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Publication Date
2016

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UNIVERSITY OF CALIFORNIA, SAN DIEGO

Essays in International Economics

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Economics

by

Gonzalo Valdes

Committee in charge:

Professor Gordon Hanson, Chair
Professor Thomas Baranga
Professor David Lagakos
Professor David Mares
Professor Marc Muendler

2016

The dissertation of Gonzalo Valdes is approved, and it is acceptable in quality and form for publication on microfilm:

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Chair

University of California, San Diego

2016

DEDICATION

To my wife, Fanny, and my daughters Javiera and Emilie.

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ACKNOWLEDGEMENTS

My PhD student days have come to an end. It has been a long beautiful tough road. It has had its ups and downs. There were moments I thought I could not make it. There were those instants that I felt the burden was too heavy to bear. In those moments, precisely when I needed it the most, I had my family to support me. I do not have enough words to express my deepest gratitude to my family. But yet, I will try my best in these lines.

I want to thank my parents, Hector Valdes and Adriana Gonzalez whom I deeply love. Even though they do not have formal education, they instructed me since I was a child to study and work hard. I cannot thank enough those calls from my mother exactly at those moments when I was in need of those kind and gentle words that just a mom can say. Or those words of wisdom from my dad, that meant the world for me. I also want to thank the best brothers in the world, my brothers!, Moises, Eliseo, and Daniel. They have always been there for me. I know that I can always count on you guys.

I want to thank my wife, Fanny Fuentes, who lovely bears with me even when I can not even stand myself. Together we dreamed of a better future for our family almost thirteen years ago. With your care, your support, and your prayers, today we have come through the other end of this road. Thank you so much, I love you.

I want to thank my daughters for their unconditional pure love, for their smiles, for bringing light and happiness to our home, for all those lessons I learned from them, for making me the happiest I could be. Definetely, my life has a greater and more beautiful meaning since they are around me.

I also want to express my deepest gratitude to my advisor, Professor Gordon Hanson, for his awesome guidance, care, and patience. I could not have completed my PhD without his help, comments, supervision, and patience. I also want to thank the other committee members, Professor Marc Muendler, Professor David Lagakos, Professor David Mares, and Professor Thomas Baranga, for guiding my

research for the past years.

I also want to thank my friends during these years, Jungbin Hwang and Henrique Romero. They have helped me to develop not only my academic abilities but also my social skills.

Last but not least, I want to thank God. I do not know where he is or his ways of doing things. All I know is that deeply in my heart I feel him next to me helping me along the way.

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ABSTRACT OF THE DISSERTATION

Essays in International Economics

by

Gonzalo Valdes

Doctor of Philosophy in Economics

University of California, San Diego, 2016

Professor Gordon Hanson, Chair

This dissertation studies dynamic models in the context of international economics and the U.S. economy. First, the focus is on the effect of commodity shocks in an small open economy in a dynamic trade model. Then, this dissertation studies the effects of monetary policy in a dynamic trade model. Finally, dynamics between government debt and bubbles are studied considering an overlapping generation model.

The first chapter, "Dutch Disease in a Dynamic International Trade Model of an Small Open Economy", models a dynamic small open economy which produces and trades final goods and a commodity. The commodity is modeled as an homogeneous good and it is demanded by the rest of the world. The dynamic system developed in this chapter relies on key parameters that characterize the small open economy. These parameters are the elasticity of substitution across final goods and the shape parameter of the Pareto distribution of productivities. In order to compute the steady-state of the economy and study the dynamics implied by the model, we estimate both the elasticity of substitution and the shape param-

eter of the Pareto distribution considering data for the Chilean economy, which satisfies the small open economy assumption as well as the commodity production.

The second chapter, "Monetary policy in a dynamic trade model with heterogeneous firms", studies the effect of monetary policy on a dynamic model of trade with heterogeneous firms. We study the dynamic implications of monetary policies that act during "normal times" and monetary policies that leave the economy at the zero lower bound. In order to do so, we craft a model which incorporates nominal rigidities. This feature generates a friction such that nominal shocks affect real allocations in the economy. To build the model, we combine nominal frictions with firm heterogeneity as in Melitz (2003) in a dynamic setting as in Ghironi and Melitz (2005).

The final chapter, "A Note on Government Debt and Bubbles", studies the interactions between government debt and bubbles in an economy. We consider a general equilibrium approach in a productive economy and we explore conditions under which government debt path in our model is consistent with the government debt path observed in the last twenty years. During that period, government debt, as share of GDP, has interacted with bubbles in a countercyclical pattern. That is, in the absence of bubbles there is an increase in the evolution of debt-to-GDP ratio, and when a bubble is traded, debt-to-GDP ratio is decreasing.

Chapter 1

Dutch Disease in a Dynamic International Trade Model of an Small Open Economy

Abstract. In this chapter, we model a dynamic small open economy which produces and trades final goods and a commodity. The commodity is modeled as an homogeneous good and it is demanded by the rest of the world. The dynamic system developed in this chapter relies on key parameters that characterize the small open economy. These parameters are the elasticity of substitution across final goods and the shape parameter of the Pareto distribution of productivities. In order to compute the steady-state of the economy and study the dynamics implied by the model, we estimate both the elasticity of substitution and the shape parameter of the Pareto distribution considering data for the Chilean economy, which satisfies the small open economy assumption as well as the commodity production.

We study the impulse response of the endogenous variables to different shocks within the stochastic dynamic system developed in the chapter. We first implement an aggregate productivity shock. This type of shock increases welfare over time and it is very persistent. Secondly, we implement a shock to the commodity price. This shock illustrates the effect of the so called "Dutch disease",

since the manufacturing sector is negatively affected although welfare in increased.

We also study the effect of government policy on the volatility induced by the Dutch disease. Our findings show that a government policy associated with corruption spending interact with the Dutch disease in such a way that volatility is amplified.

1.1 Introduction

The literature on the Dutch disease suggests that a bonanza in natural resource exports will lead to a contraction in production of tradables. In a first view, the idea behind the Dutch disease is that the extra wealth generated by the sales of natural resources induces appreciation of the real exchange rate, which makes domestic goods more expensive and less attractive, and thus, a contraction of the traded sector follows (Corden and Neary, 1982; Corden, 1984).

It seems intuitive that the market response to a resource windfall is the decline in the traded sector. However, the Dutch disease phenomenon is perceived as a negative issue. This is because a problem takes place when the traded sector is an important determinant of growth and, for example, it benefits from learning by doing. If human capital spillovers in production are generated by employment in the traded sector and it induces endogenous growth, then the negative effect of the natural resource exports on employment in the traded sector slows down learning by doing and thus it restricts growth (van der Ploeg, 2011).

Sach and Warner (1995) show that resource rich countries grow on average about one percent point less during the period 1970-89. Even after taking into account traditional growth determinants, there is a strong negative effect of resource dependence on growth[1].

Recent empirical evidence, for 135 countries during the period 1975-2007, indicates that in response to a natural resource bonanza nonresource exports de-

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[1]Even though, these results have been criticized on the econometric grounds.

crease by 35-70 percent and nonresource imports increase by 0-35 percent (Harding and Venables, 2010). These findings hold in pure cross-sections of countries, and in panel estimations including dynamics and country fixed effects. Ismail (2010) uses detailed disaggregated sectoral data for manufacturing and obtains similar results: a ten percent resource windfall is on average associated with a 3.4 percent fall in value added across manufacturing.

Beine et al (2012) find a negative relationship between natural resource windfalls and employment for the Canadian manufacturing sector.

Quasi-experimental evidence within country from Brazil shown in Caselli and Michaels (2009) offers support for the Dutch disease hypothesis, but also shows evidence of wasteful local government and corruption.

Thus, macroeconomic and sectoral evidence seems to offer support for the Dutch disease effect.

A second face of the Dutch disease is associated with corruption. Resource wealth may worsen the quality of institutions (Ishan et al, 2005). Sala-i-Martin and Subramanian (2003) argue that corruption seems to be why oil resources have ruined growth performance in the Nigerian economy.

Acemoglu et al (2004) argue that resource wealth makes it easier for incumbent politicians to buy off political challengers. Resource wealth raise the value of being in power and induce politicians to expand public sectors, bribe voters, create unproductive jobs, inefficient subsidies, etc. (Robinson et al, 2006).

Mauro (1995) shows, using a sample of fifty five countries, that resource dependence is indeed strongly positively associated with corruption perceptions which in turn is associated with lower growth.

A third face of the Dutch disease is associated with volatility. Resource revenues are highly volatile. Thus, the Dutch disease can also induce real exchange volatility and thus less investment in physical capital, worsening the contraction of the traded sector. Estimates in Van der Ploeg and Poelhekke (2009, 2010) suggest a strong negative and significant effect of macroeconomic volatility on growth

and a strong and positive effect of exports of especially point-source resources on macroeconomic volatility. They also show that with commodity price volatility liquidity constraints are more likely to happen and thus growth would fall.

In this chapter we study the dynamic effects of a commodity price boom from the perspective of an small open economy. In order to do so, we model a dynamic small open economy which produces and trades final goods and a commodity. The commodity is modeled as an homogeneous good and it is demanded by the rest of the world.

We tune the model estimating the elasticity of substitution across final goods and the shape parameter of the Pareto distribution of productivities considering Chilean data which satisfies the small open economy assumption as well as the commodity production.

The main commodity exported from Chile is Copper. Figure (1.1) shows the historical evolution of the copper price since 1960. The copper price boom can be seen starting around 2004.
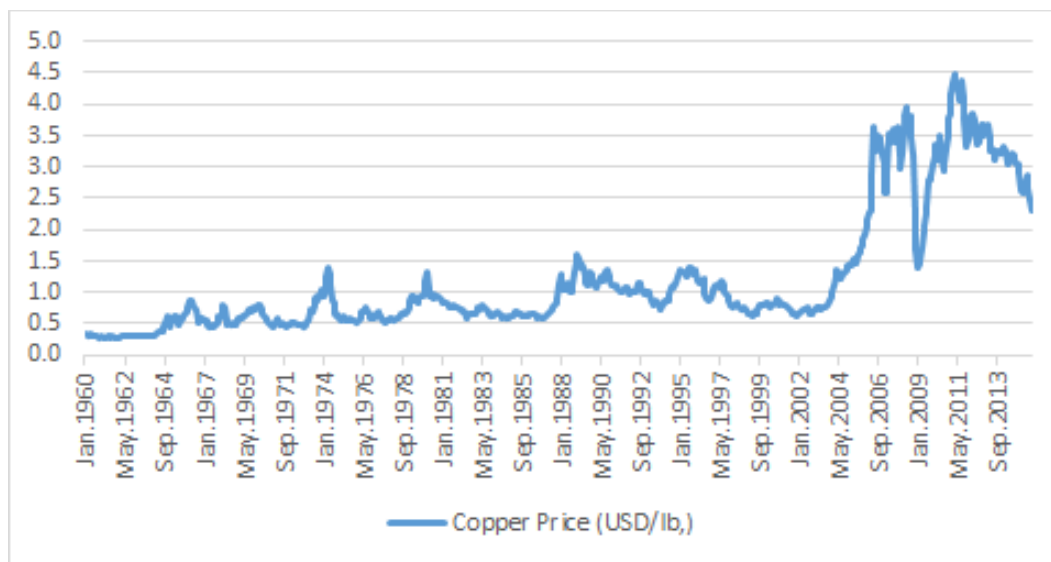
Figure 1.1: Copper Price evolucion since 1960

As discussed before in this section, there is empirical evidence that a boom in natural resources brings along a contraction in the traded sector. In particular,

Ruehle and Kulkarni (2011) find evidence in support of the Dutch disease for the Chilean economy.

In this chapter we want to build a theoretical model consistent with such findings in which we can explore the effect of the resource bonanza on the manufacture sector.

Our model considers firm heterogeneity as in Melitz (2003), in a dynamic setting as in Ghironi and Melitz (2005), in a small open economy as in Demidova and Rodriguez-Clare (2009 and 2013).

Besides the direct or traditional effect of the Dutch disease, the extra resources associated with the copper boom may bring along potential problems regarding the efficient use of such resources. Figure (1.2) shows the evolution of government income from copper as a percentage of GDP and government revenues from copper as a percentage of total government income.
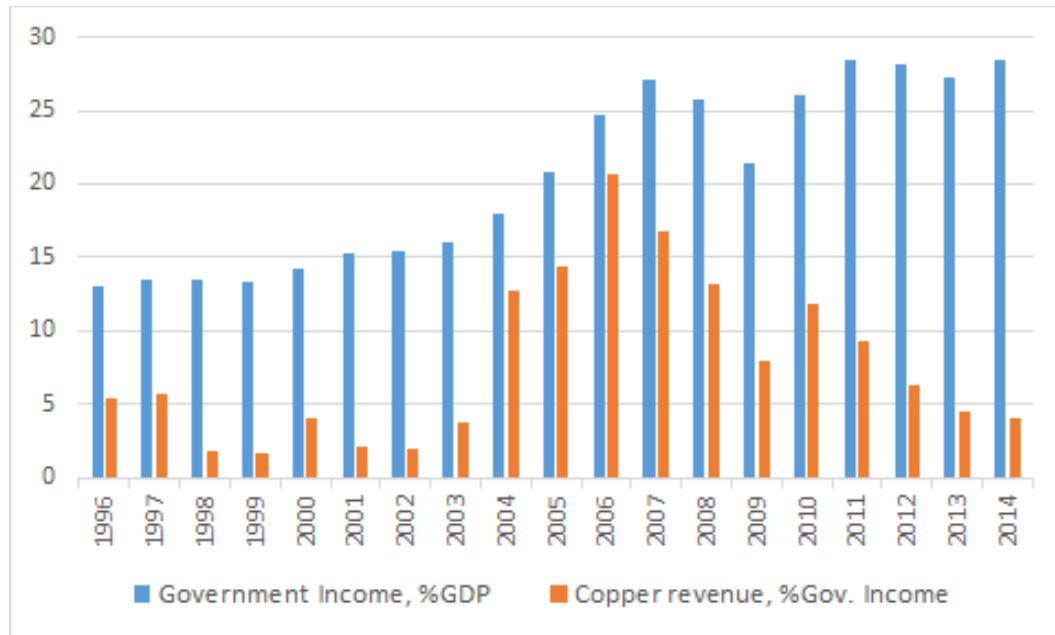
Figure 1.2: Evolution of government revenue

Both measures shown in Figure (1.2) illustrate the effect of the commodity boom on the government budget. As discussed earlier in this section, the com-

modity boom could trigger undesired effect in an economy inducing politicians to expand public sectors, bribe voters, create unproductive jobs, inefficient subsidies, etc.

Employing the model developed in this chapter, we will explore the impact of the commodity price boom on the manufacture sector as the direct or traditional effect of the Dutch disease, and the effect induced by the interaction of the commodity price boom and government policy.

The rest of the chapter is organized as follows: Section 1.2 describes the model. In Section 1.3, we delineate the macro dynamics of the model. In Section 1.4, the data employed in this chapter and the estimation of the parameters that describe the model are presented. Section 1.5 shows the steady state while section 1.6 describes the impulse responses of our model. Section 1.7 studies the effect of government policies on volatility. Finally, Section 1.8 offers some conclusions.

1.2   Model

Our model combines three distinctive characteristics. First, we consider a model of trade for a small open economy as in Demidova and Rodriguez-Clare (2009 and 2013) (DRC). Their model relies on the traditional Melitz (2003) model, as it considers a monopolistic competition environment and firm heterogeneity. Second, we include in the DRC setting a homogeneous good which is produced and traded by the small open economy. We use this homogeneous good to model a commodity which is produced in the small open economy and is traded with the rest of the world. Third, we develop a dynamic version of the small open economy with heterogeneous firms shown in DRC in which we include the commodity market.

1.2.1 Household Problem

Let us consider a small country populated by L identical individuals, each of which has a unit of labor that is supplied inelastically and earns a wage w_t. Each agent spends his income on a continuum of domestic and imported goods indexed ω and ω', respectively. The consumption of domestic and imported goods are given by $q_t(\omega)$ and $q_{x,t}^*(\omega')$.

A representative household maximizes expected intertemporal utility from consumption of the following form:

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{(C_s)^{1-\gamma}}{1-\gamma} \right]$$

where $\beta \in (0,1)$ is the subjective discount factor and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution. At time t, the household consumes a basket of goods C_t composed by domestically produced goods $q_t(\omega)$ and foreign produced goods $q_{x,t}^*(\omega')$ of the form:

$$C_t = \left[\int_{\omega \in \Omega} (q_t(\omega))^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega' \in \Omega'} (q_{x,t}^*(\omega'))^{\frac{\sigma-1}{\sigma}} d\omega' \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across goods.

The budget constraint faced by the representative agent at prices $p_t(\omega)$ and $p_{x,t}^*(\omega')$ for domestic and foreign goods, respectively, is as follows:

$$\int p_t(\omega) q_t(\omega) d\omega + \int p_{x,t}^*(\omega') q_{x,t}^*(\omega') d\omega' = I_t$$

where I_t corresponds to the income at time t.

The solutions to the household problem is given by equations shown in (2.18).

$$\begin{aligned} Q_t(\omega) &= \left(\frac{p_t(\omega)}{P_t} \right)^{-\sigma} C_t \\ Q_{x,t}^*(\omega') &= \left(\frac{p_{x,t}^*(\omega')}{P_t} \right)^{-\sigma} C_t \end{aligned} \tag{1.1}$$

In this chapter we assume a small open economy as in Demidova and Rodriguez-Clare (2009 and 2013). This translates into the following functional form for the amount produced of a variety ω for the foreign market,

$$Q_{x,t} = A_t \cdot p_{x,t}(\omega)^{-\sigma} \tag{1.2}$$

where A_t is exogenous and $p_{x,t}(\omega)$ is the price charged by a domestic exporter of a variety ω.

The price index in the home economy associated to the consumer problem is as follows:

$$P_t = \left[\int p_t(\omega)^{1-\sigma} \, d\omega + \int p_{x,t}^*(\omega')^{1-\sigma} \, d\omega' \right]^{\frac{1}{1-\sigma}}$$

1.2.2 Firm Problem

Firms are embedded in a monopolistic competition environment as in Melitz (2003). Firms pay a sunk cost f_E to draw a productivity level z. Depending on the level of z, a firm will serve the domestic economy exclusively (relatively low z with respect to a threshold), or it could serve both the domestic economy and the foreign economy (relatively high z with respect to a threshold).

The production scheme for a domestic firm z is given in (1.3). Where $Q_t(\omega)$ is the amount of goods, of a variety ω, domestically demanded, and $Q_{x,t}(\omega)$ is the amount of goods, of a variety ω, foreign demanded. τ corresponds to the traditional iceberg type of transport cost. The fixed costs of production are shown in (1.4). Then, a firm that exclusively serves the domestic economy will produce $Q_t(\omega)$ with no fixed cost of production.

$$\bar{Q}_t = \begin{cases} Q_t(\omega) + \tau \cdot Q_{x,t}(\omega) & \text{if firm is exporting} \\ Q_t(\omega) & \text{otherwise} \end{cases} \tag{1.3}$$

Also, a firm that serves the domestic economy and the foreign economy

will produce an amount equal to $Q_t(\omega) + \tau \cdot Q_{x,t}(\omega)$ incurring in fixed costs of production f_x. The fixed cost f_x is measured in effective units of labor which translates into $\frac{w_t f_x}{Z_t}$ units of consumption.

$$f_{\bar{Q}} = \begin{cases} f_x & \text{when exporting} \\ 0 & \text{otherwise} \end{cases} \tag{1.4}$$

Monopolistic firms produce final goods employing a production technology, shown in (1.5), which depends on labor l_t. The Hicks-neutral productivity $Z_t z$ is the product between an aggregate productivity shock Z_t and the firm's productivity draw z.

$$Q_t = (Z_t z) \cdot l_t \tag{1.5}$$

The conditional factor demand for a firm z is given in (1.6). The amount of labor demanded by a firm z depends on the exporting status of the firm, the productivity level, and the input cost, which is determined by the wage rate w_t.

$$l_t(z) = \frac{\alpha}{w_t} \left(\frac{w_t}{Z_t z} \right) \cdot \bar{Q}_t + f_Q \tag{1.6}$$

The price of the domestically produced non-traded goods, $p_t(z)$, and the price of the domestically produced traded goods, $p_{x,t}(z)$, are shown in (1.7). It is customary to say that the price charged by the domestic competitive firms correspond to a constant mark-up, $(1/\rho)$, over the marginal cost of production.

$$p_t(z) = \left(\frac{\sigma}{\sigma-1} \right) \left(\frac{w_t}{Z_t z} \right) = \left(\frac{w_t}{\rho Z_t z} \right), \quad p_{x,t}(z) = \tau \cdot p_t(z) \tag{1.7}$$

where $\rho = \frac{\sigma-1}{\sigma}$.

As mentioned before, the production scheme faced by a domestic firm z depends on its productivity level with respect to an endogenous threshold $z_{x,t}$. For instance, a firm z which satisfies $z \geq z_{x,t}$, will serve both the domestic economy and the foreign economy, and its production scheme will be given by $\bar{Q}_t = Q_t + \tau \cdot Q_{x,t}$.

On the other hand, whenever $z < z_{x,t}$, such firm will serve only the domestic economy and $\bar{Q}_t = Q_t$.

The productivity threshold for exporting firms corresponds to the productivity level for which profits are equal to zero. This productivity threshold is shown in equation (1.8).

$$(z_{x,t})^{1-\sigma} = \frac{Z_t}{\sigma w_t f_x} \left(\frac{w_t}{\rho Z_t} \tau \right)^{1-\sigma} A_t \tag{1.8}$$

Total profits for a monopolistic firm z, at time t, are given by $d_t(z) = d_{D,t}(z) + d_{x,t}(z)$. Where $d_{D,t}(z)$ corresponds to the profits earned by firm z from serving the domestic economy at time t, while $d_{x,t}(z)$ represents the profits earned by a firm z from serving the foreign market at time t.

$$d_{D,t}(z) = \frac{p_t(z)^{1-\sigma}}{\sigma} \frac{C_t}{(P_t)^{-\sigma}} \quad , \quad d_{x,t}(z) = \begin{cases} \frac{p_{x,t}^{1-\sigma}(z)}{\sigma} A_t - \frac{w_t f_x}{Z_t} & \text{if firm } z \text{ exports,} \\ 0 & \text{otherwise} \end{cases}$$

Using (1.8), we can write $d_{x,t}(z)$ as in (1.9).

$$d_{x,t}(z) = \frac{w_t \cdot f_x}{Z_t} \left[\left(\frac{z_{x,t}}{z} \right)^{1-\sigma} - 1 \right] \tag{1.9}$$

The definition of the small open economy employed in this chapter considers three main characteristics. First, A_t is exogenous as shown in equation (1.2). Second, the bundle cost for the foreign firms is unaffected by the small open economy. Thus, the foreign salary w_t^* is exogenous. Third, there is an exogenous number of total firms in the rest of the world M_t^*. However, a fraction of such number is going to export to the small open economy $(M_{x,t}^*)$ and that fraction is defined by the endogenous foreign productivity threshold $\bar{z}_{x,t}^*$. Thus, $M_{x,t}^*$ is endogenous while M_t^* is not.

Commodity Production

In this model, we have a sector in the home economy which produces homogeneous good for the rest of the world. The production of the homogeneous good is performed using a technology of the form:

$$\bar{Q}_{o,t} = \psi \cdot \bar{K}^{1-\zeta} \cdot l_{o,t}^{\zeta} = \Psi \cdot l_{o,t}^{\zeta}$$

The firm problem in the commodity market is to choose labor, which is priced at w_t, to produce the homogeneous good, which is traded at an exogenous price $p_{o,t}$, in order to maximize profits. Where ψ is the productivity of the sector, ζ is the labor share, and $\bar{K}$ is the long-run capital associated to the sector.

The operating profits generated in this sector have the form $IC_t = (1 - \zeta)p_{o,t}\bar{Q}_{o,t}$. Where $\bar{Q}_{o,t}$ represents the total demand of the homogeneous good. Total profits from the homogeneous sector are collected by the government and transfered to households.

Average Productivity

Firms, in the final market, draw their productivity level, after paying a sunk cost f_E, from a Pareto distribution of the form $G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^{\kappa}$. The sunk cost, f_E, is measured in effective units of labor which translates into $\frac{w_t f_E}{Z_t}$ units of consumption.

Let's define the aggregated (or average) productivity of domestically produced non-traded goods, $\widetilde{z}_D$, and the average productivity of domestically produced traded goods, $\widetilde{z}_{x,t}$, as in (1.10).

$$\widetilde{z}_D = \left[\int_{z_{\min}}^{\infty} z^{\sigma-1}dG\left(z\right)\right]^{\frac{1}{\sigma-1}} = \nu \cdot z_{\min}$$
$$\widetilde{z}_{x,t} = \frac{1}{1-G(z_{x,t})}\left[\int_{z_{x,t}}^{\infty} z^{\sigma-1}dG\left(z\right)\right]^{\frac{1}{\sigma-1}} = \nu \cdot z_{x,t}$$

(1.10)

where $z_{\min}$ is the minimum productivity level in the support of the distri-

bution G, $\nu = \left(\frac{\kappa}{\kappa+1-\sigma}\right)^{1-\sigma}$, and it is required that $k > \sigma - 1$.

