UC Berkeley Fisher Center Working Papers

Title

The Importance of Model Selection on Housing Price Indexes: Comparisons of Temporal Aggregation and Sample Selectivity

Permalink <https://escholarship.org/uc/item/66s584s8>

Authors

Englund, Peter Quigley, John M. Redfearn, Christian L.

Publication Date

1997-06-01

Peer reviewed

WORKING PAPER SERIES

WORKING PAPER NO. 97-256

THE INPORTANCE OF MODEL SELECTION ON HOUSING PRICE INDEXES: COMPARISONS OF TEMPORAL **AGGREGATION & SAMPLE SELECTIVITY**

BY

JOHN QUIGLEY PETER ENGLUND **CHRIS REDFEARN**

These papers are preliminary in nature: their purpose is to stimulate discussion and comment. Therefore, they are not to be cited or quoted in any publication without the express permission of the author.

UNIVERSITY OF CALIFORNIA AT BERKELEY

WALTER A. HAAS SCHOOL OF BUSINESS

FISHER CENTER FOR REAL ESTATE AND URBAN ECONOMICS UNIVERSITY OF CALIFORNIA AT BERKELEY

Kenneth T. Rosen, Chair Robert H. Edelstein, Co-Chair Dwight M. Jaffee, Co-Chair Nancy E. Wallace, Co-Chair

Established in 1950 to examine in depth a series of major changes and issues involving urban land and real estate markets, the Fisher Center for Real Estate and Urban Economics (FCREUE) continues to be one of the major university

centers in the United States for research on real estate markets, financial institutions, the California economy, and other urban and regional economy topics. FCREUE supports an in-house research program, research by faculty associates, and dissertation research by doctoral students. Research findings are made available in the form of books, working papers, reprints from scholarly journals, monographs and a research newsletter.

The Fisher Center for Real Estate and Urban Economic Research is an organized research unit of the University of California at Berkeley. These working papers are issued to disseminate research results to other scholars. The authors welcome comments; inquiries may be directed to the author in care of the Center.

> F602 Haas School of Business #6105 Berkeley, CA 94720 phone 510-643-6105 fax 510-643-7357 http:// haas.berkeley.edu/realestate/

The Importance of Model Selection on Housing Price Indexes: Comparisons of Temporal Aggregation and Sample Selectivity

by

Peter Englund Uppsala University Sweden

John M. Quigley University of California Berkeley

Christian L. Redfearn University of California Berkeley

June 1997

WORKING PAPER NO. 97-256

Support for this research has been provided by the Fisher Center for Real Estate and Urban Economics, University of California, Berkeley and by the Swedish Council for Building Research.

 \sim

Abstract

Housing transactions are executed and recorded daily but are routinely pooled into longer time periods for the measurement and analysis of housing price trends. This aggregation has unknown consequences for the estimation of housing prices and their volatilities. We utilize a unique and rich data set, covering essentially all arm's length housing sales in Sweden for a dozen years, in an attempt to understand the effect of temporal estimates of housing prices aggregation upon and their This rich data set also provides a unique volatilities. opportunity to extend previous research on this topic. $A11$ previous work comparing estimates of price volatility has been based on the most parsimonious model imaginable, a so-called weighted repeat sales model (WRS). In contrast, our analysis is based upon a detailed model of housing price determination in addition to the conventional WRS model.

We compare the results using the conventional WRS model to those based on a research strategy which incorporates all available information on house sales. The results indicate the clear importance of temporal disaggregation in the estimation of housing prices and volatilities -- regardless of the model employed.

The appropriately disaggregated model is then used as a benchmark to compare estimates of the course of housing prices produced by the two models during the twelve year period 1981 -1993. These results indicate that much of the difference between estimates of price movements can be attributed to the data limitations which are inherent in the repeat sales approach. **The** suggest caution in the interpretation of results, thus, government-produced price indices or those produced by private firms based on the repeated sales model.

I. Introduction

The single largest investment most households ever make is in owner-occupied housing. Most home-owning households purchase insurance to protect this asset against unexpected loss from natural disaster, but few households can shield their housing investments from real estate cycles and price declines. **Booms** and busts in residential real estate markets are well documented, but hedging mechanisms that would allow middle-income households to diversify their real estate holdings or to insure the values of their homes have yet to be established. There are many economic and legal issues in designing programs to diversify housing price risk, but none is more basic than the accurate measurement of price levels and volatilities.

A substantial literature exists on the measurement of prices for non-standard assets such as housing. There are two major problems to be overcome in constructing a price index for housing: the relative infrequency of dwelling unit sales; and the heterogeneity in characteristics across housing units. Simple price indexes based on mean or median housing prices (for example, the index produced by the National Association of Realtors) do not consider the characteristics of houses sold. They are thus unable to distinguish between movements in prices and changes in the composition of homes sold from one period to the next. Crude regression models (for example, the U.S. Bureau of the Census $C-27$ Index) are just that $-$ crude. More sophisticated repeat sales models (for example, Bailey, Muth, and

Nourse, 1963 and Case and Shiller, 1987) are based on strong assumptions about the constancy of the housing quality of any given dwelling.

