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ABSTRACT

Using analyticity and crossing, one can express the experimental quantities for the physical channel in terms of the crossed-channel amplitudes. For N-N scattering, a new set of t-channel amplitudes $\{g_i\}$ are introduced for which the s-channel experimental quantities I_0, D, D_t, C_{NN}, C_{KP}, P, R, A, R', and A' are very simple in form. By use of these it is shown that the measurement of certain spin-dependent parameters in N-N scattering can provide critical tests of the validity of the Regge pole model of high-energy scattering. In particular it is possible to test the factorization theorem, which is an immediate consequence of the simple pole Regge theory. The relationship of the new amplitudes g_i to the Wolfenstein parameters is given.

I. INTRODUCTION

Analyticity of scattering amplitudes makes it possible to compute the experimental parameters of a given channel in terms of the crossed-channel transition amplitudes.[1,2] The general crossing property is the basis of many theoretical models and phenomenological analyses.[3] In particular, models in which the scattering of two particles proceeds by the exchange of sets of quantum numbers have been especially fruitful in the discussion of experimental data. Consideration of the crossed channel often facilitates the interpretation and parameterization of the amplitudes for the exchange, and analyticity allows the use of these amplitudes in the main channel to calculate the experimental parameters.

Assuming the usual strong interaction symmetries for nucleon-nucleon scattering, there are five independent amplitudes associated with each value of isotopic spin. Since a 5-by-5 crossing matrix relates the direct and crossed-channel amplitudes, one expects unwieldy expressions for the experimental parameters when they are written in terms of the crossed-channel amplitudes. A judicious choice of crossed-channel amplitudes, however, gives these parameters an amazingly simple form.

In Sec. II, we introduce a new set of t-channel amplitudes for which the crossing matrix simplifies tremendously. Calculation of the s-channel experimental quantities is then easily performed. For the reader interested only in results, the new set of amplitudes is defined in Eq. (2.7), and the expressions for the s-channel experimental quantities in terms of these crossed-channel amplitudes are given in Eq. (2.11). We include the differential cross section, the depolarization parameters D and D_t, the correlation functions C_{NN} and C_{KP}, the polarization P , and the rotation parameters R, A, R', and A'. We emphasize that these results are model independent and rely only on a sufficiently large domain of analyticity to give physical content to crossing.

In Sec. III, we consider models in which sets of definite quantum numbers are exchanged. Obvious applications can be made to both fixed-angular-momentum pole models and Regge pole models.[4]

These results are applied to the Regge pole model in Sec. IV. Our main aim here is to devise critical tests for some of the most characteristic properties of Regge poles. In particular we examine the consequences of the "factorization theorem," the validity of which rests heavily on the simple pole assumption. Indeed, there is already some vague indication that the factorization theorem is invalid from the experimental result that the difference between the $p\bar{p}$ and pp differential cross sections changes sign at a very small value $[t \approx -0.15 (\mathrm{BeV}/c)^2]$ of the momentum transfer. A detailed analysis of the situation is given. We also discuss in detail the usual approximation of neglecting trajectories other than the P, P', ρ, ω, and R.

A general explanation of the nature of the tests in terms of the concept of the "class" of a Regge trajectory is based on the discussion in Sec. III.

In the Appendix we relate the s-channel Wolfenstein parameters[5] to the helicity amplitudes and to the new (analytically continued) crossed-channel amplitudes.

II. DERIVATION OF GENERAL RESULTS

In this section, we derive expressions for the experimental quantities in terms of the crossed-channel amplitudes. Consider $NN \to NN$ scattering in the direct channel (s channel). We shall use the Mandelstam variables,

$$\begin{aligned} s &= 4(p^2 + m^2) = 2m^2 + 2mE , \\ t &= -2p^2(1 - z) , \\ u &= -2p^2(1 + z) , \\ z &= \cos\theta = \frac{u - t}{u + t} , \end{aligned} \tag{2.1}$$

where p is the center-of-mass (c.m.) momentum of one of the nucleons, θ is the scattering angle in the c.m. system, and E is the <u>total energy</u> of the incident nucleon in the <u>laboratory system.</u>

In the t channel, $N\bar{N} \to \bar{N}N$,

$$\begin{aligned} t &= 4(p_t^2 + m^2) = 4E_t^2 , \\ u &= -2p_t^2(1 - z_t) , \end{aligned}$$

Cont.- (2.2)

$$s = -2p_t^2(1 + z_t) ,$$

$$z_t = \cos\theta_t = \frac{s - u}{s + u} = \frac{mE + \frac{1}{4}t}{-p_t^2} , \qquad (2.2)$$

where p_t and E_t are the c.m. momentum and energy of the nucleon (or antinucleon) and θ_t is the t-channel c.m. scattering angle.

We assume the usual conservation laws of strong interactions so that there are five independent helicity amplitudes for the s-channel process for each value of isotopic spin. Since the 2-by-2 isospin crossing matrix is easily included, we shall simplify the notation by omitting isospin until Sec. III.

Let $\underset{\sim}{\phi}$ denote the column matrix formed from the five helicity amplitudes ϕ_i $(i = 1,\cdots,5)$ introduced in Ref. 1, and let X be <u>any</u> experimental quantity of interest. Then it is always possible to find a 5-by-5 matrix $\mathcal{M}(X)$ such that

$$X = \underset{\sim}{\phi}^\dagger \mathcal{M}(X) \underset{\sim}{\phi} = \sum_{i,j=1}^{5} \phi_i^* \phi_j \mathcal{M}(X)_{ij} . \qquad (2.3)$$

Let $\underset{\sim}{f}$ be the column matrix of the five kinematical-singularity free t-channel amplitudes f_i $(i = 1,\cdots,5)$ defined in Ref. 1, Eq. (4.23). The partial-wave expansions of the f_i are[6]

$$f_1 = \frac{E_t}{p_t} \sum_{J=0}^{\infty} (2J + 1) f_0^J(t) P_J(z_t) ,$$

Cont. (2.4)

$$f_2 = \frac{E_t}{p_t} \sum_J (2J+1) f_{11}^J(t)\, P_J(z_t) \,,$$

$$f_3 = \frac{E_t}{p_t} \sum_J \frac{2J+1}{J(J+1)} \left[f_1^J(t) \left(z_t P_J{}'(z_t)\right)' - f_{22}^J(t)\, P_J{}''(z_t) \right] ,$$

$$f_4 = \frac{E_t}{p_t} \sum_J \frac{2J+1}{J(J+1)} \left[f_{22}^J(t) \left(z_t P_J{}'(z_t)\right)' - f_1^J(t)\, P_J{}''(z_t) \right] ,$$

$$f_5 = -\frac{m}{p_t} \sum_J \frac{2J+1}{[J(J+1)]^{1/2}} f_{12}^J(t)\, P_J{}'(z_t) \,, \qquad (2.4)$$

where $f_0^J(t)$ is the spin singlet t-channel transition amplitude and $f_1^J(t)$ the spin triplet $J = L$ amplitude. The spin triplet $J = L \pm 1$ transitions are described by $f_{11}^J(t)$, $f_{12}^J(t)$, and $f_{22}^J(t)$, and do not mix with the $J = L$ amplitudes because of parity conservation. We shall express all our t-channel amplitudes in terms of these spin amplitudes, since they are easily interpretable in models where definite sets of quantum numbers are exchanged in the t channel, e.g. fixed pole or Regge pole models.

By analytic continuation, $\not{\tilde{f}}$ is related to $\tilde{f}$ by the crossing relation[3]

$$\not{\tilde{f}} = \frac{1}{(s)^{1/2}\,(s+u)(t+u)} K \tilde{f} \,, \qquad (2.5)$$

where K is the matrix explicitly displayed in Eq. (16) of Ref. 3.