Using (1.10) we can write the price of varieties and the price index as a function of aggregate productivity. Thus, the price index becomes a function of the number of home monopolistic firms M_t, the average price for non-traded domestic goods $\widetilde{p}_t$, the fraction of foreign monopolistic firms which are productive enough to be exporters $M^*_{x,t}$, as well as the average price of imported final goods $\widetilde{p}^*_{x,t}$; where:

$$\widetilde{p}_t = p_t\left(\widetilde{z}_D\right), \widetilde{p}^*_{x,t} = p^*_{x,t}\left(\widetilde{z}_{x,t}\right)$$

$$P_t = \left[M_t \cdot (\widetilde{p}_t)^{1-\sigma} + M^*_{x,t} \cdot \left(\widetilde{p}^*_{x,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

In a similar way we can write average profits as function of average productivity as in (1.11). Where $\widetilde{d}_t$ corresponds to the average profits obtained by a domestic firm that only serves the domestic economy. $\widetilde{d}_{x,t}$ is the average profit obtained by a domestic firm from serving the foreign economy.

$$\widetilde{d}_t = d_t\left(\widetilde{z}_D\right) \quad , \quad \widetilde{d}_{x,t} = d_{x,t}\left(\widetilde{z}_{x,t}\right) \tag{1.11}$$

Then, a representative domestic firm generates dividends or profits, $\tilde{D}_t$, of the following form:

$$\widetilde{D}_t = \widetilde{d}_t + \frac{M_{x.t}}{M_t} \cdot \widetilde{d}_{x,t}$$

Profits from the manufacturing sector are transfered to households since they own the firms.

Since, M_t is the total number of domestic competitive firms and $M_{x,t}$ is the number of monopolistic firms who actually export at time t, then, $\frac{M_{x.t}}{M_t}$ represents the proportion of home firms that export at time t.

$$M_{x,t} = (1 - G(z_{x,t})) \cdot M_t$$

The small open economy assumption also imposes that the total number of foreign firm, M_t^*, is unaffected by the small economy. Therefore, M_t^* is exogenous. The fraction of foreign firms that export to the small open economy is endogenous and depends on the foreign productivity threshold $z_{x,t}^*$.

$$M_{x,t}^* = (1 - G(z_{x,t}^*)) \cdot M_t^*$$

1.2.3 Equilibrium conditions and dynamics

In equilibrium, the free entry condition must be satisfied. This condition states that the value of the representative (average) firm $\tilde{v}_t$, at time t, equalizes the value of the sunk cost f_E paid to obtain a productivity draw. This is shown in (1.12). If it were not the case, that is the free entry condition does not hold, then there would be additional firms willing to produce $\left(\text{if } \tilde{v}_t > \frac{w_t \cdot f_E}{Z_t}\right)$ or firms willing to exit $\left(\text{if } \tilde{v}_t < \frac{w_t \cdot f_E}{Z_t}\right)$. Thus, in equilibrium, it must be the case that:

$$\tilde{v}_t = \frac{w_t \cdot f_E}{Z_t} \tag{1.12}$$

The value of a representative (average) firm at time t corresponds to the expected present discounted value of future average dividends.

$$\tilde{v}_t = E_t \left(\left\{ \tilde{D}_s \right\}_{s=t+1}^{\infty} \right)$$

In this model, firms produce in every period, until they are hit with a death shock, which occurs with probability $\delta \in (0,1)$ at the very end of each period. That is, among firms that were producing in the market at $t-1$, and new firms that are ready to produce at t, $M_{E,t-1}$, only a proportion $1-\delta$ of those firms will actually produce at time t. This exit inducing shock is independent of the firm's

productivity level. Thus, M_t is the sum of the firms that where already incumbent in the previous period and survived the death shock, and the firms that were "new firms" (entrants) in the previous period (at the very end of previous period) and also survived the death shock. Therefore M_t represents the total number of firms producing during period t.

$$M_t = (1 - \delta)(M_{t-1} + M_{E,t-1})$$

The total number of firms willing to produce at time $t+1$ is shown in (1.13). Where M_t is the number of firms already operating in the market at time t, and $M_{E,t}$ is the number of new entrants.

$$M_{H,t} = M_t + M_{E,t} \tag{1.13}$$

Another important feature of the model is that profits obtained in the market for the homogeneous good are transferred to the households. That is, we have that $T_t = IC_t$, where T_t corresponds to the amount transferred to households.

Another equilibrium condition that must be satisfied is the labor market clearing condition. In this condition, the labor employed in the production of the homogeneous good and the final goods as well as the labor spent on the sunk cost f_E and the fixed cost of production f_x must equalize the labor supply L. This equation is shown in (1.14).

$$M_t \cdot \tilde{l}_t + M_{x,t} \cdot \tilde{l}_{x,t} + \frac{1}{Z_t}(M_{x,t} \cdot f_x + M_{E,t} \cdot f_E) + l_{o,t} = L \tag{1.14}$$

where $M_t \cdot \tilde{l}_t$ is the amount of labor employed to satisfy the domestic demand for domestic goods and $M_{x,t} \cdot \tilde{l}_{x,t}$ is the amount of labor employed to satisfy the foreign demand for domestic goods. $\frac{M_{x,t} \cdot f_x}{Z_t}$ corresponds to the amount of labor used to cover the fixed costs incurred by exporting domestic firms, while $\frac{M_{E,t} \cdot f_E}{Z_t}$ corresponds to the labor used to cover the sunk costs incurred by entering firms. $l_{o,t}$ is the amount of labor needed to produce the homogeneous good.

1.3 Aggregate Dynamic Problem

The dynamic problem solved by a representative agent corresponds to the maximization of an intertemporal utility function $U\left(\{C_s\}_{s=t}^{\infty}\right)$. In this section, we will consider an economy which is under financial autarky.

$$\underset{C_t, B_{t+1}, x_{t+1}}{Max} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{(C_s)^{1-\gamma}}{1-\gamma}\right] \tag{1.15}$$

A representative household holds two type of assets: shares of stock of domestic firms and domestic risk-free bonds. x_t is the share of stock of domestic firms held by the representative household entering period t, and B_t is the bond holding in period t.

The shares of stocks pay dividends every period that correspond to the average total profit of domestic firms that produce in that period. During period t, the representative household buys x_{t+1} shares of stocks of $M_{H,t}$ domestic firms. It is worth mentioning that only $M_{t+1} = (1-\delta)M_{H,t}$ firms will produce and pay dividends at time $t+1$. The household buys stocks of $M_{H,t}$, because the household does not know which firms will be hit by the exogenous exit shock δ at the very end of period t.

The representative household enters period t with bond holding B_t and shares of stock holding x_t. Thus, the income that the representative household receives, at period t, comes from: labor income, bond holdings, dividends from shares of stock plus the value of selling its initial share position, and transfers from the homogeneous good sector. The period budget constraint, in units of consumption, is the following:

$$B_{t+1} + \widetilde{v}_t M_{H,t} x_{t+1} + C_t = (1+r_t) B_t + \left(\widetilde{D}_t + \widetilde{v}_t\right) M_t x_t + w_t L + T_t \tag{1.16}$$

Equation (1.16) considers that a representative household invests in period

t on bonds, B_{t+1}, and buys x_{t+1} shares in a mutual fund of $M_{H,t}$ home firms. Also, the household spends on period t consumption, C_t. Thus the expenditure of the representative household at time t (left hand side of equation (1.16)) is composed by bonds, shares of stock, and consumption. The right hand side of equation (1.16) corresponds to the household income. The household income is given by the return on bond holding $(1 + r_t) B_t$, dividends $\widetilde{D}_t$ from the share of stocks, the value of the shares of stock, the salary from labor, $w_t L$, and the transfers from de government, T_t.

The household problem is then given by (1.15) subject to (1.16). That is, the household allocates its resources between consumption, purchase of bonds and shares of stock to be carried into next period.

The first order conditions, of the dynamic problem described above, are as follows:

$$(C_t)^{-\gamma} = \beta \left(1 + r_{t+1}\right) E_t \left[(C_{t+1})^{-\gamma}\right] \tag{1.17}$$

$$\widetilde{v}_t = \beta \left(1 - \delta\right) E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\widetilde{D}_{t+1} + \widetilde{v}_{t+1}\right)\right]$$

Equations in (1.17) describe the traditional Euler equations for bonds and stocks.

Aggregating the budget constraint across symmetric households and imposing the equilibrium condition under financial autarky[2], we obtain the aggregate accounting equation in the domestic economy as in (1.18).

$$C_t = w_t L + T_t + M_t \cdot \widetilde{D}_t - M_{E,t} \cdot \widetilde{v}_t \tag{1.18}$$

The aggregate accounting equation states that consumption, C_t, in each period must be equal to labor income $w_t L$ plus the profits from the domestic firms

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[2]In equilibrium under financial autarky it must be the case that $B_{t+1} = B_t = 0$ and $x_t = x_{t+1} = 1$.

$M_t \tilde{D}_t$ and profits from the commodity market, T_t, minus the investment in new firms $M_{E,t} \tilde{v}_t$.

1.3.1 Competitive Equilibrium

Table (1.1) summarizes the equilibrium conditions of the model presented in previous sections. The equations in the Table (1.1) establish a system of 12 endogenous variables and 12 equations. That is, the endogenous variables of the model are w_t, $\widetilde{D}_t$, $M_{E,t}$, $\widetilde{\varphi}_{x,t}$, $\widetilde{\varphi}^*_{x,t}$, M_t, $M_{x,t}$, $M^*_{x,t}$, r_t, $\widetilde{v}_t$, C_t, T_t.

Table 1.1: System of Equations

Price index	$1 = \left[M_t \cdot (\widetilde{p}_t)^{1-\sigma} + M^*_{x,t} \cdot (\widetilde{p}^*_{x,t})^{1-\sigma} \right]$
Profits	$\widetilde{D}_t = \widetilde{d}_t + \frac{M_{x,t}}{M_t} \cdot \widetilde{d}_{x,t}$
Free entry	$\widetilde{v}_t = \frac{w_t \cdot f_E}{Z_t}$
Average exports profit	$\widetilde{d}_{x,t} = \frac{w_t \cdot f_x}{Z_t} \left[v^{\sigma-1} - 1 \right]$
Share of exporting firms	$\frac{M_{x,t}}{M_t} = (v \cdot z_{\min})^{\kappa} \, \widetilde{z}_{x,t}^{-\kappa}$ $\frac{M^*_{x,t}}{M^*_t} = (v \cdot z_{\min})^{\kappa} \left(\widetilde{z}^*_{x,t}\right)^{-\kappa}$
Number of firms	$M_t = (1 - \delta)(M_{t-1} + M_{E,t-1})$
Euler equations (Bonds)	$(C_t)^{-\gamma} = \beta(1 + r_{t+1}) E_t \left[(C_{t+1})^{-\gamma} \right]$
Euler equations (Shares)	$\widetilde{v}_t = \beta(1 - \delta) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\widetilde{D}_{t+1} + \widetilde{v}_{t+1} \right) \right]$
Aggregate accounting	$C_t = w_t L + T_t + M_t \cdot \widetilde{D}_t - M_{E,t} \cdot \widetilde{v}_t$
Labor market clearing	$M_t \cdot \widetilde{l}_t + M_{x,t} \cdot \widetilde{l}_{x,t} + \frac{1}{Z_t}(M_{x,t} \cdot f_x + M_{E,t} \cdot f_E) + l_{o,t} = L$
Transfers	$T_t = I C_t$

The system of equations developed in this section characterizes the stochastic dynamics of a small open economy in a general equilibrium context. To study the dynamics implied from our model, we need to know the parameters which are important elements to determine the dynamic of the endogenous variables. These parameters are the shape parameter of the Pareto distribution κ, and the elasticity of substitution across final goods σ. In order to know these parameters, in the next section we estimate them for a small open economy. We use Chilean data, since

Chile satisfies the small open economy assumption as well as the commodity trade assumption.

1.4   Estimation of Parameters

We employ two sources of data. One is data at the plant level for Chilean firms. This data is collected by the National Institute of Statistics (INE). This data is an annual survey and it is intended to cover manufacturing firms which employ at least ten workers in the case of single-plant production firms, and all firms (with no restriction) for multi-plants production firms. The other source of data corresponds to the exports data collected by the General Direction of International Policy at the Ministry of Foreign Affairs of Chile. This data covers all exports made by Chilean firms for products at the HS-8 digits code level.

1.4.1   Plant Level Data for Chilean Manufacturers

The National Annual Industry Survey (ENIA) elaborated by INE contains data at the plant level for manufacturers firms in Chile. Each plant is individualized with an identification number which is consistent over the years. We have this data for the period between the years 1996 and 2006.

In this database, for each plant we have data on production, sales, energy consumption, labor employed, capital, etc. The data is also classified at ISIC Rev. 3. The number of plants by ISIC Rev.3 (Divisions)[3] is shown in Table (1.2).

1.4.2   Export Data

Export data are provided by the General Direction of International Policy at the Ministry of Foreign Affairs of Chile. This data contains information on trade

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[3]ISIC Rev.3 also considers Groups and Classes. However, in Table (1.2) it is shown by divisions for the tabulation category D (Manufacturing).

Table 1.2: Number of Plants by ISIC Rev. 3. Period 1996-2006

Manufacturer of:	ISIC Rev. 3	1996	1998	2000	2002	2004	2006
Food products and beverages	15	1,701	1,656	1,515	1,621	1,685	1,606
Textiles	17	344	294	269	276	268	232
Wearing apparel	18	418	322	293	300	283	245
Tanning and dressing of leather	19	241	199	162	158	145	115
Products of wood and cork	20	417	387	343	369	360	342
Paper and paper products	21	135	132	144	146	169	158
Publishing	22	236	217	222	253	281	265
Coke, refined petroleum products	23	3	3	0	0	0	0
Chemicals and chemical products	24	321	309	298	311	327	315
Rubber and plastics products	25	335	292	300	315	359	344
Other non-metallic products	26	262	287	272	277	268	286
Basic metals	27	115	117	123	127	150	159
Fabricated metal products	28	425	387	392	405	416	403
Machinery and equipment	29	303	297	290	330	335	315
Office products	30	0	0	0	0	3	3
Electrical machinery	31	99	94	104	90	91	73
Communication equipment	32	10	9	12	12	12	4
Medical instruments	33	26	25	30	31	33	34
Motor vehicles	34	91	84	76	75	34	88
Other transport equipment	35	64	62	52	56	55	51
Furniture, manufacturing	36	308	267	264	264	276	235
Total Number of Firms	-	5,854	5,440	5,162	5,416	5,600	5,273

between Chile and different countries of the world at the firm level. Exports by products are reported with a level of aggregation given by the harmonized system classification at the 8-digit level.

This data provides information of products exported, FOB value of exports, and country of destination for each exporting firm and product.

Table (1.3) shows the number of firms, number of products and percentage of total exports by country of destination for selected countries. Those trading partners import 86% of the total export made from Chile.

Table 1.3: Number of Firms, products and percentage of total exports by country destination (Selected Countries, year 2006)

Country	Number of Firms	Number of Products	% Total FOB
Germany	671	714	3%
Argentina	1,517	2,279	1%
Belgium	332	300	1%
Brazil	967	1,193	5%
Canada	641	607	2%
China	477	307	9%
South Korea	397	286	6%
Spain	797	804	2%
USA	2,084	1,993	16%
France	521	584	4%
Netherland	615	355	7%
India	108	122	3%
Italy	510	482	5%
Japan	567	463	11%
Mexico	1,023	1,285	4%
Peru	1,641	2,782	2%
UK	615	588	1%
Taiwan	355	177	3%

1.4.3 Productivity Estimation

In this section we use the information contained in ENIA to estimate the productivity of Chilean manufacturing firms. There are a number of econometric

problems that one encounters when trying to estimate unobserved productivity as the residual of the production function using the observed firm level variables.

Estimating firm level production functions is a non-trivial exercise due to simultaneity bias caused by the relationship between unobserved productivity shocks and inputs used in production. Different methods have been developed to address the simultaneity bias in production function estimation. Most of them rely on finding proxy variables for productivity shocks, which are used to invert out productivity from the regression residual in a two-step estimation. The two most popular methods in this vein were developed by Olley and Pakes (1996) (OP) and Levisohn and Petrin (2003) (LP). Wooldridge (2009) proposes a one-step estimation implemented in a generalized method of moments framework.

Let us assume a production function of the standard Cobb-Douglas form. In particular, let us consider a two-factor production function.

A simple standard estimation equation of the production function (in logs) looks as follows:

$$y_{it} = \beta_o l_{it} + \beta_k k_{it} + \zeta_{it} \tag{1.19}$$

where y_{it} is the log of some real measure of firm i's output (i.e. revenue or value added), k_{it} is the log of the level of capital, l_{it} is the log of labor, and ζ_{it} is an error term.

In equation (1.19), the error term is given by $\zeta_{it} = v_{it} + e_{it}$, where v_{it} is the productivity which is observed by the firm i, but it is unknown for the econometrician. So, the firm is able to decide the amount of the variable input (labor) when it observes the productivity level. This means that the realization of the error term affects the choice of factor input. The unobserved productivity shock v_{it} is therefore correlated with factor inputs, so that estimating (1.19) with ordinary least squares without controlling for v_{it} yields biased parameter estimates.

In order to estimate the production function at the plant level we will use three different methodologies.

OP show how, under certain assumptions, investment can be used as a proxy variable for unobserved time-varying productivity. Specifically, OP show how to invert an investment rule to express productivity as an unknown function of capital and investment, when investment is positive. OP present a two step estimation method where, in the first stage, semiparametric methods are used to estimate the coefficients on the variable inputs. In a second step, the parameters on capital inputs can be identified under assumptions on the dynamics of the productivity process.

LP propose a modification of the OP approach to address the problem of lumpy investment. LP suggest using intermediate inputs to proxy for unobserved productivity. Similarly to OP, LP contains assumption under which productivity can be written as a function of capital input and intermediate inputs (such as electricity). LP also propose a two step method to estimate the coefficients on labor and capital.

Ackerberg et al (2006) (ACF) have highlighted a potential problem with identification of the parameters in the LP first stage estimation problem. If labor is determined by the firm as a function of the unobserved productivity and state variables, then the coefficient on labor is unidentified. Wooldridge (2009) uses GMM estimation to solve the issue pointed out by ACF.

Table (1.4) shows the estimates of a production function of the Cobb-Douglas form using OP, LP and Wooldridge methods[4].

Table 1.4: Productivity Estimation

		OP (1996)	LP (2003)	Wooldridge (2009)	
				with exogenous k_{it}	with endogenous k_{it}
β_o	β_{BC}	0.535***	0.552***	0.567***	0.467***
	β_{WC}	0.143***	0.164***	0.184***	0.130***
β_k		0.137***	0.102***	0.150***	0.491***
N		31,877	29,252	21,081	17,120

[4]see more details in the Appendix A.

In Table (1.4), the estimates of the labor parameter β_o have been divided into the separate estimation of "blue collar" labor β_{BC}, and "white collar" labor B_{WC}. We do this to compare our results with the ones obtained by Levisohn and Petrin (2003) in which Chilean data (for a different period of time) is used. Our results are very similar to the findings of LP.

In Table (1.4), it seems we find decreasing returns to scale with OP and LP but increasing returns to scale using Wooldridge with endogenous capital[5]. However, the first three measures seem to yield similar results in Tables (1.6) and (1.7).

1.4.4 Trade Estimation

In this section, we estimate trade elasticities for Chilean firms who export to the rest of the world. We consider the log of the revenue equation for a firm operating in the final market, with productivity z, in an industry φ, in the home country, and exporting to country j. Thus, the equation to estimate has the following form[6]:

$$r_{z,\varphi}^{h,j} = \lambda_h + \lambda_j + \lambda_\varphi + \beta_\sigma \ln(z) + \epsilon_{z,\varphi}^j \tag{1.20}$$

where $r_{z,\varphi}^{h,j}$ is the revenue of a firm in the home economy, h, with productivity, z, in an industry, φ, that exports to country j. λ_h is the constant of the model (also home economy fixed effect), λ_j is the country of destination fixed effect, λ_φ is the industry fixed effect, $\epsilon_{z,\varphi}^j \tilde{} N\left(0, \sigma_\epsilon^2\right)$, and $\beta_\sigma = \sigma - 1$ is the coefficient of interest. Equation (1.20) specifies an equation that suffers from sample selection. This is because firms do not export to every country and we observe exports from a firm z to a country j when trade happens. In order to control for sample selection, we will first employ the traditional Heckman correction.

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[5]The estimation obtained using Wooldridge method with endogenous capital differs from the results in the first three columns in Table (1.4). This is due to the assumptions under GMM and the lag characterization of the endogenous variable.

[6]See more details in Appendix B

A firm z, in an industry φ, exports to country j if $\left(\frac{z}{z_{x,\varphi}^j}\right) > 1$, that is, firm z exports to country j if the productivity of the firm is above the industry productivity threshold. Then, we can define a latent variable $W_{z,\varphi}^j = \left(\frac{z}{z_{x,\varphi}^j}\right)$, where $\omega_{z,\varphi}^j = \log\left(W_{z,\varphi}^j\right)$. Thus, a firm z in an industry φ export to country j if $\omega_{z,\varphi}^j > 0$.

We follow Helpman, Melitz, and Rubinstein (2008) identification approach and we consider that the latent variable has the following structure:

$$\omega_{z,\varphi}^j = \zeta_h + \zeta_j + \zeta_\varphi + \beta_\rho \ln(z) + \beta_\omega \phi_{z,\varphi}^j - \eta_{z,\varphi}^j$$

where ζ_h is the home economy fixed effect, ζ_j is the country of destination fixed effect, ζ_φ is the industry fixed effect, and $\phi_{z,\varphi}^j$ is a measure that affects the probability of been an exporter.

Then, the probability of being an exporter can be written as follows:

$$\Pr\left(\omega_{z,\varphi}^j > 0\right) = \Pr\left(\zeta_h + \zeta_j + \zeta_\varphi + \beta_\rho \ln(z) + \beta_\omega \phi_{z,\varphi}^j > \eta_{z,\varphi}^j\right)$$

We assume that $\eta_{z,\varphi}^j \sim N\left(0, \sigma_\eta^2\right)$, and define the indicator variable $T_{z,j}$ to equal one when firm z export to country j and zero when it does not. Let $\rho_{z,\varphi}^j$ be the probability that a firm z in an industry φ exports to country j. Thus we can have the following Probit equation:

$$\rho_{z,\varphi}^j = \Pr\left(T_{z,j} = 1 \,|\text{observed variables }\right)$$

$$\rho_{z,\varphi}^j = \Phi\left(\zeta_h^* + \zeta_j^* + \zeta_\varphi^* + \beta_\rho^* \ln(z) + \beta_\omega^* \phi_{z,\varphi}^j\right) \tag{1.21}$$

Where $\Phi\left(.\right)$ is the cdf of the standard normal distribution, and every starred coefficient represents the original coefficient divided by σ_η.

The instrument we will employ in the Probit equation (1.21) considers the three way interaction between firm's productivity z, the capital intensity of the

Table 1.5: Probit Estimation

	LP	OP	W
$\hat{\beta}_\rho^*$	0.256***	0.267***	0.255***
	(0.009)	(0.008)	(0.008)
$\hat{\beta}_{\omega-}^*[z \cdot K_\varphi \cdot CSU_j]$	-0.221	-0.361	-0.589
	(0.781)	(0.411)	(0.669)
$\hat{\beta}_{\omega-}^*[z \cdot CSU_j]$	-0.386*	-0.215	-0.282
	(0.229)	(0.134)	(0.200)
$\hat{\beta}_{\omega-}^*[z \cdot K_\varphi]$	-0.981***	-0.418***	-0.682***
	(0.236)	(0.129)	(0.203)
$\hat{\beta}_{\omega-}^*[K_\varphi \cdot CSU_j]$	0.152	0.216**	0.002**
	(0.100)	(0.099)	(0.001)
N	151,524	151,524	151,524
R^2	0.3102	0.3153	0.3124

industry in which the firm z produces, K_φ, and the cost of business start-up procedures in country j, CSU_j, and the three pair wise interactions between z, K_φ, and CSU_j.[7]

$$\phi_{z,\varphi}^j = [z \cdot K_\varphi \cdot CSU_j, \; z \cdot CSU_j, \; z \cdot K_\varphi, \; K_\varphi \cdot CSU_j]$$

Then, we can estimate (1.20), using the following specification:

$$r_{z,\varphi}^{h,j} = \lambda_h + \lambda_j + \lambda_{\varphi,j} + \beta_\sigma \ln(z) + \beta_H \hat{\mu}_{z,\varphi}^j + e_{z,\varphi}^j \tag{1.22}$$

where $\hat{\mu}_{z,\varphi}^j = \frac{\phi(\hat{\omega}_{z,\varphi}^j)}{\Phi(\hat{\omega}_{z,\varphi}^j)}$ is the traditional inverse Mills ratio.

The results of the estimation for (1.21) are shown in Table (1.5)[8].

The results for the estimation of equation (1.22) are shown in Table (1.6).

The standard errors shown in the Table have been bootstrapped and clustered by

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[7]Although we implement the three way interaction and the three pair wise interactions what ultimately provides the identification is the pair wise interaction between z and K_φ.

[8]** Significat at 5%. *** Significant at 1%.

LP stands for Levinsohn and Petrin.

OP stands for Olley and Pakes.

W stands for Wooldridge.

Table 1.6: Estimation Results

	Benchmark			Sample Selection		
	LP	OP	W	LP	OP	We
$\hat{\beta}_\sigma$	0.702***	0.767***	0.745***	1.005***	1.206***	1.167***
	(0.038)	(0.042)	(0.033)	(0.163)	(0.157)	(0.133)
$\hat{\beta}_H$				1.749**	2.389***	2.410***
				(0.942)	(0.905)	(0.803)
$\hat{\sigma}$	1.702	1.767	1.745	2.005	2.206	2.167
N	7,519	7,519	7,519	7,519	7,519	7,519
R^2	0.2746	0.2963	0.2893	0.2754	0.2979	0.2907

industry. The first three column in Table (1.6) show a benchmark case in which equation (1.20) is estimated without considering the sample selection issue using our productivity estimates. The last three column show the result of estimates for equation (1.22) using Heckman correction.

