.45 years), and 12 as subset-knowers (M_{age}=3.61 years, SD_{age}=0.65 years) by the Give-N task.

Procedure

All children were tested individually in a small room set apart from the classroom, museum, or lab. All children were tested first on the Number Matching Task, and then their knower-level (subset-/CP-knower) was determined using the Give-N task.

Number matching task We minimally modified the set-matching task from Schneider and Barner (2020). Children were presented with two 4"x30" blue rectangular boards placed about .5" apart, with one board directly above the other, and a set of 15 identical plastic fish. The experimenter then told the child, "This is a number matching game. In this game, I want you to make sure that your pond has the *exact same* number of fish as my pond *without counting*." The experimenter then placed one fish in the center of the top board and said, "Using your fish, can you make your pond have the same exact number of fish as my pond without counting?" If the child provided the correct number of fish, the experimenter said "Great job! Our ponds have the exact same number of fish because there is one fish here and one fish here!" If the child did not provide the correct number of fish, the experimenter said, "Hmm, I don't think that's the same exact number of fish, because I have only one fish here, and there are [number of fish] in your pond. Let's try again!" Unlike Schneider and Barner, we used number language in our feedback to emphasize that children should be generating exact matches. After the child passed a training trial with one fish, they received a second training trial with two fish. Only children who could pass both trials and understood the purpose of the task were included in the study.

After training trials, children received test trials with sets of 3, 4, 8, and 10. The experimenter's set was always placed on the center of one board with about .25" separation between each fish. Although the experimenter used number language while introducing the task, they did not use any number language while conducting the test trials, and children were not permitted to count. If the experimenter observed a child counting, they quickly covered both boards and said, "This isn't a counting game! This is just a number matching game!" and restarted the trial, removing fish from the child's board if necessary. Counting was prohibited both to be consistent with other work (Gordon, 2004; Schneider & Barner, 2020), and also so that any differences between CP- and subset-knowers did not simply reflect the application of a symbolic tool (counting) that only CP-knowers could deploy. Counting attempts were extremely rare: CP-knowers attempted counts on 5/78 trials, and no subset-knowers attempted to count. Because Schneider and Barner (2020) found evidence that CP-knowers were not covertly counting to solve this task, we did not use additional measures to prevent counting, such as verbal interference.

To test the effect of drawing children's attention to number, we compared children's performance to a previously-collected dataset in which the set-matching task was presented only as a "matching" game (N = 144, M_{age} = 3.94 years, SD_{age} = 0.53 years; n CP-knowers = 70, n subset-knowers = 74). The matching task was identical to the number matching task, with the following exceptions: children were told that it was a "matching game," and that the goal of the game was to make their pond "look like" or "look the same" as the experimenter's; there were no stuffed animals; fish were glued to the experimenter's board; and children were given their own set of 15 fish to generate sets.

Give-N We assessed children's knowledge of the CP using a titrated version of the Give-N task (Wynn, 1992). Children were given a small plate and 10 plastic items (e.g., bananas), and were asked to place some number of bananas on the plate. After the child finished generating the set the experimenter asked, "Is that N? Can you count and make sure?" If the child noticed any errors they were permitted to fix the set. If the child correctly generated a set for a requested N, the next number queried was N+1; otherwise, the next number queried was N-1.

All children were first asked to generate a set of *one.* Any child who was able to generate a set of 6 (the maximum number tested) at least 2 out of 3 times was classified as a CP-knower. If children could not meet this criterion, they were classified as subset-knowers.

Results and Discussion

In Experiment 1 we asked whether set-matching performance improved relative to a neutral matching condition of the set-matching task when number was verbally highlighted. Due to the small sample size, no meaningful statistical analysis within CP- or subset-knowers was possible in Experiment 1. Instead, we first explored an overall effect of verbally highlighting number by combining both CP- and subset-knower data from Experiment 1 and testing whether overall accuracy or absolute error differed from the matching task (also with combined CP- and subset-knower data). We then report summary statistics for set-matching accuracy and error for CP- and subset-knowers relative to the matching condition.

