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Spacetime equals entanglement

Yasunori Nomura [a,b,c], Nico Salzetta [a,b,*], Fabio Sanches [a,b], Sean J. Weinberg [a,b]

[a] Berkeley Center for Theoretical Physics, Department of Physics, University of California, Berkeley, CA 94720, USA
[b] Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
[c] Kavli Institute for the Physics and Mathematics of the Universe (WPI), The University of Tokyo Institutes for Advanced Study, Kashiwa 277-8583, Japan

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A B S T R A C T

We study the Hilbert space structure of classical spacetimes under the assumption that entanglement in holographic theories determines semiclassical geometry. We show that this simple assumption has profound implications; for example, a superposition of classical spacetimes may lead to another classical spacetime. Despite its unconventional nature, this picture admits the standard interpretation of superpositions of well-defined semiclassical spacetimes in the limit that the number of holographic degrees of freedom becomes large. We illustrate these ideas using a model for the holographic theory of cosmological spacetimes.

1. Introduction

How does the semiclassical picture arise from the fundamental theory of quantum gravity? Recently it has become increasingly clear that quantum entanglement in holographic [1,2] descriptions plays an important role in the emergence of the classical spacetime of general relativity [3–8]. This raises the possibility that entanglement is indeed the defining property that controls the physics of dynamical spacetimes.

In this letter we take the view that entanglement in holographic theories *determines* gravitational spacetimes at the semiclassical level. Rather than proving this statement, we adopt it as a guiding principle and explore its consequences. This principle has profound implications for the structure of the Hilbert space of quantum gravity. In particular, it allows us to obtain a classical spacetime as a superposition of (an exponentially large number of) different classical spacetimes. We show that despite its unconventional nature, this picture admits the standard interpretation of superpositions of well-defined semiclassical spacetimes in the limit that the number of holographic degrees of freedom becomes large.

To illustrate these concepts, we use a putative holographic theory for cosmological spacetimes, in which the effects appear cleanly. Our basic points, however, persist more generally; in particular, we expect that they apply to a region of the bulk in the AdS/CFT correspondence [9]. In the context of Friedmann–Robertson–Walker (FRW) universes, we find an interesting "Russian doll" structure: states representing a universe filled with a fluid having an equation of state parameter w are obtained as exponentially many (exponentially rare) superpositions of those having an equation of state with $w' > w$ ($< w$).

While completing this work, we received Ref. [10] by Almheiri, Dong and Swingle which studies how holographic entanglement entropies are related to linear operators in the AdS/CFT correspondence. Their analysis of the thermodynamic limit of the area operators overlaps with ours. See also Ref. [11] for related discussion.

2. Holographic theory on screens

We begin by describing the holographic framework we work in. The AdS/CFT case appears as a special situation of this more general (albeit more conjectural) framework.

The covariant entropy bound [12] implies that the entropy on a null hypersurface generated by a congruence of light rays terminated by a caustic or singularity is bounded by its largest cross sectional area $\mathcal{A}$ divided by 2 in Planck units. (The entropy on each side of the largest cross sectional surface is bounded by $\mathcal{A}/4$.) This suggests that for a fixed gravitational spacetime, the holographic theory lives on a hypersurface—called the holographic screen—on which null hypersurfaces foliating the spacetime have the largest cross sectional areas [13].

The procedure of erecting a holographic screen has a large ambiguity. A particularly useful choice [14,15] is to adopt an "observer centric reference frame." Let the origin of the reference frame follow a timelike curve $p(\tau)$ which passes through a fixed spacetime point p_0 at $\tau = 0$, and consider the congruence of past-directed light rays emanating from p_0. Assuming the null energy condition,

* Corresponding author.
E-mail address: nsalzetta@berkeley.edu (N. Salzetta).

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the light rays focus toward the past, and we may identify the apparent horizon, i.e. the codimension-2 surface on which the expansion of the light rays vanishes, to be an equal-time hypersurface—called a leaf—of a holographic screen. Repeating the procedure for all τ, we obtain a specific holographic screen, with the leaves parameterized by τ, corresponding to foliating the spacetime region accessible to the observer at $p(\tau)$. Such a foliation is consonant with complementarity [16] which asserts that a complete description of a system refers only to the spacetime region that can be accessed by a single observer.

