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THE ELECTROSEISMIC WAVE THEORY OF FRENKEL

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ABSTRACT

Frenkel's 1944 theory of porous media acoustics is carefully scrutinized. After some manipulation, Frenkel's equations are seen to have nearly identical form to Biot's 1962 equations. The only difference is that Frenkel includes an extraneous fluid-pressure gradient in his bulk force balance. Frenkel also makes a slight error in the development of his effective poroelastic moduli that prevents him from being the first to obtain the so-called "fluid-substitution" relations. Outside of these two small problems, Frenkel's analysis is correct. He limits his treatment of electroseismic phenomena to explaining the electric field that accompanies a compressional seismic wave in a homogeneous material. He predicts that the electric field in a compressional wave is directly proportional to the particle acceleration and this has been verified by the recent field measurements of Garambois and Dietrich 2001. However, such electric fields are only a small part of the total electroseismic response of the earth. Accordingly, some of the additional phenomena not discussed by Frenkel are also presented and discussed.

Keywords: electroseismic, poroelasticity, electrokinetics.

INTRODUCTION

Frenkel 1944 is the first author to have developed a complete set of equations governing the acoustics of isotropic porous media. Kosten and Zwikker 1941 proposed two coupled force-balance equations for the average fluid and solid response of a porous material, and predicted the existence of two compressional-response modes. However, unlike Frenkel 1944, Kosten and Zwikker 1941 proposed a purely scalar theory that failed to allow for shear and that did not define the effective compressibility moduli in terms of drained and undrained experiments. The coefficients in the Kosten and Zwikker 1941 theory are entirely phenomenological.

Frenkel only wrote one paper on poroelasticity. His principal motivation for developing a theory of porous-media acoustics is to quantitatively explain the so-called "E effect" (a nomenclature apparently no longer used in the literature) which is the phenomenon by which a pair of electrodes attached to the earth registers a voltage difference as a seismic wave traverses the electrode pair. Ivanov 1939 measured this phenomenon in the field and suggested that the explanation of the recorded electric field was electrokinetic in nature. The stated purpose of Frenkel's 1944 article is to develop Ivanov's idea into a quantitative theory.

The purpose of the present paper is to: (1) carefully compare Frenkel's porous-media acoustics equations to those of Biot 1956 and 1962 and determine the reason for any discrepancies; (2) determine whether Frenkel's analysis of the electric field contained within a

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compressional wave is correct; (3) define and discuss electrokinetic coupling phenomena for the non-specialist reader; and (4) present results for those effects not considered by Frenkel which include relaxation in the transport coefficients and an assortment of electrokinetic couplings between seismic waves (both compressional and shear) and electric and magnetic fields.

FRENKEL'S THEORY OF POROUS-MEDIA ACOUSTICS

With some effort, the various macroscopic fields in Frenkel's theory can be identified and his governing equations rewritten in a form that is perhaps more familiar.

To achieve this, the response fields used in the present paper are slightly different from those in Frenkel's paper. The poroelastic displacements are taken to be $\mathbf{u}$ and $\mathbf{w}$ where $\mathbf{u}$ is the average displacement in the solid phase of a porous sample, and $\mathbf{w}$ is defined $\mathbf{w} = \phi(\mathbf{u}_f - \mathbf{u})$ where ϕ is porosity and $\mathbf{u}_f$ is the average displacement of the fluid in the pores. The time derivative $\partial \mathbf{w}/\partial t$ thus represents the Darcy filtration velocity induced by the wave. The stress variables are the bulk stress tensor $\boldsymbol{\tau}$ and the average fluid pressure in the pores p_f. The bulk stress tensor represents the average stress throughout both the solid and fluid phases of a porous sample and can be identified as $\boldsymbol{\tau} = (1-\phi)\boldsymbol{\tau}_s - \phi p_f \mathbf{I}$ where $\boldsymbol{\tau}_s$ is the average stress tensor of the solid phase. The confining pressure is defined as $P_c = -\mathrm{tr}\{\boldsymbol{\tau}\}/3 = (1-\phi)p_s + \phi p_f$ and both P_c and p_f are used as the two independent pressure variables in the compressibility laws. In Table 1, Frenkel's 1944 variables are given in terms of these porous-continuum variables.

Frenkel is only concerned with developing a theory of linear acoustics. As such, we forego placing a small d or δ in front of the stress and displacement variables, it being understood that all stresses and displacements are increments induced by a passing wave.

Frenkel's Stress/Strain Relations

In short, Frenkel obtains the proper form of the stress/strain relations but, because of a small error to be described below, does not obtain the so-called Gassmann 1951 "fluid substitution relations" that give the poroelastic compressibility moduli in terms of the underlying fluid bulk modulus, solid bulk modulus, and drained bulk modulus.

Frenkel obtains his macroscopic compressibility laws by focusing on two thought experiments. Adding the results of the two thought experiments yields the functions $\nabla \cdot \mathbf{u}\,(P_c, p_f)$ and $\Delta\phi\,(P_c, p_f)$ that describe how sample volume and porosity are changed due to applied increments in confining pressure P_c and fluid pressure p_f. Frenkel makes the not so obvious identification that in a porous material

$$\nabla \cdot \mathbf{u} = \Delta V / V \tag{1}$$

where $\mathbf{u}$ is the average displacement in the solid phase and V is the sample volume. Pride and Berryman 1998 demonstrate that only when the geometrical center of the grain space coincides with the geometrical center of the pore space in a porous sample does $\nabla \cdot \mathbf{u}$ become exactly the volumetric dilatation of the sample. However, most authors starting from Biot and Willis 1957 also invoke Frenkel's Eq. (1) and it can be considered a reasonable approximation.

In his first "drained" thought experiment, Frenkel applies a confining pressure to a sample with no change in the fluid pressure. Under such drained conditions, one has $P_c = (1-\phi)p_s = p_1$ (where p_1 is Frenkel's variable for the partial pressure in the solid). Frenkel introduces two poroelastic moduli K and α_F to describe the drained response [α_F is used here instead of

Frenkel's original α so as to avoid later confusion with the Biot and Willis 1957 parameter]

$$\begin{aligned} \nabla \cdot \mathbf{u}\,(P_c, 0) &= -\frac{P_c}{K} && (2) \\ \Delta\phi\,(P_c, 0) &= -\left(\frac{1}{1+\alpha_F} - \phi\right)\frac{P_c}{K}. && (3) \end{aligned}$$

Here, K is the so-called "drained" bulk modulus and α_F has no standard name ("Frenkel's alpha") but can either be defined from Eq. (3) or can be identified, as Frenkel properly does, as the ratio of the increment in solid volume ΔV_s to the increment of pore volume ΔV_ϕ under drained conditions; i.e., $\alpha_F = \Delta V_s / \Delta V_\phi|_{p_f=0}$.

In his second thought experiment, Frenkel applies a fluid pressure p_f everywhere throughout the porespace and, simultaneously, a confining pressure $P_c = p_f$ to the external surface of the sample. One way to realize this in practice is to immerse an unjacketed sample into a fluid reservoir and change the pressure of the reservoir by p_f. In this case, Frenkel notices that the effect of the applied-pressure increment is to simply scale-down (or up) the grain pack without changing its relative geometry. Accordingly, the porosity remains unchanged in this experiment. This is only exactly correct when the grains are made of an isotropic material having a bulk modulus K_s that is spatially uniform. Under this isotropic monomineral assumption (that Frenkel does not discuss), he properly obtains

$$\begin{aligned} \nabla \cdot \mathbf{u}\,(p_f, p_f) &= -\frac{p_f}{K_s} && (4) \\ \Delta\phi\,(p_f, p_f) &= 0. && (5) \end{aligned}$$

Up to this point, Frenkel has done everything correctly.