In terms of accuracy, explicitly drawing children's attention to number in the set-matching task did not produce significantly more exact matches overall. A generalized linear mixed effects model constructed with combined CP- and subset-knower data predicting accuracy (1/0) from condition (matching/number), set size, and age, with a random intercept of participant indicated no effect of condition ($\chi^2_{(1)}$ = 2.57, p = .11), as shown in Figure 1. Subset-knowers' mean accuracy was not substantially different in the number condition for either small (M = .71, SD=.46) or large (M=0.21, SD=0.42) numerosities in comparison to the matching condition (small: M = .60, SD = .49; large: M = .13, SD = .33). Although accuracy for small numerosities was slightly higher for subset-knowers in the number condition, this effect is inconclusive due to the small sample size. CP-knowers' matching behavior was similarly unaffected by the experimenter highlighting number: mean performance for small (M = .88, SD = .34) and large sets (M = .38, SD = .49) in the number condition was comparable to performance in the matching condition (small: M = .89, SD = .32; large: M = .42, SD = .50).

We also investigated differences in the magnitude of error between the two conditions as a less conservative measure of whether framing the task as a number game encouraged children to attempt exact matches. To do this, we calculated the absolute error of incorrect trials (|Target item - Response|), and again combining CP- and subset-knower data built a linear mixed effects model predicting absolute error from condition (matching/number), set size, and age, with a random intercept of participant, and again found no effect of condition ($\chi^2_{(1)}$ = .03, p = .86). Subset-knowers' mean absolute error in the number condition for both small (M=1.71, SD=1.25) and large (M=2.89, SD=2.31) numerosities was similar to the matching condition (small: M = 1.56, SD = 1.00; large: M = 3.39, SD = 2.44). CP-knowers' absolute error was also not greatly impacted by explicitly highlighting number, with roughly equivalent mean absolute error for both small (M=1.2, SD= 0.45) and large (M=3.8, SD= 4.04) numerosities in comparison to the matching condition (small: M = 1.31, SD = 0.60; large: M = 2.36, SD = 1.67).

While limited by a small sample size, these results nevertheless suggest that even when children are explicitly told to match by number, they are no more likely to make exact matches. However, one possible explanation for this failure to find an effect was that while the experimenter prompted children to focus on number, it was possible children could have interpreted this as referring to approximate, rather than exact, number. To address this concern, we framed the matching game as a sharing game to specifically highlight the importance of exact equality and one-to-one correspondence.

Experiment 2: Sharing

Methods

This study was pre-registered on OSF (https://osf.io/3wta2), and all methodological and analytical choices were pre-registered, unless stated otherwise.

Participants Our final analyzable sample included 86 children (M_{age} = 4.13 years, SD_{age} = 0.56 years) recruited from local preschools and the surrounding area in San Diego, CA. In this sample, 53 children were identified as CP-knowers (M_{age}= 4.30 years, SD_{age}= 0.56 years), and 33 as subset-knowers (M_{age}= 3.86 years, SD_{age}= 0.42 years) by the Give-N task (described below).

Procedure

Sharing task To test whether fairness concerns motivate children to generate exact matches for both large and small numerosities, we modified the set-matching task used by Schneider and Barner (2020) by framing it as a game about sharing. Children were again presented with two 4"x30" blue rectangular boards placed about .5" apart and two stuffed animals of the same kind directly next to each board (e.g., penguins). The experimenter showed children a bowl with identical plastic fish, and said, "These penguins worked together to catch these fish, but they need our help to share them in their ponds. Do you know what sharing means?" Regardless of how children responded, the experimenter said, "Sharing means to make things fair so that people have the same. Because these penguins *both* worked hard to catch these fish, they should have the same, because that's fair, right?" Based on previous research, we emphasized equal collaboration to encourage children to distribute resources equitably (Heyman, Ng, & Barner, 2011).