We now relax the normality assumption, and hence the Mills ratio functional form for the selection correction. Thus, we drop the normality assumption and we work directly with the predicted probabilities $\hat{\rho}^j_{z,\varphi}$ as in Helpman, Melitz, and Rubinstein (2008). In order to approximate an arbitrary functional form of the predicted probabilities $\hat{\rho}^j_{z,\varphi}$, we employ a relatively large set of indicator variables. Thus, we partition the predicted probabilities from the first stage into a number of bins (we use 50 bins and 100 bins) in which each bin has the same number of observations. Then, we identify each bin with an indicator variable. Now, we are able to estimate equation (1.22) replacing $\hat{\mu}^j_{z,\varphi}$ with the set of indicators. The results are shown in Table (1.7).

Table 1.7: Non-Parametric Estimation

| | Indicator Variables | | | | | |
| | 50 bins | | | 100 bins | | |
	LP	OP	W	LP	OP	W
$\hat{\beta}_\sigma$	0.890***	1.123***	1.089***	1.048***	1.398***	1.298***
	(0.106)	(0.107)	(0.109)	(0.127)	(0.128)	(0.130)
$\hat{\sigma}$	1.890	2.123	2.089	2.048	2.398	2.298
N	7,519	7,519	7,519	7,519	7,519	7,519
R^2	0.2815	0.3044	0.2958	0.2874	0.3104	0.3012

1.5 Steady State

On the steady state, we consider that any variable follows $x_{t+1} = x_t = x$. In addition we consider that $\tau = \tau^*$, $f_x = f_x^*$, $Z = 1$, $p_o = 1$, and $z_{\min} = 1$.

From the Euler equation for bonds we immediately obtain the steady state value of the return for bonds, $r = \frac{1}{\beta} - 1$.

From the definition of $\tilde{p}$ and $\widetilde{p}_x$, we can obtain:

$$\tilde{p} = \tilde{p}(w), \quad \widetilde{p}_x = \widetilde{p}_x(w, \tilde{z}_x) \tag{1.23}$$

Using the definition of $\widetilde{d}_x$ at the steady state, and the "Average exports profit" equation from Table (1.1), at the steady state, we can obtain $\tilde{p}_x^{1-\sigma} = \frac{\sigma f_x \nu^{\sigma-1}}{A} w$. Combining this expression and equation (1.23) we get $w = w(\tilde{z}_x)$.

Thus, we can write w, $\tilde{p}$, $\tilde{p}_x$, $\widetilde{d}_x$, and $\tilde{v}$ (from Free entry equation) as function of $\tilde{z}_x$.

Using the Euler equation for shares combined with the free entry equation we can write $\tilde{D}$ as $\tilde{D} = \tilde{D}(\tilde{z}_x)$. Employing $\tilde{d} = \frac{\tilde{p}^{1-\sigma}}{\sigma} C$ along with the Profits equation and the Share of exporting firms equation for the domestic economy (from Table (1.1)), we are able to solve for C as $C(\tilde{z}_x)$.

In steady state, transfers to the representative household can be written as $T = T(\tilde{z}_x)$. Using the aggregate accounting equation we can solve out for M as $M(\tilde{z}_x)$. Moreover, from the "Share of exporting firms" equation (Table (1.1)) it is

also possible to write M_x as $M_x(\tilde{z}_x)$.

The labor amount used to produce the homogeneous good as well as the final goods can be written as function of M, M_x, d, and d_x. Thus, labor employed in the domestic economy are of the form $\tilde{l} = \tilde{l}(\tilde{z}_x)$, $\tilde{l}_x = \tilde{l}_x(\tilde{z}_x)$, and $l_o = l_o(\tilde{z}_x)$.

Also, using the labor market clearing equation from Table (1.1), we can form a nonlinear equation in $\tilde{z}_x$ of the form:

$$f(\tilde{z}_x) = \left[M(\tilde{z}_x) \cdot \tilde{l}(\tilde{z}_x) + M_x(\tilde{z}_x) \cdot \tilde{l}_x(\tilde{z}_x) + (M_x(\tilde{z}_x) \cdot f_x + M_E(\tilde{z}_x) \cdot f_E) + l_o(\tilde{z}_x) - L \right]$$

$$(1.24)$$

The solution of the non-linear equation (1.24) pins down the steady state values for $p, p_x, p, w, d_x, C, \tilde{d}, M, M_x, \tilde{D}, T, \tilde{v}$.

$\tilde{p}_x^*$ can be written as $\tilde{p}_x^* = \tilde{p}_x^*(\tilde{z}_x^*)$. Also, employing the share of exporting firms for the foreign economy allows us to write $M_x^* = M_x^*(\tilde{z}_x^*)$. Substituting these expressions into the price index equation we have $M_x^*(\tilde{z}_x^*) \cdot \tilde{p}_x^*(\tilde{z}_x^*)^{1-\sigma} = 1 - M(\tilde{z}_x) \cdot \tilde{p}(\tilde{z}_x)^{1-\sigma}$ from which we solve out for $\tilde{z}_x^*$. Knowing the steady state value of $\tilde{z}_x^*$, we can now pin down the steady state values for p_x^* and M_x^*.

Figure (1.3) we plot equation (1.24), in the form $f(\tilde{z}_x)^2$, for the elasticity of substitution σ^* estimated in previous section and the estimation of the shape parameter κ^*. We also consider a sensitivity analysis for values $\sigma_1, \sigma_2, \kappa_1$ and κ_2 such that $\sigma_1 < \sigma^* < \sigma_2$, and $\kappa_1 < \kappa^* < \kappa_2$.[9]

Figure (1.3) shows uniqueness for the productivity threshold base on the parameters estimated in previous section. In Figure (1.3, a, b, and c), the productivity threshold (given by $f(\tilde{z}_x)^2 = 0$) is above $z_{\min}$ which means that a proper fraction of the monopolistic firms will export.

In our case, Figure (1.3) shows that the smaller is the value of the elasticity of substitution, the greater the productivity threshold for domestic exporting firms $\tilde{z}_x$. It also shows that, given a level for the elasticity of substitution, the

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[9]It is also necessary to consider the restriction $\kappa > \sigma - 1$.
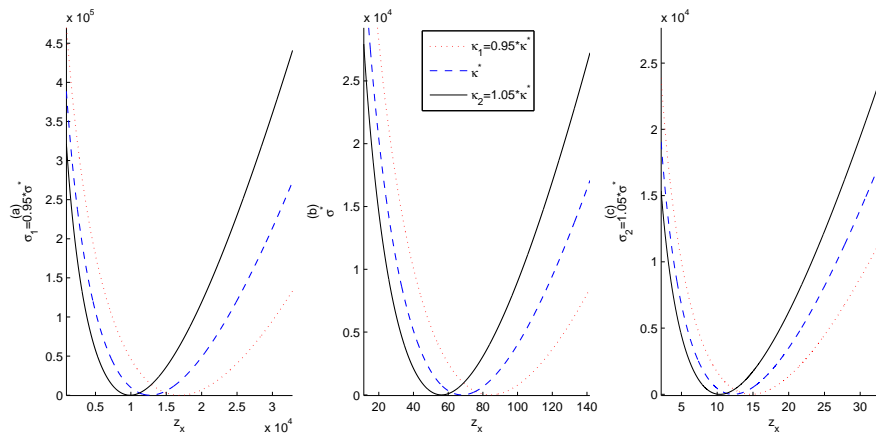
Figure 1.3: $f(\tilde{z}_x)^2$ for $k_1 < k^* < k_2$ and $\sigma_1 < \sigma^* < \sigma_2$.

productivity threshold for domestic exporting firms $\tilde{z}_x$ is negatively related to the value of the shape parameter κ.

1.6   International Trade and Macroeconomic Dynamics

1.6.1   Impulse responses

We now analyze the full response path of key variables in response to transitory shocks to aggregate productivity, Z_t, and the commodity price, $p_{o,t}$. Previously, we have estimated the parameters of the model for the Chilean economy. Using these parameters, we compute the steady-state levels of endogenous variables and numerically solve for the dynamic responses to exogenous shocks.

In this section, we now study the responses of the endogenous variables in our model to a transitory one percent increase in domestic aggregate productivity and in the commodity price. The responses are shown on the vertical axis as percent deviations from the steady-state. The number of quarters after the shock are shown on the horizontal axis.

Figure (1.4) and (1.5) show the response of the endogenous variables to a transitory productivity shock. The dynamic of the transitory productivity shock considers a process with an autocorrelation coefficient of 0.9 (following the productivity shock parametrization in Ghironi and Melitz, 2005)[10]. The temporary increase in productivity generates a temporary increase in both the average profits of serving the domestic economy, $\tilde{d}_t$, and the average profits of serving the foreign market, $\tilde{d}_{x,t}$. Thus, the profits of the representative firm, $\tilde{D}_t$, increase temporarily as well. This implies that the domestic economy becomes more attractive and the number of entrants is higher, which translates into a higher number of producing firms. The increment in producing firms operating in the market put pressure on the wage level. So, the productivity shock brings along a higher wage. The rise in wages increases the input cost, because the change in wages offsets the effect of the shock. A higher production cost leads to a higher average productivity for exporting firms. Thus, the number of domestic exporters, $M_{x,t}$, decreases even though manufacturing exports increase.

The shock in productivity has a positive effect on the price of non-traded final goods domestically produced. This is because the increase in the wage level dominates the effect of the increase in the productivity.

The effect of the productivity shock is negatively related to the price of traded goods that are domestically produced. The response of $\tilde{p}_x$, which is different from the response of $\tilde{p}_t$, is due to the combined effect of the shock and the increase in $\tilde{z}_x$. This combined effect dominates the wage increase.

From the price index equation, the average productivity of exporting firms in the foreign economy, $\tilde{z}_x^*$, is positively related to the increase of the term $M_t \cdot \tilde{p}_t^{1-\sigma}$. The knowledge of $\tilde{z}_x^*$ pins down the path for $M_{x,t}^*$ and $\tilde{p}_{x,t}^*$ which are both decreasing due to the small economy assumption and the increase in $\tilde{z}_x^*$.

The free entry condition shows the path of the value of a representative average firm, which increases as a response to the productivity shock, this is because

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[10]Although, we do not estimate the autocorrelation parameter, the value chosen for this coefficient is quite common in the macroeconomic literature.

the change in wage rate dominates the effect of the productivity shock.

The increase in wages increases the marginal cost of production in the homogeneous good sector. Thus, the supply of homogeneous good is contracted which translates into a decrease in the production of the homogeneous good. Therefore, profits and transfers from the homogeneous sector are reduced

The combined effect of the increase in wages (w_t), the increase of domestic monopolistic competitive firms (M_t), the increase of profits of the representative firm $(\tilde{D}_t)$ lead the temporary increase in consumption as a result of the shock.

In summary, a temporary productivity shock increases the average productivity of exporting firms, profits of the representative firm, consumption, and welfare in the short run. In the long run these effects disappear as the shock vanishes. However, as in Ghironi and Melitz (2005), the responses of the endogenous variables highlight the persistence of the shock on the endogenous variables.
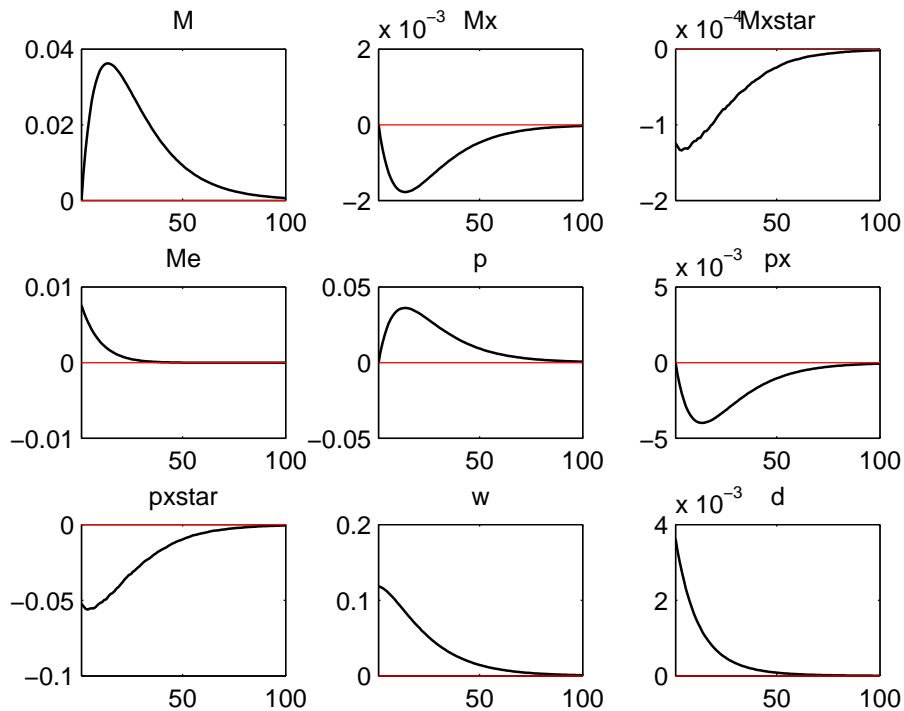
Figure 1.4: Response to a transitory productivity shock
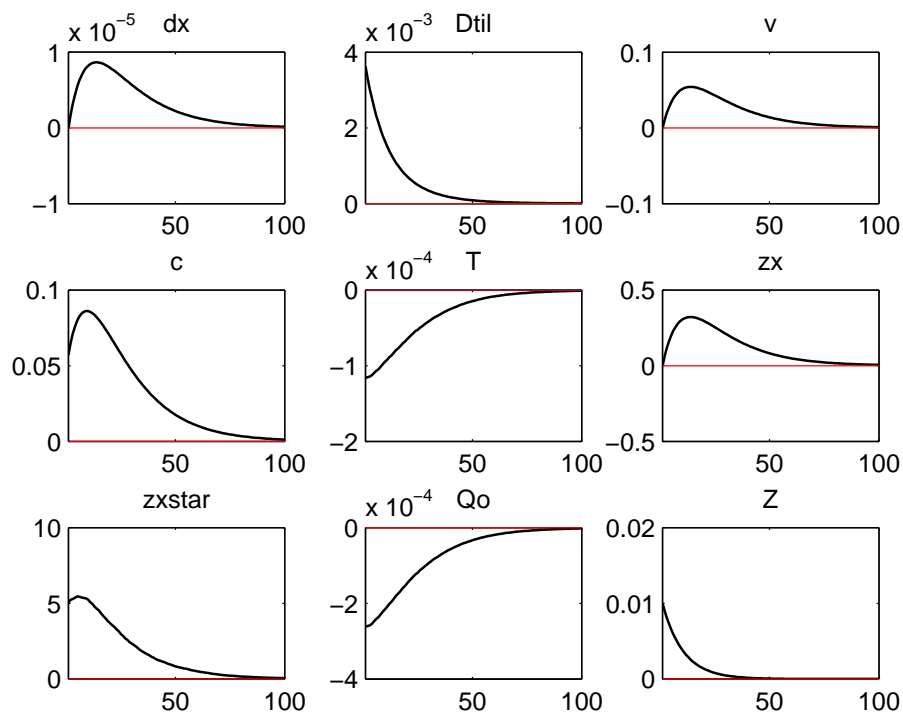
Figure 1.5: Response to a transitory productivity shock, continuation

Figure (1.6) and (1.7) show the response of the endogenous variables to a transitory one percent increase in the price of the homogeneous good. The commodity price process considers an autocorrelation coefficient of 0.8 which matches the real copper price autocorrelation for the last 50 years.

The increase in $p_{o,t}$ generates an increase in the quantity of the homogeneous good produced as well as in the profits delivered by the commodity sector. The expansion of the commodity sector drives wages up shifting the labor force from the manufacturing sector to the commodity sector. The increase in labor income as well as the boom in the profits generated in the commodity sector raise consumption. The increase in consumption is such that the number of firms, M_t, and the entrants $M_{E,t}$ increase.

The increase in wages raises the variable cost of production as well as the fixed cost. Thus, the average productivity of domestic exporters increases. This

rise in the productivity threshold generates exit in the pool of domestic exporters such that $M_{x,t}$ decreases.

The rise in wages increases the average profits of exporters (from the average exports profit equation in Table (1.1)), but decreases the average profits of domestic producers due to the increase in the average price. Therefore, by considering the profit equation in Table (1.1)), we have that the profits of the representative firm drops. This effect embodies the traditional effect of the Dutch disease in which a boom in the commodity sector negatively affects the manufacturing sector.

Welfare increses due to the commodity price shock and as is traditional in a Melitz type model such change in welfare is fully characterized by the extensive margin adjustment.

In summary, a transitory positive shock to $p_{o,t}$ temporarily increases the average productivity of domestic exporting firms, wages, consumption, as well as welfare. However, our findings illustrate the effect of the so called Dutch disease, that is the shock negatively affects the manufacturing sector.
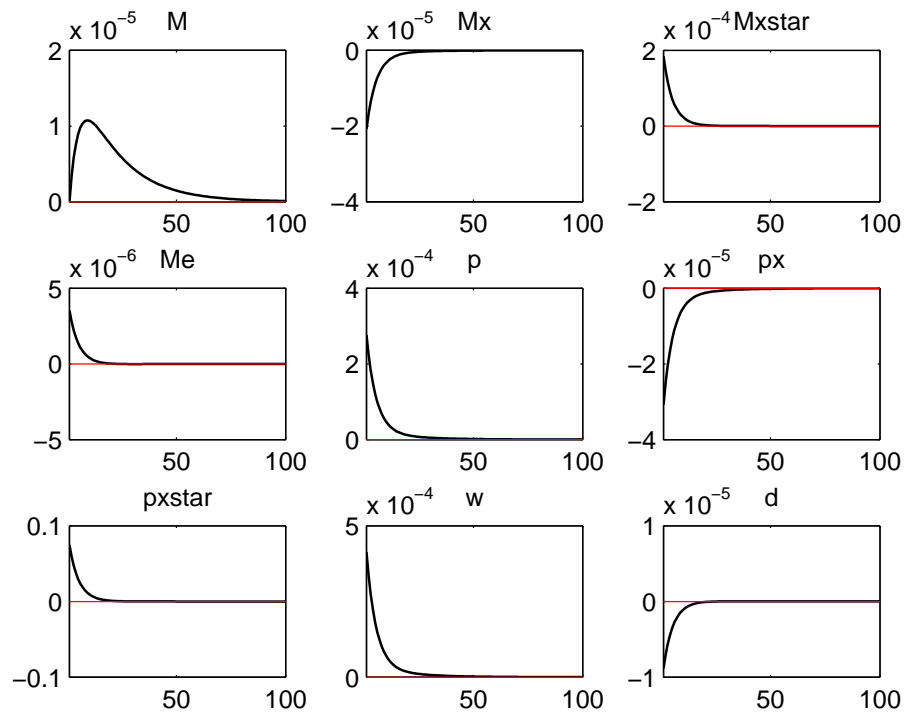
Figure 1.6: Response to a transitory commodity price shock
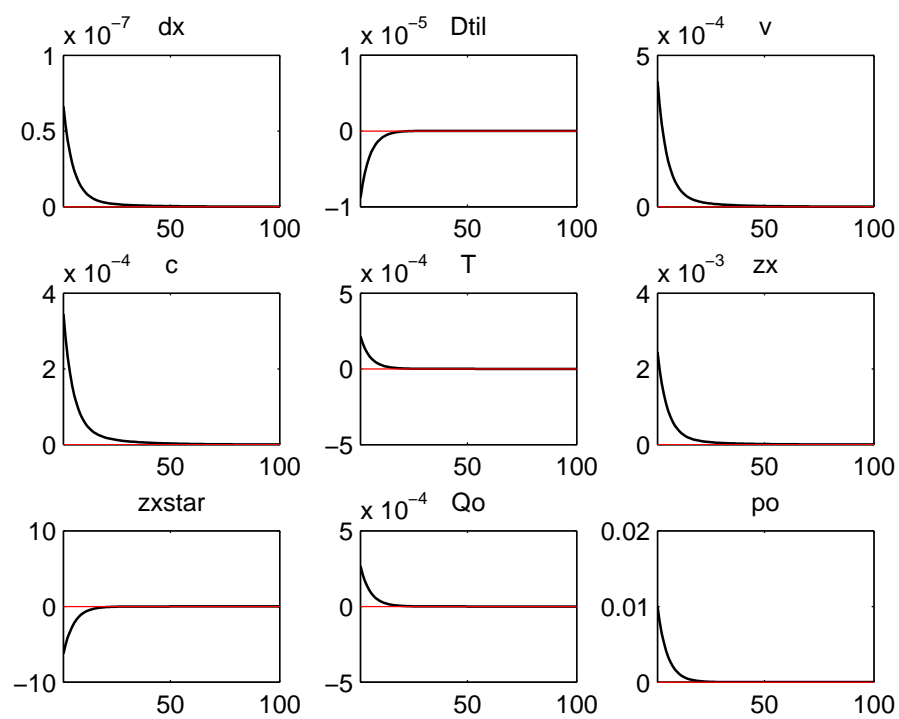
Figure 1.7: Response to a transitory commodity price shock, cotinuation

1.7 Government Policy: Spending and volatility

Economies that trade commodities have a large exposure to commodity price shocks. The dependence on commodity exports could bring along considerable aggregate volatility in the economy. That is why governments have developed ways of insuring themselves against commodity price shocks. Governments may accumulate a stock of assets in commodity stabilization funds. This asset accumulation is intended as precautionary savings against uncertainty in future commodity prices. However, there are potential issues with this strategy. For instance, those funds may be misused because of weak governance losing the initial insurance purpose.

A widely used self-insurance mechanism is the accumulation of foreign assets by the country to act as a commodity stabilization fund. For instance, a fund like this was established in Chile in 1985. During periods of high commodity prices and high export earnings, the country would accumulate foreign assets which it would draw down in periods of low commodity prices.

As mentioned before Dutch disease is also associated with corruption. Acemoglu et al (2004) argue that resource wealth makes it easier for incumbent politicians to buy off political challengers. Resource wealth raises the value of being in power and induces politicians to expand public sectors, bribe voters, create unproductive jobs, inefficient subsidies, etc. (Robinson et al, 2006).

Now, we would like to study the potential interactions between a government spending associated to corrupted behavior and the resource bonanza.

Let us consider the government budget as follows:

$$IC_t + R^D \cdot D_{t-1} = T_t + D_t$$

where IC_t are the profits from the commodity sector at time t. D_t is an international asset that only the government has access to. T_t are the transfers from the government to the HH in every period. R^D is an international constant

interest rate (SOE).

Let us consider first the case in which the government fully smooths out the fluctuation from the commodity revenue in the international market such that it keeps transfers constant and equal to the long-run level of the commodity revenue, that is $T_t = \overline{IC}$. In this case, government savings are given by $D_t = (IC_t - \overline{IC}) + R^D \cdot D_{t-1}$, and correspond to the portion of total revenue that exceeds the transfers.

In the case that the government does not smooth out revenue fluctuations such that $T_t = IC_t$, then we have that $D_t = 0$.

More generally, let us consider the case in which the government sets transfers according to $T_t = \overline{IC} + \Delta_t(IC_t - \overline{IC})$. Where Δ_t characterizes the evolution of T_t. In particular, if $\Delta_t = 0$, we are back to the case in which revenue fluctuations are fully smoothed out and transfers are $T_t = \overline{IC}$. In the case that $\Delta_t = 1$, then we are back in the case in which $D_t = 0$ and $T_t = IC_t$.

The parameter Δ_t can be used as a reduced form to capture the effect of changes in government spending (transfers). In particular, Δ_t can capture a government that incurs in higher transfers to households, for instance due to an increase in commodity revenues, looking for political gains. An increase in Δ_t due to a commodity bonanza could be rationalized as increase in subsidies, bribing voters, asigning unproductive jobs, etc. In summary, the parameter Δ_t can be used to capture the effect of corruption on the volatility of the model.

We are going to consider two cases to isolate the effect of government spending in the economy. First, we consider the case in which $\Delta_t = 0$ for all t in which case transfers to households are constant, $T_t = \overline{IC}$. Figure (1.8) and (1.9) show the result for this exercise.

We will also consider a second case in which the parameter Δ_t positively correlates with the commodity shock $p_{o,t}$. That is, the government increases spending in response to the temporary commodity bonanza. This scenario intends to show the effect of a government policy in which the government becomes more wasteful as the price of the commodity rises. Figure (1.10) and (1.11) show the

result for this exercise.

The comparison of the exercises mentioned above shows that the increase in government spending is associated with higher volatility. Consumption, average productivity and wages increase volatility when Δ_t positively correlates with the commodity shock $p_{o,t}$.

