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Essays on Strategic Pricing and Quality Decisions

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Management

by

Ji-Hung "Ryan" Choi

Dissertation Committee:
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2016

DEDICATION

To

my parents and family

in recognition of their worth

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ABSTRACT OF THE DISSERTATION

Essays on Strategic Pricing and Quality Decisions

By

Ji-Hung "Ryan" Choi

Doctor of Philosophy in Management

University of California, Irvine, 2016

Professor Rajeev K. Tyagi, Co-Chair & Assistant Professor Sreya Kolay, Co-Chair

In the first essay, we study a business-to-business (B2B) contract between truck carriers (sellers) and shippers (buyers). With the increase in the price of oil, fuel surcharges are now widely used in the transportation industry. However, there is suspicion that truck carriers use fuel surcharges to make profits beyond the cost of fuel. The purpose of this paper is to apply economic theory to investigate why freight carriers impose fuel surcharges instead of raising the freight rate. We analyze how well the formula for fuel surcharges widely used in the transportation industry mirrors truck carriers' fuel cost changes. The analysis shows that, in a market where both the seller and the buyer are risk averse against the actual fuel price volatility, imposing a fuel surcharge can prevent the buyer's utility from being overly reduced. Hence the seller can extract more of the buyer's utility. If the seller is sufficiently risk averse, implementing a fuel surcharge prevents the seller from losing expected profit due to cost uncertainty. Therefore, both the seller and the buyer are willing to make a contract even when each individual's risk averseness is sufficiently high if a fuel surcharges schedule is offered. In addition, when there are

multiple buyers, a lower type buyer is more likely to bear the burden of risk due to fuel price uncertainty.

In the second essay, we study channel member's strategic price and quality decisions on their products. It has been conventionally known that the introduction of a store brand can be used as a tool for customer segmentation or store profitability. More recent studies investigate the similarity in quality levels between store and national brands, but ignored the retailers' competitive behaviors. This paper investigates how retailers design store brand products under different market characteristics, such as the intensity of competition, consumer heterogeneity, and the manufacturer's strategic decisions. We find that symmetric retailers have an incentive to decrease the product quality of their store brands as the competition among them gets more intense, while a monopolistic retailer positions its store brand product relatively close to the national brand.

CHAPTER 1: FUEL SURCHARGE PRICING: THEORY AND PRACTICE

1. INTRODUCTION

Fuel surcharges are defined as an additional fee for travel used to cover increases in fuel cost for the type of transportation used.[1] In the transportation industry, to accommodate volatile fuel prices, several freight companies started to add fuel surcharges to freight charges during the gas crisis of the 70's rather than changing freight rates. Since then, fuel surcharges have been widely used and are considered as a common factor in the transportation industry these days. In other words, fuel surcharges are generally considered to be a major component of total freight bills.[2]

Since freight carriers directly depend on fuel costs, imposing fuel surcharges seems quite logical.[3] Firms also claim that surcharges increase the transparency of price changes and allow them to justify shifting the extra burden of their costs to their buyers. However, shippers do not have full knowledge of a carrier's surcharge structure.[4] Although many factors play a role in determining fuel surcharges, shippers prefer a rather simplified formula. Some shippers even suggest that carriers raise their freight rates without fuel surcharges. They claim that this would help both shippers and freight forwarders with their planning activities (Putzger 2004). Despite this controversy, it is still unclear why freight carriers give fuel surcharges great significance in practice.

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[1] http://thelawdictionary.org/fuel-surcharge/

[2] Terms are different in different context. Freight rate in current study is sometimes referred as transportation net cost in other context. Anyway, 'rate' and 'cost' are always switchable.

[3] "This assessment doesn't require governmental approval and you do not need to file an application with Department of Transportation (DOT) to implement a fuel surcharge." - America's Independent Truckers' Association, Inc.

[4] See Hannon (2006) "Every customer has a different idea of pricing in their base rates [freight rate] and surcharges," quoted by Douglas Duncan, former CEO of FedEx Freight.

The purpose of this paper is to apply economic theory to analyze the relationship between market parameters (e.g., fuel costs fluctuations) and fuel surcharges in practice. We investigate whether freight carriers find it optimal to offer fuel surcharges to supplement their net transportation costs (freight rate). Is raising prices instead of imposing fuel surcharges an option? Specifically, we would like to address the following research questions: (i) when does a carrier find it optimal to offer fuel surcharges? (ii) Is the purpose of a fuel surcharges to enable a total or partial "pass-through" of fuel costs from carrier to shipper? With a fuel surcharges system in place, does the carrier bear the additional risk of high fuel prices? Or can the carrier recover its additional cost due to fuel price uncertainty? (iii) If there are multiple shippers, which type of shipper between small and large shippers does primarily carry the burden of the cost of fuel? (iv) Does the carrier have an incentive to improve fuel efficiencies? If so, under what condition is the carrier incentivized to show such effort?

We also find some interesting claims from the industry press on fuel surcharges. One article observe:

> *"When diesel prices surged to over $4 a gallon in the summer of 2008, many carriers approached shippers with new schedules having a higher peg, offering to keep the total line haul price the same (or adjusting it slightly lower). For example, if a carrier raised the peg from $1.20 to $2.50 they would raise the line haul rate to account for baking in an extra $1.30 per gallon of fuel costs. The new fuel surcharge would be lower by that amounts as well. The total rate (base rate plus fuel surcharge) would remain the same. With the total rate unchanged, should a shipper be indifferent?"* [5]

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[5] http://spendmatters.com/2010/08/04/a-closer-look-at-fuel-surcharges-in-transportation-part-2/

Another interesting article we found observes multiple shippers bear different levels of burden of fuel cost uncertainty:

> *"While shippers begrudgingly accept the fact that higher fuel costs must equate to higher freight charges, what is not widely known is that it's the small shipper that primarily carries the burden of the nation's cost of fuel, not the large shipper."* [6]

To answer these questions and investigate the claims addressed above, we start by examining the pricing formula of per unit charge, which is widely used in industry practice. Using this formula, we set up and solve the problems of the players in a market with (i) one seller and one buyer, and (ii) one seller and multiple buyers. In this setup, this study generates the following findings: (1) Given that both buyers and sellers are risk averse against the actual fuel price fluctuation, imposing fuel surcharges is not always an outperforming strategy over a no-fuel-surcharges schedule. An optimal location of the base rate will determine whether a fuel surcharge will be exercised.[7] For example, the more risk-averse the buyers are, the lower the chance that the seller charges a fuel surcharge if she is also risk-averse. (2) The seller can pass through a burden of risk to the buyer. However, the buyer could also benefit from an uncertain cost of fuel and fuel surcharges. Both players share an additional cost of operation and additional social profits. This sharing is dependent upon the opponent player's level of risk averseness. (3) A fuel surcharge pricing mechanism plays its role effectively to reduce the seller's variation against an uncertain fuel cost when the actual fuel price is realized unfavorably. Instead, if the actual fuel price is realized favorably, this pricing mechanism may also bring the buyer higher utility. (4) If there are multiple buyers in the market, and they are differentiated in terms of their

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[6] http://reconlogistics.com/why-are-fuel-surcharges-penalizing-smaller-shippers
[7] The term 'base rate' we use in current study is also referred as 'price peg' or 'trigger point' in other context. This is the minimum fuel index price for the application of the fuel surcharge schedule. Fuel surcharges are calculated based on the difference between the actual fuel price and this index price.

levels of risk averseness, the low type buyer is more likely to bear the burden of risk due to fuel price uncertainty than the high type buyer.[8]

Overall, this study makes the following contributions: (i) we properly model the market phenomenon by using the dominant fuel surcharge formula that is widely used in the industry, and (ii) theorize the effect of uncertain fuel price on the seller's decision regarding their fuel surcharge schedule and on the market players' behaviors.