To ensure that children understood what a fair/unfair share looked like, all children received two demonstrations prior to a training trial. To demonstrate a fair share, the experimenter placed one fish in each penguin's "pond," and said "Let's ask the penguins if this is fair!" The experimenter made each penguin say, "Yay! That's fair!" and then told the child, "This is fair, because there is a fish in this penguin's pond and a fish in this penguin's pond, so they have the same." To demonstrate an unfair share, the experimenter placed one fish in one pond and no fish in the other pond, and said, "Now, let's ask the penguins if this is fair." The experimenter made both penguins protest, saying "That's not fair!" and then told the child, "This isn't fair, because there's a fish in this penguin's pond, but *no* fish in this penguin's pond. Let's try again, and this time you can help me make it fair."

For the training trial, the experimenter removed all fish, and then added two fish to the center of one penguin's board before offering the bowl to the child and saying, "Now it's your turn to share with the other penguin. Remember to make it fair!" After the child was done, the experimenter said, "Let's ask the penguins if this is fair," making the penguins say "Yay! It's fair!" if the child placed two fish in the pond, or "Hey! That's not fair!" if they had not. If the child failed the training trial, the experimenter repeated it a second time. Only children who could pass the training trial and demonstrated an understanding of the purpose of the task were included in the study.

After the training, children received five test trials with neutral feedback for sets of 3, 4, 6, 8, and 10. Order of trials (3, 4, 10, 6, and 8; or 4, 3, 8, 6, and 10) was counterbalanced across participants. To emphasize that fish were a common resource to be shared between the two penguins, both the experimenter and the child drew fish from the same bowl; before each trial the experimenter covertly added or subtracted fish to the bowl so that it always contained 15 fish when the child generated their set. As in Experiment 1, children were not allowed to count. If the experimenter observed a child counting they covered both boards and said, "This isn't a counting game, it's a sharing game!" Counting attempts were rare: CP-knowers attempted to count on 8/265 trials, while subset-knowers attempted counts on 9/170 trials.

Results and Discussion

Does concern for fairness motivate children to generate exact numerical matches for both small and large numerosities? Overall, the general pattern of children's responses was similar in both the sharing and matching conditions (Figure 2). To test this, we compared data from the sharing task and the previously-collected matching task data, using a generalized linear mixed effects model to predict a correct match (1/0) from condition (sharing/matching), set size, CP-knower status, and age, with a random intercept of participant. A Likelihood Ratio Test indicated that the presence of the condition (sharing/matching) term significantly improved the fit of the model ($\chi^2_{(1)} = 9.15$, $p = .002$). However, this effect was in the opposite direction of our prediction, with *less* accurate performance overall for the sharing, relative to matching, condition (β =-0.62, $p = .003$), as shown in Figure 1. A set of follow-up *t*-tests comparing mean accuracy between conditions for CP- and subset-knowers by numerosity revealed that the effect of condition was primarily driven by subset-knowers' poorer performance for small numerosities (3,4) in the sharing condition in comparison to the matching condition ($t(105) = -2.37$, $p = .02$); CP-knowers' performance did not differ significantly for either small or large numerosities (*p*s > .10). Consistent with Chernyak, et al. (2019), we found a significant effect of numeracy, with CP-knowers significantly more accurate than subset-knowers (β =1.35, $p < .001$), even controlling for age (β =0.53, $p = .002$) and set-size (β =-1.06, $p < .001$). Still, while CP-knowers were more accurate than subset-knowers, their performance remained below adult levels (Frank, Fedorenko, Lai, Saxe, & Gibson, 2012). We return to this finding in the General Discussion.

Framing the task as a sharing game also did not impact the magnitude of children's errors. A linear mixed effects model predicting absolute error from condition, set size, CP-knower status, and age, with a random intercept of participant indicated a significant effect of condition ($\chi^2_{(1)}$ = 5.2, p = 02), with *higher* absolute error for the sharing condition in comparison to matching (β =0.46, $p = .02$), opposite from our predictions. Once again, however, follow-up *t*-tests of mean performance between conditions indicated that this difference was due to significantly higher absolute error in subset-knowers for small numerosities in the sharing task ($t(67) = 2.22$, $p = .03$); there was no difference in overall absolute error for CP-knowers between the two conditions (*p*s > .3).