In order to simplify the crossing matrix K, we define a new set of amplitudes denoted by $\underset{\sim}{g}$, which are related to $\underset{\sim}{f}$ by

$$\begin{aligned} f_1 &= g_5 , \\ f_2 &= g_1 , \\ (1 - z_t^2) f_3 &= g_2 - z_t\, g_4 , \\ (1 - z_t^2) f_4 &= g_4 - z_t\, g_2 , \\ [\epsilon(1 - z_t^2)]^{1/2} f_5 &= g_3 , \end{aligned} \tag{2.6}$$

where

$$\epsilon = t/4m^2 .$$

The partial-wave expansions for the g_i are

$$\begin{aligned} g_1 &= \frac{E_t}{p_t} \sum_J (2J + 1) f_{11}^J(t)\, P_J(z_t) , \\ g_2 &= \frac{E_t}{p_t} \sum_J \frac{2J + 1}{J(J + 1)} \left\{ \left[J(J + 1) P_J(z_t) - z_t\, P_J{}'(z_t) \right] f_{22}^J(t) + P_J{}'(z_t) f_1^J(t) \right\} , \\ g_3 &= - \frac{(stu)^{1/2}}{p_t(s + u)} \sum_J \frac{2J + 1}{[J(J + 1)]^{1/2}} f_{12}^J(t)\, P_J{}'(z_t) , \\ g_4 &= \frac{E_t}{p_t} \sum_J \frac{2J + 1}{J(J + 1)} \left\{ \left[J(J + 1)\, P_J(z_t) - z_t\, P_J{}'(z_t) \right] f_1^J(t) + P_J{}'(z_t) f_{22}^J(t) \right\} , \\ g_5 &= \frac{E_t}{p_t} \sum_J (2J + 1) f_0^J(t) P_J(z_t) . \end{aligned} \tag{2.7}$$

We can now define a new crossing matrix $\mathcal{K}$ between the t-channel g_i and the s-channel ϕ_i :

$$\mathcal{K} = \frac{1}{(s+u)(t+u)} \times \begin{pmatrix} 4m^2u & -st & 4m(stu)^{\frac{1}{2}} & -(s+u)(t+u) & 0 \\ -st & 4m^2u & 4m(stu)^{\frac{1}{2}} & 0 & (s+u)(t+u) \\ 4m^2u & -st & 4m(stu)^{\frac{1}{2}} & (s+u)(t+u) & 0 \\ st & -4m^2u & -4m(stu)^{\frac{1}{2}} & 0 & (s+u)(t+u) \\ -2m(stu)^{\frac{1}{2}} & -2m(stu)^{\frac{1}{2}} & 4m^2u - st & 0 & 0 \end{pmatrix} \tag{2.8}$$

where

$$K_1 \underset{\sim}{f} = (s+u)(t+u)\mathcal{K}\underset{\sim}{g} .$$

From Eqs. (2.8), (2.5), and (2.3), the direct-channel experimental quantity, X , is given by

$$X = s^{-1} \underset{\sim}{g}^\dagger \mathcal{K}^\dagger \mathcal{M}(X) \mathcal{K} \underset{\sim}{g} . \tag{2.9}$$

The matrix product

$$M(X) = \mathcal{K}^\dagger \mathcal{M}(X) \mathcal{K} \tag{2.10}$$

is easily calculated. In Eq. (2.11), we give X in terms of $\underset{\sim}{\phi}$ [this determines $\mathcal{M}(X)$] and in terms of $\underset{\sim}{g}$, which is found by calculating the matrix product M(X):

$$I_0 \equiv \frac{d\sigma}{d\Omega_{CM}} = \frac{1}{2}\left[|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2\right]$$

$$= (s)^{-1}\left[|g_1|^2 + |g_2|^2 + 2|g_3|^2 + |g_4|^2 + |g_5|^2\right], \qquad (a)$$

$$I_0(1 - D) = \frac{1}{2}\left[|\phi_1 - \phi_3|^2 + |\phi_2 + \phi_4|^2\right]$$

$$= 2(s)^{-1}\left[|g_4|^2 + |g_5|^2\right], \qquad (b)$$

$$I_0(D_t - C_{NN}) = \mathrm{Re}\left\{(\phi_3^* - \phi_1^*)(\phi_2 + \phi_4)\right\}$$

$$= 4(s)^{-1}\,\mathrm{Re}(g_4^*\, g_5), \qquad (c)$$

$$I_0(1 - C_{NN}) = \frac{1}{2}\left[|\phi_1 - \phi_2|^2 + |\phi_3 + \phi_4|^2\right]$$

$$= (s)^{-1}\left[|g_1 - g_2|^2 + |g_4 + g_5|^2\right], \qquad (d)$$

$$I_0\, C_{KP} = \frac{1}{4}\left[-|\phi_1 + \phi_2|^2 + |\phi_3 - \phi_4|^2\right]\sin\theta$$

$$- \mathrm{Re}\left\{\phi_5^*(\phi_1 + \phi_2 - \phi_3 + \phi_4)\right\}\cos\theta \qquad (e)$$

$$= (s)^{-1}\,\mathrm{Re}\left\{(g_4 - g_5)^*\left[2\alpha(\theta)g_3 - \beta(\theta)(g_1 + g_2)\right]\right\},$$

$$I_0\, P = -\mathrm{Im}\left\{\phi_5^*(\phi_1 + \phi_2 + \phi_3 - \phi_4)\right\}$$

$$= -2(s)^{-1}\,\mathrm{Im}\left\{g_3^*(g_1 + g_2)\right\}, \qquad (f)$$

$$I_0R = \mathrm{Re}\left\{\phi_1^* \phi_3 + \phi_2^* \phi_4\right\} \cos\frac{\theta}{2} - \mathrm{Re}\left\{\phi_5^*(\phi_1 - \phi_2 + \phi_3 + \phi_4)\right\} \sin\frac{\theta}{2}$$

$$= s^{-1}\left[\mathcal{A}(\underset{\sim}{g})\, \alpha(\tfrac{\theta}{2}) + \mathcal{B}(\underset{\sim}{g})\, \beta(\tfrac{\theta}{2}) - \mathcal{C}(\underset{\sim}{g}) \cos\frac{\theta}{2}\right], \qquad (g)$$

$$I_0A = \frac{1}{2}\left[|\phi_1|^2 - |\phi_2|^2 + |\phi_3|^2 - |\phi_4|^2\right] \sin\frac{\theta}{2}$$

$$+ \mathrm{Re}\left\{\phi_5^*(\phi_1 - \phi_2 + \phi_3 + \phi_4)\right\} \cos\frac{\theta}{2} \qquad (h)$$

$$= s^{-1}\left[-\mathcal{A}(\underset{\sim}{g})\, \beta(\tfrac{\theta}{2}) + \mathcal{B}(\underset{\sim}{g})\, \alpha(\tfrac{\theta}{2}) + \mathcal{C}(\underset{\sim}{g}) \sin\frac{\theta}{2}\right],$$

$$I_0R' = -\mathrm{Re}\left\{\phi_1^* \phi_3 + \phi_2^* \phi_4\right\} \sin\frac{\theta}{2}$$

$$- \mathrm{Re}\left\{\phi_5^*(\phi_1 - \phi_2 + \phi_3 + \phi_4)\right\} \cos\frac{\theta}{2}$$

$$= s^{-1}\left[\mathcal{A}(\underset{\sim}{g})\, \beta(\tfrac{\theta}{2}) - \mathcal{B}(\underset{\sim}{g})\, \alpha(\tfrac{\theta}{2}) + \mathcal{C}(\underset{\sim}{g}) \sin\frac{\theta}{2}\right], \qquad (i)$$

$$I_0A' = \frac{1}{2}\left[|\phi_1|^2 - |\phi_2|^2 + |\phi_3|^2 - |\phi_4|^2\right] \cos\frac{\theta}{2}$$

$$- \mathrm{Re}\left\{\phi_5^*(\phi_1 - \phi_2 + \phi_3 + \phi_4)\right\} \sin\frac{\theta}{2}$$

$$= s^{-1}\left[\mathcal{A}(\underset{\sim}{g})\, \alpha(\tfrac{\theta}{2}) + \mathcal{B}(\underset{\sim}{g})\, \beta(\tfrac{\theta}{2}) + \mathcal{C}(\underset{\sim}{g}) \cos\frac{\theta}{2}\right], \qquad (j)$$