His error comes when he adds together the results of the two thought experiments. Because of the linear nature of the incremental response, the proper additions yield

$$\begin{aligned} \nabla \cdot \mathbf{u}\,(P_c, p_f) &= \nabla \cdot \mathbf{u}\,(P_c - p_f, 0) + \nabla \cdot \mathbf{u}\,(p_f, p_f) \\ &= -\frac{P_c}{K} + \left(1 - \frac{K}{K_s}\right)\frac{p_f}{K} && (6) \\ \Delta\phi\,(P_c, p_f) &= \Delta\phi\,(P_c - p_f, 0) + \Delta\phi\,(p_f, p_f) \\ &= -\left(\frac{1}{1+\alpha_F} - \phi\right)\frac{(P_c - p_f)}{K}. && (7) \end{aligned}$$

However, Frenkel performs the meaningless addition $\nabla \cdot \mathbf{u}\,(P_c - \phi p_f, 0) + \nabla \cdot \mathbf{u}\,(p_f, p_f) = \nabla \cdot \mathbf{u}\,(P_c + (1-\phi)p_f, p_f) \neq \nabla \cdot \mathbf{u}(P_c, p_f)$ and similarly for $\Delta\phi$ which results in his Eqs. (13) and (24a) being incorrect. He thus makes the misidentifications

$$\begin{aligned} \nabla \cdot \mathbf{u}\,(P_c, p_f) &= -\frac{P_c}{K} + \left(\phi - \frac{K}{K_s}\right)\frac{p_f}{K} && (8) \\ \Delta\phi\,(P_c, p_f) &= -\left(\frac{1}{1+\alpha_F} - \phi\right)\frac{(P_c - \phi p_f)}{K}. && (9) \end{aligned}$$

It is this error, that prevents Frenkel from properly identifying the poroelastic compressibility moduli.

The constitutive equation for $\Delta\phi$ can be translated into a constitutive equation for the increment in fluid content $\nabla \cdot \mathbf{w}$ by rewriting Frenkel's Eq. (20) for the conservation of fluid mass as

$$\Delta\phi + \frac{\phi}{K_f} p_f + \nabla \cdot \mathbf{w} + \phi \nabla \cdot \mathbf{u} = 0. \tag{10}$$

If the proper results of Eqs. (6) and (7) are then employed, Frenkel's "corrected" compressibility laws can be written

$$\begin{bmatrix} \nabla \cdot \mathbf{u} \\ \nabla \cdot \mathbf{w} \end{bmatrix} = -\frac{1}{K} \begin{bmatrix} 1 & -(1 - K/K_s) \\ -(1+\alpha_F)^{-1} & \phi(K/K_f - K/K_s) + (1+\alpha_F)^{-1} \end{bmatrix} \begin{bmatrix} P_c \\ p_f \end{bmatrix}. \tag{11}$$

These equations can further be used to identify the nature of the parameter α_F, an issue not addressed by Frenkel. Since the compressibility laws can also be derived by taking derivatives of a strain-energy function, one has the Maxwell relation that

$$\frac{1}{1+\alpha_F} = 1 - \frac{K}{K_s}. \tag{12}$$

Frenkel does not state this result for α_F which can also be given as $1/(1+\alpha_F) = \alpha$ where α is known as the Biot and Willis 1957 constant. As has been shown by Berge et al. 1993, α is always independent of the fluid's bulk modulus for any porous material and is only given exactly by $\alpha = 1 - K/K_s$ in the special case of isotropic monomineral grains.

In passing, we give the compressibility laws in perhaps their most useful form (applicable to porous media with anisotropy at either the grain or macro scales and with arbitrary heterogeneity in the mineral properties)

$$-\begin{bmatrix} P_c \\ p_f \end{bmatrix} = K_U \begin{bmatrix} 1 & B \\ B & B/\alpha \end{bmatrix} \begin{bmatrix} \nabla \cdot \mathbf{u} \\ \nabla \cdot \mathbf{w} \end{bmatrix} \tag{13}$$

where the coefficients K_U (undrained bulk modulus) and B (Skempton's 1954 coefficient) are defined

$$K_U = -\left.\frac{P_c}{\nabla \cdot \mathbf{u}}\right|_{\nabla\cdot\mathbf{w}=0} \tag{14}$$

$$B = \left.\frac{p_f}{P_c}\right|_{\nabla\cdot\mathbf{w}=0}. \tag{15}$$

Both K_U and B are measured on samples that have their exterior surface sealed ($\nabla \cdot \mathbf{w} = 0$). A generally-valid definition of the Biot and Willis 1957 coefficient is

$$\alpha = (1 - K/K_U)/B \tag{16}$$

so that the three fundamental compressibility constants of poroelasticity are K_U, K, and B. Under the restriction to isotropic monomineral grains, one obtains

$$\alpha = 1 - \frac{K}{K_s} \tag{17}$$

$$B = \frac{\alpha}{\alpha + \phi(K/K_f - K/K_s)} \tag{18}$$

$$K_U = \frac{K}{1 - B\alpha}. \tag{19}$$

Gassmann 1951 only considers the undrained response ($\nabla \cdot \mathbf{w} = 0$) and, accordingly, only obtains results for K_U and B. It is Biot and Willis 1957 who first properly obtain the entirety of Eqs. (13)–(19). Frenkel also obtains poroelastic compressibility laws of the form of Eq. (13). However, his error discussed earlier and his lack of use of the symmetry condition prevents him from obtaining the fluid-substitution relations of Eqs. (17)–(19).

Finally, like Gassmann 1951 and Biot and Willis 1957 who follow him, Frenkel makes the assumption that the fluid in the pores has no influence on the shear properties of an isotropic porous material. Frenkel thus assumes that the deviatoric stress tensor $\boldsymbol{\tau}^D = \boldsymbol{\tau} + P_c\mathbf{I}$ acting on the bulk material is related to the deviatoric shear by a fluid-independent constant G

$$\boldsymbol{\tau}^D = G\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T - \frac{2}{3}\nabla\cdot\mathbf{u}\,\mathbf{I}\right). \tag{20}$$

At the low frequencies used in seismic exploration, Frenkel's assumption of fluid independent shear is usually appropriate.

Frenkel's Dynamics of Porous Media

Frenkel proposes force-balance equations for the fluid in relative motion to the solid [his Eq. (22)] and for the solid skeleton [his Eq. (23)] that, except for one slight problem discussed below, can be considered equivalent to those of Biot 1956 a,b.

Frenkel states Darcy's law [his Eq. (18)] in a non-standard form. The relative velocity $\mathbf{v}_2 - \mathbf{v}_1$ (his notation) that Frenkel uses in Darcy's law represents the average *speed* at which the fluid is moving relative to the solid and is not a filtration velocity like it should be. A filtration velocity is the volume of fluid traversing unit area of porous material in unit time and is the porosity ϕ times the relative fluid speed. In the notation of the present paper, the filtration velocity is denoted $\partial\mathbf{w}/\partial t$ where $\mathbf{w} = \phi(\mathbf{u}_f - \mathbf{u})$ is called the filtration displacement. Because of this, the steady-flow permeability k_o used in the present paper (standard definition) is ϕ times the permeability used by Frenkel.