Thus, the model predicts that spendings associated to corruption increase volatility in the economy. Corruption spending may act as an amplification mechanism for volatility. More broadly, the model would predict that the more wasteful or corrupted the government, the greater the volatility of that economy.
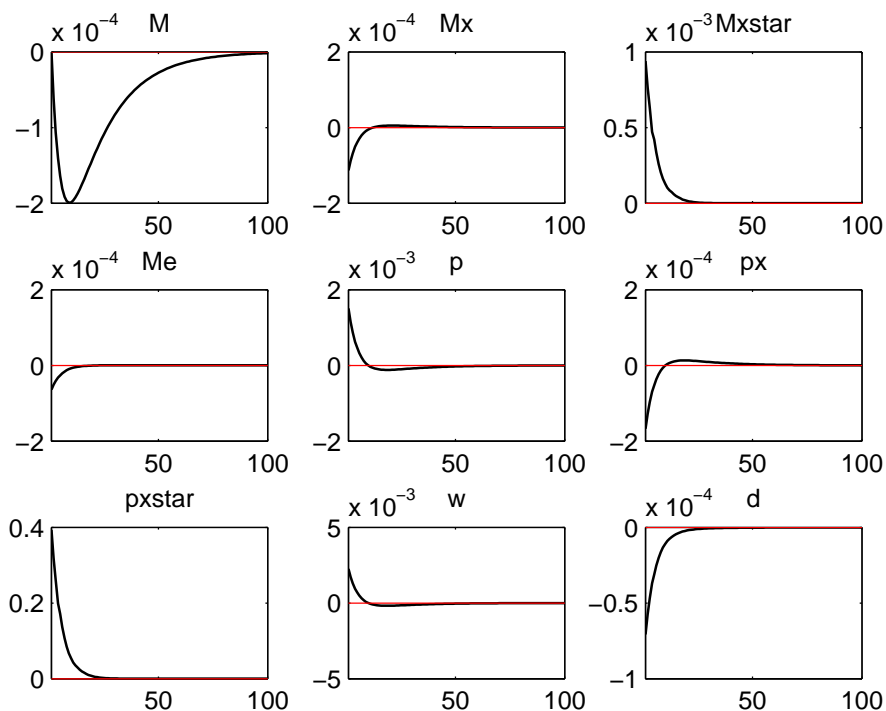
Figure 1.8: Responses to commodity priec shock and $\Delta_t = 0$

In Figure (1.12) we show correlations between Consumption volatility (a), Productivity volatility (b), and Wage volatility (c) versus an average of the Corruption Perception Index (CPI)[11] across countries. It is shown that the volatility of

[11]The CPI is an annual measure provided by the Transparency International. We use the
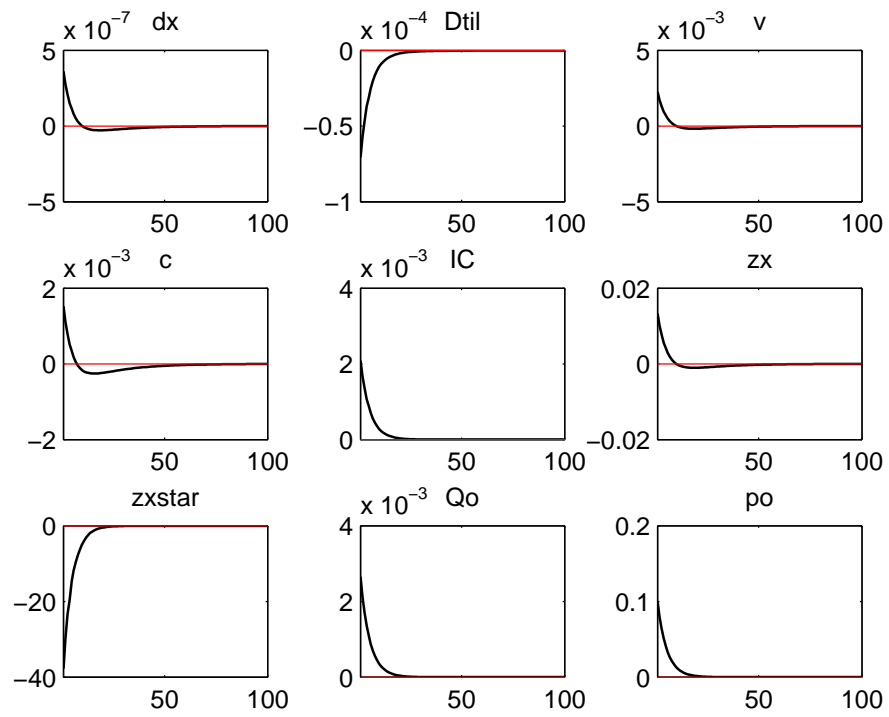
Figure 1.9: Responses to commodity priec shock and $\Delta_t = 0$, continuation
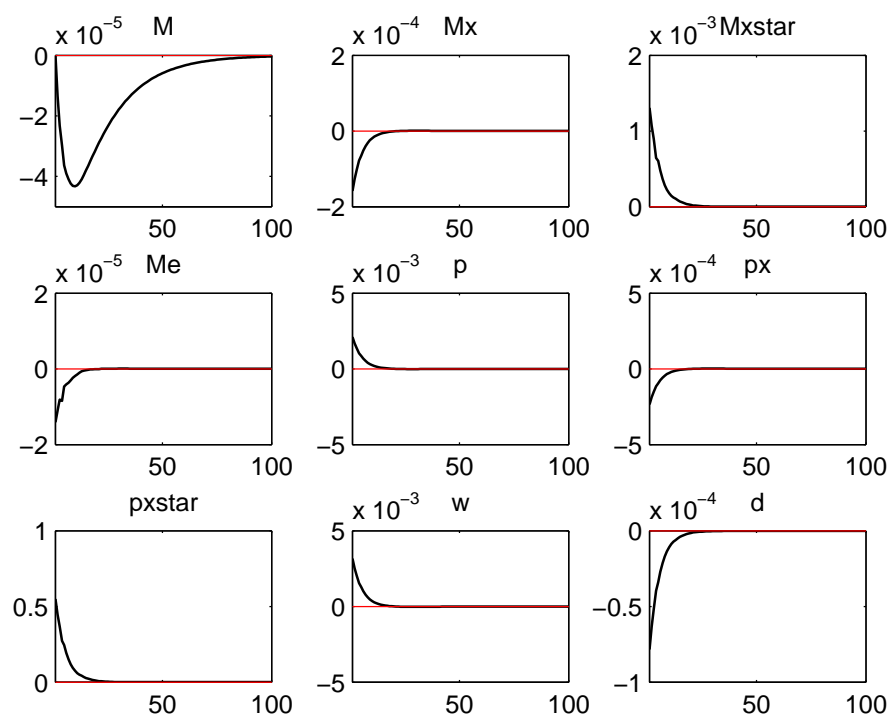
Figure 1.10: Responses to commodity priec shock, $p_{o,t}$ correlates with $\Delta_t = 0$
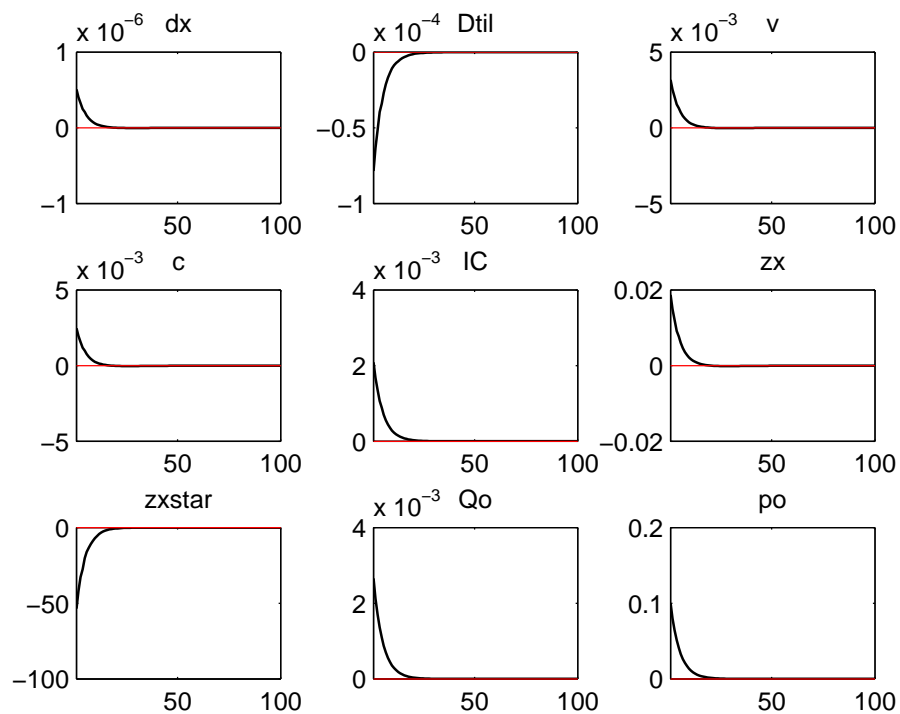
Figure 1.11: Responses to commodity priec shock, $p_{o,t}$ correlates with $\Delta_t = 0$, continuation
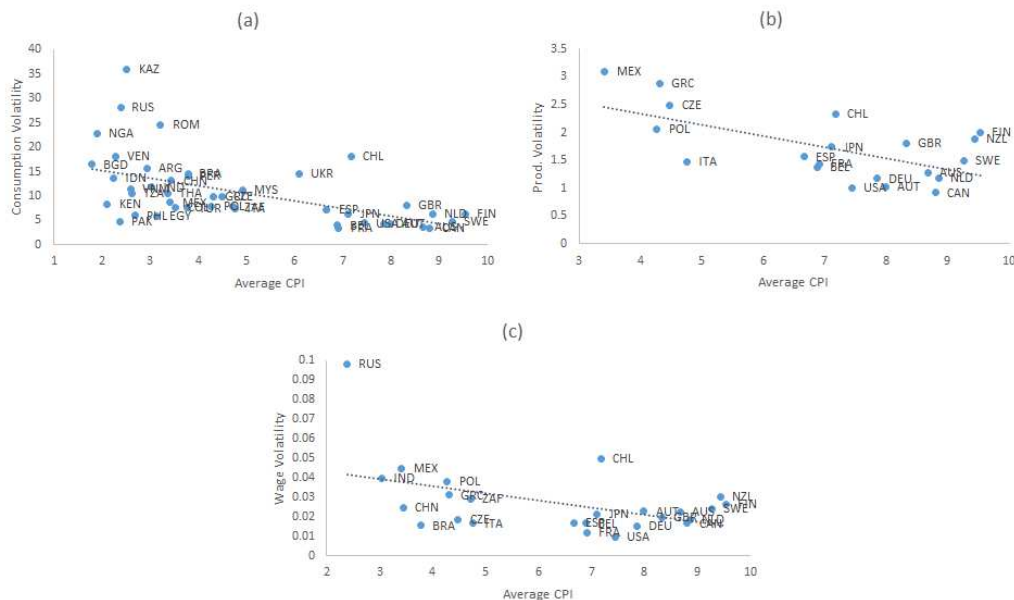
Figure 1.12: Volatility and Corruption Perception Index

consumption, productivity, and wage across countries is negatively correlated with the CPI which means that the volatility of these variables are positively correlated with corruption perceptions.

1.8 Conclusions

In this chapter, we have developed a dynamic model of a small open economy that produces and trades a commodity. We have studied the dynamic implications of productivity shocks and commodity price shocks. To do so, we have estimated the elasticity of substitution across final goods and the shape parameter of the Pareto distribution for the Chilean economy. We use Chilean data because it satisfies the small open economy assumption as well as the commodity production. Using these estimations we are able to compute the steady-state of the economy and study the dynamics implied by the model.

average between the years 1998 and 2011. The period 2012-2014 has been excluded because for that period the measure follows a different scale.

In this context, we have found that the commodity price shock generates the so called Dutch disease. That is, a positive shock in the commodity price negatively affects the manufacturing sector. Although, the commodity bonanza increase welfare.

We study the effect on volatility of spending associated to corruption during commodity boom. Our results indicate that corruption spendings act as an amplification mechanism for volatility. This prediction is in line with empirical evidence available for countries that trade commodities.

1.9 Appendix A

In general, let us write equation (1.19) as follows:

$$y_{it} = \alpha + l_{it}\beta + k_{it}\gamma + v_{it} + e_{it} \tag{1.25}$$

The theory underlying OP and LP is that there is an unknown function g such that:

$$v_{it} = g(k_{it}, m_{it})$$

where m_{it} is a proxy variable which is investment in the case considered by OP, and intermediate inputs in LP.

Under the assumption $E(e_{it}|l_{it}, k_{it}, m_{it}) = 0$, we have the following regression:

$$E(y_{it}|l_{it}, k_{it}, m_{it}) = \alpha + l_{it}\beta + k_{it}\gamma + g(k_{it}, m_{it}) \tag{1.26}$$

$$E(y_{it}|l_{it}, k_{it}, m_{it}) = l_{it}\beta + h(k_{it}, m_{it})$$

where $h(k_{it}, m_{it}) = \alpha + k_{it}\gamma + g(k_{it}, m_{it})$.

Since $g(.)$ is allowed to be any function, when it is linear in k_{it}, then γ is not identified from (1.26).

In both OP and LP, equation (1.26) is used to identify β in a first stage. For the OP case, the use of investment as a proxy variable generates problems with the identification since there is evidence that investment at the firm level is lumpy. In the LP case, the identification of β is not clear. As shown by ACF, if labor inputs are chosen at the same time as intermediate inputs, there is a fundamental identification problem in equation (1.26). That is, l_{it} is some function of (k_{it}, m_{it}) which means β is not identified.

Wooldrige (2009) proposes to estimate β and γ together. He assumes that:

$$E(e_{it}|l_{it}, k_{it}, m_{it}, l_{it-1}, k_{it-1}, m_{it-1}, ..., l_{i1}, k_{i1}, m_{i1}) = 0$$

and restricts the dynamics of productivity shocks as:

$$E(v_{it}|v_{it-1}, ..., v_{i1}) = E(v_{it}|v_{it-1}) = f(v_{it-1}) = f(g(k_{it-1}, m_{it-1}))$$

So, it is possible to express productivity innovations as:

$$a_{it} = v_{it} - f(v_{it-1})$$

where, $E(a_{it}|k_{it}, l_{it-1}, k_{it-1}, m_{it-1}, ..., l_{i1}, k_{i1}, m_{i1}) = 0$. This means that labor l_{it} and the proxy variable m_{it} can be correlated with productivity innovations a_{it}. However, k_{it} and all past values of l_{it}, k_{it}, m_{it} and the function of these are uncorrelated with a_{it}. Plugging into the production function yields:

$$y_{it} = \alpha + l_{it}\beta + k_{it}\gamma + f(g(k_{it-1}, m_{it-1})) + u_{it} \tag{1.27}$$

where $u_{it} = a_{it} + e_{it}$.

The problem is reduced now to estimate both equations, (1.25) and (1.27),

using GMM or to estimate equation (1.27) using IV.

1.10   Appendix B

Revenue for a domestic exporting firm is given by:

$$R_{x,t}(z) = p_{x,t} \cdot q_{x,t} = \left(\frac{w_t^{\alpha} \cdot p_{o,t}^{1-\alpha}}{\rho \cdot \theta \cdot Z_t \cdot z} \tau \right) \cdot \left(\frac{w_t^{\alpha} \cdot p_{o,t}^{1-\alpha}}{\rho \cdot \theta \cdot Z_t \cdot z} \tau \right)^{-\sigma} A_t$$

$$= \left(\frac{w_t^{\alpha} \cdot p_{o,t}^{1-\alpha}}{\rho \cdot \theta \cdot Z_t} \tau \right)^{1-\sigma} \frac{A_t}{z^{1-\sigma}}$$

Then, employing the threshold equation (1.8) we can rewrite the revenue equation as:

$$R_{x,t}(z) = \sigma \cdot \frac{w_t \cdot f_x}{Z_t} \left(\frac{z}{z_{x,t}} \right)^{\sigma-1}$$

Chapter 2

Monetary Policy in a Dynamic Trade Model with Heterogeneous Firms

Abstract. The main goal of this chapter is to study the effect of monetary policy on a dynamic model of trade with heterogeneous firms. We study the dynamic implications of monetary policies that act during "normal times" and monetary policies that leave the economy at the zero lower bound. In order to do so, we craft a model which incorporates nominal rigidities. This feature generates a friction such that nominal shocks affect real allocations in the economy.

To build the model, we combine nominal frictions with firm heterogeneity as in Melitz (2003) in a dynamic setting as in Ghironi and Melitz (2005).

2.1   Introduction

An important and traditional question in economics relates to the role played by monetary policy in stabilizing the economy when it faces fluctuations. Empirical evidence and economic theory seems to enjoy a relatively harmonious relationship in answering this question whenever the economy is away from the

liquidity trap at the zero bound during "normal times". On this topic, the new Keynesian model has been the standard tool to study the effect of monetary policy on macroeconomic aggregates. Although, the role played by monetary policy seems to be claer during "normal times", we would like to study the interaction between monetary policy and firm heterogeneity. In fact, the more important way in which this chapter departs from the standard new Keynesian model is firm heterogenity and the extensive margin adjusment mechanism.

We will also consider "special times" in the economy whenever it is constrained by the zero lower bound (ZLB). In a flexible price equilibrium, the ZLB will be associated with a price path that generates the same allocations that we would have in the absence of the bound. Thus, the ZLB becomes a concern when the economy faces nominal rigidities as well. That is, nominal rigidities generate a friction between the real interest rate and inflation.

The effect of the ZLB on the real side of the economy is currently a hot topic. Specially, whether the ZLB feature interacts with other policies in the economy. For example, according to the traditional new Keynesian model, policies that are contractionary according to the neoclassical model during normal times would become expansionary at the ZLB (Eggetsson, 2010 and 2012). Thus, under the ZLB and nominal rigidities the traditional New Keynesian model predicts that heterodox policies such as generating inflation, raising taxes, or making the economy less productive are expansionary (Eggertsson, 2012), while orthodox policies such as competitive-oriented structural reforms turn out to be contractionary (Villaverde, 2013; Eggertsson, Ferrero and Raffo, 2014).

Although, the scope of this chapter is limited to the effect of monetary policiy on firm heterogeneity, the literature on the ZLB mentioned above motivates the exercise of studying the effect of a monetary policy that leaves the economy at the ZLB.

In this chapter we will study monetary policies that act during "normal times" stimulating the economy and its implications toward average productivity

and price dynamics in the economy. We will also study monetary policies that leave the economy at the ZLB which have diferent implications compared to the dynamics induced by monetary policies during normal times. The endogenous dynamics implied by monetary policies are tied to the extensive magin adjustment mechanism delivered by firm heterogeneity.

From a theoretical point of view this chapter extends the dynamic model developed in Ghironi and Melitz (2005) including in it nominal rigidities which allows us to study the implications of different monetary policies.

The rest of the chapter is organized as follows: Section 2.2 describes a closed economy model with firm heterogeniety. Section 2.3 extends the model considering an open economy. Section 2.4 shows the results obtained in this chapter while Section 2.5 offers some conclusions.

2.2 A Closed Economy Model

To gain intuition, let us first consider the model in a closed economy in which monopolistic firms are engaged in infrequent price setting. Such firms are heterogeneous as in Melitz (2003).

2.2.1 Household Problem

Let us consider an economy populated by L identical individuals, each of which has a unit of labor that is supplied inelastically and earns a wage w_t. Each agent spends his income on a continuum of domestic goods indexed ω. The consumption of goods at time t is given by $q_t(\omega)$. In our notation, $q_t(\omega)$ represents the amount q of goods produced at time t, of a variety ω.

A representative household maximizes expected intertemporal utility from consumption:

$$E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \log(C_s) \right] \tag{2.1}$$

where $\beta \in (0,1)$ is the subjective discount factor. At time t, the household consumes a basket of goods C_t composed by goods $q_t(\omega)$:

$$C_t = \left[\int_{\omega} (q_t(\omega))^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across goods.

The budget constraint faced by the representative agent at prices $p_t(\omega)$ is as follows:

$$\int p_t(\omega) q_t(\omega) d\omega = I_t$$

where I_t corresponds to income at time t.

The solution to the household problem is given by the equation shown in (2.2).

$$q_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\sigma} C_t \tag{2.2}$$

The price index, P_t, associated with the consumer problem is as follows:

$$P_t = \left[\int p_t(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

2.2.2 Firm Problem

There is a continuum number of firms with a mass of one. The productivity level z is drawn every period from a Pareto distribution with shape parameter κ, $G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^{\kappa}$, where $z_{\min}$ is the minimum productivity level in the support of the distribution G. In every period, firms pay a sunk cost f_E, measured in effective units of labor, to draw a productivity level.

Firms are embedded in a monopolistic competition environment as in Melitz

(2003). Depending on the level of z, a firm will serve the economy at time t (relatively high z with respect to a threshold), or it could not produce at time t (relatively low z with respect to a threshold).

The production scheme for a domestic firm z is given in (2.3). Where $Q_t(\omega)$ is the amount of goods demanded of a variety ω. The fixed cost of production is shown in (2.4). Then, a firm that serves the economy at time t will produce $Q_t(\omega)$ with a fix cost of production f_D.

$$\bar{Q}_t = \begin{cases} Q_t(\omega) & \text{when producing} \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

The fixed cost f_D is measured in effective units of labor which translates into $\frac{w_t f_D}{Z_t}$ units of consumption, where Z_t is an aggregate productivity level.

$$f_{\bar{Q}} = \begin{cases} f_D & \text{when producing} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

Monopolistic firms produce final goods employing a production technology, shown in (2.5), in which labor l_t is the only factor of production. The Hicks-neutral productivity $Z_t z$ is the product between an aggregated productivity shock Z_t and the firm's productivity z.

$$\bar{Q}_t = (Z_t z) \cdot l_t \tag{2.5}$$

The monopolistic firms engage in infrequent price setting. In each period, a fraction $1 - \theta$ of the producers reoptimize their nominal prices. All other firms keep their old prices.

The timing of the model is the following: At the beginning of each period t, firms pay f_E, they learn productivity z, and they learn the expected price associated to z at $t - 1$, as shown in Appendix A, equation (2.40). After that, the shock is realized and firms learn the optimal price. Firms solve for their expected profits this period based on being able to charge the optimal price with probably

$1 - \theta$, and being stuck with charging the previous price with probably θ. Firms with non-negative expected profits choose to incur in the production fixed cost and produce, while firms with negative expected profits do not produce.

To simplify the dynamics, I remove expectations about future profits from firm decision-making by imposing that firms live for a single period. There are alternative approaches to the dynamics than the one taken in this chapter. For example, a Calvo type setting could characterize the firm's problem. However, this approach together with firm heterogeneity appears to be not solvable. Another approach would be to consider that the sticky price in every period is the frictionless price at t_o. Similarly, it could be considered a case in which the sticky price in every period is the optimal price picked at period $t - 1$.

The solution for the firm's pricing problem has the following form[1].:

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w_t}{Z_t z} \tag{2.6}$$

In equation (2.6) the superscript, $*$, in the optimal price makes reference to the fact that firms are actually choosing the price at period t. On the other hand, prices without the superscript, $*$, denote a price that has been reseted to the level in the previous period.

Price Index

The nominal rigidity assumed in this chapter is such that the price index of the domestic economy at time t, P_t, can be written as in (2.7).

$$P_t^{1-\sigma} = \theta \left[\int_{\bar{z}_t}^{\infty} p_t^{1-\sigma} dG(z) \right] + (1 - \theta) \left[\int_{\bar{z}_t}^{\infty} p_t^{*1-\sigma} dG(z) \right] \tag{2.7}$$

---

[1]To the best of my ability, I believe that the productivity resampling assumption is sufficient to simplify the firm problem and avoid a Calvo type setting. In case it were not sufficient, we would need a myopic assumption in the form of firms discounting the future sufficiently high such that the firm problem is reduced to the maximization of current profits.

$$1 = \theta \left(\Phi_t^{\sigma-1} \right) + (1 - \theta) \left(\Phi_t^{*\sigma-1} \right)$$

where $\Phi_t^{\sigma-1} = \frac{\int_{\bar{z}_t}^{\infty} p_t^{1-\sigma} dG(z)}{P_t^{1-\sigma}}$, and $\Phi_t^{*\sigma-1} = \frac{\int_{\bar{z}_t}^{\infty} p_t^{*1-\sigma} dG(z)}{P_t^{1-\sigma}}$. $\bar{z}_t$ is the productivity thresholds of firms serving the economy at time t. The productivity threshold is endogenous and is such that only firms whose productivity satisfies $z \geq \bar{z}_t$ will produce a positive amount at time t. The productivity threshold is formally derived later in this section.

We could also write a recursive structure for $\Phi_t^{\sigma-1}$ of the following form (See Appendix B for details):

$$\Phi_t^{\sigma-1} = \Pi_t^{\sigma-1} \left(\frac{\bar{z}_t}{\bar{z}_{t-1}} \right)^{\sigma-1} \left[\theta \Phi_{t-1}^{\sigma-1} \cdot + (1 - \theta) \Phi_{t-1}^{*\sigma-1} \right] \tag{2.8}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$.

Productivity Threshold

Firms are willing to produce at time t as long as profits, d_t, are greater or equal to zero. Thus, the productivity threshold is given by the productivity $\bar{z}_t$ of the marginal firm willing to produce:

$$\theta \cdot d_t(\bar{z}_t, p_t(\bar{z}_t)) + (1 - \theta) \cdot d_t(\bar{z}_t, p_t^*(\bar{z}_t)) = 0 \tag{2.9}$$

where $d_t(z, p_t) = \left(\frac{p_t}{P_t} \right)^{1-\sigma} - \left(\frac{w_t}{Z_t z} \right) \left(\frac{p_t}{P_t} \right)^{-\sigma} - \frac{w_t f_D}{Z_t}$.

Using the fact that the inverse of prices are distributed Pareto, we have the following[2] (See Appendix A for details):

$$\left(\frac{p_t(\bar{z}_t)}{P_t} \right)^{1-\sigma} = \left(\frac{\kappa + 1 - \sigma}{\kappa} \right) \Phi_t^{\sigma-1} \text{ and } \left(\frac{p_t^*(\bar{z}_t)}{P_t} \right)^{1-\sigma} = \left(\frac{\kappa + 1 - \sigma}{\kappa} \right) \Phi_t^{*\sigma-1}$$

---

[2]We consider that at the pre-history of the model there is a initial condition in which all firms optimize freely. That is there is no price stickiness. As we will see later, this is true at the steady state.