Beyond the issue of model selection is the appropriate measurement of time itself in analyzing trends and volatilities in prices. This paper addresses the implications of aggregating observations on housing prices across time, combining housing sales observed in continuous time into discrete time periods for statistical analysis.

The exceptional nature of the data available supports a detailed analysis of temporal aggregation and other properties of price indexes. The data we analyze cover essentially all arm's length housing sales in Sweden from 1981 to 1993. All previous work comparing volatility estimates has been based on the most parsimonious model imaginable, a so-called weighted repeat sales model (WRS). In contrast, our analysis is based upon a detailed model of housing price determination using information on a wide variety of hedonic characteristics, as well as the WRS model.

The data also provide a unique opportunity to compare the properties of repeat sales estimators with more sophisticated Following the framework offered by Calhoun, Chinloy, methods. and Megbolugbe (CCM, 1995) for the analysis of U.S. data, a comparable repeat sales price index is estimated for the three largest metropolitan regions in Sweden. We also estimate a more elaborate hybrid price index using the same data.

 $\overline{4}$

Tests for temporal aggregation are performed using both indexes. For each index, our results parallel those of CCM based on U.S. data on house sales in five census regions. Our results suggest strongly that housing price indexes should be estimated using the finest disaggregation of time available.

The research design, based on two indexes estimated from the same underlying data, also provides an opportunity to examine differences between the now-standard repeat sales estimator and a more elaborate hybrid technique. Our comparison suggests that much of the difference in estimates of price trends can be attributed to the maintained hypothesis and the data limitations inherent in the repeat sales approach to the measurement of housing prices.

Section II briefly reviews the methodology underlying the two indexes: the weighted repeat sales index (Case and Shiller, 1987) and the "hybrid" index of Englund, Quigley, and Redfearn (1996) . Section III describes the data and discusses the different samples utilized in constructing each index. Section IV discusses the results of the tests for temporal aggregation bias and provides further comparisons of the hybrid and repeat sales indexes. Section V provides a brief conclusion.

Methodologies for Estimating Housing Price Trends II.

The most widely used technique for estimating housing price trends is the repeat sales method introduced by Bailey, Muth, and

Nourse (1963). As extended by Case and Shiller (1987), the weighted repeat sales model (WRS) is widely used in academic It also forms the basis for regional housing price research. trends published by the federal government (OFHEO, 1997) and defines the methodology which underlies all proprietary indices used commercially in the $U.S.^1$ An alternative estimator, combining single sales and repeat sales, is proposed by Englund, Quigley, and Redfearn (EQR, 1996). This estimator utilizes information on all sales, as well as all available information on housing attributes, to estimate trends in housing prices. **The** two models are described in detail in Appendix A; the relevant properties of both are summarized below.

The genius of the repeat sales method is that, under appropriate assumptions, it completely controls for housing quality while requiring little data in comparison to hedonic or By confining the sample to repeat sales of hybrid methods. dwellings with unchanged characteristics, differences in observed selling prices of houses can be attributed solely to changes in housing prices. In practice, few data sets allow verification that those units sold twice are unchanged between sales (and there is no previous analysis of the topic). Typically, dwelling modifications involve improvements and corresponding increases in value -- increases that are improperly attributed to price

This includes, for example, the price series marketed by Case-Shiller-Weiss, Inc., MRAC, and TRW, Inc.

changes whenever units which have been modified are included in the analysis.

Even if the characteristics of houses were carefully matched to insure that they were unchanged between sales, two aspects of the weighted repeat sales method would remain problematic. **The** first is the inability of the WRS method to account for depreciation and normal maintenance. In the presence оf depreciation, the repeat sales index is necessarily biased, downward if the rate of depreciation exceeds normal maintenance. **The** second problem concerns interpretation and sample selectivity. The WRS index is constructed from a non-random sample of the stock of houses and the population of house sales, namely those houses that have sold more frequently during a given interval. Thus, the repeat sales index may be a poor measure of prices for the entire stock of housing and even for those which have been sold during any time interval.

The hybrid method takes advantage of the information that is present in repeat sales, but without ignoring information on single sales. The hybrid method is data intensive, but where the data are available, it represents an obvious improvement over the repeat sales method. Computed price indexes are based on far more information, and the information used is more representative of the housing stock. Within the hybrid model, repeat sales of houses permit the investigation of depreciation and vintage effects, as well as the temporal course of house prices.

 $\overline{7}$

In the next section, we describe the data used in the analysis. In section IV, we compare the implications of these techniques for the representation of time in price indexes. **We** consider the implications of the aggregation of sales reported daily into months, quarters, half years, or years for the estimation of housing prices, the returns to housing investment, and price volatilities.

