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Circuit design and exponential stabilization of memristive neural networks

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HIGHLIGHTS

- Design a class of memristive neural networks with time-varying delays and general activation functions.
- Investigate the exponential stabilization problem of such systems.
- Set up a delay-dependent criteria for the global exponential stability and stabilization of memristive neural networks.

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ABSTRACT

This paper addresses the problem of circuit design and global exponential stabilization of memristive neural networks with time-varying delays and general activation functions. Based on the Lyapunov–Krasovskii functional method and free weighting matrix technique, a delay-dependent criteria for the global exponential stability and stabilization of memristive neural networks are derived in form of linear matrix inequalities (LMIs). Two numerical examples are elaborated to illustrate the characteristics of the results. It is noteworthy that the traditional assumptions on the boundness of the derivative of the time-varying delays are removed.

1. Introduction

Since the experimental prototyping of the memristor (Chua, 1971) was announced by the HP Lab (Strukov, Snider, Stewart, & Williams, 2008), memristive neural networks (MNNs) have been widely investigated, for their immense potential applications in different areas such as brain emulation, combinatorial optimization, knowledge acquisition and pattern recognition (Chen, Li, Huang, Chen, & Wang, 2013; Guo, Wang, & Yan, 2013a, 2013b; Hu & Wang, 2010; Wang, Li, Huang, & Duan, 2013; Wen, Bao, Zeng, Chen, & Huang, 2013; Wen, Zeng, Huang, & Zhang, 2013; Wu, Wen, & Zeng, 2012; Wu & Zeng, 2012). On the other hand, delayed neural networks have attracted great attention due to their potential applications in many fields (Cao, Chen, & Li, 2008; Chen & Dong, 1998; Chua & Roska, 2002; Liang, Wang, Liu, & Li, 2008; Wen, Zeng, & Huang, 2013b; Zhang, Ma, Huang, & Wang, 2010). In some of these applications, it is a necessary step to analyze the dynamical behaviors before the practical design of neural networks (He, Li, Huang, & Li, 2013; He, Li, Huang, Li, & Huang, 2014; Liu, Dang, & Huang, 2013; Liu & Wang, 2013; Liu, Wang, Zhao, & Wei, 2012; Luo & Wu, 2012; Wen, Zeng, & Huang, 2013a; Wen, Zeng, Huang, & Yu, 2014), and the equilibrium points of the designed networks are required to be stable.

In both biological and artificial neural networks, due to integration and data communication, time delays are ubiquitous and often become a source of instability. Time delays are usually time-varying due to the finite switching speed of amplifiers and faults in the electrical circuitry. Therefore, stability analysis of memristive neural networks with time-varying delays is an important issue, and many stability criteria have been developed in the literature (see Huang, Li, Duan, & Starzyk, 2012, Li, Gao, & Yu, 2011, Wang, Zhang, Xu, & Peng, 2009, Wu, Feng, & Lam, 2013, Zhang, Zhang, & Wang, 2011, Zeng, He, Wu, & Zhang, 2011, and the references

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cited therein). Furthermore, the implementation of neuromorphic circuits and chips has long been hindered by challenges related to area and power consumption restrictions. More than tens of transistors and capacitors are needed to estimate a synapse. In particular, when neural connections become high level, a large part of neuromorphic chips are utilized for synapses, whereas neurons take only a small portion compared to that of synapses. However, shrinking the current transistor size is very difficult. Therefore, it is critical to introduce a more efficient approach to implement neuromorphic circuits and chips.

In the process to analyze the periodicity or stability of a neural network, the conditions to be imposed on the neural network are determined by the characteristics of activation functions and network parameters. When memristive neural networks are designed to solve practical problems, it is desirable for their activation functions to be general. As a result, several research work has been done on the stability of memristive neural networks with nonmonotonic activation functions. However, the stability and/or exponential stability of memristive neural networks with general activation functions which may be nonmonotonic is still an open problem, therefore, it is necessary to design a feedback controller to provide exponential stability of memristive neural networks.

As an important research focus, several interesting results on the stabilization of memristive neural networks have been proposed (see Guo et al., 2013b, Wu & Zeng, 2012). In Wu and Zeng (2012), the problem of exponential stabilization for a class of memristive neural networks with constant time delays was studied. The authors proposed some sufficient conditions in terms of linear matrix inequalities to achieve exponential stabilization. However, the authors set the time delays as constants. Utilizing the method in Phat and Trinh (2010), the results in Wu and Zeng (2012) can be extended to memristive neural networks with time-varying delays which are with upper boundness on the derivative of the time-varying delays. At the same time, we observe that available results on the stabilization of memristive neural networks have not specifically considered the global delay-dependent exponential stabilization of memristive neural networks with time-varying delays which are without upper boundness on their derivatives.

To shorten such gap, we investigate the problem of exponential stabilization for a class of delayed MNNs. The main contributions of this paper can be summarized as follows: (1) Based on the circuit design, a model of MNNs is established; (2) based on the Lyapunov–Krasovskii functional method and free weighting matrix technique, delay-dependent criteria for the global exponential stability and stabilization of memristive neural networks are derived in form of linear matrix inequalities; (3) the traditional assumptions on the boundness of the derivative of the time-varying delays are removed for memristive neural networks.

The rest of the paper is organized as follows. In Section 2, a memristive circuit is proposed and corresponding dynamical equation is established. In Section 3, the exponential stabilization problem of MNNs is discussed by using the Lyapunov–Krasovskii functional method and free weighting matrix technique. Several sufficient conditions are derived to ensure the exponential stabilization of MNNs. In Section 4, two illustrative examples are discussed to demonstrate the effectiveness of the theoretical analysis. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Notation. The notation used through the paper is fairly standard. $\mathbb{N}$ is the set of natural numbers and $\mathbb{N}^+$ stands for the set of nonnegative integers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The notation $P > 0$ (≥ 0) means that P is real positive definite (semi-definite). In symmetric block matrices or complex matrix expressions, we use an asterisk ($*$) to represent a term that is induced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $\|.\|$ refers to the Euclidean vector norm. Sometimes, when no confusion would arise, the dimensions of a function or a matrix will be omitted for convenience.

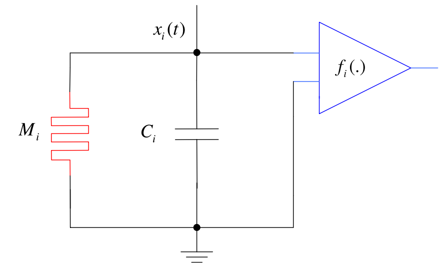

Fig. 1. Schematic diagram of memristive neuronal cell.