(2.11)

where

$$\mathcal{A}(\underset{\sim}{g}) = |g_1|^2 - |g_2|^2 , \qquad (a)$$

$$\mathcal{B}(\underset{\sim}{g}) = 2 \,\mathrm{Re} \left\{ g_3^*(g_1 - g_2) \right\}, \qquad (b)$$

$$\mathcal{C}(\underset{\sim}{g}) = |g_4|^2 - |g_5|^2 , \qquad (c)$$

(2.12)

and

$$\alpha(\theta) = \xi \cos\theta + \gamma \sin\theta , \qquad (d)$$

$$\beta(\theta) = \gamma \cos\theta - \xi \sin\theta , \qquad (e)$$

with

$$\xi = \frac{4m^2u - st}{(s + u)(t + u)} = \frac{m(1 + 3\cos\theta) - E(1 - \cos\theta)}{4m + (E - m)(1 - \cos\theta)} , \qquad (f)$$

$$\gamma = \frac{4m(stu)^{1/2}}{(s + u)(t + u)} = \frac{2[2m(m + E)]^{1/2} \sin\theta}{4m + (E - m)(1 - \cos\theta)} . \qquad (g)$$

We emphasize that: (a) The experimental parameters and helicity amplitudes are s-channel quantities; (b) The g_i amplitudes were defined in the t channel, then analytically continued by crossing to the s channel; (c) These formulae are model-independent and depend only on sufficient analyticity for crossing to be physically meaningful. They are applicable to any model in which the t-channel amplitudes are of prime importance in studying the s-channel process.

As an alternative derivation of Eq. (2.11), an appendix is included in which the Wolfenstein parameters are related to the helicity amplitudes. With the crossing relation between the Wolfenstein parameters and the g amplitudes, Stapp's Table I (Ref. 5) can be used to rederive Eq. (2.11).

III. EXCHANGE MODELS

We consider here models in which the scattering proceeds by the exchange of well-defined sets of quantum numbers P, I, I_z, B, Y, and Q. In some models, J is included among the definite quantum numbers, e.g., in "elementary particle" exchange. If angular momentum is interpolated, the exchange carries J parity instead, as in Regge pole models. Attention is confined to exchanges of $B = Y = 0$ so that the t reaction is an $N\overline{N}$ channel, and the exchange also carries definite G parity.

For each of the three main types of spin-transition amplitudes ($S = 0$, $J = L$; $S = 1$, $J = L$; and $S = 1$, $J = L \pm 1$), isotopic spin and J or J parity uniquely label the exchange. Relations among the quantum numbers determine the G parity,[4] and it turns out that there are 12 possible exchange types. We now assume that the amplitudes for these 12 possible exchange types have been calculated. There might be several contributions to each exchange type, so we will denote them by $g_{i,n}(JI;LS)$, where the $g_{i,n}$ are defined in terms of the partial-wave spin-transition amplitudes in Eq. (2.7). Note, for example, that for a pionic exchange, π, only $g_{5,\pi}$ is nonzero, and so on.

The s-channel experimental quantities for pp, $p\bar{p}$, pn, and $\bar{p}n$ elastic scattering can now be calculated and compared. The contribution of a given exchange will differ at most by a sign from process to process. The amplitude is a sum of terms which we take by convention to have positive coefficients in the s channel for elastic pp scattering. Hence, the g_i to be substituted into Eq. (2.11) for pp elastic scattering are

$$g_i^{(pp)} = \frac{1}{2} \sum_n g_{i,n}(JI;LS) \tag{3.1}$$

where $i = 1, \cdots, 5$. The factor $\frac{1}{2}$ comes from the isospin crossing matrix. The relative contribution of the nth term to the other NN and $N\bar{N}$ processes is now determined by isotopic spin and the transformation properties of the NN system. As an example, consider the process $\bar{p}n \rightarrow \bar{p}n$. Starting from the s-channel pp → pp process, cross to the t channel, perform the G-conjugation operation on the final state and charge conjugate both the final and initial states, then cross back to the s channel, which has become $\bar{p}n \rightarrow \bar{p}n$. The phase we pick up is just the G parity, $(-1)^{L+S+I}$, where, of course, L, S, and I are well-defined quantum numbers in the t channel (i.e., they are just the quantum numbers of the exchange). In general, we write

$$g_i^{(NN)} = \frac{1}{2} \sum_n c_n^{(NN)} g_{i,n}(JI;LS), \tag{3.2}$$

where

where

$$c_n^{(pp)} = 1 ,$$

$$c_n^{(\bar{p}n)} = (-1)^{L_n + S_n + I_n} ,$$

$$c_n^{(p\bar{p})} = (-1)^{L_n + S_n} ,$$

$$c_n^{(pn)} = (-1)^{I_n} .$$

These are listed explicitly in Table I.

The experimental quantities for elastic scattering are then

$$X^{(NN)} = \frac{1}{4s} \sum_{n,m} c_n^{(NN)} c_m^{(NN)} \underset{\sim}{g}_n^{+} M(X) \underset{\sim}{g}_m . \qquad (3.3)$$

From Eq. (3.1) and Table I, it is easy to verify the following statements, which hold for any experimental quantity X :

(i) The differences $(X^{pp} - X^{pn})$ and $(X^{p\bar{p}} - X^{\bar{p}n})$ depend only on the interference between exchanges with different isospin.

(ii) The differences $(X^{p\bar{p}} - X^{pp})$ and $(X^{\bar{p}n} - X^{pn})$ depend only on the interference of terms for which $(-1)^{L_n+S_n} = -(-1)^{L_m+S_m}$. (If we neglect the $S = 1$, $J = L$ amplitudes, which is reasonable in the Regge pole model, we can say that the difference depends only on the interference between exchanges with opposite J parity.)

(iii) The differences $(X^{p\bar{p}} - X^{pn})$ and $(X^{\bar{p}n} - X^{pp})$ depend only on the interference between exchanges with opposite G parity.

Similarly, the expressions $(X^{p\bar{p}} \pm X^{pp} \pm X^{pn} \pm X^{\bar{p}n})$ can be used to test the contributions of more restricted sets of interference terms.

IV. THE REGGE POLE MODEL--CRITICAL TESTS

A particularly interesting application of the foregoing formalism can be made when the exchanged system is a Regge pole. It will be our aim to find critical tests that can be used phenomenologically to estimate the validity of the Regge pole model.[8] We remind the reader that the Regge pole model has been very successful in correlating a huge amount of experimental information about scattering processes.[9] The most convincing successes are those involving forward scattering processes ($t = 0$), since very few parameters are needed to collate the large quantity of data. For nonforward processes, the situation is much more difficult to assess, since essentially arbitrary functions of t are introduced in order to fit the data.

An analysis of the overall situation[10] suggests very strongly that Regge-type "states" are being exchanged and that one is indeed seeing the effect of the rightmost singularities in the complex J plane, each of which has well-defined sets of quantum numbers. It is much less clear, however, that these singularities can be thought of as simple poles. In fact, from a theoretical point of view, it is very likely that there are cuts in the J plane, perhaps ending on the poles.

In the following we shall propose tests that might make it possible to decide on phenomenological grounds, whether or not the singularities can be treated as simple poles. The method hinges on the so-called "factorization theorem," which states that if the singularities are simple poles, then at the poles,

$$f_{11}^{J} f_{22}^{J} = (f_{12}^{J})^{2} . \tag{4.1}$$

Our object will thus be to compare predictions based on a Regge model with and without the imposition of the condition Eq. (4.1).

We are also interested in testing some of the typical assumptions usually made in Regge pole theory, such as keeping only the contributions of P, P', ρ, ω, and R, which are all trajectories of Class I type. The argument in favor of this assumption is based on a consideration of the positions of the known particles and resonances when plotted on a Chew-Frautschi diagram [Re(α) vs t]. Thus if there is no known particle or resonance with the quantum numbers of a particular one of the 12 possible Regge trajectories, then that trajectory is ignored. Also, if from the position of a particle or resonance one is led to suppose that its trajectory $\alpha(t)$, for $t < 0$, is going to lie much lower than the trajectories of the P, P', ρ, ω, and R, then again one usually ignores that trajectory. (For example, the pion is ignored for this latter reason.)