With this in mind, Frenkel's relative force balance [his Eq. (22a)] can be directly rewritten using the variables of the present paper as

$$\frac{\rho_f}{\phi}\frac{\partial^2\mathbf{w}}{\partial t^2} + \frac{\eta_f}{k_o}\frac{\partial\mathbf{w}}{\partial t} = -\nabla p_f - \rho_f\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{21}$$

where η_f is the fluid's shear viscosity. This force-balance can be considered equivalent to the one later developed by Biot 1962. In modern retrospect, we now know (Brown 1980) that the effective fluid inertia in this force balance is more generally $\rho_f F$ where F is the electrical formation factor in the porous material when surface electrical conduction is not important. In models of the pore space where the current lines are straight, one has that $F = 1/\phi$ which then reduces to the Frenkel statement. Biot 1956b, and later Johnson et al. 1987 and others, have provided models that allow for the development of viscous-boundary layers in the pores at sufficiently high frequencies. However, for most earth materials of interest and across the entire frequency band of interest in seismic exploration, the flow in the pores is well-modeled as being parabolic. As such, the inertial effect captured by the term $\rho F\partial^2\mathbf{w}/\partial t^2$, as well as all viscous boundary layer effects, can normally be neglected in Eq. (21). This is particularly true for the seismic (as opposed to ultrasonic) applications of the theory that Frenkel has in mind.

The total force balance for all the material in an averaging volume is obtained by adding Frenkel's Eqs. (22) and (23) to obtain directly

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} + \rho_f\frac{\partial^2\mathbf{w}}{\partial t^2} = \boldsymbol{\Phi}^{(1)} - \phi\nabla p_f \tag{22}$$

where $\rho = (1-\phi)\rho_s + \phi\rho_f$ is the bulk density and where Frenkel defines $\mathbf{\Phi}^{(1)}$ as the "elastic force acting on the solid skeleton". Frenkel should therefore have identified this as $\mathbf{\Phi}^{(1)} = (1-\phi)\nabla\cdot\boldsymbol{\tau}_s = \nabla\cdot\boldsymbol{\tau} + \phi\nabla p_f$ so that the right-hand side of Eq. (22) becomes simply the total elastic force $\nabla\cdot\boldsymbol{\tau}$. Written in this way, Frenkel's Eq. (22) is identical to the results of Biot 1956 and 1962.

However, Frenkel for some reason proposes that $\mathbf{\Phi}^{(1)} = (1-\phi)\nabla\cdot\boldsymbol{\tau}_s - (1-\phi)\nabla p_f$ [his Eq. (16)]. He somehow pictures an additional fluid-pressure gradient acting directly on the solid frame which is erroneous. Because of this, his equation governing the total force balance has an extra fluid-pressure gradient term present that should not be there.

Frenkel's Wave Properties

As seen above, the form of Frenkel's equations, after some manipulation, are nearly identical to those of Biot 1956 and 1962. However, because his analysis misidentifies two of the three compressibility constants and adds an erroneous fluid-pressure gradient to the total force balance, his detailed results for the nature of compressional (longtitudinal) slownesses are not correct. Since he correctly writes down the force balance governing the relative flow, he properly predicts the existence of two distinct longtitudinal modes. Unfortunately, he gives little importance to the fluid-pressure-diffusion (or "slow-wave") mode and even goes so far as to say it is "non-existent".

Something Frenkel gets entirely correct is the complex shear-wave slowness s_s

$$s_s = \sqrt{\frac{\rho + i\omega k_o \rho_f^2/\eta}{G}}. \tag{23}$$

This is because the shear-wave slowness is independent of the presence of fluid-pressure gradients and the compressibility moduli.

In the balance, Frenkel certainly deserves more recognition than he has received for his pioneering contribution to porous-media acoustics. He did many non-obvious things correctly, not the least of which is defining appropriate dependent and independent variables for a theory of porous-media acoustics.

Frenkel's Electroseismic Theory

Ivanov 1939 measured an electric field as seismic waves passed electrode pairs on the earth's surface. He proposed that such coupling is electrokinetic in nature. In soils, rocks and other porous materials, the solid grains have an excess charge fixed to their surfaces that is balanced by free counter ions diffusely distributed in the fluid layers immediately adjacent to the grain surfaces. The bound charge on the grain surfaces is immobile while the free counter ions in the fluid are able to move in response to applied forces. The bound charge and diffuse counter charge together is called the "electric double layer". Electrokinetic phenomena are, by definition, processes associated with the transport of the diffuse counter charge.

Frenkel states that his principal motivation for developing a theory of porous-media acoustics is to explain the electroseismic measurements of Ivanov. Both Ivanov and Frenkel envision that a compressional wave creates fluid-pressure gradients at the scale of the wavelength and that the associated flow from the compressions to dilatations transports the counter charge of the double layer relative to the bound charge. In this way, counter charge accumulates in the regions of dilation and the bound charge becomes "exposed" in the regions of compression so that an electric field is created perpendicular to the wavefront at the scale of the wavelength. This electric field moves along with the seismic wave as part of the material response and can be recorded as the seismic wave traverses an electrode pair.

We have seen above that Frenkel produces a theory capable of predicting (properly so if he hadn't made his small mistakes) the macroscopic fluid-pressure gradients ∇p_f associated with a compressional wave. It only remains to relate this seismic response to the associated electric field.

To do so, Frenkel simply quotes the famous result of Smoluchowski 1903 for the electric field $\mathbf{E}$ created by a static fluid-pressure gradient ∇p_f in a porous material

$$\mathbf{E} = -\frac{\varepsilon_f \zeta}{\eta_f \sigma_f} \nabla p_f \tag{24}$$

where ε_f is the fluid's dielectric permittivity, σ_f is the fluid's electrical conductivity, η_f is again the fluid's viscosity, and ζ is a property of the grain-fluid interface that quantifies the electric potential at the interface separating the bound charge from the diffuse-layer charge. The "zeta potential" ζ is thus a measure of the amount of charge separation present in the electric double layer.

For a plane P-wave having $e^{-i\omega t}$ time dependence and propagating in the x direction in a uniform porous material, Frenkel first calculates the $\partial p_f/\partial x$ associated with the wave and then determines E_x using Eq. (24). If the corrected variant of Frenkel's equations are used, one obtains

$$\frac{E_x}{-\omega^2 u_x} = \frac{\rho_f \varepsilon_f \zeta}{\eta_f \sigma_f} \left(1 + \frac{\rho}{\rho_f} \frac{C}{H} \right) \tag{25}$$

where u_x is the amplitude of the particle displacement associated with the wave in the x direction, and where C and H are elastic moduli (as defined by Biot 1962) related to the undrained bulk modulus K_U and Skempton's coefficient B as

$$C = BK_U \tag{26}$$

$$H = K_U + 4G/3. \tag{27}$$

Equation (25) for E_x is the same as the culminating Eq. (46) of Frenkel's paper except for a slighltly different definition of the elastic moduli ratio C/H due to Frenkel's errors associated with the longtitudinal response as previously discussed.