With this construction, we can view a quantum state of the holographic theory as living on a leaf of the holographic screen obtained as above. We can then consider the collection of all possible quantum states on all possible leaves, obtained by considering all timelike curves in all spacetimes. It is often convenient to consider Hilbert space $\mathcal{H}_B$ spanned by the states living on the "same" leaf B.[1] We can then write the full Hilbert space as [14,15]

$$\mathcal{H} = \sum_B \mathcal{H}_B + \mathcal{H}_{\rm sing}, \tag{1}$$

where $\mathcal{H}_{\rm sing}$ contains intrinsically quantum gravitational states that do not admit a spacetime interpretation, and we have defined the sum of Hilbert spaces by[2]

$$\mathcal{H}_1 + \mathcal{H}_2 = \{v_1 + v_2 \,|\, v_1 \in \mathcal{H}_1, v_2 \in \mathcal{H}_2\}. \tag{2}$$

This formulation is not restricted to descriptions based on fixed semiclassical spacetime backgrounds. For example, we may consider a state in which macroscopically different universes are superposed. Time evolution of a quantum gravity state occurs within the space of Eq. (1).

Recently, Bousso and Engelhardt have identified two special classes of holographic screens [17,18]: if a portion of a holographic screen is foliated by marginally anti-trapped (trapped) surfaces, then that portion is called a past (future) holographic screen. They proved that the area of leaves $\mathcal{A}(\tau)$ monotonically increases (decreases) for a past (future) holographic screen. Furthermore, Ref. [19] proved that this area law holds locally. In many regular circumstances, including expanding FRW universes, the holographic screen is a past holographic screen, so that the area of the leaves monotonically increases, $d\mathcal{A}(\tau)/d\tau > 0$. In this letter we focus on this case.

What is the structure of the holographic theory and how can we explore it? Recently, a conjecture has been made in Ref. [8] which relates geometries of general spacetimes to the entanglement entropies of states in the holographic theory. This extends the analogous statement [3,4,20] in AdS/CFT to more general cases. In particular, Ref. [8] proved that for a given region Γ of a leaf σ, a codimension-2 extremal surface $E[\Gamma]$ anchored to the boundary $\partial\Gamma$ of Γ is fully contained in the causal region D_σ of σ: the domain of dependence of an interior achronal hypersurface whose only boundary is σ. This implies that the normalized area of the extremal surface $E[\Gamma]$

$$S[\Gamma] = \frac{1}{4}\|E[\Gamma]\|, \tag{3}$$

satisfies expected properties of entanglement entropy, so that it can be identified with the entanglement entropy of the region Γ in the holographic theory. Here, $\|x\|$ represents the area of x. If there are multiple extremal surfaces in D_σ for a given Γ, then we must take the one with the minimal area.

[1] In general, the equivalence condition for the label B is not well understood. For states representing FRW universes, however, we expect from the high symmetry of the system that B is uniquely specified by the leaf area (at least in some coarse sense).

[2] Unlike Ref. [15], here we do not assume specific relations between $\mathcal{H}_B$'s or $\mathcal{H}_{\rm sing}$. In particular, $\mathcal{H}_{B_1}$ and $\mathcal{H}_{B_2}$ for different leaves B_1 and B_2 may not be orthogonal.

3. Holography for FRW universes

Adopting the above framework, we now study the holographic description of $(3+1)$-dimensional FRW universes

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\psi^2 + \sin^2\psi\, d\phi^2)\right], \tag{4}$$

where $a(t)$ is the scale factor, and $\kappa < 0$, $= 0$ and > 0 for open, flat and closed universes, respectively. We choose the origin of the reference frame, $p(\tau)$, to be at $r = 0$. The holographic theory then lives on the holographic screen at

$$r = \frac{1}{\sqrt{\dot{a}^2(t_*) + \kappa}} \equiv r_\sigma(t_*), \tag{5}$$

where the dot represents t derivative, and t_* is the FRW time on a leaf. For flat and open universes, the leaves always form a past holographic screen as long as the universe is initially expanding. Below, we focus on these cases.

For the purpose of illustrating our points, it is sufficient to consider a "single" Hilbert space $\mathcal{H}_* \in \{\mathcal{H}_B\}$ specified by a fixed leaf area $\mathcal{A}_*$. Specifically, we consider FRW universes with $\kappa \le 0$ having vacuum energy ρ_Λ and filled with various ideal fluid components. For every universe with $\rho_\Lambda < 3/2\mathcal{A}_*$, there is an FRW time t_* at which the area of the leaf is $\mathcal{A}_*$. Any quantum state representing the system at such a time is an element of $\mathcal{H}_*$.