The rest of the paper is organized as follows. Starting with reviewing the fuel surcharge literatures with some additional literatures that share the theoretical applications in the next subsection 1.1, Section 2 introduces the model. We illustrate the market where a single seller makes a service contract with a single buyer in Section 3. In Section 4, we analyze the market with one seller with multiple buyers and compare the players' behaviors with the results obtained in the previous section. We conclude and discuss future research in Section 5. All proofs and mathematical derivations are given in the Appendix.

1.1 Related Literatures

The role of fuel surcharges is a controversial subject that has been discussed often in media. Some articles claim that fuel surcharges may be another profit source for the carriers (Gilroy 2005; Busch 2010; Grossman 2010). Although fuel surcharges have been an important topic in practice, we find that this topic has not been studied rigorously in academia, especially in areas of business. There are some studies found in other areas, but there is a limitation of implementing the results in business areas. One major stream of academic studies on fuel surcharges focuses mainly on competition and collusion between carriers (Garrod 2006;

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[8] In this particular case, we define a risk-neutral buyer as the high type buyer and a risk-averse buyer as the low type buyer.

Karamychev and van Reeven 2008; Bilotkach 2009; Lehr 2009). These studies analyze how firms utilize the practice of fuel surcharges to produce less competition and more collusion.

Karamychev and van Reeven (2008) show that the practice of fuel surcharge increases firms' profits. They model a fuel surcharge scheme, which allows firms to earn higher profits and benefit more from larger variations in costs. This result is different from what we found in this study. However, they model fuel surcharges based on an additive pricing scheme rather than based on either the proportion of freight rates or mileages. Therefore, managerial implication is limited. Kent et al. (2008) also argue that fuel surcharge policies represent a significant portion of the revenue received by Less-Than-Truckload (LTL) carriers. Since they use small interview-based data, however, the results may be limited to be generalized. There are other papers that examine how firms utilize fuel surcharges to make pricing schemes, but the models used in these studies poorly reflect real world practices, especially in terms of their design of the fuel surcharge formulas (e.g., Bilotkash 2009). As a result, they conclude that there is no evidence for carriers to make profits through the use of percent based surcharges (Lehr 2009).

Theoretically, there is another set of research that sets up the markets with uncertainty. In current study, one of the major assumptions is the players' risk averseness tendencies. This is a plausible assumption because as a policy maker, people should make their decisions in a way that they can handle any uncertainty or risk, if there is any. Stiglitz (2001) argues, in his Nobel Lecture, that firms tend to act in a risk-averse manner in the face of incentive design issues. He also adds that this risk-aversion cannot be managed away and will be reflected in action. Thus, we investigate all possible status of truck carriers and shippers in terms of the levels of their risk averseness. In most of the literatures in marketing and other related areas, carriers are assumed to be risk-neutral while shippers are risk-averse entities. However, we relax this assumption by

extending the scenarios that carriers are also risk-averse. This setup fits the market situation better since the development of fuel surcharge mechanism has been originated from the carrier's risk averse tendency.

Risk aversion is not an uncommon assumption in economics, especially in game theory models. From literatures, agents are more commonly assumed to be risk-averse while principals are more likely assumed to be risk-neutral (Shavell, 1979; Alvi, 1988; Kimball, 1993). This assumption is natural because what we observe in practice is contracting offered by dominant firms who have a higher market power, provide mechanisms that specify the contract terms and thus coordinate the market. Thus, they are often considered as risk-neutral principals, compared to those risk-averse agents who decide whether to take offers or not.

However, there are still studies that investigate how risk-averse principals behave in various business and economics context (Haubrich, 1994; Lewis and Sappington, 1995; Misra, Coughlan, and Narasimhan, 2005). Lewis and Sappington (1995) also argue in their study that when the principal is sufficiently averse to risk, she affords the agent no choice among incentive schemes. Laffont and Martimort (2002) provide us a variety of risk-averse and risk-neutral combinations of the principal-agent contract models. They state that when the principal is risk-averse and the agent is risk-neutral, and contracting takes place *ex ante*, the optimal incentive contract implements the first-best outcome. This is somewhat consistent with what we observe in the later sections of the current study. It means that by making the risk-neutral agent the residual claimant for the value of trade, *ex-ante* contracting allows the risk-averse principal to have full insurance. Further, when both the principal and the agent are averse to risk, the principal will offer a contract in a way that they both split the value of trade.

Another theoretical application of our study is to further account for the players' risk aversion through a mean-variance formulation incorporating specific risk-aversion parameters for the shipper and the buyer (Dhrymes, 1964; Chavas and Pope, 1982). Although the assumption of linearity in this model is restrictive (Freund, 1956; Newberry and Stiglitz, 1981), the mean-variance model has had extensive use in agricultural economics (Robinson and Barry, 1977; Coyle, 1992). In business studies, mean-variance analysis is particularly important in finance and largely because of the central role played by the capital asset pricing model in financial theory. However, it is not exceptional for us to find studies that use this model even in marketing and operations management areas in recent days (Misra et al., 2005; Kim et al., 2007). The mean-variance approach has usually been applied in terms of a utility function that is linear in expected profits and profit variance. Any details how to model this particular function will be specified in the next section.

2. THE MODEL

Consider a market of a monopoly seller (i.e., truck carrier) with one buyer (i.e., shipper). The seller offers a set of contract variables which consists of a freight rate (p_F) and a base rate (p_B). The buyer decides whether to accept the offer or not. However, the actual price will be known after the contract is made.

The actual fuel price, p_A, has a mean of $\bar{p}$ with a price uncertainty, δ, in either side. That is, we assume that $p_A = \bar{p} - \delta$ with a probability of ½ and $p_A = \bar{p} + \delta$ with a probability of ½, without loss of generality. The condition below shows that the fuel price uncertainty should be less than or equal to the average fuel price.

Condition 1. $\delta \leq \bar{p}$.