To test these assumptions we construct experimental quantities that, so far as possible, depend solely on those trajectories which are usually ignored.

The procedure is as follows. The division into classes in Table I is based on spin and angular momentum. Thus a suitable geometrical combination of the standard experimental quantities can be found which depends only on Class III trajectories. It is not simple, however, to split up Classes I and II, and for these we have to rely on interference measurements.

An experimental quantity which depends only on a certain class will serve equally well whether we look at that experimental quantity for pp, pn, $p\bar{p}$, or $\bar{p}n$ scattering. However, if we wish to study the <u>individual members</u> of a class, then we must be able to measure linear combinations of the experimental quantity for several or perhaps all of the processes pp, pn, $p\bar{p}$, and $\bar{p}n$. The manner in which these rules arise and their specific applications will become clear in the following.

The g amplitudes are Reggeized in the usual way.[4] We define a new set of partial-wave amplitudes that contain the proper threshold behavior at $t = 0$, and that preserve the factorization theorem, Eq. (4.1), if it holds:

$$f'^{J}_{11}(t) = \frac{E_t}{p_t} (2J + 1) f_{11}^{J}(t) ,$$

$$f'^{J}_{22}(t) = \frac{m^2}{p_t E_t} \frac{2J + 1}{J(J + 1)} f_{22}^{J}(t) ,$$

$$f'^{J}_{1}(t) = \frac{m^2}{p_t E_t} \frac{2J + 1}{J(J + 1)} f_{1}^{J}(t) ,$$

$$f'^{J}_{12}(t) = \frac{m}{p_t} \frac{2J + 1}{[J(J + 1)]^{1/2}} f_{12}^{J}(t) ,$$

$$f'^{J}_{0}(t) = \frac{m^2}{p_t E_t} (2J + 1) f_{0}^{J}(t) . \tag{4.2}$$

The Sommerfeld-Watson transformation is then applied to the g_i's. Retaining only the Regge pole terms we get (putting $m = 1$, as will be done in all that follows),

$$g_1 = \sum_n \zeta_n E^{\alpha_n} \gamma_{11,n}(t) ,$$

$$g_2 = \epsilon \sum_n \zeta_n E^{\alpha_n} [\alpha_n^2 \gamma_{22,n}(t) + \frac{\alpha_n}{E} \gamma_{1,n}(t)] ,$$

$$g_3 = -(-\epsilon)^{1/2} \sum_n \zeta_n E^{\alpha_n} \alpha_n \gamma_{12,n}(t) ,$$

$$g_4 = \epsilon \sum_n \zeta_n E^{\alpha_n} [\alpha_n^2 \gamma_{1,n}(t) + \frac{\alpha_n}{E} \gamma_{22,n}(t)] ,$$

$$g_5 = \epsilon \sum_n \zeta_n E^{\alpha_n} \gamma_{0,n}(t) , \tag{4.3}$$

where terms of order $E^{\alpha_n - 2}$ are neglected, and where $\epsilon = t/4m^2 \leq 0$. The term $\gamma_{i,n}(t)$ (for $i = 0, 1, 11, 12, 22$) is defined by

$$\gamma_{i,n}(t) = -\pi \frac{2^{\alpha_n} \Gamma(\alpha_n + \frac{1}{2})}{\pi^{1/2} \Gamma(\alpha_n + 1)} (-p_t)^{-2\alpha_n} \beta_{i,n}(t) ,$$

where

$$\beta_{i,n}(t) = \lim_{J \to \alpha_n(t)} [J - \alpha_n(t)] f_0'^J(t) .$$

The term $\gamma_{i,n}(t)$ is called the "reduced" residue and is assumed to be a real analytic function with only a right-hand cut. The sum is over all poles of both values of isospin and $\tau = (-1)^J$. Also we have

$$\zeta_n^{(+)} = \frac{1}{2} \left(\cot \left(\frac{\pi\alpha_n}{2} \right) - i \right) \quad \text{for} \quad \tau = +1$$

and (4.4)

$$\zeta_n^{(-)} = \frac{1}{2} \left(\tan \left(\frac{\pi\alpha_n}{2} \right) + i \right) \quad \text{for} \quad \tau = -1 .$$

The factorization theorem now asserts that the pole residues satisfy

$$\gamma_{11,n} \gamma_{22,n} = (\gamma_{12,n})^2 . \tag{4.5}$$

It has been customary[8] to satisfy Eq. (4.5) by introducing b_1 and b_2 so that

$$\begin{aligned} \gamma_{11} &= b_1^2 , \\ \gamma_{22} &= b_2^2 , \\ \gamma_{12} &= b_1 b_2 . \end{aligned} \tag{4.6}$$

The effect of this is to simplify certain formulae, but the results can be misleading. This is because the b's are analogous to coupling constants, and we know nothing about their analyticity properties. On the other hand we believe that we have some knowledge about the properties of the γ's, namely the $\gamma_{i,n}(t)$ are real analytic functions with no left-hand cuts. It will therefore be advantageous to continue using the $\gamma_{i,n}(t)$.

We note that because of the analyticity, if $\gamma_{11}(t)$ has a zero of order r for some negative value of t, then both γ_{22} and γ_{12} must have zeroes at this point. Thus γ_{12} could have a zero of order s and then the zero of γ_{22} would have to be of order $2s - r$. In particular it is obvious that γ_{11} and γ_{22} must change signs together.

Returning now to Eq. (2.11), we give the Regge parameterization for the experimental quantities, without making use of the factorization theorem. For simplicity of writing, we define the operators

$$\sum_R \equiv \frac{1}{4s} \sum_{n,m} c_n c_m \operatorname{Re}(\zeta_n^* \zeta_m) E^{\alpha_n+\alpha_m} ,$$
$$\sum_I \equiv \frac{1}{4s} \sum_{n,m} c_n c_m \operatorname{Im}(\zeta_n^* \zeta_m) E^{\alpha_n+\alpha_m} , \qquad (4.7)$$

where the sums on n and m go over all Regge poles. The expressions for the experimental quantities in terms of the Regge parameters are

$$I_0 = \sum_R \Big\{ \gamma_{11,n}\,\gamma_{11,m} - 2\epsilon\alpha_n\alpha_m\gamma_{12,n}\gamma_{12,m} + \epsilon^2\alpha_n{}^2\alpha_m{}^2\gamma_{22,n}\gamma_{22,m}$$

$$+ \epsilon^2\,\gamma_{0,n}\,\gamma_{0,m} + \epsilon^2\,\alpha_n{}^2\,\alpha_m{}^2\,\gamma_{1,n}\,\gamma_{1,m}$$

$$+ \frac{\epsilon^2\,\alpha_n\,\alpha_m}{E}(\alpha_n + \alpha_m)(\gamma_{1,n}\,\gamma_{22,m} + \gamma_{22,n}\,\gamma_{1,m})\Big\}\,, \quad \text{(a)}$$

$$I_0(1 - D) = 2\epsilon^2 \sum_R \Big\{ \gamma_{0,n}\,\gamma_{0,m} + \alpha_n{}^2\,\alpha_m{}^2\,\gamma_{1,n}\,\gamma_{1,m}$$

$$+ \frac{\alpha_n\,\alpha_m}{E}(\alpha_n\,\gamma_{1,n}\,\gamma_{22,m} + \alpha_m\,\gamma_{22,n}\,\gamma_{1,m})\Big\}\,, \quad \text{(b)}$$

$$I_0C_{NN} = 2\epsilon \sum_R \alpha_m \Big\{ -\alpha_n\,\gamma_{12,n}\,\gamma_{12,m} + \gamma_{22,m}(\alpha_m\,\gamma_{11,n} - \frac{\epsilon}{E}\,\gamma_{0,n})$$

$$- \gamma_{1,m}(\epsilon\,\alpha_m\,\gamma_{0,n} - \frac{1}{E}\,\gamma_{11,n})\Big\}\,, \quad \text{(c)}$$

$$I_0(D_t - C_{NN}) = 4\epsilon^2 \sum_R \alpha_m \Big\{ \alpha_m\,\gamma_{1,m}\,\gamma_{0,n} + \frac{1}{E}\,\gamma_{22,m}\,\gamma_{0,n} \Big\}\,, \quad \text{(d)}$$

$$I_0P = 2(-\epsilon)^{1/2} \sum_I \alpha_n\,\gamma_{12,n}(\gamma_{11,m} + \epsilon\,\alpha_m{}^2\,\gamma_{22,m} + \frac{\epsilon\,\alpha_m}{E}\,\gamma_{1,m})\,, \quad \text{(e)}$$