2.1. Circuit of an MNN

The neuronal cell of MNNs can be implemented in an MNN as shown in Fig. 1. By Kirchoff's current law, the equation of the ith neuronal state is written as follows:

$$C_i\dot{x}_i(t) = -\Big[\sum_{j=1}^{n}\Big(\frac{1}{R'_{ij}}+\frac{1}{R''_{ij}}\Big)+W_i(x_i(t))\Big]x_i(t) + \sum_{j=1}^{n}\frac{\mathrm{sign}_{ij}f_j(x_j(t))}{R'_{ij}}+\sum_{j=1}^{n}\frac{\mathrm{sign}_{ij}g_j(x_j(t-\tau_j(t)))}{R''_{ij}}, \quad (1)$$

where f_j, g_j are the activation functions, $\tau_j(t)$ and δ_j is the discrete delay, for the ith neuron cell, $x_i(t)$ is the voltage of the capacitance C_i, $f_j(x_j(t)), g_j(x_j(t-\tau_j(t)))$ are the functions of $x_i(t)$ without or with discrete respectively, R'_{ij} is the resistance between the feedback function $f_j(x_j(t))$ and $x_i(t)$, R''_{ij} is the resistance between the feedback function $g_j(x_j(t-\tau_j(t)))$ and $x_i(t)$, M_i is the memristance parallel to the capacitance C_i, where $i, j = 1, 2, \ldots, n$, $\mathrm{sign}_{ij} = \begin{cases}1, & i \neq j;\\ -1, & i = j,\end{cases}$ is the sign function, and $W_i(.)$ is the memductance of the ith memristor M_i, and

$$W_i(x_i(t)) = \begin{cases} W'_i, & f_i(x_i(s)) - x_i(s) \downarrow \rightarrow,\ s \in (t-\Delta, t];\\ W''_i, & f_i(x_i(s)) - x_i(s) \uparrow,\ s \in (t-\Delta, t]\end{cases}$$

where $\downarrow$ means "decrease", $\rightarrow$ means "unchange", $\uparrow$ means "increase", Δ is a sufficiently small positive constant. The memductance function may be discontinuous.

Then, Eq. (1) can be rewritten as follows with control input $u_i(t)$:

$$\dot{x}_i(t) = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_j(t))) + u_i(t) \quad (2)$$

where

$$a_{ij} = \frac{\mathrm{sign}_{ij}}{C_iR'_{ij}}, \qquad b_{ij} = \frac{\mathrm{sign}_{ij}}{C_iR''_{ij}},$$

$$d_i(x_i(t)) = \frac{1}{C_i}\Big[\sum_{j=1}^{n}\Big(\frac{1}{R'_{ij}}+\frac{1}{R''_{ij}}\Big)+W_i(x_i(t))\Big] = \begin{cases} d_{1i}, & f_i(x_i(s)) - x_i(s) \downarrow, \rightarrow,\ s \in (t-\Delta, t];\\ d_{2i}, & f_i(x_i(s)) - x_i(s) \uparrow,\ s \in (t-\Delta, t].\end{cases}$$

Then we have

$$\dot{x}(t) = -D(x(t))x(t) + Af(x(t)) + Bg(x(t-\tau(t))) + u(t), \tag{3}$$

where

$$D(x(t)) = \mathrm{diag}\{d_1(x_1(t)), d_2(x_2(t)), \ldots, d_n(x_n(t))\},$$
$$A = [a_{ij}]_{n\times n}, \qquad B = [b_{ij}]_{n\times n}, \qquad u(t) = [u_1(t), \ldots, u_n(t)]^T,$$
$$f(x(t)) = \Big(f_1(x_1(t)), \ldots, f_n(x_n(t))\Big)^{\mathrm{T}},$$
$$g(x(t-\tau(t))) = \Big(g_1(x_1(t-\tau_1(t))), \ldots, g_n(x_n(t-\tau_n(t)))\Big)^{\mathrm{T}}.$$

Remark 1. With the property of multiple memristances, for a neuron cell, only one memristor is employed to store multiple states of the input information. Therefore, the synapses of the neural networks need not to be designed with memristor, which will obviously reduce the cost to produce the memristive neural networks in future. Based on the analysis above, $D(x(t))$ in system (3) is changed according to the state of the system, so this network based on memristors is a state-dependent switching system. System (3) represents a class of MNNs with time-varying delays.

2.2. Modeling of MNNs as switching systems

Considering the state-dependent switching property of the memristor, MNNs can be modeled as switching systems. Define an indicator function $\pi_p(t) = \mathrm{diag}\{\pi_{p1}(t), \ldots, \pi_{pn}(t)\}$, $p = 1, 2$, where

$$\pi_{1i}(x_i(t)) = \begin{cases} 1, & f_i(x_i(s)) - x_i(s) \downarrow, \rightarrow,\ s \in (t-\Delta, t], \\ 0, & f_i(x_i(s)) - x_i(s) \uparrow,\ s \in (t-\Delta, t], \end{cases}$$

$$\pi_{2i}(x_i(t)) = \begin{cases} 0, & f_i(x_i(s)) - x_i(s) \downarrow, \rightarrow,\ s \in (t-\Delta, t], \\ 1, & f_i(x_i(s)) - x_i(s) \uparrow,\ s \in (t-\Delta, t]. \end{cases}$$

Then,

$$\dot{x}_i(t) = -\sum_{r=1}^{2} \pi_{pi}(x_i(t)) d_{pi} x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_j(t))) + u_i(t). \tag{4}$$

Therefore, system (3) can be represented by

$$\dot{x}(t) = -\sum_{p=1}^{2} \Pi_p(x(t)) D_p x(t) + Af(x(t)) + Bg(x(t-\tau(t))) + u(t), \tag{5}$$

where $\Pi_p(x(t)) = \mathrm{diag}\{\pi_{p1}(x_1(t)), \ldots, \pi_{pn}(x_n(t))\}$, and $\sum_{p=1}^{2} \pi_{pi}(x_i(t)) = 1$, $i = 1, \ldots, n$, $p = 1, 2$, and

$$D_p = \mathrm{diag}\{d_{p1}, d_{p2}, \ldots, d_{pn}\}.$$

The output of this system is $y(t) = Ex(t)$, and the initial condition of system (5) is in the form of $x(t) = \phi(t) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$ with the uniform norm

$$\|\phi\| = \max_{t\in[-\tau,0]} \|\phi(t)\|, \tag{6}$$

where $\tau = \max\{\tau_i\}$, $\tau_i(t)$ satisfies $0 \le \tau_i(t) \le \tau_i$, $i \in \{1, 2, \ldots, n\}$. And $\dot{\tau}(t) \le \mu$, $\mu = \max\{\mu_i\}$, where μ_i satisfies $\dot{\tau}_i(t) \le \mu_i$, for $i \in \{1, 2, \ldots, n\}$.

Definition 1. Given $\beta > 0$. The zero solution of system (5) is β-exponentially stable if every solution $x(t, \phi)$ of the system satisfies

$$\exists\, \varpi > 0 : \|x(t, \phi)\| \le \varpi \|\phi\| e^{-\beta t}, \quad \forall t \ge 0,$$

where $u(t) = 0$.