How does a state in $\mathcal{H}_*$ encode information about the universe it represents? Consider an FRW universe with the energy density given by $\rho(t)$. We can then determine the FRW time t_* at which the leaf σ_* has the area $\mathcal{A}_*$. Now, consider a spherical cap region of σ_* specified by an angle γ $(0 \le \gamma \le \pi)$:

$$L(\gamma):\ \ t = t_*, \quad r = r_\sigma(t_*), \quad 0 \le \psi \le \gamma, \tag{6}$$

and determine the extremal surface $E(\gamma)$ anchored on the boundary of $L(\gamma)$. The quantity

$$S(\gamma) = \frac{1}{4}\|E(\gamma)\|, \tag{7}$$

then gives the entanglement entropy of the region $L(\gamma)$ in the holographic theory.

We focus on the case in which the expansion of the universe is dominated by a single fluid component with w or negative spacetime curvature in (most of) the region probed by the extremal surfaces anchored to σ_*. This holds for almost all t in realistic FRW universes. In this case, $S(\gamma)$ becomes extensive with respect to $\mathcal{A}_*$ [21], so that

$$\tilde{S}(\gamma) = \frac{S(\gamma)}{\mathcal{A}_*/4}, \tag{8}$$

is independent of $\mathcal{A}_*$. This effectively counts the number of Bell pairs between $L(\gamma)$ and its complement per (qubit) degree of freedom.

We may express $\tilde{S}(\gamma)$ as a function of the fractional volume that $L(\gamma)$ occupies on σ_*

$$F(\gamma) = \frac{1}{2}(1 - \cos\gamma), \tag{9}$$

i.e. $s(F) \equiv \tilde{S}(\gamma(F))$. In Fig. 1, we plot this quantity for flat universes with $w = -1$ (vacuum energy), -0.98, -0.8, 0 (matter),

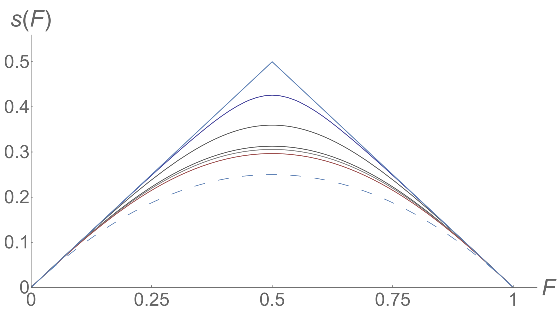

Fig. 1. The normalized entanglement entropy $s(F)$ for flat universes with $w = -1$, -0.98, -0.8, 0, 1/3, 1 (solid lines, from the top to the bottom) and for a curvature dominated open universe (dashed line).

1/3 (radiation), 1 and for a curvature dominated open universe.[3] This shows how a state in the holographic theory encodes the information about the spacetime it represents. For example, $s(F)$ decreases monotonically in w for any fixed F.

There are simple geometric bounds on $s(F)$. The maximin construction [8,22] states that the extremal surface is the one having the maximal area among all possible codimension-2 surfaces each of which is anchored on $\partial L(\gamma)$ and has minimal area on some interior achronal hypersurface bounded by σ_*. This implies that $s(F)$ obeys [21]

$$F(1-F) \le s(F) \le \frac{1}{2} - \left(\frac{1}{2} - F\right) \mathrm{sgn}\left(\frac{1}{2} - F\right). \tag{10}$$

The universes dominated by a $w = -1$ component and curvature saturate the upper and lower limits, respectively.

4. Qubit model

Unlike the case of asymptotically AdS spacetimes, entanglement entropies in the holographic theory for FRW universes obey a volume law. (From the viewpoint of the holographic theory, $\mathcal{A}_*$ is a volume.) This motivates us to consider the following toy model for holographic states representing FRW universes.