Cont. (4.8)

$$I_0R = \sum_R \left\{ \alpha(\tfrac{\theta}{2})\mathcal{A}_{mn} + \beta(\tfrac{\theta}{2})\mathcal{B}_{mn} - \cos(\tfrac{\theta}{2})\mathcal{C}_{mn} \right\}, \quad (f)$$

$$I_0A = \sum_R \left\{ -\beta(\tfrac{\theta}{2})\mathcal{A}_{mn} + \alpha(\tfrac{\theta}{2})\mathcal{B}_{mn} + \sin(\tfrac{\theta}{2})\mathcal{C}_{mn} \right\}, \quad (g)$$

$$I_0R' = \sum_R \left\{ \beta(\tfrac{\theta}{2})\mathcal{A}_{mn} - \alpha(\tfrac{\theta}{2})\mathcal{B}_{mn} + \sin(\tfrac{\theta}{2})\mathcal{C}_{mn} \right\}, \quad (h)$$

$$I_0A' = \sum_R \left\{ \alpha(\tfrac{\theta}{2})\mathcal{A}_{mn} + \beta(\tfrac{\theta}{2})\mathcal{B}_{mn} + \cos(\tfrac{\theta}{2})\mathcal{C}_{mn} \right\}, \quad (i)$$

(4.8)

where

$$\mathcal{A}_{mn} = \gamma_{11,m}\,\gamma_{11,n} - \epsilon^2 \alpha_m^{\ 2} \alpha_n^{\ 2}\, \gamma_{22,m}\,\gamma_{22,n} - \frac{\epsilon^2 \alpha_n \alpha_m}{E} (\alpha_n \gamma_{22,n} \gamma_{1,m} + \alpha_m \gamma_{22,m} \gamma_{1,n}) ,$$

$$\mathcal{B}_{mn} = -2(-\epsilon)^{1/2} \alpha_n \gamma_{12,n} (\gamma_{11,m} - \epsilon\, \alpha_m^{\ 2}\, \gamma_{22,n} - \frac{\epsilon\alpha_m}{E} \gamma_{1,m}) ,$$

$$\mathcal{C}_{mn} = \epsilon^2 \left\{ \alpha_m^{\ 2} \alpha_n^{\ 2}\, \gamma_{1,n}\, \gamma_{1,m} - \gamma_{0,n}\, \gamma_{0,m} + \frac{\alpha_n \alpha_m}{E} (\alpha_n \gamma_{1,n} \gamma_{22,m} + \alpha_m \gamma_{1,m} \gamma_{22,n}) \right\} .$$

(4.9)

We now attempt to analyze these results and to derive various tests for the Regge pole hypothesis. The tests are divided into three categories according to whether they examine: 1. general properties of the Regge theory; 2. the validity of the factorization theorem; or 3. the validity of neglecting Class II and III trajectories. We shall assume, as seems entirely reasonable, that if we limit our attention to fairly small values of momentum transfer, we can treat the trajectory functions $\alpha(t)$ as approximately linear functions of t. It then follows, from a consideration of the position of the particles associated with them, that of the three types of trajectory discussed in Table I, the Class I type with $S = 1$, $J = L \pm 1$ (i.e., those associated with the P, p', ρ, ω, and R) have trajectories that lie much higher than the types with $S = 0$, $J = L$ (Class III) or $S = 1$, $J = L$ (Class II). We shall test this conclusion by studying the behavior of experimental quantities that depend directly on the latter two trajectory types.

To make our statements more precise let us define $\alpha_{II}(t)$ and $\alpha_{III}(t)$ to be the highest-lying trajectories of the Class II and Class III respectively.

$$\alpha_{II}(t) = \text{Max}\left\{\alpha_{II_i}(t)\right\} ,$$

$$\alpha_{III}(t) = \text{Max}\left\{\alpha_{III_i}(t)\right\} , \quad \text{for } i = 1,\cdots,4 . \qquad (4.10)$$

We number the following tests with lower case Roman numerals.

1. General Properties

From the formulae of Eq. (4.8), we deduce the following expressions for the experimental quantities.

(i) The differential cross section is given by

$$I_0 = \sum_R \left\{ \gamma_{11,n}\, \gamma_{11,m} - 2\, \epsilon\, \alpha_n\, \alpha_m\, \gamma_{12,n}\, \gamma_{12,m} + \epsilon^2\, \alpha_n^{\,2}\, \alpha_m^{\,2}\, \gamma_{22,n}\, \gamma_{22,m} \right\} + 0(\epsilon^2\, E^{M(t)}) \,, \tag{4.11}$$

where

$$M(t) = \text{Max} \left\{ 2\alpha_{III}(t) - 1;\ \ 2\alpha_{II}(t) - 1;\ \ \alpha_{II}(t) + \alpha_P(t) - 2 \right\} . \tag{4.12}$$

The leading term in I_0 behaves like $E^{2\alpha_P(t)-1}$ and its exact form can be found from Eq. (4.7). This form of I_0 will be useful in discussing the factorization theorem.

(ii) For the polarization we have

$$I_0 P = 2(-\epsilon)^{1/2} \sum_I \alpha_n\, \gamma_{12,n} \left(\gamma_{11,m} + \epsilon\, \alpha_m^{\,2}\, \gamma_{22,m}\right) + 0\left((-\epsilon)^{3/2}\ E^{\alpha_P + \alpha_{II} - 2} \right) . \tag{4.13}$$

The leading terms are

$$I_0 P \approx \frac{(-\epsilon)^{1/2}}{2s} \; c_P \sum_{n \neq P} \operatorname{Im}(\zeta_P^* \, \zeta_n) \; c_n \; E^{\alpha_P + \alpha_n}$$

$$\times \left[\alpha_P \, \gamma_{12,P} (\gamma_{11,n} + \epsilon \, \alpha_n^2 \, \gamma_{22,n}) - \alpha_n \, \gamma_{12,n} (\gamma_{11,P} + \epsilon \, \alpha_P^2 \, \gamma_{22,P}) \right] . \tag{4.14}$$

The remaining formulae of Eq. (4.8) are easily discussed in the same manner, however, the rotation parameters (A, R, A', and R') do not simplify appreciably.

2. Factorization Theorem

(iii) If we apply the factorization theorem to Eq. (4.11), then we get, for the leading terms of I_0 ,

$$I_0 = \frac{1}{4s} \sum_{n,m} c_n \, c_m \; \operatorname{Re}(\zeta_m^* \, \zeta_n) E^{\alpha_m + \alpha_n} \, \gamma_{11,m} \, \gamma_{11,n} \left(1 - \epsilon \, \lambda_n \, \lambda_m\right)^2 \tag{4.15}$$

where

$$\lambda_n(t) = \frac{\alpha_n(t) \, \gamma_{12,n}(t)}{\gamma_{11,n}(t)} . \tag{4.16}$$

Using Table I, we compute the difference of the $p\bar{p}$ and pp differential cross sections to be

$$\Delta I_0 \equiv I_0(p\bar{p}) - I_0(pp) = \frac{1}{4s} \sum_{n=P,P',R}$$

$$\times \left(1 - \cot \frac{\pi\alpha_n}{2} \tan \frac{\pi\alpha_\rho}{2}\right) \gamma_{11,n} \, \gamma_{11,\rho} \left(1 - \epsilon \, \lambda_n \, \lambda_\rho\right)^2$$

$$\times E^{\alpha_n + \alpha_\rho} + (\rho \to \omega) . \tag{4.17}$$

Now, for reasonably small t, the terms $(1 - \cot\frac{\pi\alpha_n}{2}\ \tan\frac{\pi\alpha_\rho}{2})$ and $(1 - \cot\frac{\pi\alpha_n}{2}\ \tan\frac{\pi\alpha_\omega}{2})$ are certainly positive, since for them to become negative would require trajectory slopes far steeper than indicated by the above-mentioned analysis.[9] Also, in the customary form of the theory, $\gamma_{11,n}$ and $\gamma_{11,\rho}$ in Eq. (4.17) appear as $(b_{1,n})^2$ and $(b_{1,\rho})^2$ respectively so that it is tempting to conclude that the difference $I_0(p\bar{p}) - I_0(pp)$ has to be positive.