2.1 Fuel Surcharge

The Fuel surcharge (*FSC*) scheme that is used most widely in the industry is the mileage-based fuel surcharge, and it is formulated as

$$FSC = INT\left(\frac{FuelPrice(\$/gal) - BasePrice(\$/gal)}{escalator(\$/gal)}\right) * Basis(\$/mile) \approx \frac{(p_A - p_B)}{\varepsilon}.$$

where p_A is an average weekly cost of fuel price per gallon, p_B is a base cost of fuel, which is a pre-established trigger point, and ε is fuel efficiency. Note that we ignore the operator *INT* as we utilize the equation, without a loss of generality, in a way of being linear which is simpler compared to non-linear integer step function.

2.2 Functions

In practice, when the actual fuel price realized is less than or equal to the base rate ($p_A \leq p_B$), no surcharge will be charged to the buyers. Rather, a surcharge is only imposed when the actual fuel price realized is greater than the base rate ($p_A > p_B$). The main functions are formed based on this assumption.

Revenue function: Given the freight rate p_F (\$/mile), unit transfer, *T*, is the total amount per mile the buyer pays to the seller. We summarize the payment scheme as follows:

$$T = \begin{cases} p_F, & \text{if } p_A \leq p_B, \\ p_F + \dfrac{(p_A - p_B)}{\varepsilon}, & \text{if } p_A > p_B. \end{cases}$$

Cost function: The seller spends the amount it pays for the actual fuel price, which is given by

$$C(p_A) = \frac{p_A}{\varepsilon}.$$

For simplicity, we assume that there are no additional costs.

Profit function: The seller collects transfer payments and spends the fuel cost. The profit function is given by

$$\Pi = \begin{cases} p_F - \dfrac{p_A}{\varepsilon} & \text{if } p_A \le p_B, \\[2ex] p_F + \dfrac{1}{\varepsilon}\left(p_A - p_B\right) - \dfrac{p_A}{\varepsilon} & \text{if } p_A > p_B. \end{cases}$$

Utility function: Buyers have a valuation $V(m)$ for the service, where m is mileage exogenously given by the buyers. With $V(m)$ unit of delivery miles and the transfer T per mile, the buyer's utility function is given by:

$$U = \max\left\{V(m) - T, 0\right\}.$$

We assume that the buyer's valuation of the service increases with mileages ($\dfrac{\partial V}{\partial m} > 0$). For simplification for the rest of the study, we will define $V(m) = \theta m$, where θ is the valuation parameter.

Time Line. We follow the following time line: (1) Base price (p_B) is determined and announced publicly ($E\pi\left(p_B; p_F^*, m, p_A\right)$), (2) shipper comes in with mileage (m), (3) freight rate (p_F) is offered to buyers, and total payment is revealed ($E\pi\left(p_F; m, p_A\right)$), (4) buyers accept or reject the offer based on its expected utility ($EU \ge 0$), and finally (4) actual fuel price (p_A) is realized (either $\bar{p} - \delta$ or $\bar{p} + \delta$).

3 SINGLE BUYER

We consider that the seller has three fuel surcharge decision strategies depending on where the base rates are placed: (i) No fuel surcharge is imposed ('*NFS*,' in short). This happens only when

the base rate is always higher than the upper boundary of the actual fuel price distribution. That is, $\bar{p} + \delta \le p_B$. (ii) Fuel surcharges are occasionally imposed depending on the level of actual fuel price ('*OFS*'). This case happens when the seller imposes the base rate within the fuel price fluctuation range, so that fuel surcharges are charged only when the real price is higher than the base rate. That is, $\bar{p} - \delta \le p_B < \bar{p} + \delta$. (iii) Finally, the fuel surcharge is always used ('*AFS*'). The seller sets the base rate below the lower boundary of the actual fuel price distribution, and so will always impose fuel surcharges at a certain level. That is, $0 \le p_B < \bar{p} - \delta$. Figure 1 below illustrates these three cases.
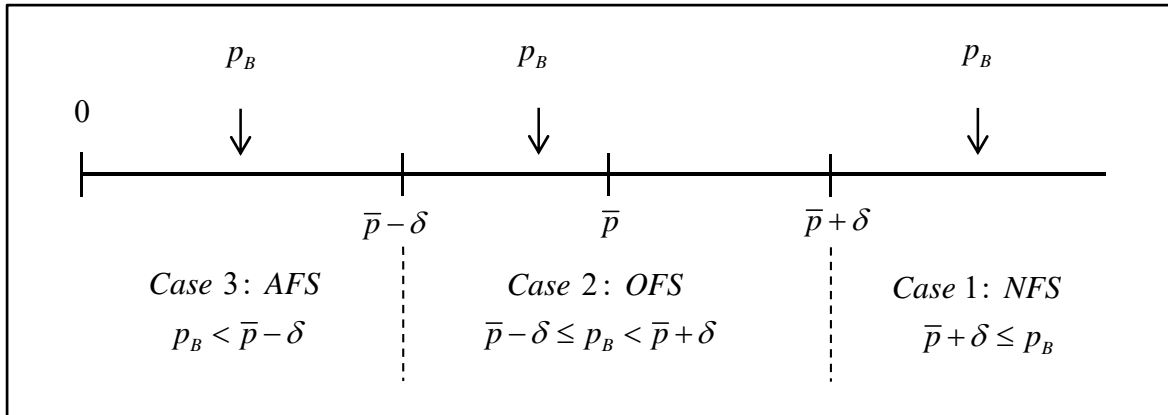

Figure 1. Location of the base rate (p_B)

3.1 Risk-Neutral Seller

Our analysis is based on whether one or both the buyer or the seller are risk averse. If they are risk averse, the seller is risk averse against the fuel price while the buyer is risk averse against the payment. To incorporate the seller and/or the buyer risk averseness into the model, we use

the mean-variance utility function.[9] The parameters to express their risk averseness are λ for the buyer and μ for the seller ($\lambda, \mu \geq 0$).[10]

3.1.1 Risk Neutral Buyer

We start with the benchmark case where both the seller and the buyer are risk-neutral.

(i) The Case of *NFS* Strategy ($\bar{p} + \delta \leq p_B$)

Given the random nature of the actual fuel price uncertainty, the buyer's expected utility function with no surcharge is as follows:

$$EU = \left(\theta - p_F \right) m .$$

The buyer accepts the seller's offer as long as his expected utility is non-negative. Otherwise, he rejects the offer. Under this scenario, the seller solves:

$$\max_{p_F} E\pi = \frac{1}{2}\left(p_F - \left(\frac{\bar{p} - \delta}{\varepsilon} \right) \right)m + \frac{1}{2}\left(p_F - \left(\frac{\bar{p} + \delta}{\varepsilon} \right) \right)m \ \text{s.t.} \ EU .$$

To solve the problem, we observe that the objective function increases linearly with the freight rate. Thus, the seller simply can choose the freight rate that leaves the buyer with zero utility. Therefore, the optimal freight rate will be given by

$$p_{F,NoRA}^{SB,NFS} = \theta ,\text{[11]}$$

where '*SB*' stands for 'Single Buyer,' and '*NoRA*' stands for 'No Risk-Averse.'