III. Data on Swedish Housing Prices

The data used in this analysis consist of essentially every arm's length sale in the three largest metropolitan regions in Sweden - Stockholm, Gothenberg, and Malmö -- during the period from January 1, 1981 through June 30, 1993. Contract data reporting the transactions price for each sale have been merged with tax assessment records containing detailed information about the characteristics of each house. Repeat sales are identified, as is the location of each unit down to the smallest geographical $unit:$ the parish (something akin to a Census tract). The data set is exceptional in its detailed description of each dwelling at the date of sale and its identification of repeat sales. Together, these characteristics of the data make possible the comparison between the hybrid and repeat sales methods discussed Moreover, they permit a comparison of results using above. different subsamples. In particular, we compare the results obtained by the WRS method using all repeat sales models with

those obtained using dwellings whose constant quality over time can be verified.

Both of the models employed in this paper rely on the use of information embodied in repeat sales. However sales of units that sell more than once during the sample period are a small fraction of all housing sales in any market run. Table 1 describes the distribution of observations on sales and dwellings by number of sales.² Almost three quarters of all units sold during the sample period were sold one time. A handful of units were sold frequently, with an average duration between sales of less than two years.

Table 2 provides a summary of the variables used to control for quality and their average values for the dwellings located in each of the three regions. The variables describe the size and quality of each dwelling, as well as numerous amenities. Size variables include floor space and lot size, and dummy variables for one- and two-car garages. Quality is measured by the age of the dwelling and the vintage of its construction, the wall and roof materials, and the standard of kitchen, bathrooms,

To insure that the observations included in the data were indeed arm's length market transactions, only the first sale of paired sales that occurred within a six-month period were retained. This filter was imposed to remove "distressed sales" and non-market transactions from the data set. Sales within the six-month period were observed to have a large negative serial This is consistent with a distressed sale shortly correlation. after an initial purchase or a pair of sales in which one serves as a familial transfer either preceded or followed by an arm's length sale.

 \sim

Table 1 Number of Sales of Dwellings, 1981:I - 1993:II \cdot

 $\ddot{}$

Table 2

Average Characteristics of House Sales by Region, 1981-1993:II (Standard Deviation in Parentheses)

Region

Table 2 (Continuted) Average Characteristics of House Sales by Region, 1981-1993:II (Standard Deviation in Parentheses)

 $\bar{\mathcal{A}}$

insulation, and heating system. The amenities recorded include the presence of a sauna, a fire place, and a laundry room.

These variables describe the physical structure and amount of land on which it sits, but there remain external influences on housing prices. The importance of location to housing prices is well established. While necessarily incomplete, we have computed several variables to measure more desirable locations. These include dummy variables for each of the 111 labor market areas defined by Sweden's Central Bureau of Statistics and the approximate distance of each dwelling to the center of the local labor market in which it is located. This variable measures the linear distance from the center of the parish in which a dwelling is located to the center of the nearest labor market area. Also included is an estimate of the present value of capital subsidies on newer dwellings.³

These regions include the three largest cities in Sweden. The primacy of Stockholm is apparent in the prices of dwellings. The average price of about 750,000 SEK in Stockholm is about sixty percent higher than the average prices in Gothenberg and Malmö.

Beginning in 1975 the government provided interest-subsidized loans to owners of newly constructed housing. The value of the subsidy depended on the construction costs and the vintage of the unit, and decayed with time. While the average estimated present value of the subsidy ("capital subsidy" in Table 2) is small, less than 2500 SEK or \$370 US, the average for transactions involving subsidized dwellings ("conditional subsidy" in Table 2) is an order of magnitude larger. During the 1980's the average subsidy on newly constructed homes was as high as twenty percent of the initial price of the dwelling.

Differences in the representative units exist across the three regions, with Stockholm having younger, and in general, higher quality dwellings. The differences are less pronounced when compared with Gothenberg than with Malmö. Age at time of sale and vintage demonstrate this well: the average age of dwellings at the time of sale in Stockholm is 24.5 years, and the year of construction is 1962. For Gothenberg the comparable figures are 28.1 and 1958, while for Malmö the average age at the time of sale is 36.3, and the average dwelling was built in 1950. The differences are not great, but the younger housing stock in Stockholm is reflected in more dwellings with access to a garage, a sauna, a fireplace, an excellent kitchen, and a laundry room. The parcel size, the dummy for single detached home, and the urban/rural dummy, together indicate the greater urbanization of the Stockholm region.

IV. Time Aggregation

Table 3 summarizes the statistical comparison of price indexes computed at four levels of aggregation -- monthly, quarterly, semi annually, and annually. The same comparison is made for each model, the WRS and the EQR. The results are reported separately for each of the three regions.