However, it is known experimentally that ΔI_0 changes sign at $t \approx -0.15(\text{BeV/c})^2$ for high energies. Thus we might conclude that there is here some evidence against the validity of the factorization theorem on which Eq. (4.17) is based.

The alternative is to assume that some of the $\gamma_{1,n}$, say $\gamma_{11,n}(t)$, change sign for $t \approx -0.15(\text{BeV/c})^2$. With respect to the analytic properties of the $\gamma_{2,n}(t)$ there is nothing against this possibility. If, however, we take the customary viewpoint that $\gamma_{11}(t)$ is the square, $[b_1(t)]^2$, of an analytically continued coupling constant, then $b_1(t)$ becomes pure imaginary, i.e., has a branch-point singularity, at this value of t. As mentioned earlier, we do not know enough about the $b_i(t)$ to preclude this possibility, so it is feasible that for some reasons of dynamical origin $b_{1,\rho}$, $b_{2,\rho}$, or $b_{1,\omega}$, $b_{2,\omega}$, or both do indeed become pure imaginary at $t \approx -0.15(\text{BeV/c})^2$.

It is worth noting that both b_1 and b_2 must become pure imaginary at the same t value in order that $\gamma_{12} = b_1 b_2$ be real real analytic for $t < 0$. It is also worth noting that it could not be, say, $b_{1,P}$, $b_{2,P}$ which become imaginary, since that would cause the leading term in both $I_0(pp)$ and $I_0(p\bar{p})$ to change sign.

If we restrict our attention to the NN system we see that there are three possibilities:

(a) The factorization theorem is valid and $\gamma_{11,\rho}$ and $\gamma_{11,\omega}$ change sign at $t \approx -0.15\ (\mathrm{BeV}/c)^2$.

(b) The factorization theorem is valid and either (i) $\gamma_{11,\rho}$ or (ii) $\gamma_{11,\omega}$ change sign at some value of t in the range $0 \geq t > -0.15$.

(c) The factorization theorem is invalid.

R. J. N. Phillips has pointed out that if we take into account πN scattering as well, we can immediately eliminate possibilities (a) and (b.i). It is well known that the charge-exchange scattering process $\pi^- p \rightarrow \pi^0 n$ depends solely on the ρ trajectory. Also the residue functions $\gamma_{1\pi,\rho}$, $\gamma_{2\pi,\rho}$ for this process, and the residue function $\gamma_{\pi\pi,\rho}$ of the $\pi\pi \rightarrow \pi\pi$ process, are related by the factorization theorem to the functions $\gamma_{11,\rho}$, $\gamma_{22,\rho}$ of the NN process. In fact we have

$$\gamma_{\pi\pi,\rho}\,\gamma_{11,\rho} = (\gamma_{1\pi,\rho})^2 ,$$

$$\gamma_{\pi\pi,\rho}\,\gamma_{22,\rho} = (\gamma_{2\pi,\rho})^2 . \qquad (4.18)$$

It is important to realize that these relations are on an equal footing with the relation Eq. (4.5) and follow directly from the factorization theorem.

It follows, then, that if possibility (a) or (b.i) holds then the differential cross section for $\pi^- p \to \pi^0 n$ will vanish at $t \approx -0.15(\text{BeV/c})^2$ or at some point t in the range $0 \geqslant t > -0.15$. However, the experimental measurements on this process show that this is manifestly not true.

We are thus forced to one of two possibilities:

(A) The factorization theorem is valid and $\gamma_{11,\omega}$ changes sign somewhere in the range $0 \geqslant t > -0.15$, or

(B) The factorization theorem is not valid.

We shall discuss these alternatives in tests (vi) and (vii).

(iv) If we now apply the factorization theorem to the leading terms for the difference between the $\bar{p}p$ and pp polarizations (Eq. (4.14)), we find

$$\Delta(I_0 P) = \frac{(-\epsilon)^{1/2}}{4s} \left(\cot \frac{\pi\alpha_P}{2} + \tan \frac{\pi\alpha_\rho}{2}\right) E^{\alpha_P + \alpha_\rho} \gamma_{11,P} \gamma_{11,\rho}$$

$$\times (\lambda_P - \lambda_\rho)(1 - \epsilon \lambda_P \lambda_\rho) + (\rho \to \omega) . \tag{4.19}$$

Thus alternative (a) of test (ii) predicts that

$$\Delta P = 0 \tag{4.20}$$

at $t \approx -0.15 \ (\text{BeV/c})^2$.

If either alternative (b) or (c) holds, then there is no reason to expect that Eq. (4.20) will be true.

(v) $I_0 C_{NN}$ is large despite the fact that it is a spin-correlation quantity, and, as such, depends on the spin-flip amplitudes. Thus

$$I_0 C_{NN} = 2\epsilon \sum_R \alpha_m(\alpha_m \gamma_{22,m} \gamma_{11,n} - \alpha_n \gamma_{12,n} \gamma_{12,m})$$

$$+ O(\epsilon^2 E^{\alpha_P+\alpha_{II}-2}) + O(\epsilon E^{\alpha_P+\alpha_{III}-2}) . \qquad (4.21)$$

Note that the leading term is

$$I_0 C_{NN} \approx \frac{\epsilon}{8s} \operatorname{cosec}^2 \frac{\pi \alpha_P(t)}{2} \alpha_P^2(t) \left\{ \gamma_{11,P} \gamma_{22,P} - \gamma_{12,P}^2 \right\} E^{2\alpha_P(t)} , \qquad (4.22)$$

which will be identically zero if the result Eq. (4.5) of the factorization theorem holds. In case it does, we would have

$$I_0 C_{NN} = O\left(\epsilon E^{\alpha_P+\alpha_H-1}\right) \qquad (4.23)$$

where $\alpha_H(t)$ is the highest-lying trajectory from among P', ρ, ω, and R.

If the factorization theorem does not hold then there is something amiss with the simple pole Regge model, and Eq. (4.21) is probably not correct. However, it is likely to be correct up to logarithmic factors in E. Thus to the extent we cannot distinguish logarithmic factors experimentally, a measurement of the variation of $I_0 C_{NN}$ with energy would help to decide the validity of the simple pole Regge model.

Next, we consider the difference between I_0C_{NN} for $p\bar{p}$ and pp. We get, for the leading terms,

$$\Delta(I_0C_{NN}) \equiv I_0C_{NN}(p\bar{p}) - I_0C_{NN}(pp)$$

$$= \frac{\epsilon}{4s}\left(1 - \cot\frac{\pi\alpha_P}{2}\tan\frac{\pi\alpha_\rho}{2}\right) E^{\alpha_P + \alpha_\rho}$$

$$\times \left[\alpha_P^{\ 2}\,\gamma_{22,P}\,\gamma_{11,\rho} + \alpha_\rho^{\ 2}\,\gamma_{22,\rho}\,\gamma_{11,P} - 2\alpha_P\,\alpha_\rho\,\gamma_{12,P}\,\gamma_{12,\rho}\right]$$

$$+ (\rho \rightarrow \omega). \tag{4.24}$$

Using the result Eq. (4.5) of the factorization theorem, we get

$$\Delta C_{NN} \approx \frac{\epsilon}{4s}\left(1 - \cot\frac{\pi\alpha_P}{2}\tan\frac{\pi\alpha_\rho}{2}\right) E^{\alpha_P + \alpha_\rho}\,\gamma_{11,P}\,\gamma_{11,\rho}(\lambda_P - \lambda_\rho)^2$$

$$+ (\rho \rightarrow \omega). \tag{4.25}$$

Again alternative (a) of test (iii) implies

$$\Delta C_{NN} = 0$$

at $\quad t \approx -0.15\ (\mathrm{BeV}/c)^2$.