Plugging this back into the seller's problem, the optimal profit is given by

$$\pi_{NoRA}^{SB,NFS} = m\left(\theta - \frac{\bar{p}}{\varepsilon} \right),$$

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[9] The expected mean-variance utility forms $E\left[U\left(X \right) \right] = E\left[X \right] - \lambda Var\left[X \right]/2$.

[10] The larger value of the risk premium factor λ (or μ) the more risk averse a firm is, whereas risk neutrality is a special case with $\lambda = 0$ (or $\mu = 0$).

[11] Summaries of the optimal values throughout the whole studies are in the appendix A (**Table 1**).

Note that this contract will be signed up only when $\theta > \dfrac{\overline{p}}{\varepsilon}$, so that the seller's profit and the

buyer's utility both become nonnegative. Since p_B is not a part of the problem in this

scenario, it can be any value that is greater than or equal to $\overline{p} + \delta$. That is, abstractly,

$$p_{B,NoRA}^{SB,NFS} = \forall p_B > \overline{p} + \delta.$$

Condition 2. *For any cases, the seller will make the contract only when $\varepsilon\theta \geq \overline{p}$.*

(ii) The Case of *OFS* Strategy ($\overline{p} - \delta \leq p_B < \overline{p} + \delta$)

Under this scenario, fuel surcharges will be occasionally imposed. Therefore, the buyer's

expected utility and the seller's expected profit functions are given, respectively, by

$$EU = \left(\theta - p_F - \frac{\overline{p} + \delta - p_B}{2\varepsilon}\right)m \text{ and } E\pi = \left(p_F + \frac{\overline{p} + \delta - p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m.$$

In the same fashion as we did in (ii) above, we solve the best responding freight rate first. It

is given by $\hat{p}_F = \theta - \dfrac{\overline{p} + \delta - p_B}{2\varepsilon}$. Substituting this into the seller's profit function, we solve

its maximization problem. However, we observe that the first-order condition is not a

function of the base rate (p_B), which leads us to define ρ_{OFS} as an arbitrary value of distance

from the upper boundary of p_B range with $0 < \rho_{OFS} \leq 2\delta$, and set $p_B = \overline{p} + \delta - \rho_{OFS}$. Then,

the optimal freight rate and profit are given, respectively, by

$$p_{F,NoRA}^{SB,OFS} = \theta - \frac{\rho_{OFS}}{2\varepsilon} \text{ and } \pi_{NoRA}^{SB,OFS} = m\left(\theta - \frac{\overline{p}}{\varepsilon}\right).$$

Note that the optimal expected profit is the same as in the previous case. In this case, the base

rate does not affect the optimal profit. Hence, we just define the optimal base rate as

$\overline{p} + \delta - \rho_{OFS}$.

(iii) The Case of *AFS* Strategy $(0 \le p_B < \bar{p} - \delta)$

Under this scenario, the seller imposes fuel surcharges all the time. The buyer's expected

utility and the seller's expected profit functions are given, respectively, by

$$EU = \left(\theta - p_F - \frac{\bar{p} - p_B}{\varepsilon} \right) m \text{ and } E\pi = \left(p_F + \frac{\bar{p} - p_B}{\varepsilon} - \frac{\bar{p}}{\varepsilon} \right) m .$$

In the same fashion, we find $\hat{p}_F = \theta - \frac{\bar{p} - p_B}{\varepsilon}$. Again, the base rate does not affect the

optimal profit. The optimal base rate can be defined as $\bar{p} - \delta - \rho_{AFS}$, where $0 < \rho_{AFS} \le \bar{p} - \delta$.

Given this, the optimal freight rate and profit are given by

$$p_{F,NoRA}^{SB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon} \text{ and } \pi_{NoRA}^{SB,AFS} = m \left(\theta - \frac{\bar{p}}{\varepsilon} \right).$$

3.1.2 Risk-Averse Buyer

In this section, we study another set of three cases that only the buyer is a risk-averse player

while the seller is risk-neutral.

(i) The Case of *NFS* Strategy $(\bar{p} + \delta \le p_B)$

In this case, where the freight rate is considered to be the only payment, no uncertainty

exists for the buyer. The buyer's utility is given by

$$EU = (\theta - p_F) m .$$

Since the seller is not risk-averse under this scenario, the profit function is just its expected

profit and is given by

$$E\pi = \left(p_F - \frac{\bar{p}}{\varepsilon} \right) m .$$

Given this, the optimal solution are the same as those in Section 3.1 (i) as follows:

$$p_{F,BORA}^{SB,NFS} = \theta \text{ and } E\pi_{BORA}^{SB,NFS} = m\left(\theta - \frac{\bar{p}}{\varepsilon}\right),$$

where '*BORA*' stands for '*Buyer Only Risk Averse.*'

(ii) The Case of *OFS* Strategy ($\bar{p} - \delta \le p_B < \bar{p} + \delta$)

The risk-averse buyer's utility function is given by

$$EU = \left(\theta - p_F - \frac{\bar{p} + \delta - p_B}{2\varepsilon}\right)m - \frac{\lambda}{2}\left(\frac{(\bar{p} + \delta - p_B)m}{2\varepsilon}\right)^2.$$

Solving this function for p_F, we find $\hat{p}_F = \theta - \frac{\bar{p} + \delta - p_B}{2\varepsilon} - \frac{\lambda(\bar{p} + \delta - p_B)^2 m}{8\varepsilon^2}$. Given this

value, the risk-neutral seller's solves her expected profit, and it is given by

$$E\pi\big|_{\hat{p}_F} = \left(\hat{p}_F + \frac{\bar{p} + \delta - p_B}{2\varepsilon} - \frac{\bar{p}}{\varepsilon}\right)m.$$

Solving this problem, we observe that the optimal base rate is derived as $\bar{p} + \delta$. However,

the base rate should be less than the optimal ($\bar{p} - \delta \le p_B < \bar{p} + \delta$). Thus, we define

$p_B = \bar{p} + \delta - \rho_{OFS}$ as the base rate where ρ_{OFS} is an arbitrary value ($0 < \rho_{OFS} \le 2\delta$). Given

the defined base rate, we find the optimal freight rate and profit as

$$p_{F,BORA}^{SB,OFS} = \theta - \frac{\rho_{OFS}}{2\varepsilon} - \frac{\lambda m \rho_{OFS}^2}{8\varepsilon^2} \text{ and } \pi_{BORA}^{SB,OFS} = \left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\lambda m^2 \rho_{OFS}^2}{8\varepsilon^2}.$$

(iii) The Case of *AFS* Strategy ($0 \le p_B < \bar{p} - \delta$)

The buyer's expected utility function is given by

$$EU = \left(\theta - p_F - \frac{\bar{p} - p_B}{\varepsilon}\right)m - \frac{\lambda}{2}\left(\frac{\delta m}{\varepsilon}\right)^2.$$