The message of Frenkel's result [Eq. (25)] is that the electric field E_x moving along with a P wave as part of the material response is directly proportional (without phase adjustments) to the particle acceleration. This has been experimentally verified by Garambois and Dietrich 2001. In Fig. 1, the recorded particle accelerations (time derivatives of the geophone response which is proportional to the particle velocities) and associated electric fields are plotted together on the same time axis for receivers on the earth's surface at different distances from the seismic source (a hammer). A horizontal geophone was at the center of each 50 cm electric-dipole antenna (two metal rods driven vertically into the earth, 50 cm apart) and the time difference between when the seismic wave arrives at the first electrode and when it arrives at a geophone have been allowed for (roughly 1 ms). The experimental field data of Fig. 1 provides the ultimate testimony to the lucidty and correctness of Frenkel's pioneering work.

Frenkel limited his electroseismic analysis to explaining the electric field that accompanies a propagating longtitudinal wave. However, there are many other aspects to the electric and magnetic fields generated by both compressional and shear waves that Frenkel did not attempt to address as we now go on to discuss.

MODERN THEORY OF ELECTROSEISMIC PHENOMENA

Governing Equations

Pride 1994 obtains a more general set of equations governing the coupled seismic and electromagnetic response of a porous material when electrokinetics is responsible for the coupling. Assuming an $e^{-i\omega t}$ time depedence, these equations are

$$-\omega^2 \rho \mathbf{u} = -\nabla P_c + \nabla \cdot \boldsymbol{\tau}^D + \omega^2 \rho_f \mathbf{w} \tag{28}$$

$$\boldsymbol{\tau}^D = G\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3}\nabla \cdot \mathbf{u}\,\mathbf{I}\right) \tag{29}$$

$$-\begin{bmatrix} P_c \\ p_f \end{bmatrix} = K_U \begin{bmatrix} 1 & B \\ B & B/\alpha \end{bmatrix} \begin{bmatrix} \nabla \cdot \mathbf{u} \\ \nabla \cdot \mathbf{w} \end{bmatrix} \tag{30}$$

$$-i\omega \mathbf{w} = \frac{k(\omega)}{\eta_f}\left(-\nabla p_f + \omega^2 \rho_f \mathbf{u}\right) + L(\omega)\mathbf{E} \tag{31}$$

$$\mathbf{J} = L(\omega)\left(-\nabla p_f + \omega^2 \rho_f \mathbf{u}\right) + \sigma(\omega)\mathbf{E} \tag{32}$$

$$\nabla \times \mathbf{H} = -i\omega\varepsilon \mathbf{E} + \mathbf{J} \tag{33}$$

$$\nabla \times \mathbf{E} = i\omega\mu \mathbf{H} \tag{34}$$

and are simply the Frenkel/Biot equations for porous media acoustics along with the Maxwell equations for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$. The electroseismic coupling occurs in the transport laws of Eqs. (31) and (32) where $\mathbf{J}$ is the electric-current density and $-i\omega\mathbf{w}$ the Darcy filtration velocity. If the coupling coefficient L were set to zero then there would be complete decoupling between the poroelastic and electromagnetic response fields.

Two types of electroseismic coupling are present in Eqs. (31) and (32). Seismic waves generate a force $-\nabla p_f + \omega^2 \rho_f \mathbf{u}$ that in addition to driving the standard Darcy fluid filtration $(k/\eta_f)(-\nabla p_f + \omega^2 \rho_f \mathbf{u})$, also transports the diffuse charge of the double layer relative to the bound charge on the grain surfaces resulting in a "streaming" electric current $L(-\nabla p_f + \omega^2 \rho_f \mathbf{u})$. Such generation of an electric current from an applied fluid-pressure gradient is known as "electrofiltration". Conversely, when an applied electric field $\mathbf{E}$ acts on a porous material, in addition to driving a conduction current given by $\sigma\mathbf{E}$, it also acts as a body force on the excess charge of the diffuse double layer resulting in a net fluid filtration given by $L\mathbf{E}$. Such generation of a fluid filtration from an electric field is known as "electro-osmosis".

As stated, Frenkel 1944 only concerns himself with the electrofiltration (or "E") effect. He makes the assumption (correctly), that within a propagating compressional wave in a homogeneous material, there is no net electric current. The streaming electric current induced by the wave causes counter charge to accumulate in the troughs of the wave and the bound charge to become exposed in the peaks. The electric field associated with this wavelength-scale charge separation drives a conduction current that exactly balances the streaming current. Thus, for a P wave propagating in a homogeneous material, $\mathbf{J} = 0$ so that Eq. (32) yields $\mathbf{E} = (L/\sigma)\nabla p_f$ which, as will be seen next, can be considered equivalent to Eq. (24).

Porous Media Transport Coefficients

The main purpose of this subsection is to address what is known about the frequency relaxation in the transport equations; an issue not addressed by Frenkel 1944.

Pride 1994 obtains analytic expression for the three porous-media transport coefficients: permeability $k(\omega)$, electrokinetic-coupling coefficient $L(\omega)$, and electric conductivity $\sigma(\omega)$. The results are valid to leading order in the dimensionless ratio d/Λ where Λ is a characteristic pore-throat radius defined by Johnson et al. 1987 and d is a skindepth measure of the diffuse-charge-layer thickness known as the Debye length given by

$$d = \sqrt{\frac{\varepsilon_f kT}{e^2 z^2 N}} \tag{35}$$

where kT is thermal energy, ez is the electric charge of each ion contributing to a symmetric electrolyte having ionic-number density N (number of ions having valence z per cubic meter). For most salinity conditions in the earth, d is on the order of nanometers and so the small d/Λ limit can be considered appropriate. The results are

$$\frac{k(\omega)}{k_o} = \left[\left(1 - i\frac{4}{m}\frac{\omega}{\omega_t}\right)^{1/2} - i\frac{\omega}{\omega_t}\right]^{-1} \tag{36}$$

$$\frac{L(\omega)}{L_o} = \left[1 - i\frac{\omega}{\omega_t}\right]^{-1/2} \tag{37}$$

$$\frac{\sigma(\omega)}{\sigma_o} = 1 \tag{38}$$

and are now described.

Equation (36) was first obtained by Johnson et al. 1987 and allows for three distinct phenomena associated with the relative flow. At low enough frequencies $\omega \ll \omega_t m/4$, there is parabolic flow in the pores resulting in a permeability k_o that depends only on the topology and size of the pores. As frequencies increase and $\omega \approx \omega_t m/4$, viscous boundary layers develop in the pores. Finally, in the high-frequency limit where $\omega \gg \omega_t$, the viscous boundary layers have a thickness that diminishes with the square root of frequency resulting in a relative motion dominated by inertial plug (ideal) flow. The real and imaginary parts of $k(\omega)$ in the high-frequency limit are

$$k(\omega) \sim \frac{\sqrt{2}}{F\Lambda}\left(\frac{\eta_f}{\rho_f \omega}\right)^{3/2} + \frac{i}{F}\frac{\eta_f}{\omega \rho_f} \tag{39}$$

where the following expressions for the viscous-to-inertial relative-flow transition frequency ω_t and the dimensionless number m have been used

$$\omega_t = \frac{\eta_f}{F k_o \rho_f} \quad \text{and} \quad m = \frac{\Lambda^2}{F k_o}. \tag{40}$$

The effective pore-throat radius Λ and the electrical formation factor F both have precise mathematical definition given by Johnson et al. 1987 in terms of the dimensionless electric field $-\nabla\Phi$ in the pore space of an averaging volume of porous material of volume V

$$\frac{1}{F} = \frac{1}{V}\int_{\Omega_p} \nabla\Phi \cdot \nabla\Phi \, dV \tag{41}$$

$$\frac{2}{\Lambda F} = \frac{1}{V}\int_{\partial\Omega_{gp}} \nabla\Phi \cdot \nabla\Phi \, dS. \tag{42}$$

Here, Ω_p is the porespace and $\partial\Omega_{gp}$ is the surface separating the grains from the pores. In clean porous rocks where secondary clay on the grain surfaces has not created a large grain-surface area, Johnson et al. 1987 suggest that a good model for Λ is to take $m = 8$ so that $\Lambda = \sqrt{8Fk_o}$.