Definition 2. Given $\beta > 0$. System (5) is globally β-exponentially stabilizable if there is a feedback control law

$$u(t) = Ky(t), \tag{7}$$
$$y(t) = Cx(t),$$

such that the augmented system

$$\dot{x}(t) = \sum_{p=1}^{2} \Pi_p(x(t)) \mathcal{D}_p x(t) + Af(x(t)) + Bg(x(t-\tau(t))), \tag{8}$$

is β-exponentially stable, where

$$x(t) = \phi(t), \quad t \in [-\tau, 0],$$
$$\mathcal{D}_p = -(D_p - KC).$$

2.3. Assumption and lemmas

The following assumption, which was first proposed in Liu, Wang, and Liu (2006a, 2006b) and Wang, Shu, Liu, Ho, and Liu (2006), will be needed throughout the paper:

A_1. For $i \in \{1, 2, \ldots, n\}$, the activation functions f, g are bounded and there exist four constant matrices $L^- = \mathrm{diag}\{l_1^-, \ldots, l_n^-\}$, $L^+ = \mathrm{diag}\{l_1^+, \ldots, l_n^+\}$, $H^- = \mathrm{diag}\{h_1^-, \ldots, h_n^-\}$ and $H^+ = \mathrm{diag}\{h_1^+, \ldots, h_n^+\}$, such that

$$l_i^- \le \frac{f_i(\alpha) - f_i(\beta)}{\alpha - \beta} \le l_i^+,$$

$$h_i^- \le \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \le h_i^+,$$

for all $\alpha, \beta \in \mathbb{R}$ and $\alpha \ne \beta$, $i = 1, \ldots, n$.

As l_i^-, l_i^+, h_i^- and h_i^+ are constants, and they can be positive, negative or zero, therefore, the activation functions may be nonmonotonic, and more general than the usual sigmoid functions as follows in Wu et al. (2012):

$$\|f_i(\alpha) - f_i(\beta)\| \le \|l_i(\alpha - \beta)\|, \quad i = 1, 2, \ldots, n.$$

By recalling the definitions of f, g, from Assumption A_1, two inequalities are first derived in Liu et al. (2006a, 2006b) and Wang et al. (2006) as follows:

$$[f_i(x_i(t)) - l_i^- x_i(t)]^T [f_i(x_i(t)) - l_i^+ x_i(t)] \le 0, \tag{9}$$

$$[g_i(x_i(t)) - h_i^- x_i(t)]^T [g_i(x_i(t)) - h_i^+ x_i(t)] \le 0. \tag{10}$$

In order to derive sufficient conditions for the exponential stabilization of system (8), we will need the following lemmas.

Lemma 1 (Zhao & Tan, 2007)**.** *It is given any real matrices X, Z, P of appropriate dimensions and a scalar $\varepsilon_0 > 0$, where $P > 0$. Then the following inequality holds:*

$$X^T Z + Z^T X \le \varepsilon_0 X^T P X + \varepsilon_0^{-1} Z^T P^{-1} Z.$$

In particular, if X and Z are vectors, $X^T Z \le \frac{1}{2}(X^T X + Z^T Z)$.

Lemma 2 (Yue, Tian, Zhang, & Peng, 2009)**.** *Suppose that $0 \le \eta_m \le \eta(t) \le \eta_M$, and P_i, $i \in \{1, 2, 3\}$ are constant matrices with appropriate dimensions, then*

$$P_1 + (\eta_M - \eta(t))P_2 + (\eta(t) - \eta_m)P_3 < 0, \tag{11}$$

holds, if and only if the following inequalities hold

$$P_1 + (\eta_M - \eta_m)P_2 < 0,$$
$$P_1 + (\eta_M - \eta_m)P_3 < 0. \tag{12}$$

Lemma 3 (Peng & Tian, 2008)**.** *For any constant positive matrix $W \in \mathbb{R}^{n\times n}$, scalar $\vartheta_1 \le \vartheta(t) < \vartheta_2$, and vector function $\dot{x}(t) : [-\vartheta_2, \vartheta_1] \to \mathbb{R}^n$, such that the following integration is well defined with*

$$
\begin{aligned}
&-(\vartheta_2-\vartheta_1)\int_{t-\vartheta_2}^{t-\vartheta_1}\dot{x}^T(s)W\dot{x}(s)ds\\
&\le \begin{bmatrix} x(t-\vartheta_1)\\ x(t-\vartheta(t))\\ x(t-\vartheta_2)\end{bmatrix}^T \begin{bmatrix} -W & W & 0\\ * & -2W & W\\ * & * & -W\end{bmatrix}\begin{bmatrix} x(t-\vartheta_1)\\ x(t-\vartheta(t))\\ x(t-\vartheta_2)\end{bmatrix}. \quad (13)
\end{aligned}
$$

Lemma 4 (Xiong & Lam, 2009). *For matrices $Q > 0$, Y and any scalar υ, the inequality $-YQ^{-1}Y \le \upsilon^2 Q - 2\upsilon Y$ holds.*

Lemma 5 (Gu, 2000). *For any constant symmetric positive definite matrix P, if there exist a number $\sigma > 0$ and vector function $x(.)$, then*

$$
\left(\int_0^{\sigma} x(s)ds\right)^T P\left(\int_0^{\sigma} x(s)ds\right) \le \sigma \int_0^{\sigma} x^T(s)Px(s)ds. \quad (14)
$$

3. Main results

In this section, we shall establish our main criterion based on the LMI approach. For convenience, we denote

$$\Gamma_1 = \mathrm{diag}\{l_1^- l_1^+, l_2^- l_2^+, \ldots, l_n^- l_n^+\},$$

$$\Gamma_2 = \mathrm{diag}\left\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right\},$$

$$\Gamma_3 = \mathrm{diag}\{h_1^- h_1^+, h_2^- h_2^+, \ldots, h_n^- h_n^+\},$$

$$\Gamma_4 = \mathrm{diag}\left\{\frac{h_1^- + h_1^+}{2}, \frac{h_2^- + h_2^+}{2}, \ldots, \frac{h_n^- + h_n^+}{2}\right\}.$$

System (8) can be rewritten as follows:

$$
\dot{x}(t) = \sum_{p=1}^{2} \Pi_p(x(t))\mathcal{M}_p\zeta(t), \quad (15)
$$

where

$$
\begin{aligned}
\zeta(t) = \big[&x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-\tau)\\
&f^T(x(t)) \quad g^T(x(t)) \quad g^T(x(t-\tau(t)))\big]^T,\\
\mathcal{M}_p = \big[&\mathcal{D}_p \quad 0 \quad 0 \quad A \quad 0 \quad B\big].
\end{aligned}
$$