(vi) If the alternative (A) of test (iii) holds, then it has the following consequence. Let $t_0 \approx -0.15(\mathrm{BeV}/c)^2$ be the value of t for which $\Delta I_0 = 0$. Then for some value $t = t_1$, with $0 > t_1 > t_0$, $\gamma_{11,\omega}$ changes sign, i.e.,

$$\gamma_{11,\omega}(t_1) = 0 . \tag{4.26}$$

Then at $t = t_1$, if we are at sufficiently high energies, we have from Eqs. (4.17), (4.19) and (4.25),

$$\Delta I_0 = \frac{1}{4s} \frac{\sin \frac{\pi}{2} (\alpha_P - \alpha_\rho)}{\sin \frac{\pi}{2} \alpha_P \cos \frac{\pi}{2} \alpha_\rho} \gamma_{11,P} \gamma_{11,\rho}(1 - \epsilon \lambda_P \lambda_\rho)^2 E^{\alpha_P + \alpha_\rho} ,$$

$$\Delta(I_0 P) = \frac{(-\epsilon)^{1/2}}{4s} \frac{\cos \frac{\pi}{2} (\alpha_P - \alpha_\rho)}{\sin \frac{\pi}{2} \alpha_P \cos \frac{\pi}{2} \alpha_\rho} \gamma_{11,P} \gamma_{11,\rho}(\lambda_P - \lambda_\rho) \times (1 - \epsilon \lambda_P \lambda_\rho) E^{\alpha_P + \alpha_\rho} ,$$

$$\Delta(I_0 C_{NN}) = \frac{\epsilon}{4s} \frac{\sin \frac{\pi}{2} (\alpha_P - \alpha_\rho)}{\sin \frac{\pi}{2} \alpha_P \cos \frac{\pi}{2} \alpha_\rho} \gamma_{11,P} \gamma_{11,\rho}(\lambda_P - \lambda_\rho)^2 E^{\alpha_P + \alpha_\rho} . \tag{4.27}$$

Since $t = t_1$ is very close to $t = 0$, we may assume that we know $[\alpha_P(t_1) - \alpha_\rho(t_1)]$ with some degree of certainty. (It would even be quite reasonable to approximate this quantity by $[\alpha_P(0) - \alpha_\rho(0)]$, since the trajectory slopes are known to be reasonably small.) We then construct the experimental quantity

$$L(t) = \frac{1}{\epsilon} \left\{ \frac{\Delta I_0 \, \Delta(I_0 C_{NN})}{\sin^2 \frac{\pi}{2} (\alpha_P - \alpha_\rho)} + \left(\frac{\Delta(I_0 P)}{\cos \frac{\pi}{2} (\alpha_P - \alpha_\rho)} \right)^2 \right\} . \tag{4.28}$$

It follows from Eq. (4.27) that at sufficiently high energies, if alternative (A) holds, then

$$L(t_1) = 0 . \tag{4.29}$$

Note that at $t = 0$, $L(0)$ has a finite value. If we use the fact that $\alpha_\rho(0) \approx \alpha_\omega(0)$ and $\alpha_P(0) \approx 1$ we have, at $t = 0$,

$$L(0) = \frac{\gamma_{11,P}^2 \ \gamma_{11,\rho} \ \gamma_{11,\omega}}{16 \ s^2 \ \cos^2 \frac{\pi}{2} \ \alpha_\rho} \ (\lambda_\rho - \lambda_\omega)^2 \ E^{2(1+\alpha_\rho)} , \tag{4.30}$$

Taking the currently accepted value $\alpha_\rho(0) \approx \frac{1}{2}$,

$$L(0) = \frac{E}{128} \ \gamma_{11,P}^2 \left[(\gamma_{22,\rho} \ \gamma_{11,\omega})^{1/2} - (\gamma_{22,\omega} \ \gamma_{11,\rho})^{1/2} \right]^2 , \tag{4.31}$$

where, of course, the $\gamma_{i,n}(t)$ are evaluated at $t = 0$.

Thus alternative (A) predicts that at high energies, $L(t)$ will become zero somewhere in the region $0 > t > t_0$.

(vii) A further test of alternative (A) can be devised by constructing a combination of processes that depend, at high energies, on $\gamma_{11,\omega}$ only. Unfortunately this takes us a little way into the experimental realm of the future, but it will be worth while to record the results.

If alternative (A) holds, then the three quantities

$$\Delta I_{\omega}(t) = I_0(pp) + I_0(pn) - I_0(\bar{p}p) - I_0(\bar{p}n) ,$$

$$\Delta P_{\omega}(t) = I_0P(pp) + I_0P(pn) - I_0P(p\bar{p}) - I_0P(\bar{p}n) ,$$

$$\Delta C_{\omega}(t) = I_0C_{NN}(pp) + I_0C_{NN}(pn) - I_0C_{NN}(\bar{p}p) - I_0C_{NN}(\bar{p}n) , \tag{4.32}$$

will all vanish at some value $t = t_1$ with $0 > t_1 \geq t_0$.

3. Class II and III Trajectories

(viii) $I_0(1 - D)$ drops quickly with energy, since it depends on the interference of trajectories of Class II and Class III. In fact,

$$I_0(1 - D) = O(\epsilon^2 \; E^{M(t)}) . \tag{4.33}$$

Thus if $M(t)$ is measured experimentally we can deduce [see Eq. (4.10) and (4.12)]

$$\alpha_{III}' \leq \frac{1}{2}(1 + M(t)) ,$$

$$\alpha_{II} \leq \frac{1}{2}(1 + M(t)) ,$$

and (4.34)

$$\alpha_{II} \leq 2 + M(t) - \alpha_P .$$

(ix) $I_0(D_t - C_{NN})$ depends on interference with Class III trajectories. It will be of order $\epsilon^2 \; E^{N(t)}$, where

$$N(t) = \text{Max}\left\{\alpha_{III}(t) + \alpha_{II}(t) - 1; \quad \alpha_{III}(t) + \alpha_P(t) - 2\right\} . \tag{4.35}$$

From a measurement of $N(t)$ we could then deduce

$$\alpha_{III} + \alpha_{II} \leqslant 1 + N(t)$$

and

$$\alpha_{III} \leqslant 2 + N(t) - \alpha_P . \tag{4.36}$$

(x) The experimental quantity Q defined as

$$Q = I_0(1 - D) + (I_0R - I_0A') \frac{1}{\cos(\theta/2)}$$

$$= I_0(1 - D) - (I_0R' + I_0A) \frac{1}{\sin(\theta/2)} \tag{4.37}$$

depends solely on Class III trajectories. It is given by

$$Q = 4\epsilon^2 \sum_R \gamma_{0,n} \gamma_{0,m} , \tag{4.38}$$

and the leading term in its energy variation is then $E^{2\alpha_{III}-1}$ [Eq. (4.10)]. A measurement of Q will thus provide us with direct evidence about the Class III trajectories.

(xi) In general if X_{III} is any experimental quantity that depends only on Class III, then the dependence on the individual members of the class can be isolated to some extent by measuring X_{III} for all of the processes pp, $\bar{p}p$, pn, and $\bar{p}n$. The results are

$$X_{III}(pp) - X_{III}(\bar{p}p) + X_{III}(pn) - X_{III}(\bar{p}n) \propto III_1\, III_2 + III_3\, III_4 \,,$$

$$X_{III}(pp) - X_{III}(\bar{p}p) - X_{III}(pn) + X_{III}(\bar{p}n) \propto III_1\, III_3 + III_2\, III_4 \,,$$

$$X_{III}(pp) + X_{III}(\bar{p}p) - X_{III}(pn) - X_{III}(\bar{p}n) \propto III_1\, III_4 + III_2\, III_3 \,, \qquad (4.39)$$

where the symbol $\propto$ is used here to mean "depends on." The symbol III_j means the sum of the amplitudes arising from all trajectories of type III_j.