The table reports F tests of the restrictions inherent in representing time in the computation of price indexes by aggregate measures. For example, the entry in the first row and column provides a test of the hypothesis that, for the WRS model

Table 3 Tests of Disaggregation of Price Trends over Time (F-ratios comparing sets of coefficients)

Note: The critical values of the F statistic, F(U-R, infinite), where U is the number of parameters estimated in the unrestricted model and R is the number of parameters in the restricted model, are:

applied to data from the Stockholm region, the coefficients on monthly prices within quarters are identical. According to the entry $(F = 1.470)$, the hypothesis can be rejected at the five percent level (where the critical value is 1.27) and also at the one percent level of confidence (where the critical value is 1.42).

The table presents a complete set of tests, comparing more restricted models (columns) and less restricted models (rows) for all four aggregations of time initially measured in days. At the five percent level, the hypothesis that a semi-annual series can be represented by an annual series is rejected in all three data sets. The hypotheses that a quarterly series can be represented by a semi-annual series is also rejected. Similarly, the hypothesis that a monthly series can be represented by a quarterly series can also be rejected.

These results are consistent across regions both for a simple and parsimonious model, the WRS, and for a more complete The results for the WRS model, reported in model, the EQR. columns 1, 2, and 3 are quite consistent with those reported by CCM (1995, Tables 1 through 5) for five census regions in the U.S. The results reported in columns 4, 5, and 6 provide further confirmation, using a different model. The conclusion is clear: in the estimation of housing price indexes, time should generally be represented using the lowest level of aggregation possible. Arbitrary aggregations into broader representations of time are generally unwarranted.

Table 4 indicates some \circ f the implications of the unwarranted aggregation of time in these three bodies of data. Again we present estimates based on both the WRS and the EQR models for different representations of time. Panel A reports the mean values of the housing price index reported for the entire period. As the entries indicate, the average values of the indexes (normalized at $1981:I = 100$) vary little with the representation of time, although there is a slight trend upward the level of aggregation increases. as Furthermore, the estimated evolution of nominal prices, including their acceleration beginning in 1986, their peak in 1991, and their rapid decline thereafter, is consistent regardless of the degree of temporal aggregation. This evolution is illustrated below.

The mean value of the price index is slightly higher for the EQR models than for the WRS models. This is true for each city and for each of the various aggregations of time. This finding is discussed further below.

Panel B compares the estimated mean returns to investment in owner occupied housing, for a one year holding period. There is no consistent difference in the estimated rates of return between the two methods. Estimated returns are generally within ten percent of each other; there is some tendency for returns estimates to be smaller for larger aggregations of time.

Panel C reports the estimated volatilities in annual returns implied by the various models. The volatilities are computed from annual returns estimated using each model. The estimated

Table 4 $\langle \cdot \rangle$ Estimates of House Prices, Returns, and Volatilities

 \sim

variance in annual returns is lower, but not consistently so, when estimated from larger aggregations of time. The variance in annual returns is estimated to be smaller when based on the EQR models than on the WRS models.

Table 5 reports the coefficients of variation in returns estimated from the two models. Panel A is based on the units used in the regression estimates (i.e., the entry 0.779 is the ratio of the variance in monthly returns for Stockholm, estimated using the monthly model and the WRS procedure, to the mean monthly return for Stockholm estimated in the same way). **The** coefficient of variation is usually, though not always, smaller when estimated from larger aggregations of time. The coefficient of variation is always smaller when estimated using the EQR model.

Panel B uses the regressions based on months, quarters, half years, or years, but converts the returns to annual series before computing the coefficient of variation. As the panel shows clearly, when the returns are computed using a common one-year holding period, the variation is reduced considerably. However, the coefficient of variation is considerably smaller when estimated by the EQR method.

V. Comparing Methodologies

Figures 1, 2, and 3 compare the estimates of the course of housing prices for the three regions. The figures report the estimated prices for a standardized house (the average Stockholm

Table 5 Coefficients of Variation in Housing Returns
(Variance/Mean)

 \mathcal{L}

 \blacksquare

Figure 2

Figure 3

dwelling, according to the many characteristics measured) as well as the 95 percent confidence intervals for those prices. $A11$ prices are estimated using the most appropriate representation of time -- in months. Results are presented for both methodologies, the WRS and the EOR methods.

Inspection of the figures for each region reveals two regularities: first, the 95 percent confidence intervals are substantially narrower for the EQR model than for the WRS model; second, in each of the three regions the paths of the two indexes diverge toward the end of the period. The two indexes track each other closely during the period of stable nominal prices, from 1981 to 1986, and are similar through the first half of the rise in prices, to the beginning of 1990. At this point, the WRS index levels off and then declines, while the index based on the EQR method continues to climb before starting to fall in 1991. Table 6 illustrates these differences.

The first regularity, the much narrower confidence intervals in the price series for the EQR estimates, is to be expected. EQR model incorporates much more **The** information in the estimation of the price index. It relies upon single sales as well as multiple sales, and it utilizes extensive information about the qualitative and quantitative attributes of dwellings. Estimates of price trends "should" be known with greater precision, and true confidence intervals "should" be narrower.