Together, these results provide evidence that children's failures to establish exact matches in the set-matching task are not improved by a sharing manipulation. Despite emphasizing the importance of generating fair shares, children were not more likely to create exact matches in comparison to a condition that did not draw their attention to numerical equality. In fact, subset-knowers were significantly *less* likely to equally distribute resources equally for small numerosities in the sharing condition, in line with other findings that the ability to distribute resources equitably for even small sets is linked to CP acquisition (Chernyak, Harris, & Cordes, 2019). Instead, we replicate the finding that an understanding of exact equality is significantly related to numeracy: as in previous work, we found that while subset-knowers struggled to create exact matches for sets >3 items, CP-knowers were significantly more likely to do so for all set sizes. This suggests that, while numerical competence improves the ability to share fairly, being in a context that requires sharing fairly may not make children any more likely to establish exact equality between the shares.

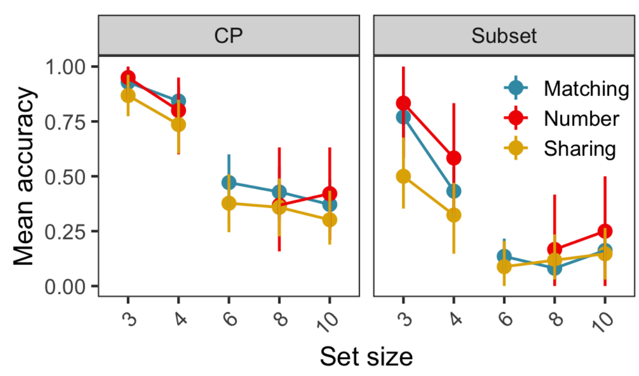

Figure 1. Mean accuracy on the set-matching task for CP- and subset-knowers, grouped by condition. Error bars represent 95% confidence intervals computed by nonparametric bootstrap.

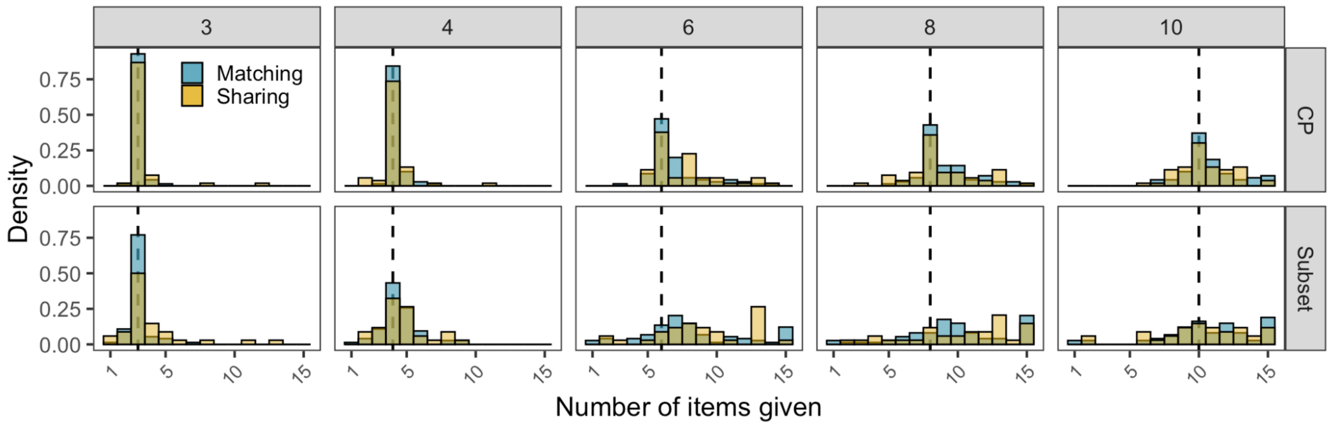

Figure 2. Density of set-size response (x-axis) for each target size in the sharing and matching conditions, grouped by CP knowledge.