Theorem 1. *For a given parameter τ and feedback gain K, system (8) is exponentially stable, if there exist matrices $P > 0$, $Q_i > 0$, M_i and N_i, $(i = 1, 2)$ with appropriate dimensions, and positive scalars $\rho, \varphi, \epsilon > 0$, such that $p = 1, 2$,*

$$
\Psi_i^p = \begin{bmatrix} \Xi^p & \Theta_1^p & \Theta_{2i}\\ * & -\tau Q_2 & 0\\ * & * & -\tau Q_2\end{bmatrix} < 0, \quad (16)
$$

where

$$
\Xi^p = \begin{bmatrix} \Xi_{11}^p & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16}\\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0\\ * & * & \Xi_{33} & 0 & 0 & 0\\ * & * & * & \Xi_{44} & 0 & 0\\ * & * & * & * & \Xi_{55} & 0\\ * & * & * & * & * & \Xi_{66}\end{bmatrix},
$$

$$\Theta_1^p = \begin{bmatrix}\tau Q_2\mathcal{D}_p & 0 & 0 & \tau Q_2 A & 0 & \tau Q_2 B\end{bmatrix}^T,$$

$$\Theta_{21} = \begin{bmatrix}\tau M_1^T & \tau M_2^T & 0 & 0 & 0 & 0\end{bmatrix}^T,$$

$$\Theta_{22} = \begin{bmatrix}0 & \tau N_1^T & \tau N_2^T & 0 & 0 & 0\end{bmatrix}^T,$$

$$
\begin{aligned}
\Xi_{11}^p =\ & P\mathcal{D}_p + \mathcal{D}_p^T P + 2\beta P - \frac{1}{\tau}\exp\{-2\beta\tau\}Q_2\\
&+ \exp\{-2\beta\tau\}(M_1^T + M_1) - \rho\Gamma_1 - \varphi\Gamma_3 - \epsilon\Gamma_3,
\end{aligned}
$$

$$\Xi_{12} = \frac{1}{\tau}\exp\{-2\beta\tau\}Q_2 + \exp\{-2\beta\tau\}(M_2^T - M_1),$$

$$\Xi_{14} = PA - \rho\Gamma_2, \qquad \Xi_{15} = -\varphi\Gamma_4, \qquad \Xi_{16} = PB - \epsilon\Gamma_4,$$

$$
\begin{aligned}
\Xi_{22} =\ & -\frac{1}{\tau}\exp\{-2\beta\tau\}(Q_2^T + Q_2) + \exp\{-2\beta\tau\}\\
&\times(-M_2^T - M_2 + N_1^T + N_1),
\end{aligned}
$$

$$\Xi_{23} = \frac{1}{\tau}\exp\{-2\beta\tau\}Q_2 + \exp\{-2\beta\tau\}(N_2^T - N_1),$$

$$\Xi_{33} = -\frac{1}{\tau}\exp\{-2\beta\tau\}Q_2 - \exp\{-2\beta\tau\}(N_2^T + N_2),$$

$$\Xi_{44} = -\rho I, \qquad \Xi_{55} = -\varphi I + Q_1,$$

$$\Xi_{66} = -\epsilon I - (1-\mu)\exp\{-2\beta\tau\}Q_1.$$

Proof. Choose the Lyapunov–Krasovskii functional candidate to be $V(t) = \sum_{i=1}^3 V_i(t)$, and

$$V_1(t) = x^T(t)Px(t),$$

$$V_2(t) = \int_{t-\tau(t)}^{t} \exp\{2\beta(s-t)\}g^T(x(s))Q_1 g(x(s))ds,$$

$$V_3(t) = \int_{t-\tau}^{t}\int_{\eta}^{t} \exp\{2\beta s\}\dot{x}^T(s)Q_2\dot{x}(s)dsd\eta,$$

where $P > 0, Q_i > 0$, $(i = 1, 2)$ are positive definite matrices with appropriate dimensions to be determined.

It is easy to verify that

$$
\hat{\alpha}\|x(t)\|^2 \le V(t) \le \breve{\alpha}\|x(t)\|^2, \quad t \in \mathbb{R}^+, \quad (17)
$$

where $\hat{\alpha} = \lambda_{\min}(P)$, $\breve{\alpha} = \lambda_{\max}(P) + \lambda_{\max}(Q_1)l^2\tau + \lambda_{\max}(Q_2)\tau^2$, $l^2 = \max\{(l_i^-)^2, (l_i^+)^2\}$.

Taking the derivative of $V_i(t)$, $i = 1, 2, 3$ along the trajectory of system (15), we can get

$$
\dot{V}_1(t) = 2\sum_{p=1}^{2}\Pi_p(x(t))x^T(t)P\mathcal{M}_p\zeta(t), \quad (18)
$$

$$
\begin{aligned}
\dot{V}_2(t) =\ & g^T(x(t))Q_1 g(x(t)) - \exp\{-2\beta\tau(t)\}(1-\dot{\tau}(t))\\
&\times g^T(x(t-\tau(t)))Q_1 g(x(t-\tau(t))) - 2\beta V_2(t)\\
\le\ & g^T(x(t))Q_1 g(x(t)) - \exp\{-2\beta\tau\}(1-\mu)\\
&\times g^T(x(t-\tau(t)))Q_1 g(x(t-\tau(t))) - 2\beta V_2(t), \quad (19)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) =\ & \tau\dot{x}^T(t)Q_2\dot{x}(t) - \int_{t-\tau}^{t}\exp\{2\beta(s-t)\}\\
&\times \dot{x}^T(s)Q_2\dot{x}(s)ds - 2\beta V_3(t)\\
\le\ & \tau\dot{x}^T(t)Q_2\dot{x}(t) - \exp\{-2\beta\tau\}\int_{t-\tau}^{t}\dot{x}^T(s)\\
&\times Q_2\dot{x}(s)ds - 2\beta V_3(t). \quad (20)
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
&\dot{V}(t) + 2\beta V(t) \le 2\sum_{p=1}^{2}\Pi_p(x(t))x^T(t)P\mathcal{M}_p\zeta(t) + 2\beta x^T(t)Px(t)\\
&+ g^T(x(t))Q_1 g(x(t)) - \exp\{-2\beta\tau\}(1-\mu)\\
&\times g^T(x(t-\tau(t)))Q_1 g(x(t-\tau(t))) + \tau\dot{x}^T(t)Q_2\dot{x}(t)\\
&- \exp\{-2\beta\tau\}\int_{t-\tau}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds. \quad (21)
\end{aligned}
$$

Based on equality (20) and Lemma 3, we can get

$$
\begin{aligned}
&-\int_{t-\tau}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds\\
&\le \frac{1}{\tau}\begin{bmatrix} x(t)\\ x(t-\tau(t))\\ x(t-\tau)\end{bmatrix}^T \begin{bmatrix} -Q_2 & Q_2 & 0\\ * & -2Q_2 & Q_2\\ * & * & -Q_2\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-\tau(t))\\ x(t-\tau)\end{bmatrix}. \quad (22)
\end{aligned}
$$