The reasons for the other differences in the estimated series -- which are disturbingly large in the 1990's -- are not

Table 6

 γ

 $\sigma_{\rm e}$

 $\frac{1}{2}$

Average Monthly Rates of Return on Housing Investment (in percent)

 \bar{z}

 \sim

immediately obvious. Upon closer inspection, however, it is apparent that these differences could easily arise from an implicit and unverified assumption used in computing "standard" WRS housing price indexes.⁴ As noted in Appendix B, by basing the index upon the time interval between sales of the "same" house (and only the time interval between sales), the computation of the index assumes that each house is really identical at each The rich sample of Swedish housing data permits this sale. implicit assumption to be tested.

Table 7 reports the number of single and multiple sales in the three housing markets. It also reports the number of paired sales in which the attributes of the house at the second sale are, in fact, the same as those recorded at the time of the first sale. The table shows that only about half the time the second of a pair of sales is identical in measured characteristics to that sold the first time.⁵

In addition, the two series are estimated from different populations of dwellings. The repeat sales in these data are located on smaller lots, are more likely to be in urban areas, and are less expensive than units selling only once. Thus, estimates from the WRS method may be based on housing submarkets which are not representative of the stock as a whole.

⁵ The only other study which compares paired sales with changes in attributes with paired sales of dwellings with unchanged attributes, by Meese and Wallace (1997), uses data from Oakland and Fremont, California over an 18 year period. For Oakland, Meese and Wallace found that 59 percent of paired sales had changes in measured attributes, while for Fremont, they reported that 47 percent of the dwelling had changed attributes.

Table 7 Number and Distribution of Second Sales

B. Paired Sales

Number of paired sales of identical houses and the total number of paired sales (in parentheses), by year of second sale

 \sim

 $\sim 10^{-1}$

The information in Table 7 probably overestimates the fraction of dwellings that have been altered.⁶ Generally speaking our data set registers "too little" quality change before 1989 and "too much" quality change thereafter. This is illustrated in panel B of Table 7, which shows that about three quarters of the repeat sales before 1989 had identical measured quality while less than one quarter of repeat sales in 1990 and 1991 were identical, according to our measurements.⁷

As demonstrated in Appendix Table B1, the changes in recorded housing quality between the first and second sale of each pair are generally consistent with improvements in housing quality over time. The amount of living area, the availability of a garage, the quality of bathrooms and materials, and the likelihood of a sauna and fireplace all increase. Several measures are not easily interpreted as they have been affected by

The fractions of identical houses are lower in 1981-1983 than in later years. This may reflect that houses sold shortly after

⁶ The raw data measuring housing attributes is obtained from questionnaires filled out by home owners in connection with the tax assessments made in 1981 and 1989. Alterations made between assessment years should in principle be reported to the tax authorities, but often reports are only made for major additions to dwellings. Thus, for houses, where both transactions are before or after 1989, some quality changes are unreported in our In 1989 new information was collected on all housing data. units, and there was some redesign of the questionnaire. Many variables (e.g., the presence of sauna, sewage connection, beachfront property, fireplace, and roofing material) did not change definition, but others did, reflecting that the standards for kitchens apparently rise with time, as does the standard for "furnished" basements and laundry rooms. At the margin some units "change" in 1989 simply because the standard has changed. For houses that were sold once before and once after 1989, comparison would report "too much" quality change.

a rising standard (see footnote 5), but even for these variables the average quality has improved between sales of dwellings. If these improvements are ignored in the computation of the price index, the WRS will overestimate housing price appreciation. As demonstrated in Appendix B, any data set which consists of all houses sold between two fixed dates will, in the presence of continuous improvement in housing quality, lead to WRS price indices which overestimate the price appreciation for a house of constant quality during the period.

Figure 4 indicates the dramatic change in the computed WRS index when the sample is confined to dwellings whose characteristics are unchanged between sales, as demanded by the underlying theory. The differences are quite large, averaging six percent for Stockholm. Without measuring and controlling for housing quality directly, the WRS methodology leads to much higher estimates of housing prices at the end of the period.

VI. Conclusion

In this paper, we have considered the aggregation of housing sales reported in continuous time to discrete periods for the computation of indexes of house prices, investment returns, and the volatility of returns. We have also considered the properties of repeat sales estimators and hybrid estimators of the price indexes.

the date of purchase are more often subject to major renovations than houses sold after more normal holding periods.

Figure 4

The analysis indicates quite clearly that house price estimates ought to be undertaken using the finest disaggregation of time available. On statistical grounds, price indices based on monthly aggregations dominate those based on quarterly data. Quarterly data, in turn, dominates semi-annual data for the computation of price indices. In this conclusion, we reinforce that made by Callhoun, Chinloy, and Megbolugbe using U.S. data. Volatilities in returns are substantially higher when estimated using monthly time intervals. However, our results also suggest that for a consistently defined holding period, returns and volatilities do not differ very much, at least for this data set. Volatilities do appear to be consistently over-estimated by the repeat sales method as compared to the hybrid method.