Thus, we are able to write equation (2.9) as in (2.10).

$$\bar{z}_t = \frac{\left(\frac{w_t}{Z_t}\right)\left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{\sigma}{\sigma-1}}\left(\theta\Phi_t^\sigma + (1-\theta)\Phi_t^{*\sigma}\right)}{\left(\frac{\kappa+1-\sigma}{\kappa}\right) - \frac{w_t f_D}{Z_t C_t}} \tag{2.10}$$

2.2.3 Aggregate Dynamic Problem

Let us consider the total profits generated in the economy at time t as follows:

$$D_t = \int d_t dG(z) \tag{2.11}$$

Let us also consider the market value of all firms at time t, v_t. This value corresponds to the expected present discounted value of future expected profits or dividends.

$$v_t = E_t\left(\{D_s\}_{s=t+1}^\infty\right)$$

The dynamic problem solved by a representative agent corresponds to the maximization of an intertemporal utility function $U\left(\{C_s\}_{s=t}^\infty\right)$ shown in equation (2.1).

A representative household holds two type of assets: shares of stock of domestic firms and domestic risk-free bonds. x_t is the share of stock of firms held by the representative household entering period t, and B_t is the bond holding in period t.

The shares of stocks pay dividends every period that correspond to the total profits from firms. During period t, the representative household buys x_{t+1} shares of stocks.

The representative household enters period t with bond holding B_t and shares of stock holding x_t. Thus, the income that the representative household receives, at period t, comes from: labor income, bond holdings, dividends from shares of stock plus the value of selling its initial share position.

The period budget constraint, in units of consumption, is the following:

$$\frac{B_{t+1}}{P_t} + v_t x_{t+1} + C_t = R_t \frac{B_t}{P_t} + (D_t + v_t) x_t + w_t L \tag{2.12}$$

That is, the household allocates her resources between consumption, purchases of bonds and shares of stock to be carried into next period.

The first order conditions of the dynamic problem described above, are as follows:

$$\frac{1}{C_t} = \beta E_t \left[\frac{R_{t+1}}{\Pi_{t+1}} \frac{1}{C_{t+1}} \right] \tag{2.13}$$

$$v_t = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right) (D_{t+1} + v_{t+1}) \right]$$

Equations in (2.13) describe the traditional Euler equations for bonds and stocks.

Aggregating the budget constraint across symmetric households and imposing the equilibrium condition for the close economy[3], we obtain the aggregate accounting equation in the domestic economy as in (2.14).

$$C_t = w_t L + D_t - \frac{w_t f_E}{Z_t} \tag{2.14}$$

The equilibrium in the labor market requires:

$$l_t + \frac{1}{Z_t} \left[(1 - G(\bar{z}_t)) f_D + f_E \right] = L \tag{2.15}$$

where l_t is the total amount of labor employed for production.

Finally, we have that all domestic firms pay a sunk cost per period of f_E to unveil their productivities which is measured in effective units of labor. Then a necessary condition is that in equilibrium:

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[3]In equilibrium, it must be the case that $B_{t+1} = B_t = 0$ and $x_t = x_{t+1} = 1$.

$$v_t = w_t \frac{f_E}{Z_t} \tag{2.16}$$

2.2.4 The Government

The government sets the nominal interest rates according to:

$$I_t = R \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{C_t}{C}\right)^{\phi_c} m_t$$

$$R_t = \max[I_t, 1]$$

where Π represents the steady state level of inflation and R is the steady state nominal gross interest rate. The term m_t is a random shock to monetary policy.

The policy rule is the maximum of two terms. The first term, I_t, follows a conventional Taylor rule that depends on the deviation of inflation with respect to its steady state, and the output gap. The second term is the ZLB. Thus, the gross nominal interest rate cannot be lower than one.

2.2.5 Equilibrium

The equilibrium is given by the sequence of endogenous and exogenous variables in our model.

The endogenous variables are $\{C_t, w_t, \Pi_t, \Phi_t, \Phi_t^*, \bar{z}_t, l_t, v_t, I_t, R_t, D_t\}$. The exogenous variables are $\{Z_t, m_t\}$. In total, our model has 11 endogenous variables and 2 exogenous variables. These variables are determined by:

Price Index and evolution:

$$1 = \theta \left(\Phi_t^{\sigma-1}\right) + (1-\theta) \left(\Phi_t^{*\sigma-1}\right)$$

$$\Phi_t^{\sigma-1} = \Pi_t^{\sigma-1} \left(\frac{\bar{z}_t}{\bar{z}_{t-1}}\right)^{\sigma-1} \left[\theta \Phi_{t-1}^{\sigma-1} \cdot + (1-\theta) \Phi_{t-1}^{*\sigma-1}\right]$$

First order condition for household's problem:

$$\frac{1}{C_t} = \beta E_t \left[\frac{R_{t+1}}{\Pi_{t+1}} \frac{1}{C_{t+1}}\right]$$

$$v_t = \beta E_t \left[\left(\frac{C_t}{C_{t+1}}\right)(D_{t+1} + v_{t+1})\right]$$

Firms optimization problem:

$$\Phi_t^{*\sigma-1} = \left(\frac{\kappa}{\kappa+1-\sigma}\right)\left[\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{\bar{z}_t}\right)\left(\frac{w_t}{Z_t}\right)\right]^{1-\sigma}$$

Productivity Threshold:

$$\bar{z}_t = \frac{\left(\frac{w_t}{Z_t}\right)\left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{\sigma}{\sigma-1}}\left(\theta \Phi_t^\sigma + (1-\theta)\Phi_t^{*\sigma}\right)}{\left(\frac{\kappa+1-\sigma}{\kappa}\right) - \frac{w_t f_D}{Z_t C_t}}$$

Labor market clearing:

$$l_t + \frac{1}{Z_t}\left[(1 - G\left(\bar{z}_t\right)) f_D + f_E\right] = L$$

Market equilibrium:

$$v_t = w_t \frac{f_E}{Z_t}$$

Aggregate Accounting:

$$C_t = w_t L + D_t - \frac{w_t f_E}{Z_t}$$

Government Policy:

$$I_t = R \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{C_t}{C}\right)^{\phi_c} m_t$$

$$R_t = \max[I_t, 1]$$

In order to build intuition, let us consider a monetary policy shock in the closed economy such that the interest rate is lowered by the monetary authority. This shock embodies the traditional spirit of monetary policy during "normal times" in which the monetary authority is trying to stimulate the economy. Here is where the nominal friction plays a role, otherwise, under flexible prices, movement of the nominal interest rate would not affect at all the real side of the economy.

The decrease in the nominal interest would push down the real interest rate. The drop of the real interest rate incentivizes current consumption against future consumption (from Euler equation). The increment in current consumption puts pressure on the real wage rate which increases as well. Therefore, the cost of production increases. That increase in marginal cost and fixed cost of production affects the productivity threshold. The marginal firm willing to produce is a firm with higher productivity since the shock negatively affects profits through the increase in production costs. Firms are able to incorporate the changes in marginal cost to prices with probability $(1 - \theta)$, but with probability θ firms have to drag prices from the previous period.

The productivity threshold will adjust to accommodate the increase in production costs. Therefore, $\bar{z}_t$ will increase meaning that the average productivity of active firms in the economy will increase and fewer firms will serve the economy. This adjustment embodies the extensive margin adjustment. There is also an intensive margin adjustment since, as a result of the shock, there is a fewer number of firms $(\Delta \bar{z}_t > 0)$ that are producing more $(\Delta C_t > 0)$.

The traditional expansionary effect of monetary policy during normal times is captured by the model since the decrease in nominal interest rate leads to an increase in current consumption. There is an important adjustment feature in the model that differs from the traditional Standard New Keynesian model (SNK). This is the extensive margin. This adjustment can play a passive role if the increase

in marginal costs offsets the increment in average productivity. That is, if aggregate price increases, then the extensive margin adjustment is spanning the same results that the SNK model does. However, the extensive margin plays an active role if the extensive margin adjustment offsets the increment in marginal costs such that the aggregate price decreases. We are interested in the latter case since it provides features that differ from the SNK model, and it fact it cannot replicate. The intuition behind such adjustment is that during booms the wage rate increases making it impossible for less productive firms to stay active. The new marginal firm willing to produce is a firm with higher productivity. Thus, the new price index is going to be the aggregation across a more productive set of firms that, with probability $(1-\theta)$, choose prices that incorporate the higher marginal costs, and with probability θ, reset prices that were set under lower marginal costs.

In summary, during normal times, a decrease in the nominal interest rate would decrease the real interest rate, and thus would increase current consumption and decrease the price index.

Let us consider now a monetary shock such that the economy is left at the ZLB. Such shock is a kink for the economic dynamics. There is no smooth transition between the normal times and the ZLB. In fact, the shock explored here triggers a regime switch from an economy under the Taylor rule to an economy in which $R_t = 1$. To begin, let us conjecture that expected inflation decreases as a result of the shock, such that current consumption decreases via the relationship established by the Euler equation. The decrease in current consumption weakens the economic activity and wage rate goes down. Therefore, production costs decrease. In this case, the marginal firm willing to produce is a firm with a lower productivity which implies that $\bar{z}_t$ decreases. Following the same reasoning from the previous example, the decrease in the productivity threshold triggered by the decrease in the marginal cost come at a cost of higher prices since the average productivity of the firms serving the economy decreases. Moreover, there are more firms in the economy producing less than before which translates in lower aggre-

gate profits. The decrease in the wage rate along with the decrease in aggregate profits reinforce the equilibrium in which consumption decreases (from the aggregate accounting equation). Moreover, the increase in the price index is consistent with the initial conjecture which established the equilibrium.

In summary, this second exercise shows that a monetary shock that leaves the economy at the ZLB can be contractionary. The key adjustment here is the extensive margin movement. The decrease in the productivity thresholds is such that the decrease in average productivity offsets the decrease in marginal costs such that the price index increases. Further, this increase in the price index is consistent with deflationary expectations. which is leading the contractionary dynamic.

The extensive margin movements leading the price index adjustment is the fundamental feature of the dynamics described in this chapter. Thus, an increase in the productivity threshold (higher marginal costs) will be associated to a lower price index, and conversely a decrease in the productivity threshold (lower marginal costs) will be associated to a higher price index. Next section builds upon these results and we explore the effect of monetary policy in a model that consider firm heterogeneity and nominal frictions in an open economy.

2.3   An Open Economy Model

The model developed in this section employs the set up used first in Ghironi and Melitz (2005) together with nominal frictions. We will consider monopolistic firms engaged in infrequent price setting.

2.3.1   Household Problem

Let's consider a home economy (h) populated by L_h identical individuals, each of which has a unit of labor that is supplied inelastically and earns a wage $w_{h,t}$. Each agent spends his income on a continuum of domestic and imported goods

indexed ω_h and ω_f, respectively. The consumption of domestic and imported goods is given by $q_{h,t}(\omega_h)$ and $q_{f,x,t}(\omega_f)$. In our notation, $q_{i,t}(\omega)$ represents the amount q of goods produced in country i, at time t, of a variety ω. Similarly $q_{i,x,t}(\omega)$ corresponds to a quantity q of a variety ω, produced in country i, at time t, and it is exported (x).

A representative domestic household maximizes expected intertemporal utility from consumption:

$$E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\log(C_{h,s})\right] \tag{2.17}$$

, where $\beta \in (0,1)$ is the subjective discount factor. At time t, the household of the home economy consumes a basket of goods $C_{h,t}$ composed by domestically produced goods $q_{h,t}(\omega_h)$ and foreign produced goods $q_{f,x,t}(\omega_f)$ of the form:

$$C_{h,t} = \left[\int_{\omega_h}(q_{h,t}(\omega_h))^{\frac{\sigma-1}{\sigma}}d\omega + \int_{\omega_f}(q_{f,x,t}(\omega_f))^{\frac{\sigma-1}{\sigma}}d\omega_f\right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across goods.

The budget constraint faced by the representative agent at prices $p_{h,t}(\omega_h)$ and $p_{f,x,t}(\omega_f)$ for domestic and foreign goods, respectively, is as follows:

$$\int p_{h,t}(\omega_h)\, q_{h,t}(\omega_h)\, d\omega_h + \int p_{f,x,t}(\omega_f)\, q_{f,x,t}(\omega_f)\, d\omega_f = I_{h,t}$$

, where $I_{h,t}$ corresponds to the income at time t.

The solutions to the household problem is given by equations shown in (2.18).

$$\begin{aligned}q_{h,t}(\omega) &= \left(\frac{p_{h,t}(\omega)}{P_{h,t}}\right)^{-\sigma}C_{h,t}\\q_{f,x,t}(\omega) &= \left(\frac{p_{f,x,t}(\omega)}{P_{h,t}}\right)^{-\sigma}C_{h,t}\end{aligned} \tag{2.18}$$

The price index, $P_{h,t}$, in the home economy associated to the consumer problem is as follows:

$$P_{h,t} = \left[\int p_{h,t} \left(\omega \right)^{1-\sigma} d\omega + \int p_{f,x,t} \left(\omega \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

2.3.2   Firm Problem

There is an continuum number of firms with a mass of one. The firm productivity level z is drawn every period from a Pareto distribution with shape parameter κ, $G(z) = 1 - \left(\frac{z_{\min}}{z} \right)^{\kappa}$, where $z_{\min}$ is the minimum productivity level in the support of the distribution G. In every period, firms pay a sunk cost f_E, measured in effective units of labor, to draw a productivity level.

Firms are embedded in a monopolistic competition environment as in Melitz (2003). Depending on the level of z, a firm will serve the domestic economy exclusively (relatively low z with respect to a threshold), or it could serve both the domestic economy and the foreign economy (relatively high z with respect to a threshold).

The production scheme for a domestic firm z is given in (2.19). Where $Q_{h,t} \left(\omega \right)$ is the amount of goods, of a variety ω, domestically demanded, and $Q_{h,x,t} \left(\omega \right)$ is the amount of goods, of a variety ω, foreign demanded. τ_h corresponds to the traditional iceberg type of transport cost. The fix costs of production are shown in (2.20). Then, a firm that exclusively serves the domestic economy will produce $Q_{h,t} \left(\omega \right)$ with fix cost of production $f_{h,D}$.

$$\bar{Q}_t = \begin{cases} Q_{h,t} \left(\omega \right) + \tau_h \cdot Q_{h,x,t} \left(\omega \right) & \text{if firm is exporting} \\ Q_{h,t} \left(\omega \right) & \text{otherwise} \end{cases} \tag{2.19}$$

Also, a firm that serves the domestic economy and the foreign economy will produce an amount equal to $Q_{h,t} \left(\omega \right) + \tau_h \cdot Q_{h,x,t} \left(\omega \right)$ incurring in fix costs of production $f_{h,x}$.

The fix costs $f_{h,D}$ and $f_{h,x}$ are measured in effective units of labor which translates into $\frac{w_{h,t} f_{h,i}}{Z_{h,t}}$ for $i \in \{x, D\}$ units of consumption, where $Z_{h,t}$ is an aggregated productivity level.

$$f_{h,\bar{Q}} = \begin{cases} f_{h,x} & \text{when exporting} \\ f_{h,D} & \text{otherwise} \end{cases} \tag{2.20}$$

Monopolistic firms produce final goods employing a production technology, shown in (2.21), in which labor l_t is the only factor of production. The Hicks-neutral productivity $Z_t z$ is the product between an aggregated productivity shock Z_t and the firm's productivity z.

$$\bar{Q}_t = (Z_t z) \cdot l_t \tag{2.21}$$

The solution for the firm's pricing problem of domestic producers of non-traded goods has the following form:

$$\left(\frac{p_{h,t}^*}{P_{h,t}}\right) = \left(\frac{\sigma}{\sigma-1}\right) \frac{1}{z} \frac{w_{h,t}}{Z_{h,t}} \tag{2.22}$$

Similarly, the solution for the firm's pricing problem of domestic producers of traded goods has the following form:

$$\left(\frac{p_{h,x,t}^*}{P_{f,t}}\right) = \left(\frac{\sigma}{\sigma-1}\right) \frac{1}{\epsilon_t z} \frac{w_{h,t}}{Z_{h,t}} \tau_h \tag{2.23}$$

The optimal price among firms serving the same market differs across firms just in the productivity z. The inverse of prices is distributed Pareto with shape parameter κ.

In equations (2.22) and (2.23) the superscript, $*$, in the optimal price makes reference to the fact that firms actually choosing the price at period t. On the other hand, prices without the superscript, $*$, denote a price that has been reseted to the level in the previous period.

Price Index

The nominal rigidity assumed in this chapter is such that the price index of the domestic economy at time t, $P_{h,t}$, can be written as in (2.24).

$$P_{h,t}^{1-\sigma} = \theta \left[\int_{\bar{z}_{h,t}}^{\infty} p_{h,t}^{1-\sigma} dG(z) + \int_{\bar{z}_{f,x,t}}^{\infty} p_{f,x,t}^{1-\sigma} dG(z) \right] + (1-\theta) \left[\int_{\bar{z}_{h,t}}^{\infty} p_{h,t}^{*1-\sigma} dG(z) + \int_{\bar{z}_{f,x,t}}^{\infty} p_{f,x,t}^{*1-\sigma} dG(z) \right]$$

(2.24)

$$1 = \theta \left(\Phi_{h,t}^{\sigma-1} + \Phi_{f,x,t}^{\sigma-1} \right) + (1-\theta) \left(\Phi_{h,t}^{*\sigma-1} + \Phi_{f,x,t}^{*\sigma-1} \right)$$

where $\Phi_{h,t}^{\sigma-1} = \dfrac{\int_{\bar{z}_{h,t}}^{\infty} p_{h,t}^{1-\sigma} dG(z)}{P_{h,t}^{1-\sigma}}$, $\Phi_{f,x,t}^{\sigma-1} = \dfrac{\int_{\bar{z}_{f,x,t}}^{\infty} p_{f,x,t}^{1-\sigma} dG(z)}{P_{h,t}^{1-\sigma}}$, $\Phi_{h,t}^{*\sigma-1} = \dfrac{\int_{\bar{z}_{h,t}}^{\infty} p_{h,t}^{*1-\sigma} dG(z)}{P_{h,t}^{1-\sigma}}$,

and $\Phi_{f,x,t}^{*\sigma-1} = \dfrac{\int_{\bar{z}_{f,x,t}}^{\infty} p_{f,x,t}^{*1-\sigma} dG(z)}{P_{h,t}^{1-\sigma}}$. $\bar{z}_{h,t}$ and $\bar{z}_{f,x,t}$ are the productivity thresholds of domestic firms and foreign firms serving the domestic economy respectively. These productivity thresholds are endogenous and are such that only the domestic firms whose productivity satisfies $z \geq \bar{z}_{h,t}$ will produce a positive amount at time t. Similarly, foreign firms whose productivity satisfies $z \geq \bar{z}_{f,x,t}$ will produce a positive amount at time t. Productivity thresholds are formally derived later in this section.

We could also write a recursive structure for $\Phi_{h,t}^{\sigma-1}$ and $\Phi_{f,x,t}^{\sigma-1}$ of the following form (See Appendix B for details):

$$\Phi_{h,t}^{\sigma-1} = \Pi_{h,t}^{\sigma-1} \left(\frac{z_{h,t}}{z_{h,t-1}} \right)^{\sigma-1} \left[\theta \Phi_{h,t-1}^{\sigma-1} \cdot +(1-\theta) \Phi_{h,t-1}^{*\sigma-1} \right]$$

(2.25)

$$\Phi_{f,x,t}^{\sigma-1} = \Pi_{h,t}^{\sigma-1} \left(\frac{z_{f,x,t}}{z_{f,x,t-1}} \right)^{\sigma-1} \left[\theta \Phi_{f,x,t-1}^{\sigma-1} + (1-\theta) \Phi_{f,x,t-1}^{*\sigma-1} \right]$$

where $\Pi_{h,t} = \dfrac{P_{h,t}}{P_{h,t-1}}$.

Productivity Threshold

Domestic firms producing non-traded goods are willing to produce at time t as long as profits, $d_{h,t}$, are greater or equal to zero. Thus, the productivity threshold for a domestic firms producing non-traded goods is given by:

$$\theta \cdot d_{h,t}(\bar{z}_{h,t}, p_{h,t}(\bar{z}_{h,t})) + (1-\theta) \cdot d_{h,t}(\bar{z}_{h,t}, p_{h,t}^{*}(\bar{z}_{h,t})) = 0$$

(2.26)

Using the fact that the inverse of prices are distributed Pareto, we have the following[4]:

$$\left(\frac{p_{h,t}\left(\bar{z}_{h,t}\right)}{P_{h,t}}\right)^{1-\sigma} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)\Phi_{h,t}^{\sigma-1} \text{ and } \left(\frac{p_{h,t}^{*}\left(\bar{z}_{h,t}\right)}{P_{h,t}}\right)^{1-\sigma} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)\Phi_{h,t}^{*\sigma-1}$$

(2.27)

Thus, we are able to write equation (2.26) as in (2.28).

$$\bar{z}_{h,t} = \frac{\left(\frac{w_{h,t}}{Z_{h,t}}\right)\left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{\sigma}{\sigma-1}}\left(\theta\Phi_{h,t}^{\sigma} + (1-\theta)\Phi_{h,t}^{*\sigma}\right)}{\left(\frac{\kappa+1-\sigma}{\kappa}\right)\left(\theta\Phi_{h,t}^{\sigma-1} + (1-\theta)\Phi_{h,t}^{*\sigma-1}\right) - \frac{w_{h,t}f_{h,D}}{Z_{h,t}C_{h,t}}}$$

(2.28)

Additionally, the total amount of labor employed by domestic firms producing non-traded goods can be written as follows:

$$l_{h,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{C_{h,t}}{Z_{h,t}\bar{z}_{h,t}}\right)\left(\theta\Phi_{h,t}^{\sigma} + (1-\theta)\Phi_{h,t}^{*\sigma}\right)$$

(2.29)

Domestic exporting firms are willing to produce at time t as long as profits, $d_{h,x,t}$, are greater or equal to zero. Thus, the productivity threshold for a domestic exporting firms is given by:

$$\theta \cdot d_{h,x,t}(\bar{z}_{h,x,t}, p_{h,x,t}\left(\bar{z}_{h,x,t}\right)) + (1-\theta) \cdot d_{h,x,t}(\bar{z}_{h,x,t}, p_{h,x,t}^{*}\left(\bar{z}_{h,x,t}\right)) = 0$$

(2.30)

Using the fact that the inverse of prices are distributed Pareto, we have the following:

$$\left(\frac{p_{h,x,t}\left(\bar{z}_{h,x,t}\right)}{P_{h,t}}\right)^{1-\sigma} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)\Phi_{h,x,t}^{\sigma-1} \text{ and } \left(\frac{p_{h,x,t}^{*}\left(\bar{z}_{h,t}\right)}{P_{h,t}}\right)^{1-\sigma} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)\Phi_{h,x,t}^{*\sigma-1}$$

(2.31)

Thus, we are able to write equation (2.30) as in (2.32).

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[4]We consider that at the pre-history of the model there is a initial condition in which all firms optimize freely. That is there is no price stickiness. As we will see later, this is true at the steady state.

$$\bar{z}_{h,x,t} = \frac{\left(\frac{w_{h,t}}{Z_{h,t}}\tau_h\right)\left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{\sigma}{\sigma-1}}\left(\theta\Phi_{h,x,t}^{\sigma} + (1-\theta)\Phi_{h,x,t}^{*\sigma}\right)}{\epsilon_t\left(\frac{\kappa+1-\sigma}{\kappa}\right)\left(\theta\Phi_{h,x,t}^{\sigma-1} + (1-\theta)\Phi_{h,x,t}^{*\sigma-1}\right) - \frac{w_{h,t}f_{h,x}}{Z_{h,t}C_{f,t}}} \tag{2.32}$$

The total amount of labor employed by domestic exporting firms is as follows:

$$l_{h,x,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{\tau_h C_{f,t}}{Z_{h,t}\bar{z}_{h,x,t}}\right)\left(\theta\Phi_{h,x,t}^{\sigma} + (1-\theta)\Phi_{h,x,t}^{*\sigma}\right) \tag{2.33}$$

2.3.3 Aggregate Dynamic Problem

Let us consider the total profits generated at the home economy at time t as follows:

$$D_{h,t} = \int d_{h,t}dG(z) + \int d_{h,x,t}dG(z) \tag{2.34}$$

Let us also consider the market value of all domestic firms at time t, $v_{h,t}$. This value corresponds to the expected present discounted value of future expected profits or dividends.

$$v_{h,t} = E_t\left(\{D_{h,s}\}_{s=t+1}^{\infty}\right)$$

The aggregated dynamic problem solved by a domestic representative agents correspond to the maximization of an intertemporal utility function $U\left(\{C_{h,s}\}_{s=t}^{\infty}\right)$ shown in equation (2.17).

A representative household holds two type of assets: shares of stock of domestic firms and domestic risk-free bonds. x_t is the share of stock of domestic firms held by the representative household entering period t, and B_t is the bond holding in period t.

The shares of stocks pay dividends every period that correspond to the total profit of domestic firms. During period t, the representative household buys x_{t+1}

shares of stocks of domestic firms.

The representative household enters period t with bond holding B_t and shares of stock holding x_t. Thus, the income that the representative household receives, at period t, comes from: labor income, bond holdings, dividends from shares of stock plus the value of selling its initial share position.

The period budget constraint, in units of consumption, is the following:

$$\frac{B_{h,t+1}}{P_{h,t}} + v_{h,t}x_{t+1} + C_{h,t} = R_{h,t}\frac{B_{h,t}}{P_{h,t}} + (D_{h,t} + v_{h,t})\, x_t + w_{h,t}L_h \tag{2.35}$$

That is, the household allocates her resources between consumption, purchase of bonds and shares of stock to be carried into next period.

The first order conditions, of the dynamic problem described above, are as follows:

$$\frac{1}{C_{h,t}} = \beta E_t \left[\frac{R_{h,t+1}}{\Pi_{h,t+1}} \frac{1}{C_{h,t+1}} \right] \tag{2.36}$$

$$v_{h,t} = \beta E_t \left[\left(\frac{C_{h,t}}{C_{h,t+1}} \right) (D_{h,t+1} + v_{h,t+1}) \right]$$

Equations in (2.36) describe the traditional Euler equations for bonds and stocks.