The quantity Q in test (x) is an X_{III} type quantity. If only the $\pi(III_4)$ and the $\eta(III_1)$ trajectories are important, we get the prediction that the combination of quantities in the last line of Eq. (4.39) should be much larger than the other two combinations.

If it is taken for granted that α_{III_2} and α_{III_3} are negligible, then a simpler method can be used to estimate the interference between π- and η-like trajectories, since then we have

$$X_{III}(pp) - X_{III}(pn) \sim III_1\, III_4 \,. \qquad (4.40)$$

Explicitly for Q we have

$$Q_{pp} - Q_{pn} = \frac{4\epsilon^2}{s} \mathrm{Re}(\zeta_\pi^* \zeta_\eta)\, \gamma_{0,\pi}\, \gamma_{0,\eta}\, E^{\alpha_\pi + \alpha_\eta} \,. \qquad (4.41)$$

(xii) If an experimental quantity Y depends only on Class I and II, then in practice it is more difficult than in test (xi) to isolate the dependence on individual members I_j or II_k. A typical equation of the type Eq. (4.39) would now read

$$Y_{pp} - Y_{\bar{p}p} + Y_{pn} - Y_{\bar{p}n} \propto (I_1 + II_1)(I_2 + II_2) + (I_3 + II_3)(I_4 + II_4), \tag{4.42}$$

and similarly for the other combinations.

In practice, however, the Class II trajectories are completely ignored, so that equations of the type Eq. (4.42) yield some knowledge about the individual I_j members.

The tests discussed in (vii) are of this type and the validity of the conclusion that the quantities listed in Eq. (4.32) will vanish at $t = t_0$ is, strictly speaking, true only if the Class II trajectories can indeed be ignored.

It is therefore of great importance to measure further experimental quantities which depend directly on Class II trajectories. We now suggest a possible check.

(xiii) The experimental quantity $\bar{Q}$ defined as

$$\begin{aligned} \bar{Q} &= I_0(1 - D) + I_0(A' - R)\frac{1}{\cos(\theta/2)} \\ &\equiv I_0(1 - D) + I_0(A + R')\frac{1}{\sin(\theta/2)} \end{aligned} \tag{4.43}$$

depends solely on Class I and Class II trajectories. It is given by

$$\bar{Q} = 4\epsilon^2 \sum_R \left\{ \alpha_n^2 \alpha_m^2 \gamma_{1,n} \gamma_{1,m} + \frac{\alpha_n \alpha_m}{E} \left(\alpha_n \gamma_{1,n} \gamma_{22,m} + \alpha_m \gamma_{1,m} \gamma_{22,n} \right) + \frac{\alpha_n \alpha_m}{E^2} \gamma_{22,n} \gamma_{22,m} \right\} . \tag{4.44}$$

The quantity $\bar{Q}$ should be quite sensitive to the Class II trajectories, since the Class I trajectories always enter in Eq. (4.44) with an extra factor of E in the denominator.

The leading term is

$$Q = O(\epsilon^2 E^{V(t)}) , \tag{4.45}$$

where

$$V(t) = \text{Max} \left\{ 2\alpha_{II} - 1; \quad \alpha_P + \alpha_{II} - 2; \quad 2\alpha_P - 3 \right\} . \tag{4.46}$$

Thus Q is expected to drop rapidly with energy. From a measurement of $V(t)$ we could then conclude that

$$\alpha_{II} \leq \frac{1}{2}\left(1 + V(t)\right)$$

and

$$\alpha_{II} \leq V(t) + 2 - \alpha_P . \tag{4.47}$$

CONCLUSION

The measurement and analysis of the various experimental quantities discussed in Sec. IV would provide the basis for a critical examination of the general validity of the Regge pole theory and of some of the customary assumptions made in the application thereof. In particular the measurements discussed in tests (iii) through (vii) are designed to test the validity of the factorization theorem that follows from the assumption of a simple pole Regge model. There is already some weak indication from the difference of the $p\bar{p}$ and pp differential cross sections that the factorization theorem is not valid.

The tests described in (viii) through (xiii) are designed to examine the validity of the customary assumption that Regge trajectories of Class II and III can be ignored.

It is hoped that in the not too distant future there will be available sufficient experimental evidence to clinch the case pro or contra the Regge pole theory.

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APPENDIX

It is useful to relate the ϕ and the g amplitudes to the Wolfenstein parameters $a(\theta)$, $c(\theta)$, $m(\theta)$, $g(\theta)$, and $h(\theta)$ as defined by Stapp, Eq. (2.10) of Ref. 5. Using Raynal's[11] Eq. (16) and Appendix 2, we write the Wolfenstein parameters in terms of the helicity amplitudes of Ref. 1:

$$4a(\theta) = \phi_1 - \phi_2 + \phi_3 + \phi_4 + (\phi_1 + \phi_2 + \phi_3 - \phi_4)\cos\theta - 4\phi_5 \sin\theta$$

$$4ic(\theta) = (\phi_1 + \phi_2 + \phi_3 - \phi_4)\sin\theta + 4\phi_5 \cos\theta$$

$$4m(\theta) = -(\phi_1 - \phi_2 + \phi_3 + \phi_4) + (\phi_1 + \phi_2 + \phi_3 - \phi_4)\cos\theta - 4\phi_5 \sin\theta$$

$$4g(\theta) = -\phi_1 + \phi_2 + \phi_3 + \phi_4$$

$$4h(\theta) = -\phi_1 - \phi_2 + \phi_3 - \phi_4 .$$

X in terms of ϕ [Eq. (2.11)] can now be derived from Stapp's Table I.

The crossing relation between the g's of Eq. (2.7) and the Wolfenstein parameters is

$$\begin{pmatrix} a(\theta) \\ m(\theta) \\ c(\theta) \\ g(\theta) \\ h(\theta) \end{pmatrix} = \frac{1}{2(s)^{1/2}} \begin{pmatrix} 1+\alpha(\theta) & -1+\alpha(\theta) & 2\beta(\theta) & 0 & 0 \\ -1+\alpha(\theta) & 1+\alpha(\theta) & 2\beta(\theta) & 0 & 0 \\ i\beta(\theta) & i\beta(\theta) & -2i\alpha(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{pmatrix}$$

where the Wolfenstein parameters are now expressed in terms of the analytically continued g's . $\alpha(\theta)$ and $\beta(\theta)$ were given in Eq. (2.12).

The crossing relation and Stapp's Table I can be used to redrive Eq. (2.11), and the triple correlation parameters, if needed.

Table I. Classification of the 12 possible exchanges giving their relative contributions to pp, $p\bar{p}$, pn, and $\bar{p}n$ elastic scattering amplitudes. Not all the listed quantum numbers are independent. The particle names corresponding to the quantum numbers are taken from Rosenfeld, et al.[7]

Type of exchange	Quantum numbers I	$(-1)^J$	P	G	Label	Possible particles	Value of the coefficients $c^{(p\bar{p})}$	$c^{(pp)}$	$c^{(pn)}$	$c^{(\bar{p}n)}$
Class I										
s=1; J=L±1	0	+	+	+	I_1	p, p', f	+	+	+	+
	0	-	-	-	I_2	ω, ϕ	-	+	+	-
	1	-	-	+	I_3	ρ	-	+	-	+
	1	+	+	-	I_4	A_2(R)	+	+	-	-
Class II										
s=1; J=L	0	-	+	+	II_1		+	+	+	+
	0	+	-	-	II_2		-	+	+	-
	1	+	-	+	II_3		-	+	-	+
	1	-	+	-	II_4	A_1	+	+	-	-
Class III										
s=0; J=L	0	+	-	+	III_1	η, X^0	+	+	+	+
	0	-	+	-	III_2		-	+	+	-
	1	-	+	+	III_3		-	+	-	+
	1	+	-	-	III_4	π	+	+	-	-

FOOTNOTES AND REFERENCES

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