Consider a Hilbert space for N ($\gg 1$) qubits $\mathcal{H} = (\mathbf{C}^2)^{\otimes N}$. Let Δ ($\le N$) be a nonnegative integer and consider a typical superposition of 2^Δ product states

$$|\psi\rangle = \sum_{i=1}^{2^\Delta} a_i \, |x_1^i\rangle |x_2^i\rangle \cdots |x_N^i\rangle, \tag{11}$$

where $\{a_i\}$ is a normalized complex vector, and $x^i_{1,\cdots,N} \in \{0,1\}$. Given an integer n with $1 \le n < N$, we can break the Hilbert space into a subsystem Γ for the first n qubits and its complement $\bar{\Gamma}$. We are interested in computing the entanglement entropy of Γ, $S_{\rm EE}(\Gamma)$.

Suppose $n \le N/2$. If $\Delta \ge n$, then i in Eq. (11) runs over an index that takes many more values than the dimension of the Hilbert space for Γ, so that Page's argument [23] tells us that Γ has maximal entanglement entropy: $S_{\rm EE}(\Gamma) = n \ln 2$. On the other hand, if $\Delta < n$ then the number of terms in Eq. (11) is much less than both the dimension of the Hilbert space of Γ and that of $\bar{\Gamma}$, which limits the entanglement entropy: $S_{\rm EE}(\Gamma) = \Delta \ln 2$.

We therefore obtain

$$S_{\rm EE}(\Gamma) = \begin{cases} n & n \le \Delta, \\ \Delta & n > \Delta, \end{cases} \tag{12}$$

for $\Delta < N/2$, while

$$S_{\rm EE}(\Gamma) = n, \tag{13}$$

for $\Delta \ge N/2$. Here and below, we drop the irrelevant factor of $\ln 2$. The value of $S_{\rm EE}(\Gamma)$ for $n > N/2$ is obtained from $S_{\rm EE}(\Gamma) = S_{\rm EE}(\bar{\Gamma})$ since $|\psi\rangle$ is pure.

The behavior of $S_{\rm EE}(\Gamma)$ in Eqs. (12), (13) is reminiscent of that of $s(F)$ in Fig. 1. The correspondence is given by

$$\frac{n}{N} \leftrightarrow F, \tag{14}$$

$$\frac{\Delta}{N} \leftrightarrow s\left(\frac{1}{2}\right), \tag{15}$$

for $\Delta \le N/2$.[4] In fact, we can consider the $N = \mathcal{A}_*/4$ qubits to be distributed over σ_* with each qubit occupying a volume of 4 in the holographic theory. The identification of Eq. (14) is then natural. The quantity Δ controls what universe a state represents. For fixed Δ, different choices of the product states $|x_1^i\rangle|x_2^i\rangle \cdots |x_N^i\rangle$ and the coefficients a_i give e^N independent microstates for the FRW universe with $w = f(\Delta/N)$. The function f is determined by Eq. (15); in particular, $f = -1$ (> -1) for $\Delta/N = 1/2$ ($< 1/2$).[5]

Below, we assume that this model captures essential features of the holographic theory.[6] An important point is that the set of states with a fixed Δ does not comprise a Hilbert space. Moreover, the set of states with any fixed Δ spans the *entire* Hilbert space, containing all FRW universes corresponding to all values of Δ. For example, we may obtain a state with any $w' < w$ by superposing $e^{\Delta_{w'} - \Delta_w}$ states with Δ_w, where $\Delta_w \equiv N f^{-1}(w)$. We may also obtain a state with $w' > w$ as a superposition of carefully chosen e^{Δ_w} states with Δ_w.[7] These statements do not depend on the details of the model and are manifestations of the fact that entanglement, and thus spacetime geometry, cannot be represented by a linear operator at the microscopic level. Fig. 2 depicts a sketch of the Hilbert space structure described here.

5. Superpositions

One might worry that the Hilbert space structure described above may not allow for a consistent interpretation of superpositions of semiclassical universes. For example, consider two states representing universes with w_1 and w_2 ($w_1 > w_2$), which contain respectively e^{Δ_1} and e^{Δ_2} product states ($\Delta_1 < \Delta_2$). Their superposition contains $e^{\Delta_1} + e^{\Delta_2} \approx e^{\Delta_2}$ product states. Does this mean that a superposition of w_1 and w_2 universes always leads to a w_2 universe, making any reasonable many worlds interpretation of spacetime impossible?

[3] In the case of a strictly single $w = -1$ component or an exactly empty open universe, the leaf is located at an infinite affine distance from p_0. We view this as a mathematical idealization. Realistic universes are obtained, e.g., by introducing an infinitesimally small amount of additional matter.