Aggregating the budget constraint across symmetric households and imposing the equilibrium condition under financial autarky, we obtain the aggregate accounting equation in the domestic economy as in $(2.37)^5$.

$$C_{h,t} = w_{h,t}L_h + D_{h,t} - \frac{w_{h,t}f_{h,e}}{Z_{h,t}} \tag{2.37}$$

The equilibrium in the labor market requires:

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[5]In equilibrium under financial autarky It must be the case that $B_{t+1} = B_t = 0$ and $x_t = x_{t+1} = 1$. Capital account is closed

$$l_{h,t} + l_{h,x,t} + \frac{1}{Z_{h,t}} \left[(1 - G(\bar{z}_{h,t})) f_{h,D} + (1 - G(\bar{z}_{h,x,t})) f_{h,x} + f_{h,e} \right] = L_h \qquad (2.38)$$

Finally, we have that all domestic firms pay a sunk cost per period of $f_{h,e}$ to unveil their productivities which is measured in effective units of labor. Then a necessary condition is that in equilibrium:

$$v_{h,t} = w_{h,t} \frac{f_{h,e}}{Z_{h,t}} \qquad (2.39)$$

2.3.4   The Government

The home government sets the nominal interest rates according to:

$$I_{h,t} = R \left(\frac{\Pi_{h,t}}{\Pi} \right)^{\phi_\pi} \left(\frac{C_{h,t}}{C} \right)^{\phi_c} m_{h,t}$$

$$R_{h,t} = \max[I_{h,t}, 1]$$

where Π represents the steady state level of inflation and R is the steady state nominal gross interest rate. The term m_t is a random shock to monetary policy. The policy rule is the maximum of two terms. The first term, $I_{h,t}$, follows a conventional Taylor rule that depends on the deviation of inflation with respect to its steady state, and the output gap. The second term is the ZLB, that is the gross nominal interest rate cannot be lower than one.

I have an underlying behavioral assumption which comes from the exclusion of the foreign Taylor rule under Walras' law. This assumption materializes in comovements between domestic and foreign interest rates.

2.3.5 Equilibrium

The equilibrium is given by the edogenous variables and the exogenous variables in our model considering both the home economy and the foreign economy.

The endogenous variables are $\{C_{h,t},\ w_{h,t},\ \Pi_{h,t},\ \Phi_{h,t},\ \Phi^*_{h,t},\ \Phi_{h,x,t},\ \Phi^*_{h,x,t},\ \bar{z}_{h,t},$ $\bar{z}_{h,x,t},\ l_{h,t},\ l_{h,x,t},\ v_{h,t},\ I_{h,t},\ R_{h,t}, D_{h,t}\}$ for the home economy and $\{C_{f,t},\ w_{f,t},\ \Pi_{f,t},$ $\Phi_{f,t},\ \Phi^*_{f,t},\ \Phi_{f,x,t},\ \Phi^*_{f,x,t},\ \bar{z}_{f,t},\ \bar{z}_{f,x,t},\ l_{f,t},\ l_{f,x,t},\ v_{f,t},\ R_{f,t}, D_{f,t}\}$ for the foreign economy. In addition we have the exchange rate $\{\epsilon_t\}$.

The exogenous variables are $\{Z_{h,t}, m_{h,t}\}$ for the home economy. In total our model has 30 endogenous variables and 2 exogenous variables. This variables are determined by:

Price evolution (home and foreign economy):

$$1 = \theta\left(\Phi_{h,t}^{\sigma-1} + \Phi_{f,x,t}^{\sigma-1}\right) + (1-\theta)\left(\Phi_{h,t}^{*\sigma-1} + \Phi_{f,x,t}^{*\sigma-1}\right)$$

$$\Phi_{h,t}^{\sigma-1} = \Pi_{h,t}^{\sigma-1}\left(\frac{z_{h,t}}{z_{h,t-1}}\right)^{\sigma-1}\left[\theta\Phi_{h,t-1}^{\sigma-1}\cdot +(1-\theta)\Phi_{h,t-1}^{*\sigma-1}\right]$$

$$\Phi_{f,x,t}^{\sigma-1} = \Pi_{h,t}^{\sigma-1}\left(\frac{z_{f,x,t}}{z_{f,x,t-1}}\right)^{\sigma-1}\left[\theta\Phi_{f,x,t-1}^{\sigma-1} + (1-\theta)\Phi_{f,x,t-1}^{*\sigma-1}\right]$$

$$1 = \theta\left(\Phi_{f,t}^{\sigma-1} + \Phi_{h,x,t}^{\sigma-1}\right) + (1-\theta)\left(\Phi_{f,t}^{*\sigma-1} + \Phi_{h,x,t}^{*\sigma-1}\right)$$

$$\Phi_{f,t}^{\sigma-1} = \Pi_{f,t}^{\sigma-1}\left(\frac{z_{f,t}}{z_{f,t-1}}\right)^{\sigma-1}\left[\theta\Phi_{f,t-1}^{\sigma-1}\cdot +(1-\theta)\Phi_{f,t-1}^{*\sigma-1}\right]$$

$$\Phi_{h,x,t}^{\sigma-1} = \Pi_{f,t}^{\sigma-1}\left(\frac{z_{h,x,t}}{z_{h,x,t-1}}\right)^{\sigma-1}\left[\theta\Phi_{h,x,t-1}^{\sigma-1} + (1-\theta)\Phi_{h,x,t-1}^{*\sigma-1}\right]$$

First order condition of household (home and foreign economy):

$$\frac{1}{C_{h,t}} = \beta E_t\left[\frac{R_{h,t+1}}{\Pi_{h,t+1}}\frac{1}{C_{h,t+1}}\right]$$

$$\frac{1}{C_{f,t}} = \beta E_t \left[\frac{R_{f,t+1}}{\Pi_{f,t+1}} \frac{1}{C_{f,t+1}} \right]$$

$$v_{h,t} = \beta E_t \left[\left(\frac{C_{h,t}}{C_{h,t+1}} \right) (D_{h,t+1} + v_{h,t+1}) \right]$$

$$v_{f,t} = \beta E_t \left[\left(\frac{C_{f,t}}{C_{f,t+1}} \right) (D_{f,t+1} + v_{f,t+1}) \right]$$

Non exporters optimization problem (home and foreign economy):

$$\Phi_{h,t}^{*\sigma-1} = \left(\frac{\kappa}{\kappa+1-\sigma} \right) \left[\left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{\bar{z}_{h,t}} \right) \left(\frac{w_{h,t}}{Z_{h,t}} \right) \right]^{1-\sigma}$$

$$\Phi_{f,t}^{*\sigma-1} = \left(\frac{\kappa}{\kappa+1-\sigma} \right) \left[\left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{\bar{z}_{f,t}} \right) \left(\frac{w_{f,t}}{Z_{f,t}} \right) \right]^{1-\sigma}$$

Exporters optimization problem (home and foreign economy):

$$\Phi_{h,x,t}^{*\sigma-1} = \left(\frac{\kappa}{\kappa+1-\sigma} \right) \left[\left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{\bar{z}_{h,x,t}} \right) \left(\frac{w_{h,t}}{\epsilon_t Z_{h,t}} \tau_h \right) \right]^{1-\sigma}$$

$$\Phi_{f,x,t}^{*\sigma-1} = \left(\frac{\kappa}{\kappa+1-\sigma} \right) \left[\left(\frac{\sigma}{\sigma-1} \right) \left(\frac{1}{\bar{z}_{f,x,t}} \right) \left(\frac{\epsilon_t w_{f,t}}{Z_{f,t}} \tau_f \right) \right]^{1-\sigma}$$

Productivity Thresholds (home and foreign economy):

$$\bar{z}_{h,t} = \frac{\left(\frac{w_{h,t}}{Z_{h,t}} \right) \left(\frac{\kappa+1-\sigma}{\kappa} \right)^{\frac{\sigma}{\sigma-1}} \left(\theta \Phi_{h,t}^{\sigma} + (1-\theta) \Phi_{h,t}^{*\sigma} \right)}{\left(\frac{\kappa+1-\sigma}{\kappa} \right) \left(\theta \Phi_{h,t}^{\sigma-1} + (1-\theta) \Phi_{h,t}^{*\sigma-1} \right) - \frac{w_{h,t} f_{h,D}}{Z_{h,t} C_{h,t}}}$$

$$\bar{z}_{f,t} = \frac{\left(\frac{w_{f,t}}{Z_{f,t}} \right) \left(\frac{\kappa+1-\sigma}{\kappa} \right)^{\frac{\sigma}{\sigma-1}} \left(\theta \Phi_{f,t}^{\sigma} + (1-\theta) \Phi_{f,t}^{*\sigma} \right)}{\left(\frac{\kappa+1-\sigma}{\kappa} \right) \left(\theta \Phi_{f,t}^{\sigma-1} + (1-\theta) \Phi_{f,t}^{*\sigma-1} \right) - \frac{w_{f,t} f_{f,D}}{Z_{f,t} C_{f,t}}}$$

$$\bar{z}_{h,x,t} = \frac{\left(\frac{w_{h,t}}{Z_{h,t}} \tau_h \right) \left(\frac{\kappa+1-\sigma}{\kappa} \right)^{\frac{\sigma}{\sigma-1}} \left(\theta \Phi_{h,x,t}^{\sigma} + (1-\theta) \Phi_{h,x,t}^{*\sigma} \right)}{\epsilon_t \left(\frac{\kappa+1-\sigma}{\kappa} \right) \left(\theta \Phi_{h,x,t}^{\sigma-1} + (1-\theta) \Phi_{h,x,t}^{*\sigma-1} \right) - \frac{w_{h,t} f_{h,x}}{Z_{h,t} C_{f,t}}}$$

$$\bar{z}_{f,x,t} = \frac{\left(\frac{w_{f,t}}{Z_{f,t}}\tau_f\right)\left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{\sigma}{\sigma-1}}\left(\theta\Phi_{f,x,t}^{\sigma} + (1-\theta)\Phi_{f,x,t}^{*\sigma}\right)}{\frac{1}{\epsilon_t}\left(\frac{\kappa+1-\sigma}{\kappa}\right)\left(\theta\Phi_{f,x,t}^{\sigma-1} + (1-\theta)\Phi_{f,x,t}^{*\sigma-1}\right) - \frac{w_{f,t}f_{f,x}}{Z_{f,t}C_{h,t}}}$$

Labor employed by non-exporter firms (home and foreign economy):

$$l_{h,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{C_{h,t}}{Z_{h,t}\bar{z}_{h,t}}\right)\left(\Phi_{h,t}^{\sigma} + (1-\theta)\Phi_{h,t}^{*-\sigma}\right)$$

$$l_{f,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{C_{f,t}}{Z_{f,t}\bar{z}_{f,t}}\right)\left(\Phi_{f,t}^{\sigma} + (1-\theta)\Phi_{f,t}^{*\sigma}\right)$$

Labor employed by exporter firms (home and foreign economy):

$$l_{h,x,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{\tau_h C_{f,t}}{Z_{h,t}\bar{z}_{h,x,t}}\right)\left(\Phi_{h,x,t}^{\sigma} + (1-\theta)\Phi_{h,x,t}^{*\sigma}\right)$$

$$l_{f,x,t} = \left(\frac{\kappa+1-\sigma}{\kappa}\right)^{\frac{1}{\sigma-1}}\left(\frac{\tau_f C_{h,t}}{Z_{f,t}\bar{z}_{f,x,t}}\right)\left(\Phi_{f,x,t}^{\sigma} + (1-\theta)\Phi_{f,x,t}^{*\sigma}\right)$$

Market equilibrium (home and foreign economy):

$$v_{h,t} = w_{h,t}\frac{f_{h,e}}{Z_{h,t}}$$

$$v_{f,t} = w_{f,t}\frac{f_{f,e}}{Z_{f,t}}$$

Aggregate Accounting (home and foreign economy):

$$C_{h,t} = w_{h,t}L_h + D_{h,t} - \frac{w_{h,t}f_{h,e}}{Z_{h,t}}$$

$$C_{f,t} = w_{f,t}L_f + D_{f,t} - \frac{w_{f,t}f_{f,e}}{Z_{f,t}}$$

Total Dividends (home and foreign economy):

$$D_{h,t} = \left(\theta\Phi_{h,t}^{\sigma-1} + (1-\theta)\,\Phi_{h,t}^{*\sigma-1}\right) C_{h,t} + \epsilon_t \left(\theta\Phi_{h,x,t}^{\sigma-1} + (1-\theta)\,\Phi_{h,x,t}^{*\sigma-1}\right) C_{f,t}$$
$$- w_{h,t} \left(l_{h,t} + l_{h,x,t} + \frac{(1 - G\left(\bar{z}_{h,t}\right)) f_{h,D} + (1 - G\left(\bar{z}_{h,x,t}\right)) f_{h,x}}{Z_{h,t}} \right)$$

$$D_{f,t} = \left(\theta\Phi_{f,t}^{\sigma-1} + (1-\theta)\,\Phi_{f,t}^{*\sigma-1}\right) C_{f,t} + \frac{1}{\epsilon_t} \left(\theta\Phi_{f,x,t}^{\sigma-1} + (1-\theta)\,\Phi_{f,x,t}^{*\sigma-1}\right) C_{h,t}$$
$$- w_{f,t} \left(l_{f,t} + l_{f,x,t} + \frac{(1 - G\left(\bar{z}_{f,t}\right)) f_{f,D} + (1 - G\left(\bar{z}_{f,x,t}\right)) f_{f,x}}{Z_{f,t}} \right)$$

Government Policy:

$$S_{h,t} = R \left(\frac{\Pi_{h,t}}{\Pi}\right)^{\phi_\pi} \left(\frac{C_{h,t}}{C}\right)^{\phi_c} m_{h,t}$$

$$R_{h,t} = \max[S_{h,t}, 1]$$

Exogenous variables:

$$Z_{h,t} = (1 - \rho_{zh}) + \rho_{zh} Z_{h,t-1} + \varepsilon_{zh,t}, \quad \varepsilon_{zh,t} \tilde{\ } N(0, \sigma_{z,h}^2)$$

$$m_{h,t} = (1 - \rho_m) + \rho_m m_{h,t-1} + \varepsilon_{m,t}, \quad \varepsilon_{m,t} \tilde{\ } N(0, \sigma_m^2)$$

2.3.6   Calibration

We calibrate the model to standard choices in the literature. We set $\beta = 0.994$ as in Villaverde et al (2012) and Guerrieri and Iacoviello (2015). Also, as it is common in the New Keynesian literature, we set the parameter $\theta = 0.75$ as in Christiano et al (2005) and Eichhenbaum and Fisher (2007).

The Taylor rule and its parameters are conventional in the literature $\phi_\pi = 1.5$ and $\phi_c = 0.25$ as in Christiano et al (2011).

The volatility of the monetary shock is set to $\sigma_m = 0.0025$, which is in line with the calibration in Villaverde et al (2012) and Guerron-Quintana (2010).

We set the elasticity of substitution among good varieties to $\sigma = 2$ and the shape parameter of the Pareto distribution to $\kappa = 8$.

2.4   Results

The idea in this section is to study the effect of monetary policy on the variables that describe the trade dynamics as well as the macroeconomic dynamics. We study the responses of the endogenous variables to a monetary shock during normal times and to a monetary shock that leaves the economy at the ZLB.

2.4.1   Monetary policy shock in the home economy, normal times

First, we implement a monetary shock that decreases the domestic nominal interest rate as shown in Figure (2.1). The dynamic of the nominal interest rate and the inflation induced by the shock generate a decrease in the real interest rate with respect to the steady state value. As the effect of the temporal shock vanishes, the real interest rate goes back to the initial steady state level.

In the home economy, the movement of the real interest rate induces a positive change in the consumption path with respect to the steady state level. This is a traditional outcome that follows from the consumption Euler equation. The increase in the current consumption against future consumption strengthens the economic activity and the wage rate rises. The increment in the wage rate increases the marginal cost for domestic non-exporter firms. Similar to the mechanism explained in the closed economy case, the increase in the marginal cost will increase

the productivity threshold that characterizes active domestic non-exporter firms. Thus, $\bar{z}_{h,t}$ increases.

In the foreign economy, the increase in the foreign interest rate will decrease current consumption (foreign Euler equation). The decrease in the current foreign consumption weakens the economic activity and the foreign wage rate drops. The real price of foreign exporter firms is set in units of domestic consumption. Therefore, the real marginal cost for foreign exporter firms increase due to the effect of the real exchange rate. Thus, the increment in the marginal cost for foreign exporter firms (in units of domestic consumption) will trigger an increase in the productivity threshold for foreign exporter firms, $\bar{z}_{f,x,t}$.

As a result, the monetary shock generates an extensive margin adjustment in both the number of domestic non-exporter firms and the number of foreign exporter firms. Thus, $\bar{z}_{h,t}$ and $\bar{z}_{f,x,t}$ increase generating an increase in the average productivity of firms serving the domestic economy. This extensive margin adjustment reduces the aggregate price of the domestic economy, $P_{h,t}$.

The outcome of the monetary shock in the domestic economy is that current consumption increases and inflation decreases. The extensive margin adjustment is such that the average productivity of domestic non-exporter firms and foreign exporter firms increases. The aggregate domestic profits, $D_{h,t}$, increase. Using equation (2.39), we see that the path followed by the value of domestic firms, $v_{h,t}$, is induced by the change in the path of wages, $w_{h,t}$.

Similarly, looking at Figure (2.2) and using the same line of reasoning than in the domestic economy, we see that the drop in the foreign wage negatively affects the marginal cost for foreign non-exporter firms. Therefore, the productivity threshold for foreign non-exporter firms, $\bar{z}_{f,t}$, decreases. Also, because of the effect of the real exchange rate, the marginal cost for domestic exporters serving the foreign economy is reduced, and thus the productivity threshold for domestic exporters, $\bar{z}_{h,x,t}$, decreases as well. The combined effect of the drop in the average productivity of domestic and foreign firms serving the foreign economy is
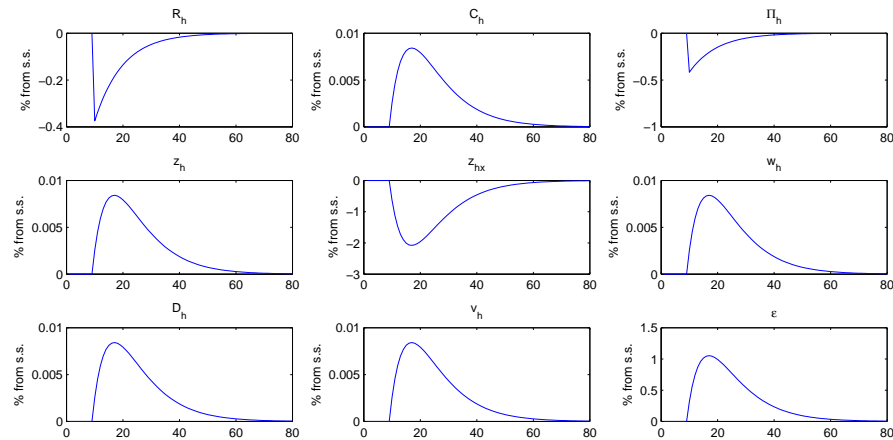
Figure 2.1: Response of Domestic endogenous variables to a monetary shock during "Normal Times"

associated to the increase in foreign inflation, $\Pi_{f,t}$.

The total profits generated by foreign firms, that is foreign exporter and non-exporter firms, decrease and the value of foreign firms decrease as well. The path followed by the value of foreign firms, $v_{f,t}$, is induced by the change in the path of wages, $w_{f,t}$.

In summary, during normal times, a monetary shock in the home economy that reduces the nominal interest rate has a positive effect in the domestic economy, stimulating the domestic consumption path, $C_{h,t}$. This is exactly what is expected from such policy, and thus our model generates consistent predictions during normal times.

2.4.2 Monetary policy shock, special times

Now we look at the effect generated by a monetary shock that leaves the economy at the ZLB as shown in Figure (2.3). As mentioned before, what generates a concern towards the ZLB issue is the existence of a nominal rigidity. Otherwise, the ZLB is meaningless with regards to the real allocations in equilibrium.

At the ZLB, the relationship between inflation and nominal interest rate
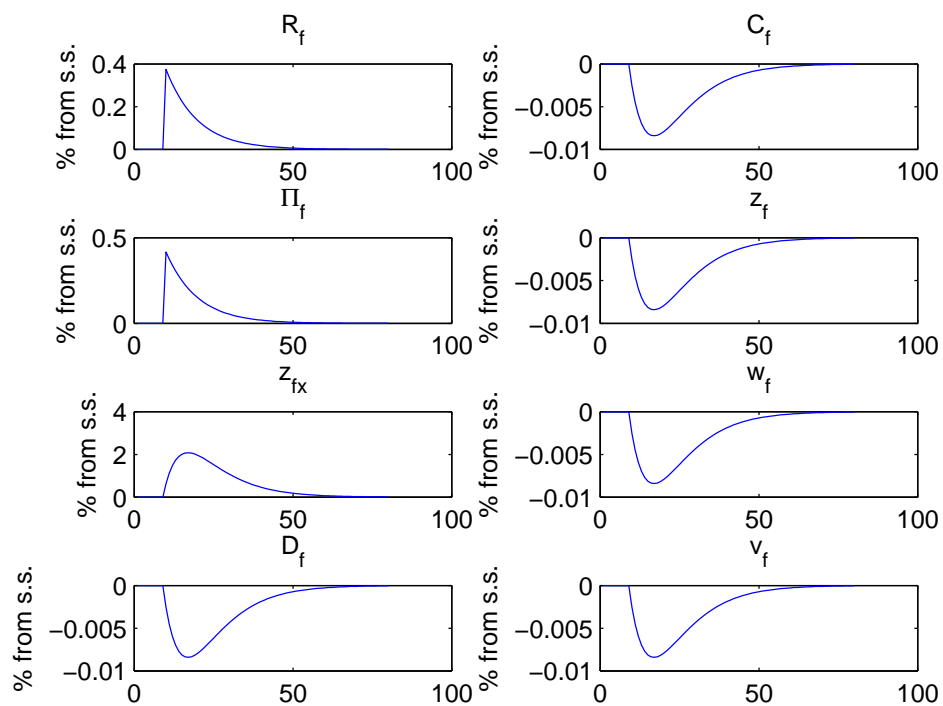
Figure 2.2: Response of Foreign endogenous variables to a monetary shock during "Normal Times"

is not smooth because the Taylor rule is not active. The nominal interest rate is equal to zero at the bound where it hits a kink. In fact, the shock explored here triggers a regime switch from an economy under the Taylor rule to an economy in which $R_t = 1$.

To begin the analysis of a monetary policy that leaves the economy at the zero bound, let us first conjecture that expected inflation decreases, as a result of a shock, such that current consumption decreases via the relationship established by the Euler equation. The decrease in current consumption weakens the economic activity and the wage rate goes down, and thus the marginal cost of production for domestic non-exporter firms goes down. Therefore, the marginal domestic non-exporter firm willing to produce is a firm with a lower productivity level which implies that $\bar{z}_{h,t}$ decreases with respect to the steady state level.

Exporting firms set prices in the denomination of the destination. Thus, the home appreciation in the exchange rate lowers the marginal cost for foreign exporter firms causing a drop in the productivity threshold that characterize active exporting foreign firms. As a result $\bar{z}_{f,x,t}$ decreases.

The decrease in both $\bar{z}_{h,t}$ and $\bar{z}_{f,x,t}$ imply that the average productivity of domestic and foreign firms serving the domestic economy is reduced. This extensive margin adjustment generates an increase in the price level, and thus domestic inflation, $\Pi_{h,t}$, rises. This increase in domestic prices is consistent with the initial conjecture in which expected inflation decreases.

Conversely, as shown in Figure (2.4) and following the same line of reasoning as in the domestic economy, we see a rise in foreign consumption and foreign wages. Thus, the marginal cost of production increases and so does the productivity threshold for foreign non-exporter firms. So, $\bar{z}_{f,t}$ goes up.

The home appreciation in the exchange rate increases the marginal cost of production for domestic exporter firms. Therefore, the marginal domestic firm willing to export is a firm with a higher productivity level. So, $\bar{z}_{h,x,t}$ goes up.

The increase in both $\bar{z}_{f,t}$ and $\bar{z}_{h,x,t}$ imply that the average productivity of
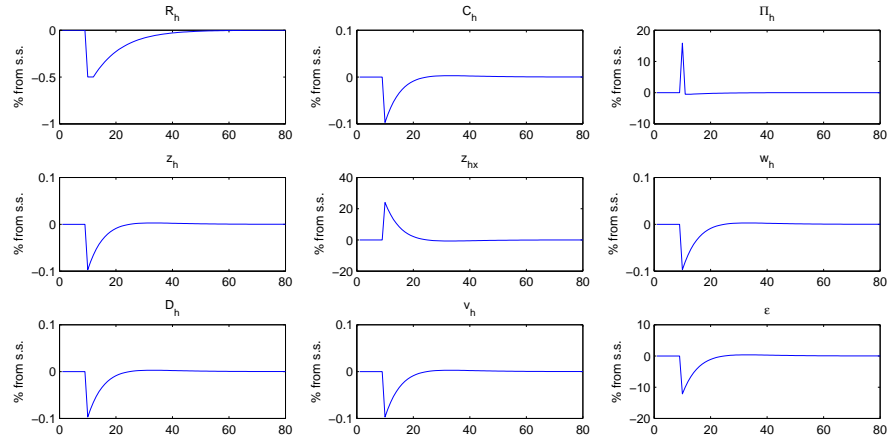
Figure 2.3: Response of Domestic endogenous variables to a monetary shock (ZLB)

domestic and foreign firms serving the foreign economy increases. This extensive margin adjustment generates a decrease in the foreign price level, and thus foreign inflation, $\Pi_{f,t}$, drops.