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Solving this utility function for p_F, we find $\hat{p}_F = \theta - \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\lambda \delta^2 m}{2\varepsilon^2}$. Substituting this value

into her profit problem, the seller maximize her objective function of

$$E\pi\big|_{\hat{p}_F} = \left(\hat{p}_F + \frac{\overline{p} - p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon} \right) m .$$

Again, the F.O.C. of this profit function is not a function of p_B, we define the optimal base

rate of $p_B = \overline{p} - \delta - \rho_{AFS}$, where $0 < \rho_{AFS} \le \overline{p} - \delta$. Given this, the optimal freight rate and

profit are given by

$$p_{F,BORA}^{SB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda \delta^2 m}{2\varepsilon^2}, \quad \text{and} \quad \pi_{F,BORA}^{SB,AFS} = \left(\theta - \frac{\overline{p}}{\varepsilon} \right) m - \frac{\lambda \delta^2 m^2}{2\varepsilon^2} .$$

In profit comparison, we first find that $\pi_{BORA}^{SB,NFS}$ is strictly greater than profits in the other

two cases. For comparing $\pi_{BORA}^{SB,OFS}$ to $\pi_{BORA}^{SB,AFS}$, we find $\pi_{BORA}^{SB,OFS} \ge \pi_{BORA}^{SB,AFS}$ if $\rho_{OFS}^2 \le 4\delta^2$. This

always holds since $0 < \rho_{OFS} \le 2\delta$. Therefore, we conclude that $\pi_{BORA}^{SB,AFS} \le \pi_{BORA}^{SB,OFS} < \pi_{BORA}^{SB,NFS}$.

3.2 Risk Averse Seller

In this section, let us assume that the seller is risk averse. This is more generalized scenario since

fuel surcharge mechanism has been originated from the risk averseness of the service providers.

3.2.1 Risk Neutral Buyer

Under this scenario, we investigate three cases where only the seller is a risk-averse player while

the buyer is risk-neutral.

(i) The Case of *NFS* Strategy ($\overline{p} + \delta \le p_B$)

The risk-neutral buyer's expected utility and the risk-averse seller's mean-variance profit

function are given by, respectively,[12]

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[12] Derivations of Mean-Variance Functions of utilities and profits for all cases are in Appendix B2.

$$EU = \left(\theta - p_F\right)m \text{ and } E\pi = \left(p_F - \frac{\overline{p}}{\varepsilon}\right)m - \frac{\mu}{2}\left(\frac{\delta m}{\varepsilon}\right)^2,$$

where μ is the seller's risk averseness parameter.

The optimal freight rate and profit are given by

$$p_{F,SORA}^{SB,NFS} = \theta \text{ and } \pi_{SORA}^{SB,NFS} = m\left(\theta - \frac{\overline{p}}{\varepsilon}\right) - \frac{\mu\delta^2 m^2}{2\varepsilon^2},$$

where '*SORA*' stands for '*Seller Only Risk Averse.*'

(ii) The Case of *OFS* Strategy ($\overline{p} - \delta \le p_B < \overline{p} + \delta$)

The buyer's expected utility function is given by

$$EU = \left(\theta - p_F - \frac{\overline{p} + \delta - p_B}{2\varepsilon}\right)m.$$

The seller's problem is given by

$$E\pi = \left(p_F + \frac{\overline{p} + \delta - p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m - \frac{\mu}{2}\left(\left(\frac{\overline{p} + \delta - p_B}{2\varepsilon} - \frac{\delta}{\varepsilon}\right)m\right)^2.$$

Given $\hat{p}_F$, $\dfrac{\partial E\pi}{\partial p_B} = 0$ derives us $p_B = \overline{p} - \delta$ as an optimal base rate. Given this base rate, we

solve the seller's problem. The optimal freight rate and profit are given by

$$p_{F,SORA}^{SB,OFS} = \theta - \frac{\delta}{\varepsilon}, \text{ and } \pi_{F,SORA}^{SB,OFS} = m\left(\theta - \frac{\overline{p}}{\varepsilon}\right).$$

(iii) The Case of *AFS* Strategy ($0 \le p_B < \overline{p} - \delta$)

The risk-neutral buyer's expected utility and the risk-averse seller's mean-variance profit

function are given by, respectively,

$$EU = \left(\theta - p_F - \frac{\overline{p} - p_B}{\varepsilon}\right)m \text{ and } E\pi = \left(p_F + \frac{\overline{p} - p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m.$$

Solving the buyer's utility function, we find $\hat{p}_F = \theta - \dfrac{\overline{p} - p_B}{\varepsilon}$. However, when we plug this

value into the seller's problem, it is not a function of p_B any more. It means that the seller

can choose any p_B within the range ($0 \le p_B < \overline{p} - \delta$). Therefore, we define ρ_{AFS} as any

arbitrary number that satisfies $p_B = \overline{p} - \delta - \rho_{AFS}$ with $0 < \rho_{AFS} \le \overline{p} - \delta$. The optimal freight

rate and profit are given by

$$p_{B,SORA}^{SB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon}, \text{ and } \pi_{SORA}^{SB,AFS} = m\left(\theta - \frac{\overline{p}}{\varepsilon}\right).$$

By comparing profits, we observe that $\pi_{SORA}^{SB,AFS} = \pi_{SORA}^{SB,OFS}$, but these profits are strictly greater

than the profit earned under the *NFS* case.

Given the profit comparisons under the three strategies, if only the seller is risk-averse, it

is identically optimal for the seller to (i) impose fuel surcharges all the time by choosing the *AFS*

strategy, or (ii) impose the fuel surcharge occasionally by choosing the *OFS* strategy. However,

we should note that the seller also can choose the *OFS* strategy under a specific circumstance.

Even though the seller chooses the *OFS* strategy, she will setup the optimal base rate at the lower

boundary of the fuel price fluctuation, that is, $p_B = \overline{p} - \delta$, so that a fuel surcharge is always

imposed unless the actual fuel price is exactly at the lower boundary. For the current study, the

model is setup as the fuel price falls in either $\overline{p} + \delta$ and $\overline{p} - \delta$ with 50-50 percent chances

respectively, for mathematical simplicity. However, assuming another probability distributions

of actual fuel price such as a normal or a uniform distribution, the chance that an actual fuel price

falls at the lower boundary is minimal. It means that theoretically the *OFS* strategy is a part of

the best decisions for the seller when the seller is the only one who is risk-averse in the market in

current study, the seller will not take a risk of not imposing fuel surcharges by choosing the *OFS* strategy but will choose to impose fuel surcharges all the time.

3.2.2 Risk Averse Buyer

Now we examine another three cases where both the seller and the buyer are risk-averse.