This analysis also suggests, however, that extreme caution should be exercised in interpreting the WRS indices of housing prices as they are typically computed for academic and business The implicit assumption of constant quality is applications. difficult to verify, but is essential to the method. In the housing markets analyzed here, dwelling improvements are undertaken frequently and are widespread. These changes in physical structure violate the maintained hypothesis of the WRS Furthermore, the results indicate that correctly method. implementing the WRS method greatly restricts the set of observations that are utilized, perhaps narrowing the sample to observations drawn from non-representative dwellings. In these conclusions, we reinforce those made by Meese and Wallace (1997),

using U.S. data which was more limited in geographical scope, sample size, and in the measurement of housing prices. While further research is needed to clarify the relationship between repeat sales indexes and price movements in the remainder of the housing stock, it appears that the widespread use of the WRS indexes in the U.S. provides an inadequate picture of housing price movements.

REFERENCES

Bailey, Marin J., Richard Muth, and Hugh O. Nourse, "A Regression Method for Real Estate Price Index Construction," Journal of the American Statistical Association, 1963(4): $933 - 942.$

Case, Karl E. and Robert J. Shiller, "Prices of Single-Family Homes Since 1970: New Indexes from Four Cities," New England Economic Review, Sep/Oct 1987: 45-56.

Calhoun, Charles A., Peter Chinloy, and Isaac Megbolugbe, "Temporal Aqqreqation and House Price Index Construction," Journal of Housing Research, 6(3) 1995: 419-438.

Englund, Peter, John M. Quigley, and Christian L. Redfearn, "Improved Price Indexes for Real Estate: Measuring the Course of Swedish Housing Prices," University of California, Berkeley, Fisher Center for Real Estate and Urban Economics Working Paper 96-248, Oct 1996 (forthcoming, Journal of Urban Economics).

Hill, R. Carter, J.R. Knight, and C.F. Sirmans, "Estimating Capital Asset Prices," Review of Economics and Statistics, forthcoming.

Meese, Richard A. and Nancy F. Wallace, "The Construction of Residential Housing Price Indices: A Comparison of Repeat-Sales, Hedonic-Regression, and Hybrid Approaches," Journal of Real Estate Finance and Economics, $14(12)$, $1997: 51-74$.

Office of Federal Housing Enterprise Oversight (OFHEO), 1997 Report to Congress, Washington, DC, OFHEO, 1997.

Quigley, John M., "A Simple Hybrid Model for Estimating Real Estate Price Indices," Journal of Housing Economics, 4(1), 1995: $1 - 12$.

Appendix A

Models of Housing Price Trends

The results reported in the text are based on two methods applied to the same data set to estimate housing price series. The first is the so-called weighted repeat sales (WRS) index developed by Case and Shiller (1989) as an extension of a method originally suggested by Bailey, Muth, and Nourse (1963). The WRS index is used extensively, for example by Calhoun, Chinloy, and Megbolugbe (1995) in their comparison of statistical models of temporal aggregation. The second is a so-called hybrid method developed Englund, Quigley, and Redfearn (EQR, 1996). It is an extension of work on hybrid indexes developed in Quigley (1995) and Hill, Carter, Knight, and Sirmans (forthcoming). The two methods differ in their approach to controlling for quality and, as a result, differ in the sub-population of dwellings whose prices are actually analyzed.

The methods address the problem that observations on the selling prices of housing are in the units of price times Additional assumptions are needed to identify quantity. separately price and quantity.

Α. The Weighted Repeat Sales Method

Assume, following Case and Shiller (1989), that the log price of the i th house at time t , P_{it} , is given by

(A1) $P_{it} = I_t + H_{it} + N_{it}$

where I_t is the logarithm of the price level at t, H_{it} is a Gaussian random walk, such that

 $(A2)$ E(H_{it} - H_{it}) = 0 $E(H_{it} - H_{it})^2 = \sigma_N^2 + (t - \tau) \sigma_H^2$

and N_{it} is white noise, such that

(A3)
$$
E(N_{it}) = 0
$$

 $E(N_{it})^2 = \sigma_N^2/2$

Let $V_{it} = P_{it} + Q_{it}$ be the sale price of house i at time t. For houses sold at time t and time t (i.e., repeat sales) during the interval (0,S), the index is computed in three steps. In the first step, equation (A4) is estimated and the residuals, μ_{it} , saved for use in the second step.

(A4) $V_{it} - V_{it} = \sum_s \phi_s D_{is} + \mu_{it}$, $s = (0, 1, ..., S)$,

where $D_{is} = 1$ if $s = \tau$, $D_{is} = -1$ if $s = t$, and $D_{is} = 0$ otherwise. ϕ_s is the estimate of I_s, the log of the price level at time s.