[4] States with $\Delta > N/2$ cannot be discriminated from those with $\Delta = N/2$ using $S_{\rm EE}(\Gamma)$ alone. The identity of these states is not clear. Below, we focus on the states with $N/4 \le \Delta \le N/2$.

[5] For the present purpose, the curvature dominated universe can be regarded as the universe filled with a fluid having $w = +\infty$: $f = +\infty$ for $\Delta/N = 1/4$.

[6] The holographic theory may have degrees of freedom representing the region outside D_{σ_*}, which may be entangled with those described here. We assume that this does not affect the analyses below based on $x^i_{1,\cdots,N}$ unless we probe more than half of them; the extra degrees of freedom may become relevant if we consider $n \ge N/2$. This property indeed appears if the extra degrees of freedom are modeled by an additional N qubits, and the FRW states are taken as typical superpositions of 2^Δ product states in the enlarged Hilbert space.

[7] These superpositions must also change the matter content filling the universe.

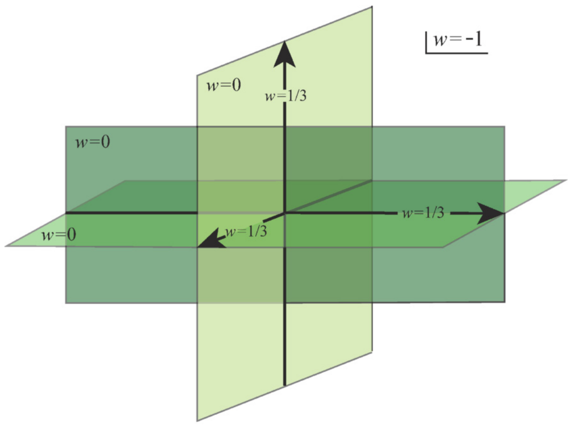

Fig. 2. A sketch of the Hilbert space for FRW universes, where we have chosen three values of $w = -1, 0, 1/3$ for illustrative purposes. Note that while the figure is drawn in 3-dimensional space, the actual dimension of the Hilbert space (as well as that of a region with a fixed w) is exponentially large. In particular, to go outside a region with a fixed w, an exponentially large number of microstates must be superposed. The same is true to go to a smaller region with $w' > w$.

Below we show that this is not the case. In particular, information about each semiclassical spacetime is contained in the exponentially differing size of the coefficients when the state is expanded in the product state basis.

We regard universes with the equation of state parameters falling in a range $\delta w \ll 1$ to be macroscopically identical. Here, δw is a small number that does not scale with $\mathcal{A}_*$. We then find that a superposition of less than $e^{O(\delta w \mathcal{A}_*)}$ microstates representing a universe with some w leads only to another microstate representing the same w universe. In other words, $e^N = e^{\mathcal{A}_*/4}$ microstates of the universe with *any* w form an "effective vector space" unless we consider a superposition of an exponentially large, $\gtrsim e^{O(\delta w \mathcal{A}_*)}$, number of microstates.

How about a superposition of states representing universes with different w's? Consider two *normalized* microstates of the form given in Eq. (11):

$$|\psi_1\rangle = \sum_{i=1}^{e^{\Delta_1}} a_i |x_1^i x_2^i \cdots x_N^i\rangle, \tag{16}$$

$$|\psi_2\rangle = \sum_{i=1}^{e^{\Delta_2}} b_i |y_1^i y_2^i \cdots y_N^i\rangle, \tag{17}$$

where $N/4 \le \Delta_1 \ne \Delta_2 \le N/2$, and the coefficients a_i and b_i are random as are the binary values $x^i_{1,\cdots,N}$ and $y^i_{1,\cdots,N}$. We are interested in understanding the physical meaning of the normalized superposition

$$|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle. \tag{18}$$

The reduced density matrix for the first n qubits ($n < N/2$) is

$$\rho_n = \mathrm{tr}_{n+1\cdots N}|\psi\rangle\langle\psi|. \tag{19}$$

Because of the normalization conditions

$$\sum_{i=1}^{e^{\Delta_1}} |a_i|^2 = \sum_{i=1}^{e^{\Delta_2}} |b_i|^2 = 1, \tag{20}$$