Utilizing the free weighting matrix method, it is obvious to derive that

$$0 = 2\zeta^T M\Big[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds\Big], \tag{23}$$

$$0 = 2\zeta^T N\Big[x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} \dot{x}(s)ds\Big], \tag{24}$$

where

$M = \begin{bmatrix} M_1^T & M_2^T & 0 & 0 & 0\end{bmatrix}^T,$

$N = \begin{bmatrix} 0 & N_1^T & N_2^T & 0 & 0\end{bmatrix}^T.$

From (23) and (24), it follows

$$-2\zeta^T(t)M\int_{t-\tau(t)}^{t}\dot{x}(s)ds$$
$$\le \tau(t)\zeta^T(t)MQ_2^{-1}M^T\zeta(t) + \int_{t-\tau(t)}^{t}\dot{x}^T(s)Q_2\dot{x}(s)ds, \tag{25}$$

$$-2\zeta^T(t)N\int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds$$
$$\le (\tau-\tau(t))\zeta^T(t)NQ_2^{-1}N^T\zeta(t) + \int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)Q_2\dot{x}(s)ds. \tag{26}$$

Under Assumption A_1, we have

$$\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix}^T \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_2^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix} \le 0, \tag{27}$$

$$\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix}^T \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ \Gamma_4^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix} \le 0. \tag{28}$$

Then, for any given $\rho, \varphi, \epsilon > 0$, there exist

$$-\rho\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix}^T \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_2^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix} \ge 0, \tag{29}$$

$$-\varphi\begin{bmatrix} x(t) \\ g(x(t))\end{bmatrix}^T \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ \Gamma_4^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ g(x(t))\end{bmatrix} \ge 0, \tag{30}$$

$$-\epsilon\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix}^T \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ \Gamma_4^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix} \ge 0. \tag{31}$$

From (18)–(29),

$$\begin{aligned}
\dot{V}(t) + 2\beta V(t) &\le 2\sum_{p=1}^{2}\Pi_p(x(t))x^T(t)P\mathcal{M}_p\zeta(t) + 2\beta x^T(t)Px(t) \\
&+ g^T(x(t))Q_1 g(x(t)) - (1-\mu)\exp\{-2\beta\tau\} \\
&\times g^T(x(t-\tau(t)))Q_1 g(x(t-\tau(t))) \\
&+ \tau\sum_{p=1}^{2}\sum_{q=1}^{2}\Pi_p(x(t))\Pi_q(x(t))\zeta^T(t)\mathcal{M}_p^T Q_2\mathcal{M}_q\zeta(t) \\
&+ \frac{1}{\tau}\exp\{-2\beta\tau\}\begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau)\end{bmatrix}^T \\
&\times \begin{bmatrix} -Q_2 & Q_2 & 0 \\ * & -2Q_2 & Q_2 \\ * & * & -Q_2\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\tau)\end{bmatrix} \\
&+ 2\exp\{-2\beta\tau\}\zeta^T M\Big(x(t) - x(t-\tau(t))\Big) \\
&+ 2\exp\{-2\beta\tau\}\zeta^T N\Big(x(t-\tau(t)) - x(t-\tau)\Big) \\
&+ \exp\{-2\beta\tau\}\tau(t)\zeta^T(t)MQ_2^{-1}M^T\zeta(t) \\
&+ \exp\{-2\beta\tau\}(\tau-\tau(t))\zeta^T(t)NQ_2^{-1}N^T\zeta(t) \\
&- \rho\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix}^T \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_2^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ f(x(t))\end{bmatrix} \\
&- \varphi\begin{bmatrix} x(t) \\ g(x(t))\end{bmatrix}^T \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ \Gamma_4^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ g(x(t))\end{bmatrix} \\
&- \epsilon\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix}^T \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ \Gamma_4^T & I\end{bmatrix}\begin{bmatrix} x(t) \\ g(x(t-\tau(t)))\end{bmatrix}.
\end{aligned}$$

By the Schur Complement and Lemma 2, from (16), we have the following inequality

$$\sum_{p=1}^{2}\sum_{q=1}^{2}\Pi_p(x(t))\Pi_q(x(t))\Big(\Xi^p + \tau\mathcal{M}_p^T Q_2\mathcal{M}_q\Big)$$
$$+ \tau(t)MQ_2^{-1}M^T + (\tau-\tau(t))NQ_2^{-1}N^T < 0, \tag{32}$$

which implies

$$\dot{V}(t) + 2\beta V(t) \le \lambda_{\max}(\Psi_i^p)|\zeta(t)|^2, \tag{33}$$

where $\lambda_{\max}(\Psi_i^p) < 0,\ i, p = 1, 2$. Therefore

$$V(t) \le V(0)\exp\{-2\beta t\}, \quad t \ge 0. \tag{34}$$

With the condition (17), we have

$$\|x(t,\phi)\| \le \sqrt{\frac{\bar{\alpha}}{\underline{\alpha}}}\|\phi\|\exp\{-\beta t\}, \quad t \ge 0, \tag{35}$$

which implies that system (15) is β-exponentially stabilizable. □

Based on Theorem 1, the following result can be obtained for the feedback control design of the augmented system (8).

Theorem 2. *For a given parameter τ, system (8) is exponentially stable, if there exist matrices $P > 0$, $Q_i > 0$, M_i and N_i, $(i = 1, 2)$ with appropriate dimensions, and scalars $\rho > 0$, $\varphi, \epsilon > 0$, such that $p = 1, 2$,*

$$\tilde{\Pi}_i^p = \begin{bmatrix} \overline{\Xi}^p & \tilde{\Theta}_1^p & \Theta_{2i} \\ * & -2\tau P + \tau Q_2 & 0 \\ * & * & -\tau Q_2\end{bmatrix} < 0, \tag{36}$$

where

$$\overline{\Xi}^p = \begin{bmatrix} \tilde{\Xi}_{11}^p & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -\rho I & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 \\ * & * & * & * & * & \Xi_{66}\end{bmatrix},$$

$$\begin{aligned}
\tilde{\Xi}_{11}^p &= -PD_p - D_p^T P + YC + C^T Y^T + 2\beta P \\
&\quad - \frac{1}{\tau}\exp\{-2\beta\tau\}Q_2 + \exp\{-2\beta\tau\}(M_1^T + M_1) \\
&\quad - \rho\Gamma_1 - \varphi\Gamma_3 - \epsilon\Gamma_3,
\end{aligned}$$

$$\tilde{\Theta}_1^p = \begin{bmatrix} -\tau PD_p + \tau YC & 0 & 0 & \tau A & 0 & \tau B\end{bmatrix}^T,$$

the other parameters are defined as in Theorem 1. Moreover, a desired controller gain matrix in (7) is given by $K = P^{-1}Y$.