In the second stage, the squared residuals from (A4), $(\mu_{it} - \mu_{it})^2$, are regressed upon a constant and the elapsed time between sales, $(t - \tau)$, yielding estimates of the variances $\sigma_{\rm H}^2$ and σ_{n}^2 . In the third stage, equation (A4) is reestimated by generalized least squares with diagonal elements $\sqrt{\hat{\sigma}_N^2 + (t-\tau)\hat{\sigma}_\pi^2}$.

Note the assumption about dwelling quality implicit in this The left hand side of (A4) can be interpreted formulation. unambiguously as a log price change if $Q_{i\tau} = Q_{i\tau}$.

That is,

(A6) $P_{i\tau} + Q_{i\tau} - P_{it} - Q_{it} = \sum_s \phi_s D_{is}$, s = (0,1,.., S).

The estimates of the price index are therefore functions of dwelling quality unless quality remains constant across time. Clearly $Q_{i\tau} = Q_{i\tau}$ is a maintained hypothesis in adopting this procedure. This is discussed further in Appendix B.

B. The Hybrid Method

Following Englund, Quigley, and Redfearn (1996), assume

(A7) $V_{it} = \beta X_{it} + P_{it} + \xi_i + \varepsilon_{it} = \beta X_{it} + P_{it} + \gamma_{it}$

where X_{it} represents the logarithm of observable characteristics of dwelling i, and P_{it} and V_{it} are defined above. ξ_i represents an error due to the unmeasured, individual-specific characteristics of dwelling i and is distributed with zero mean and variance σ_{ξ}^2 . ε_{it} is a well-behaved error term. Components of X_{it} include the vintage (y_i, year built) of the dwelling and the accumulated depreciation at age $(t-y_i)$ of the dwelling. In a cross section, y_i , (t- y_i), and P_t cannot be separately identified, but from a subsample of repeat sales at various ages and years, the parameters can be recovered.

To implement the model, estimate (A7) using the subsample of paired repeat sales at time t and time τ . Use the residuals from the regression to estimate

(A8) $\gamma_{it} - \gamma_{it} = \beta_d [t-\tau] + \varepsilon_{it} - \varepsilon_{it}$

and

$$
(A9) \quad \varepsilon_{it} = \rho^{(t-\tau)} \varepsilon_{i,t-\tau} + v_{it},
$$

where ε_{j_t} and ε_{j_t} are defined as above, and ρ is the serial correlation coefficient. v_{it} is the residual, and is distributed

with zero mean and variance σ_v^2 . Equation (A8) provides an estimate of depreciation⁸, β_d .

Together, these parameters yield an estimate of the variance-covariance matrix of the errors in equation (A7). From equation (A9) we obtain an estimate of σ_v^2 . An estimate of σ_{ϵ}^2 is constructed from the residuals in the first-step estimation of $(1, 7)$. β_d is obtained from $(1, 8)$. Together these parameters describe completely the variance-covariance matrix of the errors for equation (A7).

where A_i is the age of the house in year j.

The final step is the re-estimation of (A7) by generalized least squares including all observations in the sample, not merely repeat sales. The inverse of the estimated variancecovariance matrix (A10) is the GLS matrix.

This hybrid method is more data intensive than the repeat It relies upon qualitative and quantitative sales method. information about each housing unit at the time of sale to control for housing quality in an explicit manner. The method

⁸ Data on repeat sales allows the identification of vintage, age, and depreciation effects. Subtracting the estimate of the effect of depreciation obtained in equation (A8) from the coefficient on the age estimated in (A7) yields an estimate of the vintage effect. That is, $\beta_v = \beta_v - \beta_d$.

also capitalizes on the unique information provided by repeat sales of individual units. This permits us to separate the effects of time on housing prices from depreciation and vintage effects and to improve the efficiency of parameter estimates by explicit attention to the components of the error structure.

Appendix B

As indicated in Appendix A, as long as the housing quality of each dwelling remains constant, i.e., $Q_{it} = Q_{it}$, the WRS method remains an elegant and parsimonious method of estimating changes in relative housing prices. This is because the quantity terms on the left hand side of equation (A6) simply cancel out.

If $Q_{it} \neq Q_{it}$, then the interpretation of the estimated price series, the ϕ 's, becomes problematic; the effect of time on housing prices is confounded with the changing quality of the dwellings over time. For a unit to remain at a constant quality over time, the accumulated effects of depreciation over time must be exactly offset by maintenance. If homeowner investments and upgrading are an ongoing process leading to improvement in quality over time, then the weighted repeat sales method will systematically overestimate housing price levels. Moreover, this overstatement will increase with the elapsed time between the sale and all subsequent sales, and will yield the first divergence observed in Figure 4.

This can be shown analytically. If we assume that quality improvements are undertaken sporadically, and that maintenance offsets depreciation, we can derive the expected bias. The true price index number at time t is I_t , and is the ratio of the price level at time t, P_t and time 0, P_0 ,

(B1)
$$
I_t = P_t/P_0
$$
.