In summary, a monetary policy shock that leaves the economy at the ZLB generates outcomes in the economy which are exactly, qualitatively speaking, the opposite to the results exhibit when the economy is not at the ZLB.
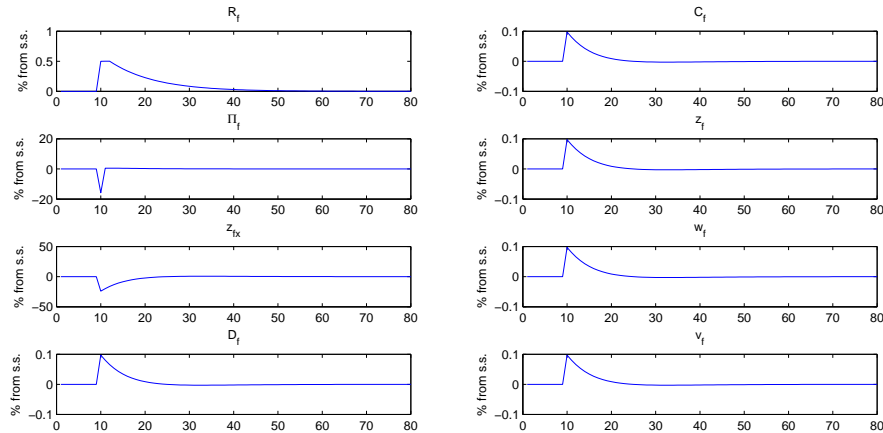
Figure 2.4: Response of Foreign endogenous variables to a monetary shock (ZLB)

2.5   Conclusions

In this chapter we develop a model that incorporates the main components of the international trade literature (firm heterogeneity) and nominal rigidities. The model crafted in this chapter allows us to study the role played by monetary policy on trade variables during normal times and at the ZLB. It is also the case that our model allows us study some macro implications of firm heterogeneity at normal times and at the ZLB.

The model is also able to describe the traditional role played by monetary policy in the short-run when it is a feasible tool during normal times. The exercises explored in this chapter show the potential opposite effects on the economy from monetary policies leaving the economy away from the zero bound and monetary policies that leave the economy at the ZLB.

The dynamics studied in the chapter are tied to the extensive margin adjusment mechanism which is the main charecteristic of firm heterogeneity.

2.6 Appendix A

For a general nominal price p_t, and a general productivity threshold z_t, we have that:

$$p_t^{1-\sigma} = \theta \cdot p_{t-1}^{1-\sigma} + (1-\theta) \cdot p_{t-1}^{*1-\sigma} \tag{2.40}$$

where p_t is the price charged by a firm z that is not able to adjust price at t. Similarly, p_t^* is the price charged by a firm z that is able to adjust price at t.

Following the recurrence relation shown in (2.40) in addition to the assumption that at the origin of time every firm is able to choose price such that $p_o = p_o^*$, we have that:

$$p_t^{1-\sigma} = \theta^{t-1} p_o^{*1-\sigma} + (1-\theta) \sum_{s=1}^{t-1} \theta^{t-1-s} p_s^{*1-\sigma} \tag{2.41}$$

We know from equation (2.22) that:

$$p_s^*(z)^{1-\sigma} = \left[P_s \left(\frac{\sigma}{\sigma-1} \right) (mcc_s) \frac{1}{z} \right]^{1-\sigma} \tag{2.42}$$

where mcc_s correspond to the component of the marginal cost that is independent of z at time s.

Then, we can re-write (2.41) as:

$$p_t(z)^{1-\sigma} = (1-\theta) \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left[\frac{\theta^{t-1}}{1-\theta} \left(P_o \left(mcc_o \right) \right)^{1-\sigma} + \sum_{s=1}^{t-1} \theta^{t-1-s} \left(P_s \left(mcc_s \right) \right)^{1-\sigma} \right] z^{\sigma-1} \tag{2.43}$$

Thus,

$$\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z) = \left(\frac{(1-\theta)\kappa z_t^{\sigma-1}}{\kappa+1-\sigma} \right) \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left[\frac{\theta^{t-1}}{1-\theta} \left(P_o \left(mcc_o \right) \right)^{1-\sigma} + \sum_{s=1}^{t-1} \theta^{t-1-s} \left(P_s \left(mcc_s \right) \right)^{1-\sigma} \right] \tag{2.44}$$

$$\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z) = \left(\frac{\kappa}{\kappa + 1 - \sigma}\right) p_t(z_t)^{1-\sigma} \qquad (2.45)$$

Thus, finallay we have:

$$\frac{\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z)}{P_t^{1-\sigma}} = \left(\frac{\kappa}{\kappa + 1 - \sigma}\right) \frac{p_t(z_t)^{1-\sigma}}{P_t^{1-\sigma}}$$

Also, since $\frac{\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z)}{P_t^{1-\sigma}} = \Phi_t^{\sigma-1}$, then $\left(\frac{p_t(z_t)}{P_t}\right)^{1-\sigma} = \left(\frac{\kappa+1-\sigma}{\kappa}\right) \Phi_t^{\sigma-1}$.

2.7 Appendix B

From Appendix A, we have that:

$$\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z) = \left(\frac{(1-\theta)\kappa z_t^{\sigma-1}}{\kappa+1-\sigma}\right) \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[\frac{\theta^{t-1}}{1-\theta} \left(P_o\left(mcc_o\right)\right)^{1-\sigma} + \sum_{s=1}^{t-1} \theta^{t-1-s} \left(P_s\left(mcc_s\right)\right)^{1-\sigma}\right]$$

$$(2.46)$$

$$\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z) = \frac{1}{\theta} \left(\frac{(1-\theta)\kappa z_t^{\sigma-1}}{\kappa+1-\sigma}\right) \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[\frac{\theta^{t}}{1-\theta} \left(P_o\left(mcc_o\right)\right)^{1-\sigma} + \sum_{s=1}^{t-2} \theta^{t-s} \left(P_s\left(mcc_s\right)\right)^{1-\sigma} + \theta \left(P_{t-1}\left(mcc_{t-1}\right)\right)^{1-\sigma}\right]$$

$$(2.47)$$

Conversely:

$$\int_{z_{t-1}}^{\infty} p_{t-1}(z)^{1-\sigma} dG(z) = \frac{1}{\theta^2} \left(\frac{(1-\theta)\kappa z_{t-1}^{\sigma-1}}{\kappa+1-\sigma}\right) \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[\frac{\theta^{t}}{1-\theta} \left(P_o\left(mcc_o\right)\right)^{1-\sigma} + \sum_{s=1}^{t-2} \theta^{t-s} \left(P_s\left(mcc_s\right)\right)^{1-\sigma}\right]$$

$$(2.48)$$

Thus,

$$\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z) = \left(\frac{z_t}{z_{t-1}}\right)^{\sigma-1} \left[\theta \int_{z_{t-1}}^{\infty} p_{t-1}(z)^{1-\sigma} dG(z) + (1-\theta)P_t^{1-\sigma}\Phi_{t-1}^{*\sigma-1}\right]$$

$$\frac{\int_{z_t}^{\infty} p_t(z)^{1-\sigma} dG(z)}{P_t^{1-\sigma}} = \Pi_t^{\sigma-1} \left(\frac{z_t}{z_{t-1}}\right)^{\sigma-1} \left[\theta \frac{\int_{z_{t-1}}^{\infty} p_{t-1}(z)^{1-\sigma} dG(z)}{P_{t-1}^{1-\sigma}} + (1-\theta)\Phi_{t-1}^{*\sigma-1}\right]$$

$$\Phi_t^{\sigma-1} = \Pi_t^{\sigma-1} \left(\frac{z_t}{z_{t-1}}\right)^{\sigma-1} \left[\theta\Phi_{t-1}^{\sigma-1} + (1-\theta)\Phi_{t-1}^{*\sigma-1}\right]$$

Chapter 3

A Note on Government Debt and Bubbles

Abstract. The goal of this chapter is to study the interactions between government debt and bubbles in an economy. We consider a general equilibrium approach in a productive economy and we explore conditions under which government debt path in our model is consistent with the government debt path observed in the last twenty years. During that period, government debt, as share of GDP, has interacted with bubbles in a countercyclical pattern. That is, in the absence of bubbles there is an increase in the evolution of debt-to-GDP ratio, and when a bubble is traded, debt-to-GDP ratio is decreasing.

In this chapter we characterize bubble cycles under exogenous investors sentiments and we study the interaction of these cycles with the government debt path. In equilibrium, our model shows that countercyclical path between bubbles and debt are possible. However, it is also possible to find equilibrium conditions under which debt and bubbles move along.

3.1   Introduction

There is empirical evidence for the U.S. economy that since the early 90s until now there exist a counter cyclical relationship between government debt and bubbles in the economy (Figure 1). Kraay and Ventura (2007) explore this phenomenon suggesting a close relationship between government behavior and bubbly assets.

Figure (3.1) shows the evolution of the $S\&P500$ series from 1980 to 2012 (quarterly) and the evolution of the government debt as a ratio of the GDP. Since early 90s it is possible to see a clear counter cyclical pattern between these two series. The correlation between these two series from 1990 to 2012 is -0.167. During the entire period the correlation is -0.096. This relationship motivates our research in the sense that, given that pattern, we may conjecture the existence of an endogenous relationship that describes co-movements between government debt and bubbles.

Figure (3.1) shows the most popular bubble events in the U.S. economy: the dot-com bubble and the subprime bubble. The dot-com bubble takes place in the economy from the mid 90s to the early 2000s. During that period government debt over GDP showed a decreasing path since the onset of the bubble (approximately third quarter of 1994) until reach the minimum level when the stock index showed its pick in the second quarter of 1999. After that, the bubble burst, the stock index decreases sharply, and the government debt increases sharply as well.

A similar pattern is shown for the sub-prime bubble. In the mid 2000s, changes in government debt is decreasing while the stock index is increasing. After the subprime bubble burst, both government debt and the price index changed sharply in opposite directions.

It is possible to rationalize this counter cyclical pattern between government debt and bubbles in, at least, a couple of ways. One view on this matter is related to the government behavior when a bubble bursts. Following the burst,
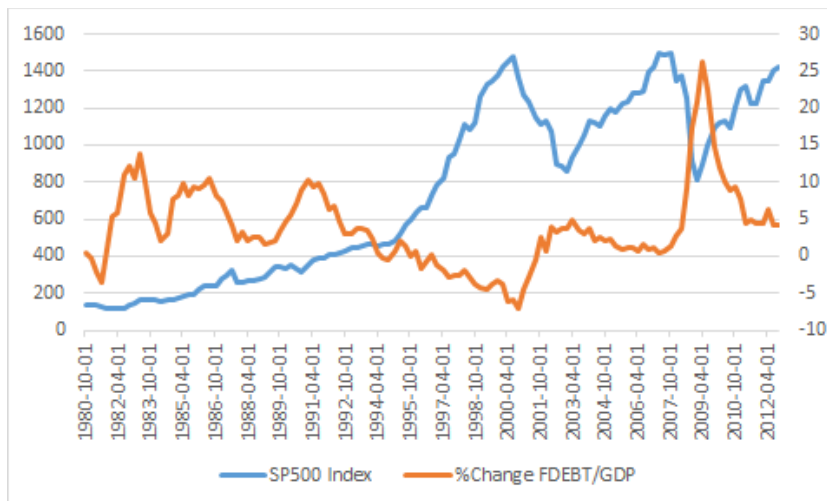
Figure 3.1: Stock index and Government Debt

the economy may face a crisis which may cause the government to intervene by implementing welfare programs or transfers in the classical counter cyclical spirit role of the government. Those type of government interventions naturally increase government debt. It is also possible to think that when there is a crisis in the economy, income and wealth may be reduced and the amount of taxes collected by the government are negatively affected.

The two possible consequences of the bubble bursting (namely, the increase in welfare programs (transfers) and a reduction in taxes collected) have an unambiguous effect on debt. That is, there is an increase in government debt. The relationship between crisis and government debt has been analyzed extensively (Barro, 1979; Aiyagari et al, 2002; Barro and Redlick, 2011). However, we will consider in this chapter an alternative approach to the counter cyclical pattern between debt and bubbles. The mechanism that we will employ to analyze the interaction between government debt and the bubbles is more related to the literature of asset shortages (Caballero, 2006, 2009; Gourinchas, 2012; Caballero et al, 2008).

We will consider stochastic bubble processes in our model. Using this set up, we will also consider investors sentiments to characterize investors attitude

towards a bubbly asset. In equilibrium, we are able to replicate the pattern shown in Figure (3.1) in which government debt follows an increasing path if a bubble is not present in the economy and a decreasing path otherwise. There are reasons to consider that this type of government behavior is an optimal decision for the economy. The main reason is that the government may be concerned about the dynamic inefficiency in the economy and these policies may help the economy to crowd out inefficient investment via government debt (Kraay and Ventura, 2007; Martin and Ventura 2012). Another explanation is that government debt, in the absence of bubbles, provides liquidity to the economy (Woodford, 1990). In the case in which a bubble is traded in the economy, the bubble can play that role of crowding out inefficient investment and it provides liquidity by its own, and thus government debt can be reduced without compromising the liquidity needs.

From the asset shortages perspective we look at the hypothesis considering that the global economy has been experiencing asset supply shortages over the last twenty years (Caballero, 2006, 2009; Gourinchas, 2012). In this context, it is plausible that fast growing economies are not able to produce enough financial assets to store value and transfer wealth across periods. There are many reasons why it could be the case. For example weak bankruptcy laws, chronic macroeconomic volatility, sheer expropriation risk, etc.

Asset supply shortages generate mechanisms for price adjustment. This price adjustment is essentially the compensation that markets would create to store the excess of wealth by increasing the real value of existing assets. The market-mechanisms through which the real value of existing assets rise may be the decline in long run real interest rates, speculative bubbles, or deflationary pressures.

The main evidence consistent with asset shortages is related to the decline in world real interest rate over the last twenty years. There is also evidence of a disconnection between the short and long interest rate. This is particularly evident from the early 2000s, which suggests that the long rates are driven by some additional economic characteristic other than the short rates tendency. We

may also look at the literature that has explored global imbalances, especially the imbalance between the asset position of Asia and the U.S. Those imbalances also support the asset shortages hypothesis.

During the recent subprime crises, part of the story behind the subprime bubble was closely related to the amount of synthetic assets in the economy. This type of assets were created to increase the average price of the assets involved in the composition of those synthetic assets. The idea is to increase the average rate qualification of some assets by repackaging them. This story is also consistent with the need of additional assets or the increase in value of the existing ones to deal with asset shortages.

The asset shortages evidence inspires a central feature of our research which is non-fundamental valuation. We understand the non-fundamental valuation of an asset as the real difference between the real price of the asset and the economic value of the asset (the fundamental value). The difference between these two values is the bubble component in the valuation of a particular asset.

Figure (3.2) shows the evolution of the real housing price for the period 1990-2011. The subprime episode is shown in the period between 1998 and the late 2000s. During that period, we may argue that there was a disconnection between the price value and the fundamental value. That disconnection is employed in this chapter to model a bubbly asset. In particular we use for simplicity an asset whose fundamental value is zero. That means, the price of the asset has just the bubble component.

The idea of a bubbly asset with non fundamental valuation is employed in Kraay and Ventura (2007) and Martin and Ventura (2012).

Our chapter follows the traditional approach of the theory of rational bubbles first described by Samuelson (1958) and Tirole (1985). We include a financial friction into the model as in Caballero and Krishnamurthy (2006) or Farhi and Tirole (2011). However, we follow the set up employed in Kraay and Ventura (2007) and Martin and Ventura (2012).
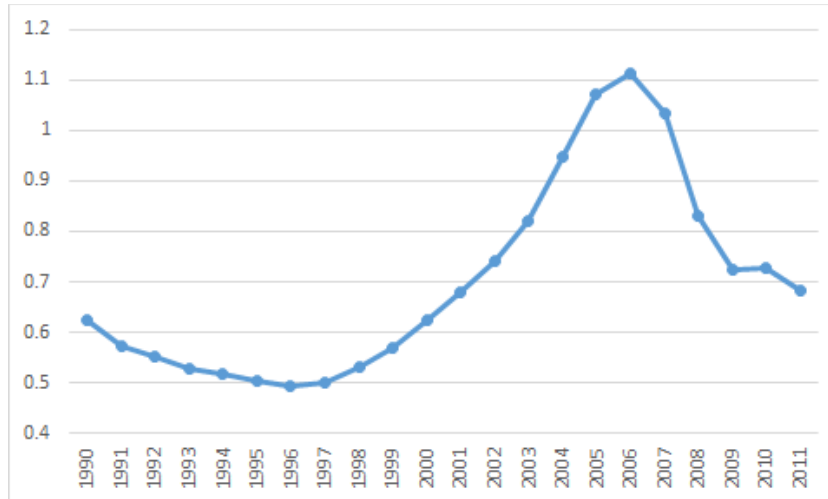
Figure 3.2: Evolution of U.S. real housing price Index, 1990-2011, Case-Shiller 10-city composite home price index adjusted by U.S. Consumer Price Index

Kraay and Ventura (2007) study an economy in which exogenous investor sentiments define whether a bubbly asset is traded in the economy. They also study the interaction of government debt and the presence of a bubbly asset in the economy. We share a similar setup of the model with Kraay and Ventura. However this chapter differs from their work in two dimensions. The first one is that in our model we consider a productive economy in which firms are modeled as function of capital and it accumulates endogenously. Another difference is that in our work a steady state with bubble exists, and the dynamics described here are on the equilibrium path.

Martin and Ventura (2012) provide a very insightful model to study economic growth in an economy with bubbles. A bubbly asset is central in their paper, but they are focused on the interaction between the bubble and capital. We share with their paper many characteristics of the model. In fact, we borrow from them the model for the productive economy, but we extend their work by considering the interaction of the bubble with government debt as well as capital.

The organization of the chapter is as follow: Section 3.2 describes the theoretical model employed in the chapter. Section 3.3 contains the equilibrium con-

dition when there is no bubbles in the economy, and in Section 3.4, we show the equilibrium condition for government debt when a bubble is traded in the economy. In Section 3.5, we provide some conclusions.

3.2 Model

In this chapter we consider a productive economy which is populated by overlapping generations. Each generation has a mass of agents equal to one, indexed by $i \in I_t$, and agents live for two periods. In the first period agents are young and in the second period agents are old. At any given time t, we have co-existing young agents who belong to generation t, and old agents who belong to generation $t - 1$.

Individuals maximize expected old-age consumption, i.e. $U_{it} = E_t\{c_{it+1}\}$, where U_{it} and c_{it+1} are the utility and the expected old-age consumption of individual i from generation t. Since they are risk-neutral, agents choose the portfolio that maximizes the expected return to their savings.

The technology employed in this economy is a Cobb-Douglas production function $F(l_t, k_t) = l_t^{1-\alpha} k_t^{\alpha}$ with $\alpha \in (0, 1)$, where l_t and k_t are the labor force and the capital stock, respectively. Markets are competitive and agents supply inelastically one unit of labor. The factors of production are paid the value of their marginal product, that is $w_t = (1 - \alpha)k_t^{\alpha}$ and $r_t = \alpha k_t^{\alpha-1}$. Thus, we have that w_t is the wage in this economy and r_t is the rental rate.

The dynamic for capital is defined by the investment made by young agents. In this economy there are two types of young agents. Productive young agents (P) of mass ϵ, and unproductive young agents (U) of mass $1 - \epsilon$. In particular, young agents can produce one unit of capital with one unit of output, while the rest only have access to an inferior technology that produces $\delta < 1$ units of capital with one unit of output.

There is a financial friction in this economy. Productive agents are not

allowed to invest on behalf of less productive ones. Thus, less productive agents invest on their own. The dynamic for capital accumulation is then given by:

$$k_{t+1} = A \cdot s \cdot k_t^\alpha \tag{3.1}$$

where $s = 1 - \alpha$ is a constant fraction that characterizes savings, and $A = \epsilon + (1 - \epsilon)\delta$ is the average efficiency of investment.

In this economy we will also consider a market for government debt. Agents are also able to save by purchasing government bonds. In the case an agent invests in government debt at time t, that agent will enjoy a return r_{t+1}^d at $t + 1$. There is no difference in the return obtained by investing in government debt whether the investment is made by a productive young agent or not. The dynamic for government debt is given by:

$$D_{t+1} = r_{t+1}^d D_t + T_{t+1} \tag{3.2}$$

where r_{t+1}^d is the return associated with buying government debt at time t, and T_{t+1} are the transfers made by the government.

In this economy, we will also have a market for bubbles. That is, agents will be able to include in their portfolios a bubbly asset.

3.2.1 Equilibrium with Bubbles

We introduce now a market for bubbles. Bubbles start randomly and without cost, they do not produce any output and the only reason to purchase them is to resell them later. A bubbly asset has no fundamental valuation. That is, there is no capital behind this asset, and there is no dividends associated with holding this asset. The bubbly asset is traded by the agents (young agents buy it) just because they expect to resell it at a higher price when they are old.

There are three type of bubbles: b_t which is the market price of the portfolio

that contains all old bubbles, i.e. already existing before period t or created by earlier generations. b_t^P and b_t^U are the market prices of the portfolios that contains all new bubbles created by productive and unproductive agents respectively, i.e. bubbles created in period t by generation t.

Bubbles are supplied by old agents who bought them when they were young, and young agents who are lucky to create new bubbles. Bubbles are demanded just by young agents, since old agents do not save.

Let $\{b_t, b_t^P, b_t^U\}_{t=0}^\infty$ be a non-negative stochastic process for the bubble. Let's define $h_t = \{b_t, b_t^P, b_t^U\}$ as the realization of the bubble shock in period t, and h^t as a history of bubble shocks until period t; and H_t as the set of all possible histories. Then, a stochastic process $\{b_t, b_t^P, b_t^U\}_{t=0}^\infty$ is an equilibrium if $b_t + b_t^P + b_t^U > 0$ for some t and there exist a non-negative sequence $\{k_t(h^t), D_t(h^t)\}_{t=0}^\infty$ that satisfies individual maximization and market clearing condition for all t.

The marginal buyer of the bubble changes as the bubble grows. If the bubble is small, the marginal buyer is the unproductive agents and the expected return in holding the bubble is equal to the return to the unproductive investment in capital. If the bubble is large, the marginal buyer is the productive agent, so the expected return in holding the bubble must be equal to the return to the productive investment in capital.

$$E_t\left\{\frac{b_{t+1}}{b_t + b_t^P + b_t^U}\right\} = \begin{cases} \delta\alpha k_{t+1}^{\alpha-1} & \text{if } b_t + b_t^P < (1-\epsilon)sk_t^\alpha - D_t \\ \in [\delta k_{t+1}^{\alpha-1}, \alpha k_{t+1}^{\alpha-1}] & \text{if } b_t + b_t^P = (1-\epsilon)sk_t^\alpha - D_t \\ \alpha k_{t+1}^{\alpha-1} & \text{if } b_t + b_t^P > (1-\epsilon)sk_t^\alpha - D_t \end{cases} \quad (3.3)$$

There is free disposal so bubbles must be positive. Bubbles and debt cannot exceed the savings of the young. So, it must be the case that:

$$0 \le b_t \le sk_t^\alpha - D_t \quad (3.4)$$

Equations (3.3) and (3.4) summarize two main characteristics for bubbles to exist: The bubble need to grow fast enough in order to be desirable for young agents (equation (3.3)). However, the bubble cannot grow too fast or it will become unsustainable (equation (3.4)).

The dynamic for capital is shown in equation (3.5).

$$k_{t+1} = \begin{cases} Ask_t^\alpha - \delta(b_t + D_t) + (1 - \delta)b_t^P & \text{if } b_t + b_t^P < (1 - \epsilon)sk_t^\alpha - D_t \\ sk_t^\alpha - b_t - D_t & \text{if } b_t + b_t^P \geq (1 - \epsilon)sk_t^\alpha - D_t \end{cases} \quad (3.5)$$

If the bubble is small, capital accumulation corresponds to the saving of the unproductive minus the bubbles they purchase weighted by the efficiency of unproductive investors to build capital. If the bubble is large, then unproductive investors are not building capital and capital accumulation corresponds to the savings of the productive agents minus what they invest in bubbles.

In equation (3.5), we can see the crowding out effects shown in Martin and Ventura (2012). First, old bubbles slow down capita accumulation. At the beginning, the bubble crowds out inefficient capital and then, as the bubble grows, it crowds out efficient capital. Second, there is a reallocation effect that increases the average efficiency in the economy when the productive young agents sell bubbles to the unproductive ones. This is because investment made by productive agents replace the one made by unproductive ones. Then b_t^P speeds up capital accumulation.

In our setting there is a third effect on capital accumulation characterized by the role played by government debt. Government debt competes with the realization of bubbles. In equation (3.5) government debt also crowds out inefficient investment but do not speed up capital accumulation.

3.2.2 Existence of Bubbles

Following Martin and Ventura (2012) we rewrite the variables in our model as a share of the saving of the young. That is, we can define $x_t = \frac{b_t}{ak_t^\alpha}$, $x_t^P = \frac{b_t^P}{ak_t^\alpha}$, $x_t^U = \frac{b_t^U}{ak_t^\alpha}$, and $d_t = \frac{D_t}{ak_t^\alpha}$. Then, we can rewrite equations (3.3) and (3.4) as follows:

$$E_t x_{t+1} = \begin{cases} \frac{\alpha}{s} \frac{\delta\left(x_t + x_t^P + x_t^U\right)}{A - \delta(x_t + d_t) + (1-\delta)x_t^P} & \text{if } x_t + x_t^P < 1 - \epsilon - d_t \\ \in [\frac{\alpha}{s} \frac{\delta\left(x_t + x_t^P + x_t^U\right)}{A - \delta(x_t + d_t) + (1-\delta)x_t^P}, \frac{\alpha}{s} \frac{x_t + x_t^P + x_t^U}{1 - x_t - d_t}] & \text{if } x_t + x_t^P = 1 - \epsilon - d_t \\ \frac{\alpha}{s} \frac{x_t + x_t^P + x_t^U}{1 - x_t - d_t} & \text{if } x_t + x_t^P \geq 1 - \epsilon - d_t \end{cases} \quad (3.6)$$

$$0 \leq x_t \leq 1 - d_t \quad (3.7)$$

Additionally, we have that government debt as a share of savings can be written as:

$$d_{t+1} = r_{t+1} d_t + g$$

where g is a constant.