(i) The Case of *NFS* Strategy ($\bar{p} + \delta \leq p_B$)

The buyer's expected utility function with no surcharge is as follows:

$$EU = (\theta - p_F)m.$$

The buyer accepts the seller's offer as long as his expected utility is non-negative. Otherwise, he rejects the offer. The seller solves:

$$E\pi = \left(p_F - \frac{\bar{p}}{\varepsilon} \right)m - \frac{\mu}{2}\left(\frac{\delta m}{\varepsilon} \right)^2.$$

The first term of right-hand side of the above equation represents the seller's mean profit and the second term represents the variance due to its risk averseness. By solving this problem with the constraint of the expected utility, we find the optimal freight rate and the profit, which are given, respectively, by[13]

$$p_{F,RA}^{SB,NFS} = \theta \text{ and } \pi_{RA}^{SB,NFS} = m\left(\theta - \frac{\bar{p}}{\varepsilon} \right) - \frac{\mu\delta^2 m^2}{2\varepsilon^2},$$

where '*RA*' stands for '(both players are) Risk-Averse.'

Since p_B is not a part of the problem in this scenario, it can theoretically be any value greater than or equal to $\bar{p} + \delta$. That is, $p_{B,RA}^{SB,NFS} = \forall p_B > \bar{p} + \delta$. We also note that, for non-negative

profit, $\mu \leq \tilde{\mu}$, where we define $\tilde{\mu} = \frac{2\varepsilon(\varepsilon\theta - \bar{p})}{\delta^2 m}$.

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[13] Derivations of the optimal values for all cases are shown in Appendix B2.

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(ii) The Case of *OFS* Strategy ($\bar{p} - \delta \le p_B < \bar{p} + \delta$)

The buyer's expected utility function is given by

$$EU = \left(\theta - p_F - \frac{\bar{p} + \delta - p_B}{2\varepsilon} \right) m - \frac{\lambda}{2} \left(\frac{(\bar{p} + \delta - p_B)m}{2\varepsilon} \right)^2.$$

Solving this function for p_F, we find $\hat{p}_F = \theta - \dfrac{\bar{p} + \delta - p_B}{2\varepsilon} - \dfrac{\lambda(\bar{p} + \delta - p_B)^2 m}{8\varepsilon^2}$. Given this

value, the seller's solves her expected profit, and it is given by

$$E\pi\big|_{\hat{p}_F} = \left(\hat{p}_F + \left(\frac{\bar{p} + \delta - p_B}{2\varepsilon} \right) - \frac{\bar{p}}{\varepsilon} \right) m - \frac{\mu}{2} \left(\left(\frac{\bar{p} + \delta - p_B}{2\varepsilon} - \frac{\delta}{\varepsilon} \right) m \right)^2.$$

Solving a simple optimization problem, we have the optimal rates and profit as

$$p_{B,RA}^{SB,OFS} = \bar{p} + \frac{\delta(\lambda - \mu)}{\lambda + \mu}, \quad p_{F,RA}^{SB,OFS} = \theta - \frac{\delta\mu}{\varepsilon} - \frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda + \mu)^2}, \text{ and } \pi_{RA}^{SB,OFS} = m\left(\theta - \frac{\bar{p}}{\varepsilon} \right) - \frac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda + \mu)}.$$

Note that we should check whether the optimal base rate falls in the feasible region; i.e.,

$\bar{p} - \delta \le p_{B,RA}^{SB,OFS} < \bar{p} + \delta$. The optimal base rate under this case should be greater than or equal to

$\bar{p} - \delta$ and should be less than $\bar{p} + \delta$. Otherwise, it is infeasible. Therefore, the following

analysis will follow:

(a) First, let us check $p_{B,RA}^{SB,OFS} < \bar{p} + \delta$: $\bar{p} + \dfrac{\delta(\lambda - \mu)}{\lambda + \mu} < \bar{p} + \delta \Leftrightarrow \dfrac{\lambda - \mu}{\lambda + \mu} < 1$. This is always true.

(b) Second, $\bar{p} - \delta \le p_{B,RA}^{SB,OFS}$: $\bar{p} - \delta \le \bar{p} + \dfrac{\delta(\lambda - \mu)}{\lambda + \mu} \Leftrightarrow -1 \le \dfrac{\lambda - \mu}{\lambda + \mu}$. This is always true as well.

Therefore, we confirm that $p_{B,RA}^{SB,OFS}$ is the optimal base rate.

(iii) The Case of *AFS* Strategy ($0 \le p_B < \bar{p} - \delta$)

The buyer's expected utility function is given by

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$$EU = \left(\theta - p_F - \frac{\overline{p} - p_B}{\varepsilon} \right) m - \frac{\lambda}{2} \left(\frac{\delta m}{\varepsilon} \right)^2 .$$

First, we solve the buyer's utility function for the freight rate. It is given by

$\hat{p}_F = \theta - \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\lambda \delta^2 m}{2\varepsilon^2}$. Then given this value, we solve the seller's profit maximization

problem as described below to get the optimal base rate.

$$E\pi \big|_{\hat{p}_F} = \left(\hat{p}_F + \frac{\overline{p} - p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon} \right) m$$

However, since the driven profit function is not a function of p_B in this particular case, but

the freight rate is still a function of the base rate, it leads us to define ρ_{AFS} ($0 < \rho_{AFS} \le \overline{p} - \delta$)

and observe the limiting case by setting the optimal base rate as $p_B = \overline{p} - \delta - \rho_{AFS}$. Then, we

have the optimal rates and profit as

$$p_{B,RA}^{SB,AFS} = \overline{p} - \delta - \rho_{AFS}, \ p_{F,RA}^{SB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda \delta^2 m}{2\varepsilon^2}, \text{ and } \pi_{RA}^{SB,AFS} = m \left(\theta - \frac{\overline{p}}{\varepsilon} \right) - \frac{\lambda \delta^2 m^2}{2\varepsilon^2} .$$

Lemma 1. *Under an assumption that both the seller and the buyer are risk-averse, no contract*

will be made unless $\theta \ge \dfrac{\overline{p}}{\varepsilon} + \dfrac{\lambda \mu \delta^2 m}{2\varepsilon^2 (\lambda + \mu)} .$

Proof. Straightforward from the optimal profits.

Recall Condition 2. There is a minimum level of the buyer's service valuation for the

seller to be able to offer a contract. This minimum level service valuation is now raised due to

the market players' risk averseness from that was shown in Condition 2. Even if the seller can

extract all the buyer's expected utility, unless it is sufficient to cover the seller's disutility from

the risk uncertainty, she will not make a positive profit, and thus will not offer the buyer a

contract. Therefore, there must be a threshold on the buyer's service valuation, as specified in Lemma 1 above, whether the seller decides to sell her service.

3.3 Discussions

3.3.1 Equilibrium

We summarize the optimal solutions in the following proposition as an equilibrium.