Proof. By Schur complement and Theorem 1, $\Psi_i^p < 0$, $(i, p = 1, 2)$ can be rewritten as follows:

$$\sum_{p=1}^{2}\sum_{q=1}^{2} \Pi_p(x(t))\Pi_q(x(t))\Big(\Xi^p + \tau \mathcal{M}_p^T Q_2 \mathcal{M}_q\Big) + \tau M Q_2^{-1} M^T < 0,$$

$$\sum_{p=1}^{2}\sum_{q=1}^{2} \Pi_p(x(t))\Pi_q(x(t))\Big(\Xi^p + \tau \mathcal{M}_p^T Q_2 \mathcal{M}_q\Big)$$
$$+ \tau N Q_2^{-1} N^T < 0. \tag{37}$$

By Lemma 2,

$$\Psi_i^p = \begin{bmatrix} \overline{\Xi}^p & \overline{\Theta}_1^p & \Theta_{2i} \\ * & -\tau Q_2^{-1} & 0 \\ * & * & -\tau Q_2 \end{bmatrix} < 0, \tag{38}$$

where

$$\overline{\Theta}_1^p = \begin{bmatrix} \tau \mathcal{D}_p & 0 & 0 & \tau A & 0 & \tau B \end{bmatrix}^T.$$

Then, performing congruence transformation of $\overline{\Gamma} = \text{diag}\{I, P, I\}$ to (38), we have

$$\tilde{\Psi}_i^p = \begin{bmatrix} \overline{\Xi}^p & \tilde{\Theta}_1^p & \Theta_{2i} \\ * & -\tau P Q_2^{-1} P & 0 \\ * & * & -\tau Q_2 \end{bmatrix} < 0. \tag{39}$$

In view of the inequality

$$-PQ_2^{-1}P \le -2P + Q_2, \tag{40}$$

we can readily arrive at (36) from (39). □

4. Numerical examples

In this section, two numerical examples are discussed to demonstrate the obtained results.

Example 1. Consider memristive system (8) with

$$A = \begin{bmatrix} 1.8 & 10 \\ 0.1 & 1.8 \end{bmatrix}, \qquad B = \begin{bmatrix} -1.5 & 0.1 \\ 0.1 & -1.5 \end{bmatrix}, \qquad C = I,$$

$$f_i(x_i) = g_i(x_i) = \frac{1}{2}\Big(|x_i + 1| - |x_i - 1|\Big), \quad i = 1, \ldots, n.$$

It can be verified that the activation functions $f(.), g(.)$ satisfy Assumption A_1 with

$$l_i^- = h_i^- = -1, \qquad l_i^+ = h_i^+ = 1.$$

Let

$$d_1(x_1(t)) = \begin{cases} 0.9, & f_1(x_1(s)) - x_1(s) \downarrow, \rightarrow, \ s \in (t - \Delta, t]; \\ 1.0, & f_1(x_1(s)) - x_1(s) \uparrow, \ s \in (t - \Delta, t], \end{cases}$$

$$d_2(x_2(t)) = \begin{cases} 1.0, & f_2(x_2(s)) - x_2(s) \downarrow, \rightarrow, \ s \in (t - \Delta, t]; \\ 0.9, & f_2(x_2(s)) - x_2(s) \uparrow, \ s \in (t - \Delta, t]. \end{cases}$$

If the time delay is set as $0.8\sin(1.5t)$, neither the sufficient conditions on exponential stabilization of neural networks with $0 \le \dot{\tau} \le \mu < 1$ in Phat and Trinh (2010), nor the sufficient conditions on exponential stabilization of memristive neural networks with constant time delay in Wu and Zeng (2012) can be applied to solve the stabilization problem of memristive neural networks with time-varying delays which are without upper boundness of the time derivatives.

Given $\tau(t) = 0.12\sin(10t)$ and $\beta = 0.1$ by Theorem 2, a feasible solution can be obtained as follows:

$$P = \begin{bmatrix} 1.7362 & -0.5224 \\ -0.5224 & 4.7202 \end{bmatrix},$$

$$Y = \begin{bmatrix} -24.9671 & -1.3641 \\ -1.3641 & -17.3225 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 1.0158 & 0.0238 \\ 0.0238 & 0.9255 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.5866 & -0.2398 \\ -0.2398 & 1.9507 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -0.0229 & -1.2659 \\ -1.2659 & 7.2083 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.1338 & 0.6356 \\ 0.6356 & -3.4931 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.0914 & -1.0555 \\ -1.0555 & 6.1099 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.1811 & 0.7573 \\ 0.7573 & -4.1310 \end{bmatrix},$$

$$\rho = 5.9508, \qquad \epsilon = 5.6621, \qquad \varphi = 4.4532.$$

Then the desired control gain K in (7) can be obtained as follows:

$$K = \begin{bmatrix} -14.9659 & -1.9551 \\ -1.9454 & -3.8863 \end{bmatrix}. \tag{41}$$

The simulation results are shown in Figs. 2 and 3.

As $\dot{\tau}(t) = 1.2 > 1$, the presented results in Phat and Trinh (2010) and Wu and Zeng (2012) cannot be utilized to calculate the control gain as well as the upper bounds τ. Table 1 gives the upper bounds of derived τ by Theorem 2 for various β where $\mu = 1.2$.

Example 2. Consider memristive system (8) with

$$A = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix}, \qquad B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix},$$

$$f_i(x_i) = g_i(x_i) = \tanh(x_i), \quad i = 1, \ldots, n.$$

Let

$$d_1(x_1(t)) = \begin{cases} 0.9, & f_1(x_1(s)) - x_1(s) \downarrow, \rightarrow, s \in (t - \Delta, t]; \\ 1.1, & f_1(x_1(s)) - x_1(s) \uparrow, s \in (t - \Delta, t], \end{cases}$$

$$d_2(x_2(t)) = \begin{cases} 1.1, & f_2(x_2(s)) - x_2(s) \downarrow, \rightarrow, s \in (t - \Delta, t]; \\ 0.9, & f_2(x_2(s)) - x_2(s) \uparrow, s \in (t - \Delta, t]. \end{cases}$$

Given $\beta = 0.2$ and $\tau(t) = 0.1\sin(10t)$ by Theorem 2, a feasible solution can be obtained as follows:

$$P = \begin{bmatrix} 4.6461 & 0.2654 \\ 0.2654 & 1.7714 \end{bmatrix},$$

$$Y = \begin{bmatrix} -10.8288 & -0.3887 \\ -0.3887 & -12.0142 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 2.2467 & 0.0235 \\ 0.0235 & 2.0305 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 2.1975 & 0.0852 \\ 0.0852 & 1.1575 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 6.5432 & 0.3208 \\ 0.3208 & 2.3756 \end{bmatrix},$$

Table 1
Upper bounds of derived τ by Theorem 2 for various β, where $\mu = 1.2$.