Proposition 1 *For $0 < g < 1 - \epsilon, \exists d^*$ in which $g < d^* < 1 - \epsilon$ such that bubbly episodes are possible if and only if:*

$$\alpha < \frac{s}{\delta} \min\{\epsilon, 1 - \epsilon + g - 2\sqrt{(1-\epsilon)g}\}$$

Proof. We want to show the existence of positive values for a bubble and government debt in steady state.

In steady state, we have the following relationships, where for any variable v_t we have in steady state $v_t = v_{t-1} = v$. Without loss of generality, we set $x_t^U = 0$ for all t.

$$
x = \begin{cases}
\frac{\alpha}{s} \frac{\delta\left(x + x^P\right)}{A - \delta(x+d) + (1-\delta)x^P} & \text{if } x + x^P < 1 - \epsilon - d \\[2ex]
\in \left[\frac{\alpha}{s} \frac{\delta\left(x + x^P\right)}{A - \delta(x+d) + (1-\delta)x^P}, \frac{\alpha}{s} \frac{x_t + x^P}{1 - x - d}\right] & \text{if } x + x^P = 1 - \epsilon - d \\[2ex]
\frac{\alpha}{s} \frac{x_t + x^P}{1 - x - d} & \text{if } x + x^P \geq 1 - \epsilon - d
\end{cases}
\tag{3.8}
$$

We consider the same bubble process employed in Martin and Ventura (2012). That is:

$$
x^P = 1 - \epsilon - d - x \tag{3.9}
$$

Then, from equation (3.8), we have that:

$$
x_1 = \frac{\alpha \delta}{s}\left(\frac{1 - \epsilon - d}{1 - x_1 - d}\right) \tag{3.10}
$$

Also, in steady state we have that:

$$
d = rd + g \quad \text{and} \quad r = \frac{x_2}{1 - \epsilon - d}
$$

Thus,

$$
x_2 = (1 - \epsilon - d)\left(1 - \frac{g}{d}\right) \tag{3.11}
$$

From equation (3.10), we can solve for x_1:

$$
x_1 = \frac{(1 - d) - \sqrt{(1 - d)^2 - 4\frac{\alpha\delta}{s}(1 - \epsilon - d)}}{2} \tag{3.12}
$$

For x_1 to be real and positive we need $\Delta \geq 0$ in equation (3.12). This condition leads us to:

$$
\alpha < \frac{s}{\delta}\epsilon \tag{3.13}
$$

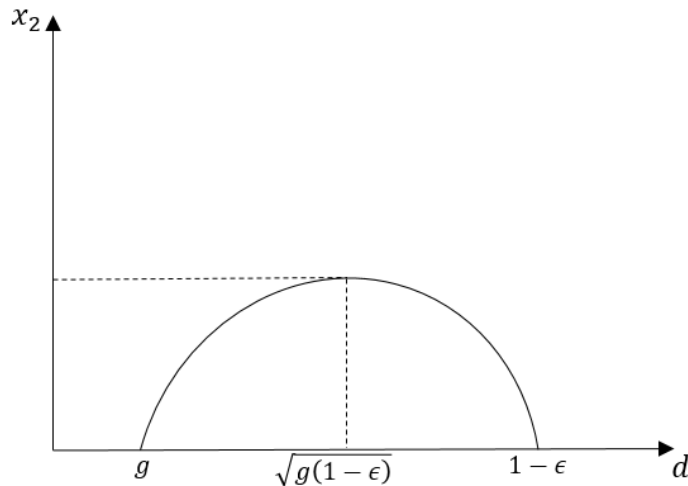From equation (3.11), we have that x_2 is positive as long as $g < d < 1 - \epsilon$.

Figure 3.3: Steady state existance

Since x_1 and x_2 are continuous, we need to show that:

- $x_1(g) > x_2(g)$, and

- $x_1(\sqrt{(1-\epsilon)g}) < x_2(\sqrt{(1-\epsilon)g})$

Using the condition shown in equation (3.13), we have that $x_1 > 0$ for any level of debt. Also, from equation (3.11) we have that $x_2(g) = 0$. Therefore, we have that $x_1(g) > x_2(g)$.

Using equation (3.12), we can show that:

$$x_1 < \frac{(1-d) - \sqrt{\left[(1-d) - 2\frac{\alpha\delta}{s}\right]^2}}{2} = \frac{\alpha\delta}{s} \tag{3.14}$$

So, in order to ensure that $x_1(\sqrt{(1-\epsilon)g}) < x_2(\sqrt{(1-\epsilon)g})$, we need that:

$$\frac{\alpha\delta}{s} < (1-\epsilon+g) - 2\sqrt{(1-\epsilon)g} \tag{3.15}$$

Then, if equations (3.13) and (3.15) hold, it must me the case that there exist $d^* \in (g, 1-\epsilon)$ such that $x_1(d^*) = x_2(d^*)$. ∎

3.3 Equilibrium debt without Bubbles

Whenever investors sentiments are low ($S_t = L$), the bubble is not traded in the economy. The bubbly asset has no fundamental valuation and the only reason to trade it is to resell it later. So, whenever investors sentiments are low, young agents are not willing to buy bubbly assets and the bubble bursts. Investors sentiments embody the intertemporal coordination mechanism that make bubbles feasible.

In this state of the economy, there is no bubble. That is, at any time t investors sentiments are low ($S_t = L$) and the bubbly asset is not traded. Young agents at time t have to decide whether they would invest in capital and/or government debt.

In an equilibrium in which government debt is crowding out inefficient investment in capital it must be the case that government debt offers a return greater or equal to the marginal productivity of capital for the unproductive agents. Otherwise, government debt would not be attractive and it would not be demanded.

In particular we look at the equilibrium in which government debt offers to agents a return equal to the marginal productivity of capital for the unproductive agents.

$$r_{t+1} = \delta \alpha k_{t+1}^{\alpha - 1} \tag{3.16}$$

Employing equation (3.2), government debt path can be written as in equation (3.17). This equation is a general equilibrium solution when there is no bubble in the economy or investors sentiments are low ($S_t = L$).

$$d_{t+1} = \left(\frac{\delta \alpha}{s} \right) \frac{d_t}{A - s d_t} + g_{NB} \tag{3.17}$$

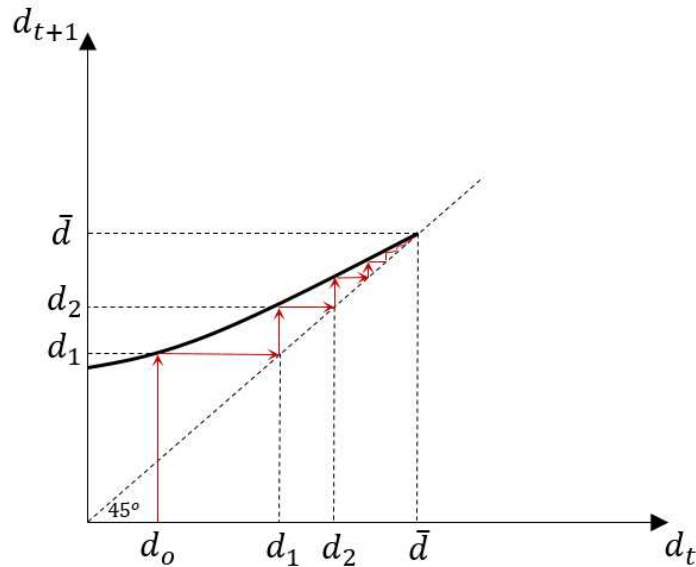where g_{NB} is such that when $d_t = \bar{d}$, then $d_{t+1} = d_t = \bar{d}$. Which means

Figure 3.4: Government debt path $(S_t = L)$

that debt path has reached a steady state level, where $\bar{d}$ is that steady state level[1].

Proposition 2 *In an equilibrium without bubbles, the debt path is convex. That is, d_{t+1} is increasing at increasing rates in d_t, and reaches a steady state $\bar{d}$.*

Proof. See Appendix for details. ∎

Figure (3.4) show the implication of Proposition 2. d_{t+1} is convex in d_t and reaches the steady state at $\bar{d}$. The evolution of debt is described by the path connecting a initial debt d_o to the point A which corresponds to d_1. The point B is the projection of debt d_1 to the 45 degree line. From d_1, debt path evolves to point C which correspond to point d_2. Government debt continue evolving under the pattern described before until it reaches a steady state level $\bar{d}$.

---

[1] $g_{NB} = \bar{d}\left(1 - \frac{\delta\alpha}{s\left(A - \delta\bar{d}\right)}\right).$

3.4   Equilibrium debt with Bubbles

In this state of the economy, investors sentiments are high $(S_t = H)$ which implies that the bubbly asset is traded. Young agents have to decide their investment portfolio. Agent's portfolio may contain capital, government debt and/or the bubbly asset.

We study an equilibrium in which the bubbly asset coexists with government debt[2]. That is why a no arbitrage condition establishes in equilibrium that government debt and the bubbly asset have to offer the same return. Otherwise, the asset with lower returns would not be demanded. So, it must be the case that, in equilibrium, the return on government debt r_{t+1}^d, is the same as the return on the bubbly asset, r_{t+1}^b.

$$r_{t+1}^d = r_{t+1}^b = r_{t+1} \tag{3.18}$$

As seen in Section 2, both government debt and the bubble crowd out inefficient investment. In fact, the economy would behave as a complete market when all inefficient investment is crowded out.

Debt and the bubbly asset will compete for the resources coming from the savings of the unproductive agents. The bubble has to grow fast enough such that it remains competitive for every generation. If it does not remain competitive, agents will not demanded it. Also, the bubble cannot grow too fast such that it becomes unsustainable at some point in time. If that were the case, the bubble becomes unsustainble for some generation $\tilde{t} > t$, then that generation would not demand the bubbly asset. By backward induction it can be shown that the bubble would not be demanded in the first place at t.

In Proposition 1, we show that a steady state exists for a bubbly asset that coexists with government debt. This result allows us to study the dynamic of bubbles and debt on the equilibrium path to the steady state. We will focus

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[2]Capital is always demanded in equilibrium. Otherwise, the marginal return in capital would be infinite.

in an economy in which the return for the bubbly asset and debt is given by $r_{t+1} \in [\delta \alpha k_{t+1}^{\alpha-1}, \alpha k_{t+1}^{\alpha-1}]$.

In particular we will consider the stochastic bubble generation process of new bubbles as in (3.19).

$$x_t^P = 1 - \epsilon - d_t - x_t \quad \text{and} \quad x_t^U = 0 \tag{3.19}$$

Martin and Ventura show that the fact that we consider $x_t^U = 0$ is without loss of generality, since the existence of the results does not rely on the particular realization of x_t^U.

We will consider a transition probability λ, which is the probability of a change in investor sentiment. This probability embodies the coordination needed among generations for the bubble to be traded.

The debt path shown in (3.2), now can be written as in (3.20)[3].

$$d_{t+1} = (1 - \lambda) \frac{(1 - x_{t+1}^k)}{1 - \epsilon - \lambda d_t} d_t + \frac{1 - \epsilon - d_t}{1 - \epsilon - \lambda d_t} g_B \tag{3.20}$$

where g_B is such that when $d_t = \bar{d}$, then $d_{t+1} = d_t = \bar{d}$. Which means that debt path has reached a steady state level, where $\bar{d}$ is that steady state level[4].

Proposition 3 *In an equilibrium with bubbles, the debt path is convex. That is, d_{t+1} is increasing at increasing rates in d_t, and reaches a steady state $\bar{d}$.*

Proof. See Appendix for details. ∎

As shown in Proposition 1, the steady state equilibrium for an economy with bubbles does not need to be unique. This is shown in Figure (3.5), where the points A' and B' illustrate such equilibrium points. The implications of Proposition 3 are shown in Figure (3.5), where d_{t+1} is convex in d_t and reaches the steady state at $\bar{d}$.

---

[3]where $x_t^k = \frac{k_t}{s k_t^\alpha}$.

[4]$g_B = \bar{d} \left(\frac{\lambda(1-\bar{d}) + (1-\lambda) x^k - \epsilon}{(1-\lambda)(1-x^k)} \right)$.
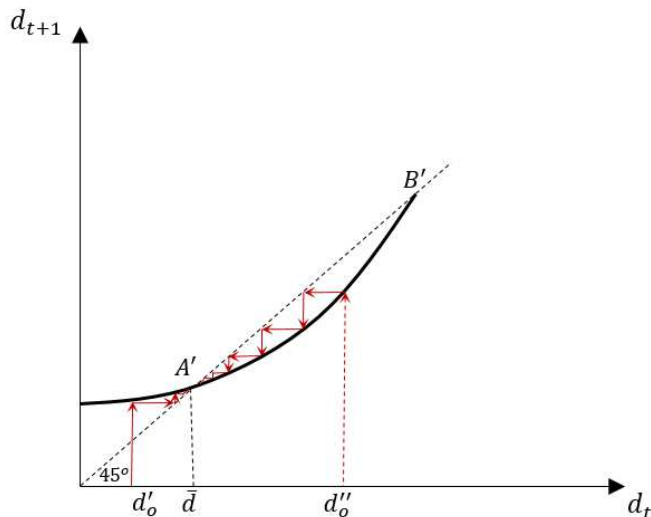
Figure 3.5: Government debt path $(S_t = H)$

The dynamic of debt is going to be different depending on the value of initial debt. If initial debt is d'_o (relatively small), then debt path is increasing. This occurs when the level of debt is small and there is enough room in the economy for both the bubble and debt to grow. This dynamic converges to the steady state equilibrium point A'. When initial debt is d''_o, then the debt follows a decreasing path. This tells us that the bubble is big enough such that in order to remain competitive, government debt needs to leave room and not compete with the bubbly asset for the resources of the unproductive agent. This dynamic also converges to the equilibrium steady state A'.[5]

Now, we study the dynamic of government debt in the economy considering shocks to investor sentiments. That is, whenever investors' sentiment is low $S_t = L$, the equilibrium dynamic is the one described in Section 3 where no bubbles are traded. Moreover, the debt path behaves according to Proposition 2. On the other hand, whenever investors' sentiments are high, $S_t = H$, the equilibrium that characterizes the economy is the one described in Section 4 and government debt behaves as shown in Proposition 3.

---

[5]The equilibrium point B' is an unstable equilibrium.

Figure (3.6) describes the case in which the initial levels of both the bubble and government debt are small. As long as investors sentiments are low ($S_t = L$), the debt path is described by the red line in Figure (3.6). Government debt is increasing and it is crowding out inefficient investment in capital. As soon as investors sentiments change from low to high ($S_t = H$) the dynamic of debt is described by the blue line which shows the equilibrium debt path when the bubbly asset is traded. The level of debt and the bubble are small such that the dynamic of debt describes an increasing path as the bubble grows. The economy reaches a steady state in which the bubbly asset and government debt are traded and they coexist in equilibrium.

According to our model, it is possible to find equilibrium dynamics of debt under which government debt and the bubbly asset follow an increasing path and together crowd out inefficient investment. This situation requires two conditions. First, the bubble needs to start small such that a sustainable growth allow it to stay competitive. Second, debt increases slowly such that the bubble has room to stay competitive and sustainable. Then, a steady state equilibrium is achieved with positive levels of debt and bubbles.

In Figure (3.7), we show the situation described in Kraay and Ventura (2007). In this case, when $S_t = L$ government debt crowds out inefficient investment in capital and it follows an increasing path until $S_t = H$. From that point on, debt has to be reduced to leave room for the bubbly asset to grow in such a way that it can remain competitive and sustainable. It is the case that when the economy is shocked with high sentiments the level of debt has reached a level that is too high and would not allow the bubble to grow fast enough. That is why when investors sentiments move from low to high, the equilibrium condition describing the debt path shifts from the red line (no bubble is traded in the economy) to the blue one (the bubble is traded in the economy). Thus, the debt dynamic describes a decreasing debt path until it reaches the steady state level $\bar{d}$.

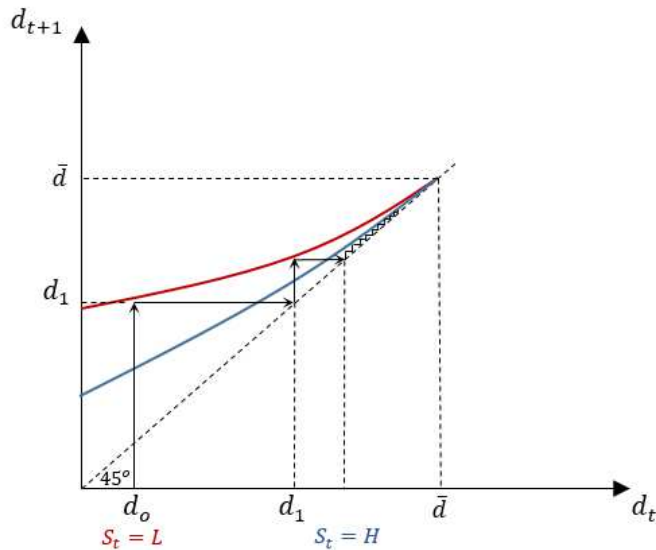The main difference between our exercise and the one shown in Kraay and

Figure 3.6: Pro-cyclical debt path

Ventura (2007) is that in our model the steady state exists. This mean that the dynamics described in this section occur on the equilibrium path. In the model presented by Kraay and Ventura (2007) the steady state does not exist. In that case, agents would not trade the bubbly asset in the first place since such dynamic is not part of the equilibrium.

Figure (3.7) describes the type of government debt path shown in the data. Government debt during dot-com bubble exhibited a counter cyclical pattern with respect to the bubble cycle.

As we saw in Figure (3.6), it is also possible to have government debt following a procyclical path along with the bubble. Then, naturally we may infer that a combination of such patterns might be possible in equilibrium. That is, in equilibrium, given investors sentiments we may have procyclical government debt behavior and countercyclical. This type of equilibrium is shown in Figure (3.8).

At t_o investors sentiment is high and the bubbly asset is traded in the economy along with a increasing debt path. Both, the bubble and government debt are crowding out inefficient investment. At t_1 the economy is shocked and the new condition for investors' sentiment, $S = L$, bursts the bubble. At time t_1,
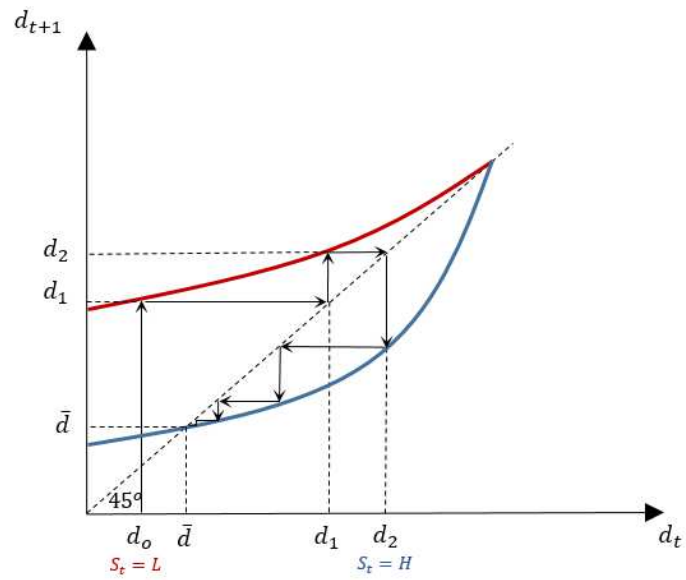
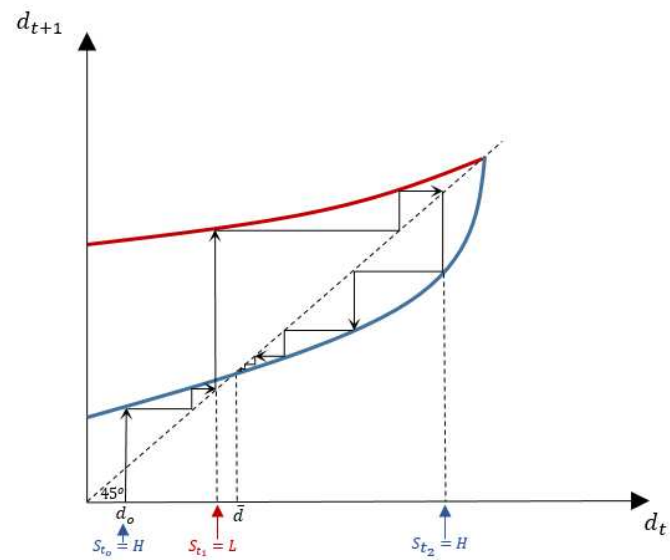Figure 3.7: Counter-cyclical debt path

Figure 3.8: Pro-cyclical and Counter-cyclical debt path

government debt shifts from the blue curve (bubbles is traded) to the red curve in which the bubble is not traded. The increase in the level of debt, after t_1, is due to the room left by the bubble when it bursts. Government debt increases along the red curve until the economy is shocked again with $S = H$ at t_2. At this point, the level of government debt is too high, which means that there is no enough room for the bubble to grow and stay competitive and affordable. That is why at t_2, debt begins to follow a decreasing path that allows the coexistence between the bubbly asset and government debt in equilibrium. An equilibrium steady state is possible at $d_t = \bar{d}$.

Under the model developed in this chapter we are able to find conditions under which bubbles and government debt coexist in equilibrium. We find an equilibrium dynamic in which government debt exhibits a countercyclical patter with respect to the bubbly asset (Figure 3.7). This is consistent with the debt behavior shown in the data (Figure 3.1). We also find an equilibrium dynamic in which government debt and the bubbly asset move in a procyclical pattern on the equilibrium path (Figure 3.6).

3.5   Conclusions

In this chapter we study the interactions between government debt and bubbles in an economy. In a general equilibrium context, we explore conditions under which government debt path in our model is consistent with the government debt path observed in the last twenty years. In particular, the model is able to show that it is optimal for the debt to move in a countercyclical pattern with respect to a bubble process. The model is also able to show that a pro-cyclical behavior between government debt and bubbles is possible in equilibrium.

The key aspect of the coexistence between government debt and bubbles is the feasibility of the bubble to grow at a rate fast enough such that the bubble stays competitive across generations and this process is sustainable given the resources

of the unproductive agents. This is important because the bubble feasibility will establish the optimal response of the government debt such that both assets coexist in equilibrium. If there is enough room in the economy, it is possible that the bubbly asset and government debt grow in equilibrium until they reach a steady state level. In the case that the level of debt reached is too high (when $S = L$), a bubble process would not be able to stay competitive and sustainable. Then, when the economy is shocked, such that $S = H$, the optimal response of the government debt is to follow a decreasing path.

In Figure (3.7), we examine the exercise proposed by Kraay and Ventura (2007). In that situation, government debt follows a counter-cyclical pattern with respect to bubbles. In our model, the steady state equilibrium with bubble exists (Proposition 1) and thus the dynamics shown in this chapter are on the equilibrium path.

3.6 Appendix

Proof for Proposition 2.

$$d_{t+1} = \left(\frac{\delta\alpha}{s}\right)\frac{d_t}{A - sd_t} + g_{NB}$$

$$\frac{\partial d_{t+1}}{\partial d_t} = \left(\frac{\delta\alpha}{s}\right)\frac{A}{(A - sd_t)^2} > 0$$

$$\frac{\partial^2 d_{t+1}}{\partial d_t^2} = \left(\frac{\delta\alpha}{s}\right)\frac{2sA}{(A - sd_t)^3} > 0$$

Proof for Proposition 3.

$$d_{t+1} = (1 - \lambda)\frac{(1 - x_{t+1}^k)}{1 - \epsilon - \lambda d_t}d_t + \frac{1 - \epsilon - d_t}{1 - \epsilon - \lambda d_t}g_B$$

$$\frac{\partial d_{t+1}}{\partial d_t} = (1-\lambda)\left[\frac{\left(1 - x_{t+1}^k - d_t\frac{\partial x_{t+1}^k}{\partial d_t}\right)(1 - \epsilon - \lambda d_t) + \left(1 - x_{t+1}^k d_t\right)\lambda - (1 - \epsilon)g_B}{(1 - \epsilon - \lambda d_t)^2}\right]$$

$$\frac{\partial d_{t+1}}{\partial d_t} > 0$$

$$\frac{\partial^2 d_{t+1}}{\partial d_t^2} = (1-\lambda) \left[\frac{\left(1 - 2\frac{\partial x_{t+1}^k}{\partial d_t} - d_t \frac{\partial^2 x_{t+1}^k}{\partial d_t^2}\right)}{(1 - \epsilon - \lambda d_t)} \right]$$

$$+ (1-\lambda) \left[\frac{\left(1 - x_{t+1}^k - d_t \frac{\partial x_{t+1}^k}{\partial d_t}\right)\lambda}{(1 - \epsilon - \lambda d_t)^2} \right]$$

$$+ (1-\lambda) \left[\frac{\left(-x_{t+1}^k - d_t \frac{\partial x_{t+1}^k}{\partial d_t}\right)}{(1 - \epsilon - \lambda d_t)^2} \right]$$

$$+ (1-\lambda) \left[\frac{2\lambda\left(1 - x_{t+1}^k d_t\right) - 2\lambda\left(1 - \epsilon\right)g_B}{(1 - \epsilon - \lambda d_t)^3} \right]$$

$$\frac{\partial^2 d_{t+1}}{\partial d_t^2} > 0$$

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