General Discussion

Are failures to establish non-symbolic exact equality in semi-numerate individuals due to pragmatic failures to understand the goals of set matching tasks? Previous work addressing this question in children has suggested that learning how to generate sets through counting is implicated in learning how one-to-one correspondence is related to knowledge of exact numerical equality (Schneider & Barner, 2020). While this work found that children with limited counting knowledge did not establish exact matches for large sets, it left open why, and whether learning to count might simply have made number more salient to children. For example, other work has found that low-numeracy individuals (including children) frequently fail numerical tasks not because they lack the appropriate concepts, but because they either do not understand that they are being asked to deploy them, or because they are less likely to privilege number as the relevant dimension for solving these tasks (Mix, 1999; Negen & Sarnecka, 2015).

In the current work we investigated whether additional pragmatic and motivational cues affected the ability of 3- to 5-year-old children to create exactly equal sets at varying stages of symbolic number development. We did this by adapting the set-matching task previously used with both innumerate adults (Gordon, 2004) and semi-numerate children (Schneider & Barner, 2020). In Experiment 1, we explicitly told children to match sets on the basis of number. In Experiment 2 we built on previous findings showing early-developing concerns for equitable resource distribution (Sloane et al., 2012), and that symbolic number knowledge supports fair (equal) sharing (Chernyak et al., 2019), and framed the set-matching task as a sharing game between two stuffed animals.

Across these experiments we found that neither explicit instructions to attend to number nor a concern for fairness increased children's likelihood of establishing exact matches. Instead, related to the hypothesis that numeracy plays a pivotal role in facilitating representations of exact number (Gordon, 2004; see Núñez, 2017, for discussion), we replicated previous findings that subset-knowers appeared to approximate, rather than exactly match, sets greater than 3 items. Subset-knowers' failures to establish exact numerical equality in these two experiments suggest that their struggles lie not in insufficient motivation or pragmatic difficulties in understanding the purpose of the task. Instead, subset-knower's failures are compatible with other research showing that they do not understand the numerical significance of one-to-one correspondence in establishing equality (Izard, Streri, & Spelke, 2014).

We also found further evidence that the acquisition of symbolic systems plays an important role in learning to reason about exact equality. While subset-knowers struggled to create exact set matches for sets >3, children who had learned how to generate larger sets through counting — i.e., CP-knowers — were significantly more likely to exactly match sets, even when controlling for age. However, despite the motivational and pragmatic manipulations, many CP-knowers struggled on this task, indicating that previously-observed failures in this group (Schneider & Barner, 2020) were likely not due to failure to understand the task. Instead, these results suggest that even proficient counters may not fully understand the logical basis of exact equality, and that additional learning is required.

Together with previous work (Schneider & Barner, 2020), these findings are compatible with the hypothesis that children's initial understanding of the CP is procedural rather than conceptual (Barner, 2017); that is, CP-knowers may understand what the count routine is *doing* (i.e., blindly deploying one-to-one correspondence in the count routine to answer the question "How many?") before they fully grasp what it is *accomplishing* (i.e., creating a summary set representation which is dependent on the one-to-one correspondence between items and number labels; Heck, 2000). This account is in line with a growing body of evidence which has shown that children demonstrate full knowledge of other properties of the natural numbers — such as the successor function (Cheung, Rubenson, & Barner, 2017; Spaepen et al., 2019) — several years after acquiring the CP and using the count routine in numeric contexts. It is possible that learning the numerical significance of one-to-one correspondence may be similarly protracted, requiring further conceptual development or experience with numerical routines.

The current work addresses key limitations in previous research showing set-matching failures in innumerate and semi-numerate individuals, strengthening the hypothesis that numeracy may be implicated in reasoning about exactness. Future work should investigate the processes through which children abstract one-to-one correspondence from using the count routine, and whether this abstraction is implicated in the acquisition of later numerical knowledge.

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