The relaxation in $L(\omega)$ is also due to the development of viscous boundary layers in the pores. Pride 1994 allows for an additional relaxation in $L(\omega)$ as the viscous skindepth $\sqrt{\eta_f/(\rho_f\omega)}$ associated with the viscous boundary layers becomes smaller than the Debye-length; however, over the range of frequencies and salinities normally encountered in the earth, this relaxation never takes place and as such has not been included in Eq. (37). The steady-state coupling coefficient L_o is given by

$$L_o = -\frac{\varepsilon_f \zeta}{\eta_f F} \tag{43}$$

to leading order in d/Λ where F is the same formation factor defined by Eq. (41).

There is no important relaxation in $\sigma(\omega)$ over frequencies where $\sqrt{\eta_f/(\rho_f\omega)} > d$ (c.f., Pride 1944). To leading order in d/Λ, one simply has

$$\sigma(\omega) = \sigma_o = \frac{\sigma_f}{F} \tag{44}$$

with the formation factor again defined by Eq. (41).

An immediate consequence of these results is that at frequencies where viscous boundary layers have not yet developed, one has that when there is no net current (e.g., in a longtitudinal wave)

$$\left.\frac{E_x}{-\partial p_f/\partial x + \omega^2 \rho_f u_x}\right|_{\mathbf{J}=0} = \frac{\varepsilon_f \zeta}{\eta_f \sigma_f} \tag{45}$$

to leading order in d/Λ which is the famous proposition of Smoluchowski 1903. It is through Eq. (45) that the zeta potential is often measured. If the left-hand side is directly measured after applying a pressure drop across a porous sample, and if the fluid properties ε_f, η_f and σ_f are known, ζ is determined. For quartz surfaces in contact with a saline solution having a given salinity C (moles/liter) and pH, ζ is observed to obey the emperical formula

$$\zeta(\text{ in volts }) = (0.01 + 0.025 \log_{10} C)\frac{(\text{pH} - 2)}{5} \tag{46}$$

which gives the measured dependence on C when pH=7, and gives the appropriate general trend for the pH dependence.

In conclusion, the important relaxation in the porous-media transport coefficients is associated with the onset of viscous boundary layers in the pores. However, since the transition frequency $\omega_t = \eta_f/(\rho_f F k_o)$ normally lies above the frequency band of 10 to 10^3 Hz used in seismic exploration, it is usually appropriate to ignore the frequency dependence in these coefficients (as did Frenkel 1944) and simply take $k(\omega) = k_o$, $L(\omega) = \varepsilon_f\zeta/(\eta_f F)$ and $\sigma(\omega) = \sigma_f/F$. In this case, only two porespace topology parameters k_o and F must be specified to model the electroseismic transport.

Diverse Electroseismic Wave Phenomena

Equations (28)–(34) allow for a wide range of electroseismic phenomena beyond the electric field contained in a compressional wave that was the sole focus of Frenkel 1944. A few of the many possible phenomena/applications are now discussed.

Magnetic field in a shear wave

A shear wave in a homogeneous material does not cause the counter charge in the double-layer to accumulate. Because of this, the grain accelerations produce a non-zero net current **J** in the plane of the wavefront and such current sheets produce magnetic fields as part of the material response. There is a small electric field produced via induction but this is extremely small compared to the electric field in a compressional wave of similar amplitude and and does not produce a conduction current that significantly opposes the streaming current. For a shear wave propagating in the x direction with a displacement amplitude u_y, Pride and Haartsen 1996 have determined the exact expression for the magnetic field which to leading order in the dimensionless number $\omega k_o \rho_f^2/(\rho\eta_f)$ reduces to

$$\frac{H_z}{-i\omega u_y} = \sqrt{\frac{G}{\rho}}\,\frac{\rho_f \varepsilon_f \zeta}{\eta_f F}. \tag{47}$$

This result states that the magnetic field in a seismic shear wave is directly proportional to the particle velocity which is the material response recorded by a geophone. The magnetic field measured by a magnetometer is $B_z = \mu H_z$ where μ is the magnetic susceptibility of the material (only different from that of vacuum when iron is significantly present in the soil). If the particle velocities of a shear wave are on the order of 10^{-3} m/s (a large amplitude but linear seismic response) and if characteristic values appropriate to water are used, one obtains that B_z is on the order of 10^{-2} nT which is at the extreme limit of being measurable. Using magnetometers as shear-selective recording devices at seismic frequencies is, therefore, probably not a commercially interesting possibility.

Converted seismic-to-EM fields at an interface

When a compressional (or shear) wave traverses an interface in which any of the transport properties or elastic moduli change, there is a dynamic imbalance of the streaming current that results in additional charge separation across the interface. Because this charge separation is concentrated in space and has a strong dipolar component, particularly so as the first Fresnel zone traverses the interface, there are electric (and magnetic) fields created that have extent outside the support of the seismic waves and that can be recorded at the earth's surface. Over the seismic band width of 10 Hz $< f <$ 10^3 Hz, the electromagnetic skindepth $\delta = 1/\sqrt{2\pi\mu_o\sigma f}$ varies as 10^3 m $> \delta >$ 10^2 m. So if the interface of interest is less than 100 m from the electrical antennas, the recorded response is in the electrostatic and magnetostatic near field of the charge separation at the interface and induction effects may be neglected.

A numerical example of the electric and magnetic fields generated at an interface is now given for the situation depicted in Fig. 2. An isotropic explosion is located at $z = 5$ m, a material-property interface is at $z = 50$ m, and a line of geophones and electric-dipole antennas is at $z = 0$ m. A numerical reflectivity algorithm (Haartsen and Pride 1997, Garambois and Dietrich 2002) is used to solve the complete set of governing equations [Eqs. (28)– (34) with no terms neglected] for this geometry. The interface separates a higher-porosity more-compressibile sandstone upper layer, from a lower-porosity stiffer lower layer. The results of the numerical simulation are displayed in Fig. 3.

Of interest from an exploration perspective are the "flat" events shown on the electric and magnetic sections that are generated at $t = 21$ ms as the P wave traverses the interface. Such converted fields provide information about the interface that is distinct from the information in the seismic reflection since they are also sensitive to fluid chemistry changes while seismic waves are not. Field data that have actually recorded such interface conversions (from P to EM) have been reported by Martner and Sparks 1959, Thompson and Gist 1993 and Garambois and Dietrich 2001. However, much further work is required before electrical antenna data can be routinely processed to obtain the no-moveout interface fields. A principal challenge is that the electric fields created at an interface are far smaller than the fields contained within the seismic waves and so the electrical section has to be filtered to remove all events that moveout as a function of time (since these are due to seismic waves traversing the antennae).