However, we observe only selling prices V_{it} , amalgams of the price level and the quality of the dwelling at the time of sale. The WRS estimate of the price index at time t is ϕ_t , which is the ratio of observed selling prices at time t and time 0,

(B2) $\phi_t = P_t Q_{it}/P_0 Q_{i0} + \mu_{it}$

where μ_{it} has zero mean and variance σ_{μ}^{2} .

If we characterize the changes in the quality of a unit by discrete improvements in the structure, perhaps a new kitchen, garage, etc., and assume that ongoing maintenance offsets the effect of depreciation, then we can calculate the bias as a function of time since the original sale. Let the homeowner improve the quality of his or her unit by x percent with probability p each year. At time 1 the expected quality of the unit is

(B3)
$$
E[Q_{i1}] = Q_{i0} + Q_{i0} * xp
$$
,

and more generally,

 $(B4)$ E[Q_{it}] = Q_{i0}* $(1+xp)^t$.

Substituting (5) into (3) yields

(B5)
$$
E[\phi_t] = P_t E[Q_{it}]/P_0 Q_{i0} = P_t Q_{i0} (1+xp)^t / P_0 Q_{i0} = I_t (1+xp)^t
$$
.

This implies that the expected bias is an increasing function of the size and frequency of the improvements and of the time interval between the first sale and all subsequent sales. Equation (B5) indicates that the interval that governs the bias is not the average time between sales, rather the cumulative elapsed time since the first sale: the time since initial quality is established. For any repeat sales data set, this interval increases with the sample period, implying that repeat sales observations drawn from the latter stages of the sample period will have longer average intervals between first and last sale than repeat sales from earlier in the sample. Equation (B5) that, under reasonable characterization of shows \mathbf{a} home improvement, the bias will be correspondingly greater with elapsed time, generating the phenomenon observed in Figure 4.

The crucial assumption made when employing the WRS price index is that quality is constant over time. The assumption is often made as an artifact of the data. The same lack of data that precludes the use of a hybrid or hedonic method also precludes verification of the assumption that quality remains constant between sales. If quality rises over time, the WRS index will overstate the price level, as higher sales prices are

attributed to overall housing price increases and not the underlying quality improvements.

 $\sim 10^7$

Percentage of units exhibiting changes in the level of an attribute Changes in the Attributes of the Second Sale of Paired Sales Appendix Table B1

Increase 11.0 $\ddot{\circ}$ $\ddot{}$ $\overline{1}$.0 7.7 1.3 4.5 3.1 2.9 $\frac{1}{6}$ 5.7 $\ddot{0}$. 5.2 3.6 8.5 7.9 0.4 5.3 Malmö Same 89.9 89.0 97.5 98.8 94.9 89.1 90.2 94.7 91.3 87.5 99.8 90.9 84.1 91.9 84.0 83.9 99.5 90.7 98.5 Decrease 8.2 \ddot{o} . 5.0 1.2 4.1 0.2 $\frac{9}{1}$ 5.3 2.3 2.6 6.8 3.9 0.1 4.5 7.5 8.2 $\overline{0}$. 3.8 0.4 **Increase** 12.0 13.1 ه .
ه $\ddot{2} \cdot \ddot{0}$ 5.6 $\ddot{5}$. 0 4.0 5.5 4.2 6.3 5.9 $\frac{9}{3}$ $\frac{1}{1}$. 0.1 7.8 7.7 0.2 4.2 0.8 Gothenberg Same 76.0 88.4 86.9 90.8 98.6 91.4 91.8 92.0 99.8 89.9 96.4 89.1 87.7 90.7 84.8 84.7 99.6 92.7 98.9 Decrease 12.0 \ddot{o} . 5.0 $\frac{6}{1}$ 3.6 0.3
 3.7 $\begin{array}{c} 2 \ 4 \ 6 \ 7 \ 8 \ 9 \ 1 \end{array}$ 6.7 0.1 4.2 5.4 7.4 7.6 0.2 3.0 0.3 Increase 6.5 8.5 4.5 0.7 7.3 0.4 5.3 2.7 3.0 3.8 $\frac{8}{3}$ 0.1 4.7 2.8
7.0 5.4 $\ddot{0}$. 1 4.2 0.6 **Stockholm** Same 98.0 86.5 91.5 9.09 99.5 94.0 92.1 91.4 93.3 94.0 90.1 99.8 92.4 92.6 87.7 87.7 99.9 93.9 99.2 Decrease 1.9 7.0 0.0 3.4 $\frac{3}{1}$. $\ddot{0}$. 3.3 3.3 3.7 2.2 2.9 $\frac{1}{6}$ 0.1 4.6 5.3 6.9 0.0 1.9 0.2 Isolation Quality Walls Winter Quality Walls Waterfront Property Furnished Basement Wall Construction Excellent Kitchen Sewer Connection Two-Car Garage One-Car Garage Excellent Roof Living Space Laundry Room Good Kitchen Tiled Bath Plot Size Fireplace Good Roof Sauna Heat