β	0	0.01	0.02	0.1	0.2	0.5	0.8	1.6
Theorem 2	0.2943	0.2938	0.2932	0.2887	0.2832	0.2677	0.2536	0.2215

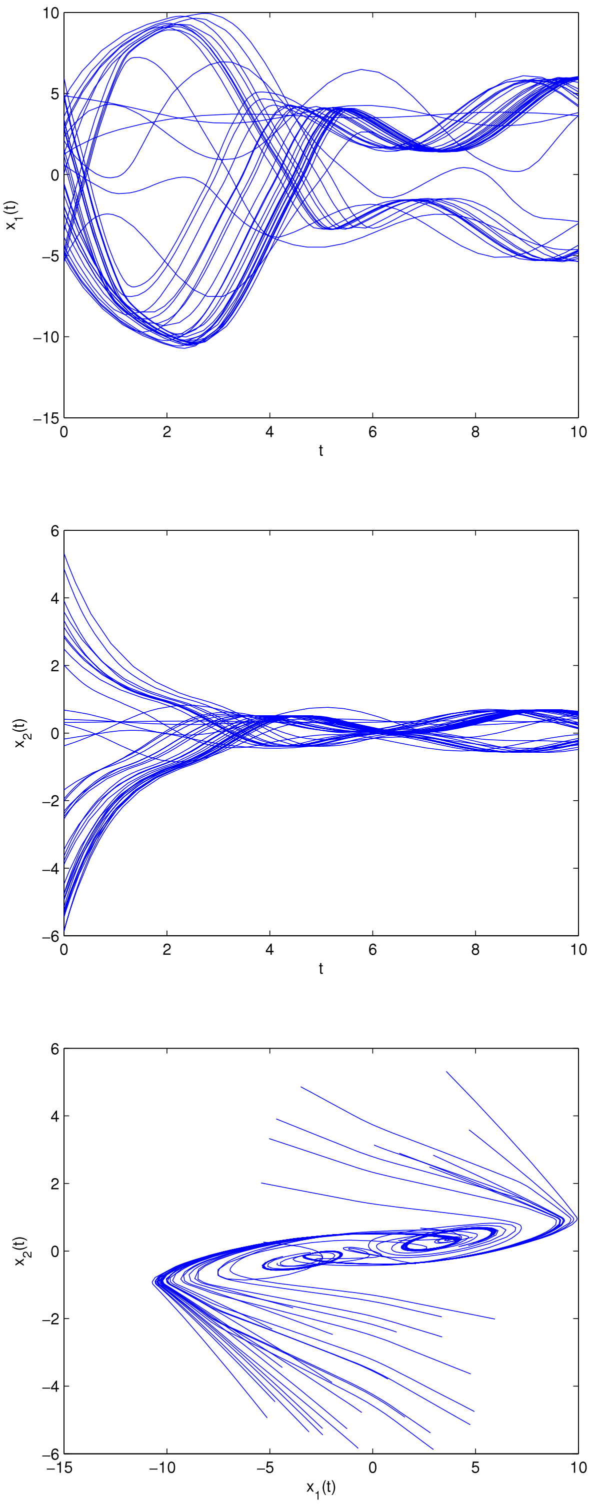

Fig. 2. State trajectories of system (8) without the feedback controller.

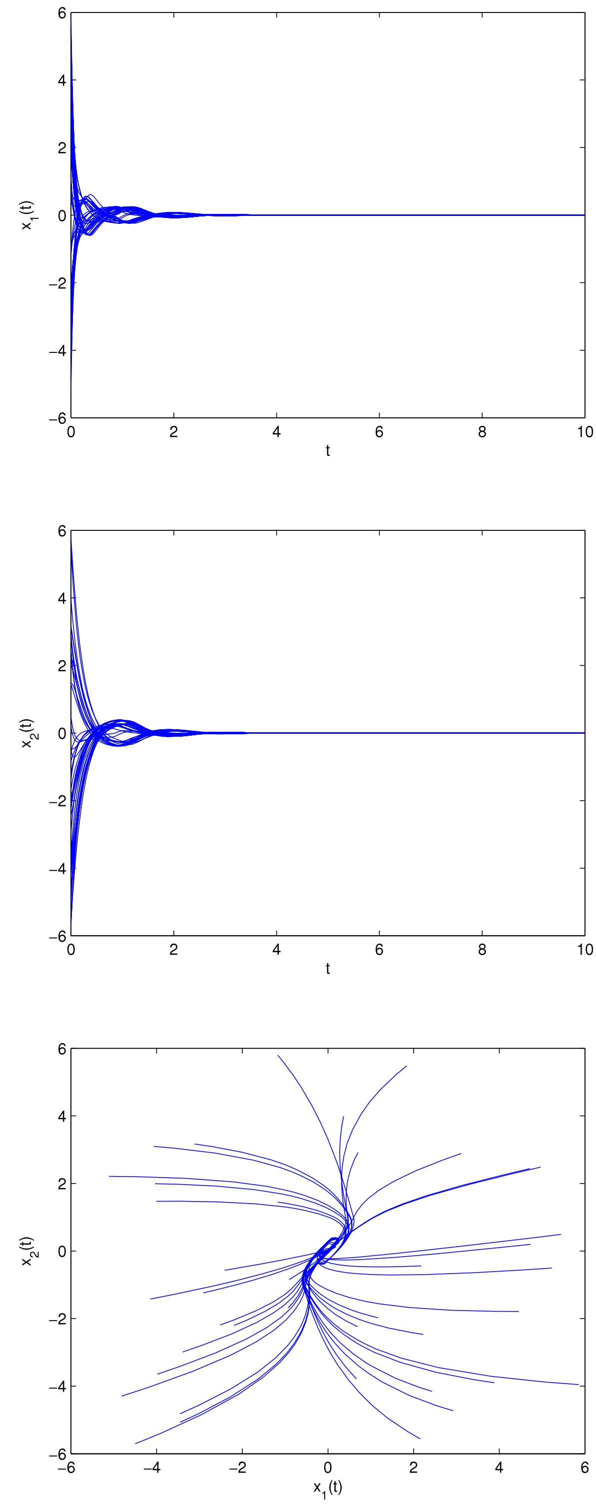

Fig. 3. State trajectories of system (8) with the feedback control gain (41).

$$M_2 = \begin{bmatrix} -2.7070 & -0.1792 \\ -0.1792 & -0.3049 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 5.6683 & 0.3141 \\ 0.3141 & 2.0506 \end{bmatrix},$$