ρ_n takes the form

$$\rho_n = |c_1|^2 \rho_n^{(1)} + |c_2|^2 \rho_n^{(2)}, \tag{21}$$

up to corrections exponentially suppressed in N. (For a detailed derivation, see Ref. [21].) Here, $\rho_n^{(1)}$ ($\rho_n^{(2)}$) are the reduced density matrices we would obtain if the state were genuinely $|\psi_1\rangle$ ($|\psi_2\rangle$):

$$\rho_n^{(1)} = \sum_{i=1}^{e^{\Delta_1}} |a_i|^2 |x_1^i \cdots x_n^i\rangle\langle x_1^i \cdots x_n^i|, \tag{22}$$

with $\rho_n^{(2)}$ given by $\Delta_1 \to \Delta_2$, $a_i \to b_i$, and $x^i_{1,\cdots,n} \to y^i_{1,\cdots,n}$. The matrix ρ_n thus takes the form of an incoherent classical mixture. Moreover, the simple form of the matrices in Eqs. (21), (22) implies that the entanglement entropies are also incoherently added

$$S_n = |c_1|^2 S_n^{(1)} + |c_2|^2 S_n^{(2)} + S_{n,\mathrm{mix}}, \tag{23}$$

where $S_n^{(1,2)}$ are the entanglement entropies obtained if the state were $|\psi_{1,2}\rangle$, and

$$S_{n,\mathrm{mix}} = -|c_1|^2 \ln|c_1|^2 - |c_2|^2 \ln|c_2|^2, \tag{24}$$

is the entropy of mixing (classical Shannon entropy), suppressed by factors of $O(\mathcal{A}_*)$ compared with $S_n^{(1,2)}$.[8] This indicates that unless $|c_1|$ or $|c_2|$ is exponentially small in N, the state $|\psi\rangle$ admits the usual interpretation of a superposition of macroscopically different universes with $w_{1,2}$ corresponding to $\Delta_{1,2}$.

Similarly, unless a superposition involves exponentially many microstates, we find

$$|\psi\rangle = \sum_i c_i|\psi_i\rangle \quad \Rightarrow \quad \begin{array}{l} \rho_n = \sum_i |c_i|^2 \rho_n^{(i)}, \\ S_n = \sum_i |c_i|^2 S_n^{(i)} + S_{n,\mathrm{mix}}, \end{array} \tag{25}$$

for $n < N/2$, up to exponentially suppressed corrections. Here, $S_{n,\mathrm{mix}} = -\sum_i |c_i|^2 \ln|c_i|^2$ and is suppressed by a factor of $O(\mathcal{A}_*)$ compared with the first term in S_n. We thus conclude that the standard many worlds interpretation applies to classical spacetimes under any reasonable measurements (only) in the limit that e^{-N} is regarded as zero, i.e. unless a superposition involves exponentially many terms or an exponentially small coefficient. This picture is consonant with the observation that classical spacetime has an intrinsically thermodynamic nature [24], supporting the idea that it consists of a large number of degrees of freedom.

We expect that the picture given here persists in the existence of excitations on semiclassical backgrounds. These excitations can be represented by non-linear/state-dependent operators at the microscopic level, along the lines of Ref. [11]. (For earlier work, see Refs. [25–27].) In fact, since entropies associated with the excitations are typically subdominant in $\mathcal{A}_*$ [1,28], they have only minor effects on the overall picture. Therefore, we effectively obtain a direct sum structure [15] for the Hilbert space

$$\mathcal{H}_{B=\mathrm{FRW},\mathcal{A}_*} \approx \bigoplus_w \mathcal{H}_w^{\mathcal{A}_*}, \tag{26}$$

despite the much more intricate structure of the fundamental Hilbert space.

Finally, this fundamental Hilbert space structure suggests that the time evolution operator leading to the change of the leaf area is also non-linear at the fundamental level. This does not require the time evolution of semiclassical degrees of freedom to be non-linear, since the definition of these degrees of freedom would also be non-linear at the fundamental level. Detailed discussions on the time evolution in the holographic theory of cosmological spacetimes will be presented in Ref. [21].

[8] In the present model, this term is absent for $n < \Delta_{1,2}$. This is an artifact of the specific toy model adopted here, arising from the fact that two universes cannot be discriminated unless n is larger than one of $\Delta_{1,2}$; see Eq. (12).

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