A particularly interesting aspect of the electroseismic conversion at an interface is that the amplitude of the converted electric field can be drastically increased if there is a thin layer of third material present at the interface. In Fig. 4, the same example as in Fig. 3 is considered but with the addition of a thin 1 cm layer of very-low permeability material (a shale aquitard) sandwhiched between the two half spaces. The amplitude of the converted electric field is increased by roughly a factor of 10 in the case where the thin shale layer is present, while the reflected seismic wave is essentially unaffected by the presence of the thin layer.

Applications to electric fields created by earthquakes

A fault that has undergone a shear dislocation creates lobes of compression and dilation in the crust surrounding the limits of the fault segment that slipped. Immediately after the earthquake, the poroelastic response is undrained (no fluid mass enters or leaves each mass element of the crust). However, the fluid pressure created by such undrained compression will equilibrate by diffusion (the slow wave) producing electric fields in the process. Modeling the complete poroelastic and electric response as the fluid-pressure equilibrates through time in a uniform crust after an earthquake as been the subject of a recent study (Pride et al. 2003).

An example is given in Fig. 5. The earthquake is modeled as 50 cm of uniform slip on a normal fault dipping at 45°. The slip surface is 5 km in the direction of slip and 20 km wide and is denoted in the top panel of Fig. 5 by the dashed-white line. This corresponds to a magnitude 6 earthquake and the stress changes created immediately after the slip are numerically determined using the algorithm of Okada 1992. The subsequent fluid-pressure variations through time and space were numerically calculated using finite differences. The displayed vertical-component of the electric field has an amplitude on the order of mV/m which is 100 to 1000 times larger than the telluric fields that are routinely measured (the telluric fields are electric fields that enter the earth from the atmosphere each day as the ionosphere is heated by the sun). Direct measurement of the decay of the crust's electric field through time after an earthquake has not yet been attempted.

Electro-osmotic coupling

One of the greatest potential applications of Eqs. (28)–(34), concerns injecting time-varying current into the earth with the goal of generating seismic waves at interfaces at depth. The idea is that an applied electric field will drive an electro-osmotic flow of fluid $L\mathbf{E}$. At interfaces where either L or $\mathbf{n} \cdot \mathbf{E}$ change ($\mathbf{n}$ being the normal to the interface), there is an accumulation of fluid and an associated volumetric dilatation of the porous material that generates a compressional wave having the same time signature as the applied current.

Although this possibility has been discussed by Thompson and Gist 1993, no studies have ever been published in which such electrically generated seismic waves have been measured. Numerical simulations indicate that for safe levels of injected current, the generated seismic waves will have a very small amplitude. However, because the source is a non-destructive electric current that can be applied for long durations with a controlled time signature, it should be possible to extract even small-amplitude seismic waves from the ambient noise by cross-correlating the seismic data with the known time signature of the electric current. This is the subject of ongoing work.

CONCLUSIONS

Frenkel 1944 produced equations having nearly identical form to those of Biot 1956a and

TABLE 1. Identification of the Frenkel variables

Frenkel 1944 variable	Equivalent definition in present paper
$\mathbf{T}$ (partial stress tensor associated with solid)	$(1-\phi)\boldsymbol{\tau}_s = \boldsymbol{\tau} + \phi p_f \mathbf{I}$
f (porosity)	ϕ
p_2 (fluid pressure)	p_f
$p_1 = -\mathrm{tr}\{\mathbf{T}\}/\mathbf{3}$ (partial pressure associated with solid)	$(1-\phi)p_s = P_c - \phi p_f$
$\mathbf{u}$ (average displacement of solid phase)	$\mathbf{u}$
$\theta = \Delta V/V = \nabla \cdot \mathbf{u}$ (dilatation of porous sample)	$\nabla \cdot \mathbf{u}$
$\varphi = -\Delta\rho_2/\rho_2$ (dilatation of fluid volume)	$-p_f/K_f$
$\mathbf{v}_1$ (average velocity of solid phase)	$\partial\mathbf{u}/\partial t$
$\mathbf{v}_2$ (average velocity of fluid phase)	$\phi^{-1}\partial\mathbf{w}/\partial t + \partial\mathbf{u}/\partial t$
Δf (porosity change)	$\Delta\phi = -\phi\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{w} - \phi p_f/K_f$

1962. The only difference in the form of Frenkel's equations is an extra fluid pressure gradient term in the total conservation of momentum equation that should not be present. As was seen here, Frenkel also made a slight error in developing his effective poroelastic compressibility coefficients with the result that he did not obtain the fluid-substitution relations of Gassmann 1951 and Biot and Willis 1957. Frenkel's estimate of the electric field contained in a compressional wave (the focus of his electroseismic investigation) is correct if the proper compressibility constants are used and if the extra fluid-pressure gradient in the bulk-force balance is neglected. He did not attempt to analyze any of the other electroseismic phenomena that were discussed above.

Frenkel covered a lot of ground in his only paper on poroelastic phenomena, and the vast majority of his intuitive approach to obtaining the governing equations is correct. He certainly deserves a far greater recognition for his pioneering contribution than he has so far been accorded. From the present author's perspecitve, it seems entirely appropriate to refer to the equations governing the acoustics of isotropic porous materials as the "Frenkel/Biot equations" instead of just the "Biot equations" as is the current fashion.

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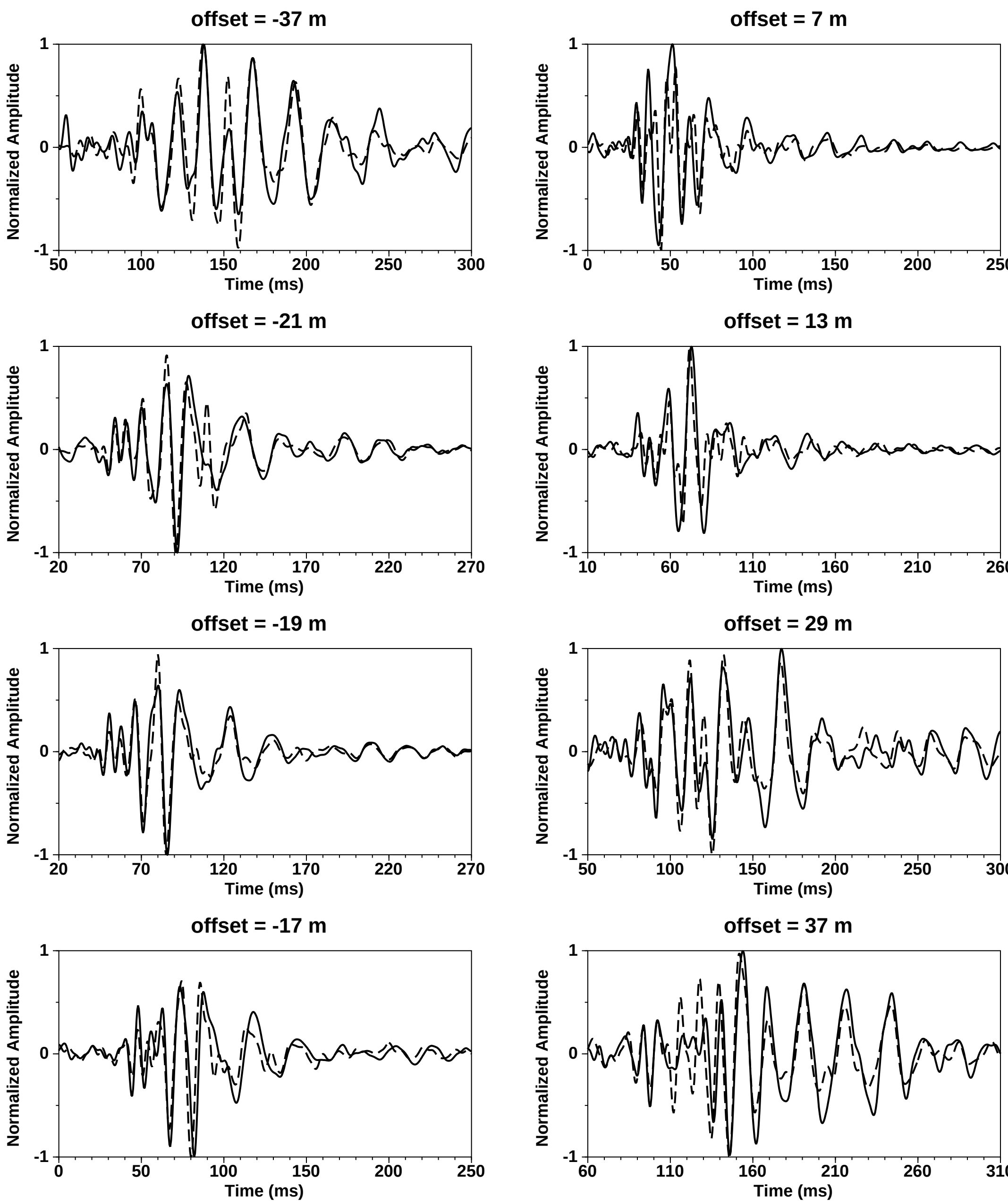