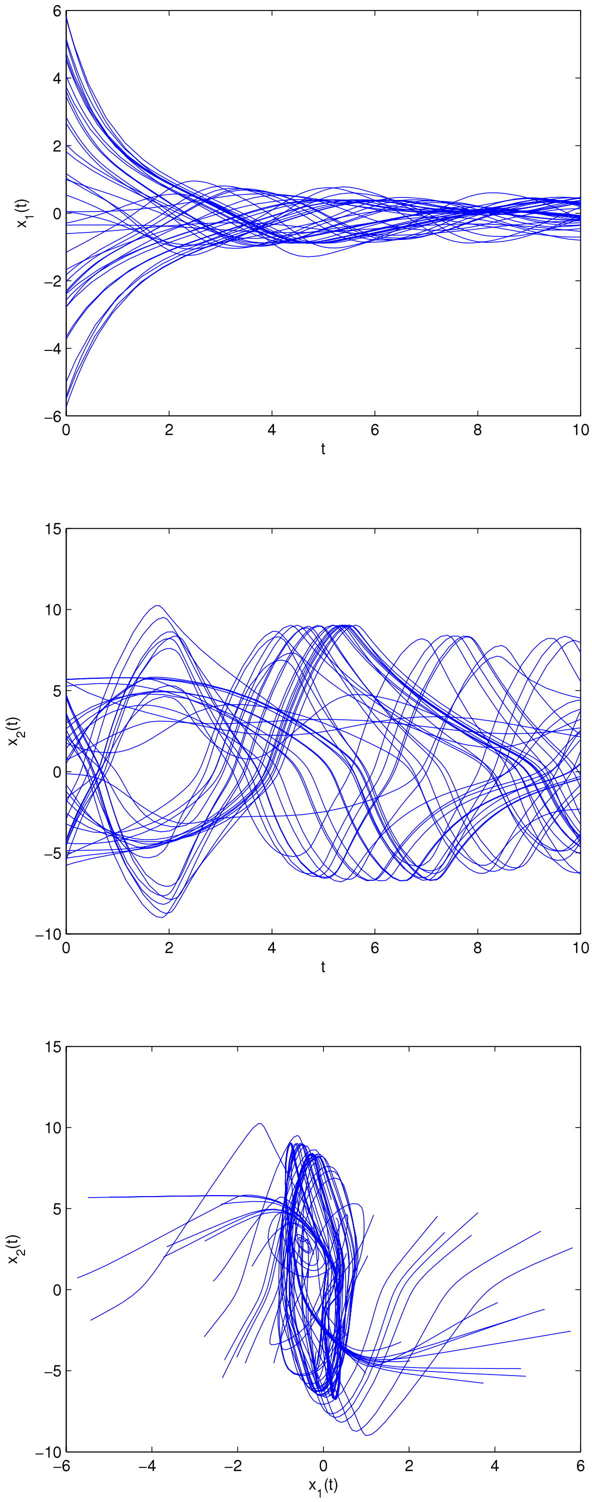

Fig. 4. State trajectories of system (8) without the feedback controller.

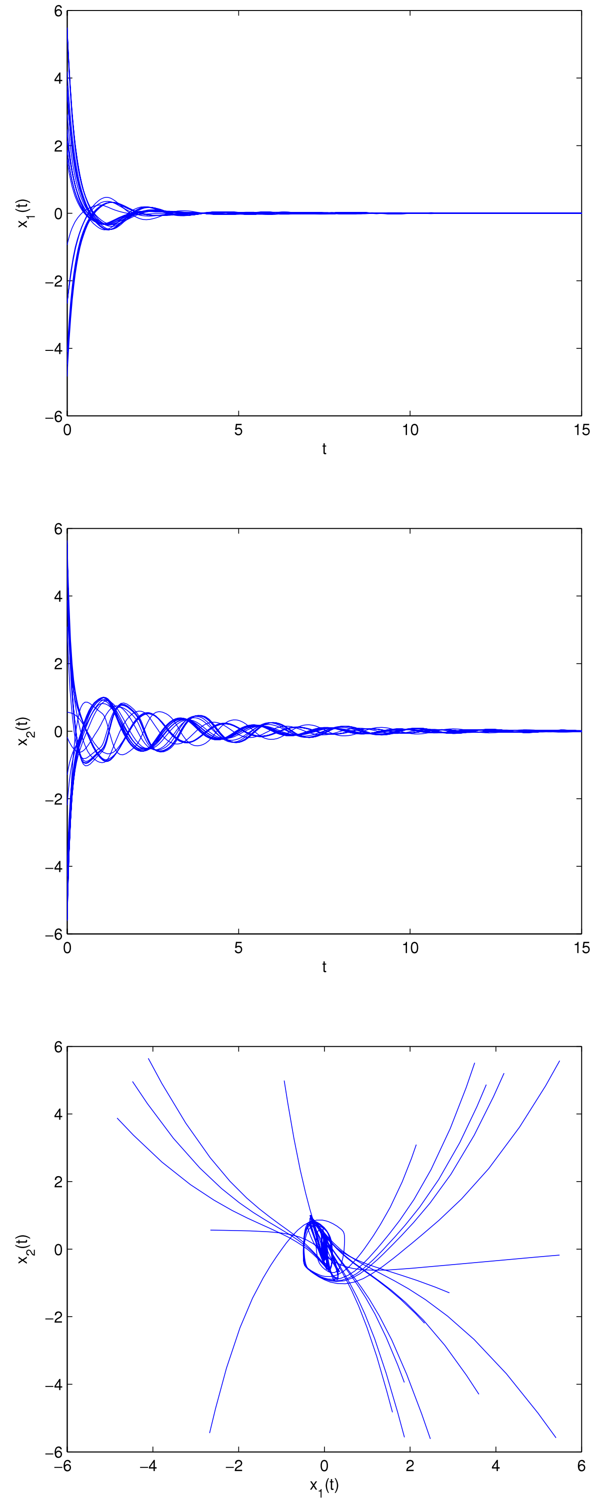

Fig. 5. State trajectories of system (8) with the feedback control gain (42).

$$N_2 = \begin{bmatrix} -3.4205 & -0.2286 \\ -0.2286 & -0.6119 \end{bmatrix},$$

$$\rho = 8.2032, \qquad \epsilon = 11.1507, \qquad \varphi = 9.9823.$$

Then the desired control gain K in (7) can be obtained as follows:

$$K = \begin{bmatrix} -2.3382 & 0.3064 \\ 0.1309 & -6.8281 \end{bmatrix}. \tag{42}$$

The simulation results are shown in Figs. 4 and 5.

As $\dot{\tau}(t) = 1.0$, the presented results in Guo et al. (2013b) and Wu and Zeng (2012) cannot be suitable to obtain the control gain as well as the upper bounds τ. Table 2 gives the upper bounds of derived τ by Theorem 2 for various β where $\mu = 1.0$.

5. Conclusion

In this paper, the problem of circuit design and global exponential stabilization has been investigated for a class of memristive

Table 2
Upper bounds of derived τ by Theorem 2 for various β, where $\mu = 1.0$.

β	0	0.01	0.02	0.1	0.2	0.5	0.8	1.6
Theorem 2	0.5237	0.5218	0.5199	0.5052	0.4878	0.4413	0.4020	0.3222

neural networks with time-varying delays and general activation functions. Based on the Lyapunov–Krasovskii functional method and free weighting matrix technique, delay-dependent criteria for the global exponential stability and stabilization of memristive neural networks were established in terms of linear matrix inequalities. Two numerical examples were discussed to substantiate the effectiveness of the obtained results.

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