Proposition 1: (Equilibrium) *In a single-buyer-single-seller contract, the optimal contract $\left(p_F^*, p_B^* \right)$ specifies solutions to the seller's problem as follows:*

(1) When the seller is risk-neutral, (i) if buyer is also risk-neutral, the seller offers any of three strategies with $\left\{ p_{F,NoRA}^{SB}, p_{B,NoRA}^{SB} \right\}$ as specified above in Section 3.1.1. (ii) If buyer is risk-averse, the seller offers the NFS strategy with $\left\{ p_{F,BORA}^{SB,NFS}, p_{B,BORA}^{SB,AFS} \right\}$ as specified above in Section 3.1.2.

(2) However, when the seller is risk averse, (i) if buyer is risk-neutral, the seller offers either of the OFS or the AFS strategy with $\left\{ p_{F,SORA}^{SB,AFS}, p_{B,SORA}^{SB,AFS} \right\}$ or $\left\{ p_{F,SORA}^{SB,AFS}, p_{B,SORA}^{SB,AFS} \right\}$, respectively, as specified above in Section 3.2.1. (ii) Instead, if buyer is risk-averse, the seller offers the OFS strategy with $\left\{ p_{F,RA}^{SB,OFS}, p_{B,RA}^{SB,OFS} \right\}$ as specified above in Section 3.2.2.

Proof. See the Appendix.

When the seller is risk-neutral, the *NFS* strategy weakly dominates. It means that when the risk-neutral seller faces with a risk-neutral buyer, all of three strategies work indifferently to the seller. She will have the same profit from utilizing any of those. Even when the buyer is risk averse, the best strategy for the seller is the *NFS*, so that the seller can earn the same profit as much as she would have from the risk-neutral buyer. Suppose that the buyer is risk averse against his payment, and the seller impose a fuel surcharge. Then, the buyer's disutility will be created, and this will affect the total amount that the seller can extract from the buyer. Thus, for a

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risk-neutral seller, imposing nothing but the fixed freight rate will be the best strategy for her to keep her profit level as much as the benchmark case.

However, when the seller is risk averse, we observe the different optimal sets of strategies. First, when the seller is risk averse while the buyer is risk neutral, the best strategy for the seller is to impose fuel surcharges all the time. Under this strategy, the buyer is expected to pay more than the amount that he would pay without the fuel surcharges, and it make the buyer's utility decreased. However, it does not affect the amount that the seller extracts from the buyer's utility because the seller will receive the payments from the buyer exactly how much the buyer loses due to the fuel surcharges. For the seller, this additional amount of payment from the buyer will compensate what she additionally spends on the fuel costs. In this particular strategy, since the seller always can charge a fuel surcharge, even if she is risk-averse against the fuel price fluctuation, there will not be a disutility for the seller due to a variance of the expected profit.

On the other hand, when both the seller and the buyer are risk-averse, choosing the *OFS* strategy is the best option for the seller. Given the results from other cases, the seller decides her best decision based on which player in the market has a risk averseness. However, in this case the seller and the buyer are willing to share the burden of risks from the fuel price uncertainty. That is, from the seller's perspective, she wants to partially transfer her burden of the risk of uncertainty to the buyer. Like we discussed above, the seller wants to keep as much as of what she can extract from the buyer, and at the same time the seller wants to compensate what she additionally spends on fuel consumption as much as she can. Therefore, the seller need a device to make a balance between them. Thus, the *OFS* strategy works as an insurance for the seller to recover her loss due to the uncertainty. In addition, the level of fuel surcharge is determined by

how much risk averse each of players are. The seller's pricing schedule will be determined depending on which player has more risk averse about the uncertainty.

Corollary 1. *Fuel Surcharge pricing is more profitable when the seller is risk-averse.*

We summarize the seller's pricing strategies in Table 2 below. It shows which strategy is the best option the seller will take in each of scenarios based on their profit comparisons from all different cases.

Table 2. Profit Comparisons.

		Seller	
		Risk-Neutral	Risk-Averse
Buyer	Risk-Neutral	$NFS = OFS = AFS$	$NFS < OFS = AFS$
	Risk-Averse	$AFS \leq OFS < NFS$	$NFS < OFS$ or $AFS < OFS$

When both the buyer and seller are risk-neutral, the expected profits are indifferent across the seller's strategies ($\pi = m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$). When only the buyer is risk-averse, choosing the *NFS* strategy makes the seller most profitable than choosing other strategies would. Therefore, the seller will never impose fuel surcharges to buyers all the time given a fixed pricing strategy. The buyer is risk averse about his uncertain payment due to an actual fuel price fluctuation and thus a fuel surcharge. This risk averseness makes a disutility from the buyer's total expected utility. Compared to risk-neutral seller's profit, the seller now has less amount of the buyer's utility that she can extract from if she decided to impose a fuel surcharge. Therefore, given an assumption that the seller can extract all the buyer's utility, it is always more profitable for the seller keep the buyer's utility as high as possible by not charging a fuel surcharge. In this case, a fixed freight

rate is the only source of the seller's profit. In sum, regardless whether the buyer is risk averse or not, the seller will choose the *NFS* as a weakly dominant strategy.

3.3.2 Comparative Statics

Table 3 shows comparative statics of optimal values with respect to various market parameters.[14]

Mileage (m). For the cases where only one of two players is risk averse while the other is risk-neutral, the seller's profit increases linearly with the buyer's mileage. However, the mileage is a fixed amount that the buyer asks the seller to serve when he comes to the table. Thus, we consider it as a unit of a product regardless of how long or short it is. If only the buyer is risk averse, the seller's best strategy is not to impose any surcharges by keeping the buyer has no utility decreased. It makes the seller to have the same profit level ($\frac{\partial \pi_{BORA}^{SB,AFS}}{\partial m} > 0$). On the contrary, if only the seller is risk averse, the seller will decide to always impose a fuel surcharge by transferring the burden of risk to the buyer, so that she has no disutility on her profit ($\frac{\partial \pi_{SORA}^{SB,AFS}}{\partial m} > 0$). Therefore, in either way, the seller will not suffer from her disutility on her profit by achieving the benchmark profit ($\pi = \left(\theta - \frac{\bar{p}}{\varepsilon} \right) m$).

However, when both players in the market are risk averse, there will be a restriction of mileages served by the seller. Recall that the seller uses the *OFS* strategy under this scenario. The seller's profit increases initially as the buyer's mileage increase, but profit decreases once the mileage goes over a certain threshold. This is because an increase in disutility due to a fuel price uncertainty exceeds the amount of an increase in profit. Therefore the total profit will start

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[14] Table 3 is in the Appendix A.

decreasing beyond the threshold that is defined as $\hat{m} = \dfrac{\varepsilon\left(\varepsilon\theta - \bar{p}\right)\left(\lambda + \mu\right)}{\lambda\mu\delta^2}$. Then, the seller's

profit eventually becomes negative if the service mileage is over a certain point. This point is

indicated in Corollary 2. If the seller is sufficiently risk-averse, dealing with higher mileage is

too risk since she needs to spend too much uncertain fuel costs. Therefore, a high level of risk

averseness can give the seller a chance that she does not even make a contract with buyers.