FIG. 1. The field data of Garambois and Dietrich 2001. The horizontal component of the electric field (solid lines) along with the horizontal component of the particle acceleration (dashed lines) as recorded at the earth's surface at different offsets from a seismic source which is a hammer impacting a plate on the earth's surface.

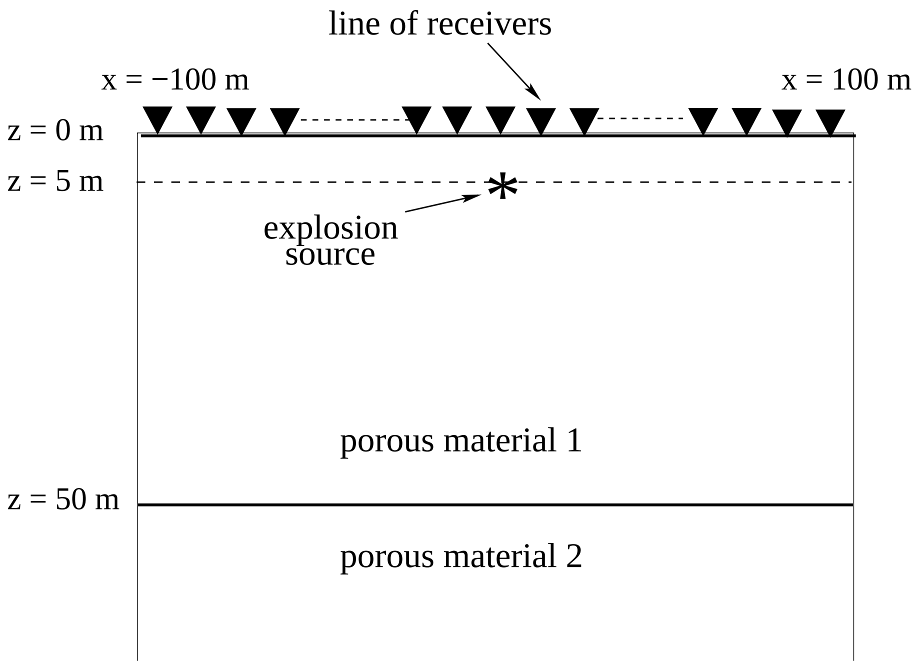

FIG. 2. The receiver, source, and interface geometry for the numerical example given in Fig. 3.

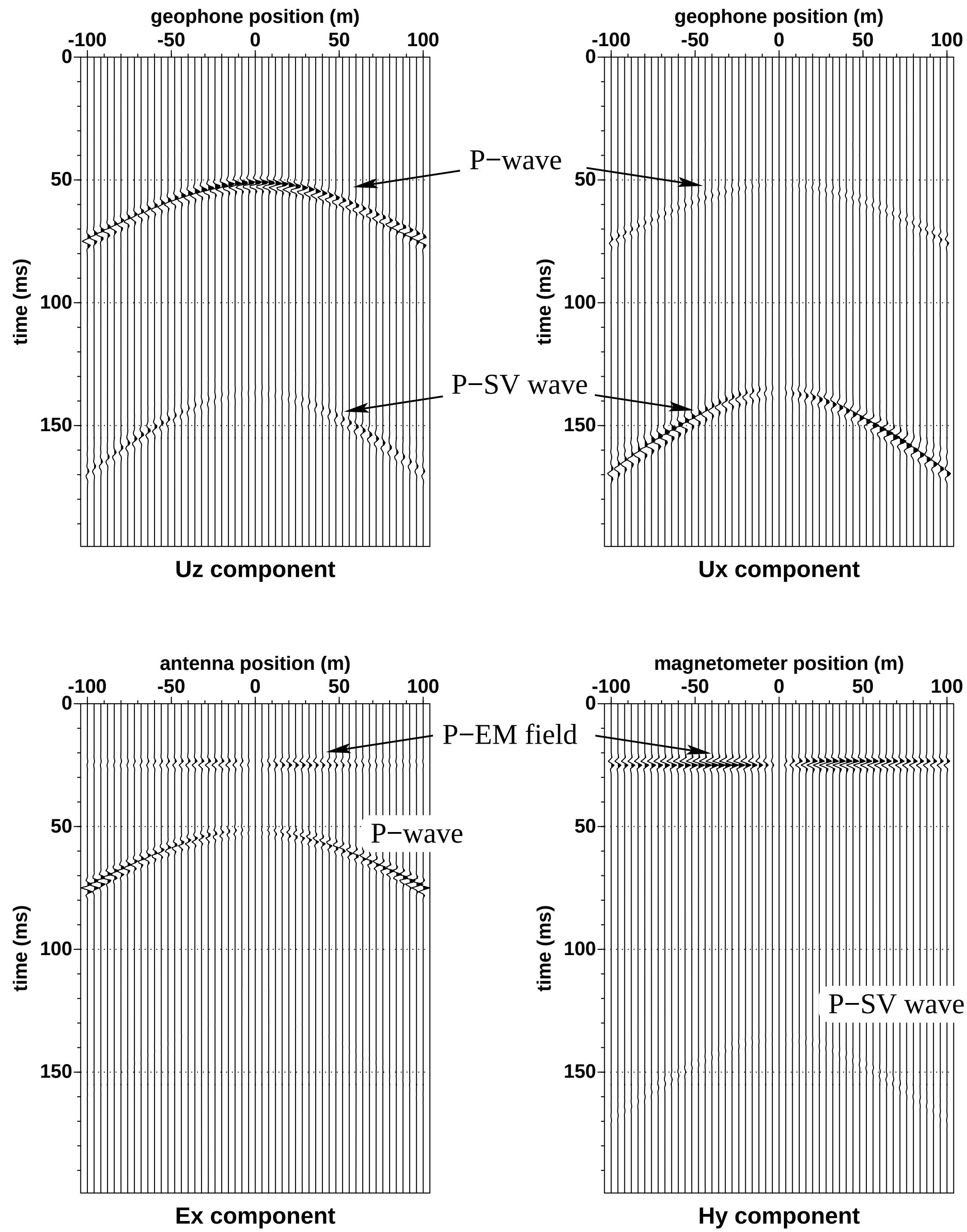

FIG. 3. An electroseismic example for the geometry depicted in Fig. 2. The interface is located at a P-wave travel time of 21 ms. The no-moveout P-EM converted fields have been multiplied by 500 in both the electric and magnetic sections. The P-SV conversion is just barely visible on the magnetic section (the hyperbolic arrival at 130 ms). The direct P wave from the source to receivers is not shown.

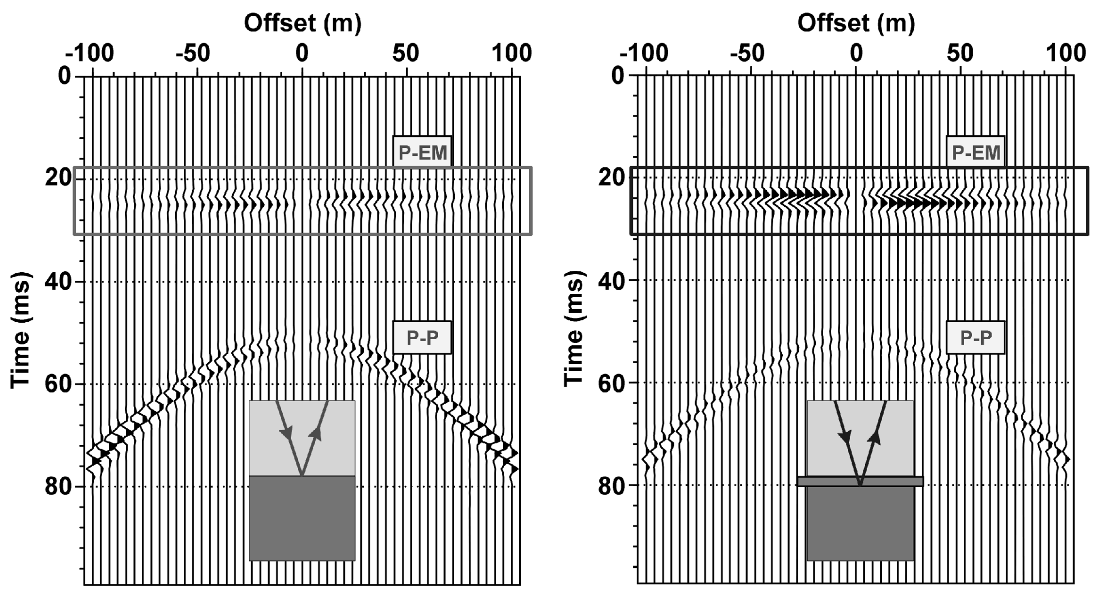

FIG. 4. The electrical section to the left corresponds to the same situation shown in Fig. 3 while the electrical section to the right corresponds to when a thin (1 cm) impermeable layer has been sandwhiched between the two half spaces. The converted electric field is roughly 10 times larger in the situation in which the thin layer is present. Stephane Garambois kindly ran his numerical code (Garambois and Dietrich 2002) to produce this figure.

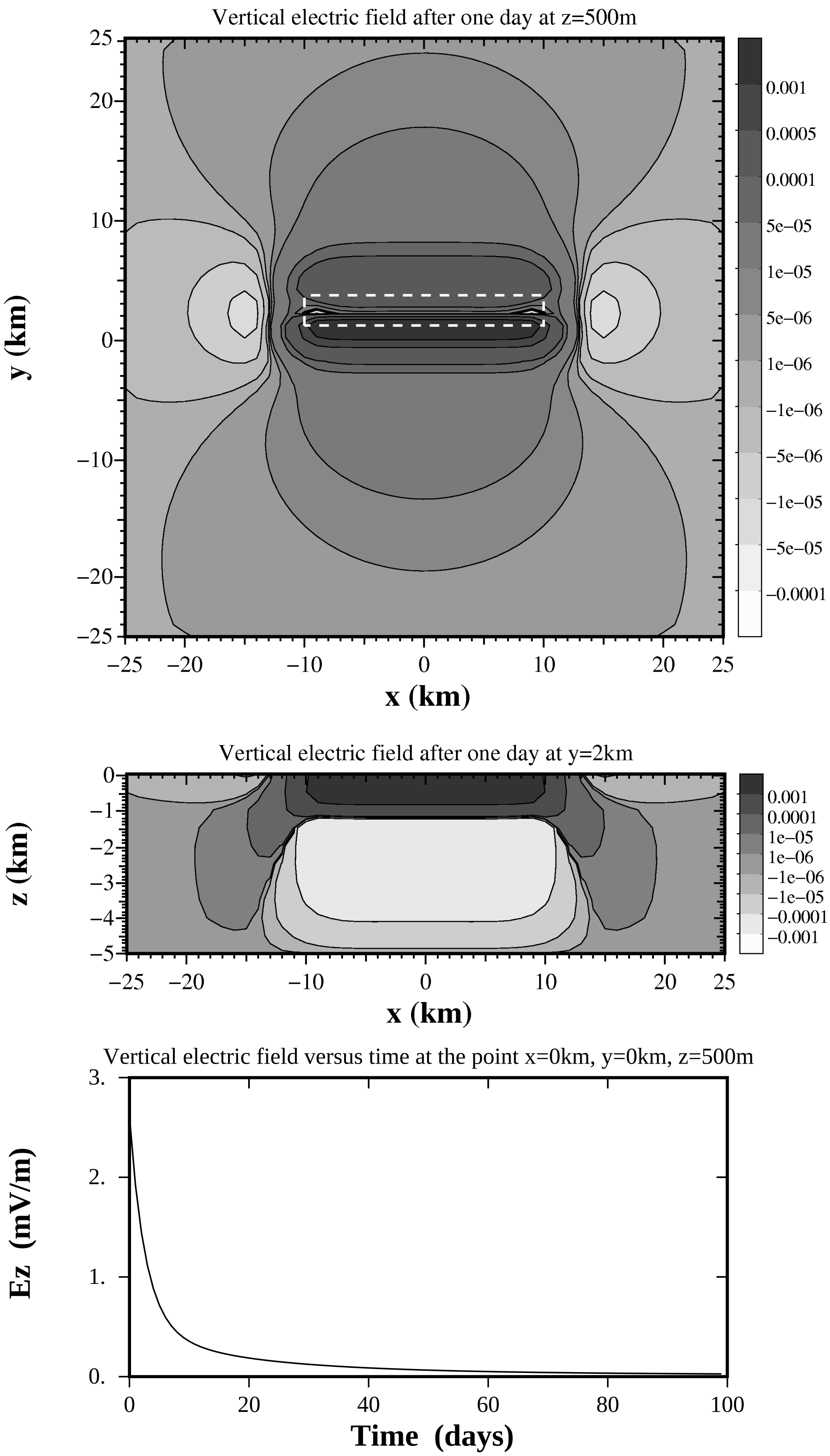

FIG. 5. The electrical field generated in a uniform crust after a magnitude 6 earthquake on an inverse fault dipping at 45 degrees and positioned as shown in the top panel with the dashed-white line. The vertical-depth axis is z.