Corollary 2. *The OFS strategies has an upper feasible limit of service mileages as*

$$m \leq \bar{m} = \frac{2\varepsilon\left(\varepsilon\theta - \bar{p}\right)\left(\lambda + \mu\right)}{\lambda\mu\delta^2} \ .$$

 Fuel efficiency (ε). For all cases that are applied, the freight rates and profits increases as

the fuel efficiency parameter ε increases ($\dfrac{\partial p_F^{SB}}{\partial \varepsilon} > 0$ and $\dfrac{\partial \pi^{SB}}{\partial \varepsilon} > 0$). Fuel surcharge and fuel cost

decrease with fuel efficiency. Hence, in a case that the buyer pays a fuel surcharge, he can save

his total utility since an increase in fuel efficiency makes the buyer to have less fuel surcharge

and less disutility from the uncertain payment risks. It also leads the seller to be able to have

more profit with less fuel costs. An increase in fuel efficiency directly decreases the seller's fuel

cost consumption and her disutility from the fuel price fluctuation. Therefore, the seller has an

incentive to increase the fuel efficiency if her long term profit exceeds the short term investment

on improvement of fuel efficiencies, such as vehicle upgrades or driver training. Overall, an

increase in this parameter ε independently makes the whole system more efficient, and thus it

brings higher social welfare.

 Service Valuation (θ). The service valuation parameter θ works in a similar way as ε

plays in the model. Higher values of this parameter always brings higher seller's profit in all

cases ($\dfrac{\partial \pi^{SB}}{\partial \theta} > 0$). Since the seller's service that the buyer value is the maximum amount what

the seller can extracts, the optimal freight rate starts with θ when there is no risk aversion, and then it starts decreasing from it depending on who has a risk aversion and how much that risk aversion is. Therefore, the freight rate always increases as the service valuation increases ($\frac{\partial p_F^{SB}}{\partial \theta} > 0$).

     Average Fuel Price ($\bar{p}$). The seller's profit decreases when the average fuel price increases ($\frac{\partial \pi^{SB}}{\partial \bar{p}} < 0$). Since the fuel price directly affects the seller's cost of operation and thus her profit, it is obvious that more fuel consumption with per unit fuel price brings the seller less profit. However, an increase in the average fuel price raise the level of the base rate ($\frac{\partial p_B^{SB}}{\partial \bar{p}} > 0$). In each cases where a fuel surcharge exists, if the average fuel price increases, the base rate also should increase accordingly since it is determined within a range for each strategy.

     Fuel Price Fluctuation (δ). Given an average fuel price, if the level of fuel price fluctuation increases, the base rate may decreases or may increases ($\frac{\partial p_B^{SB}}{\partial \delta} < 0$ or > 0). The sign changes depending on whose risk averseness level is larger. If the seller is more risk-averse than the buyer, the base rate decreases, so the seller tends to depend more on the fuel surcharge pricing (the *AFS* or the *OFS* schemes). Regarding the freight rate, this price decreases as well when the fuel price uncertainty increases ($\frac{\partial p_F^{SB}}{\partial \delta} < 0$). It is a plausible pricing scheme since the seller needs to bake in the decreased freight rate with an increasing fuel surcharge. However, even with this reaction, an increase in δ hurts the seller's profit for the case of the *OFS* strategy ($\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \delta} < 0$). Even if the seller uses a fuel surcharge pricing to compensate her in some way,

her profit cannot reach the benchmark profit level. Along with other parameters, the fuel price uncertainty level creates a disutility on the seller's profit and thus on the social welfare. We also should note that this disutility does not occur when the seller is the only one who is risk averse on the market. She will use a full surcharge scheme, which is the *AFS* strategy, and this, without the buyer's risk aversion, brings the seller the benchmark level of profit.

However, under the same situation such that the uncertainty parameter (δ) increases, if the buyer is more risk-averse than the seller, the base rate increases. It means that the seller tries to depend less on the fuel surcharge. Since the buyer's risk averseness is large, it is reasonable for the seller to raise the level of the base rate to accommodate the buyer's risk averseness tendency into the contract. However, what is interesting is that the seller still reduce the freight rate as well ($\frac{\partial p_F^{SB}}{\partial \delta} < 0$). Note that this will not happen when the *AFS* strategy is practiced since the buyer is risk-neutral ($\lambda = 0$) regardless of how high the fuel price uncertainty is. Since the total payment decreases, the seller's profit decreases as well as δ increases ($\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \delta} < 0$). Therefore, we find that the seller is not always able to respond positively against the changes in fuel price uncertainty.

Buyer Risk Averseness (λ). Buyer's risk averseness affects optimal variables and profit only when the *OFS* strategy is practiced. It means when only one player in this game is risk averse, the seller can achieve the benchmark profit regardless of who is risk averse by offering either the *NFS* or the *AFS* strategies. Under the *OFS* strategy, the base rate increases as the buyer's risk averseness increases ($\frac{\partial p_B^{SB,OFS}}{\partial \lambda} > 0$). As we discussed above, an increase in the base rate generates a smaller fuel surcharge if it is imposed. That is, when the buyer is more risk

averse, the seller tries to impose less fuel surcharges. This additionally supports what we found earlier. When both the buyer and seller are risk averse, the seller pick the *OFS* as her optimal strategy, so she can reduce a chance of imposing a fuel surcharge. In addition to that, even within this optimal strategy, the seller reduces the magnitude of fuel surcharges by increasing the level of the base rate along with the level of the buyer's risk averseness.

Given this change on the base rate, the freight rate should increase at the same time to compensate the seller's fuel cost. This assumption is true for the most of the parameter values of λ ($\frac{\partial p_F^{SB,OFS}}{\partial \lambda} > 0$). However, we observe that if the seller's risk averseness is large enough compared to that of the buyers, that is $\mu - \lambda$ is positive and large, the freight rate decreases as the buyer's risk averseness increases even if the base rate increases ($\frac{\partial p_F^{SB,OFS}}{\partial \lambda} < 0$). Therefore, we observe that p_F and p_B do not move in the same direction. It is because, even if the optimal strategy is the *OFS*, the seller is likely to act like she offers the *AFS* when her own risk averseness is strong. The seller's dependency on the fuel surcharge is large when the buyer's risk averseness is sufficiently small. However, it is a temporary reaction when the seller is highly risk averse while the buyer's risk averseness is minimal. As the buyer's risk averseness increases, a decrease in the freight rate turns out to an increase, and this fuel freight rate increasing can be substituted with a decrease in the fuel surcharge. In order to do that, the base rate should be increased.

Finally, the seller's profit decreases as the buyer's risk averseness level increases ($\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \lambda} < 0$). An increase in λ makes the seller's profit being away from the benchmark profit level. When this happens, as discussed above, the seller tries to keep the base rate high, so that

she could charge a lower fuel surcharge to the buyer. Instead, the seller raise the freight rate. It is a way for the seller not to drop her profit too low.

Seller Risk Averseness (μ). Just like the buyer risk averseness parameter does, the seller's risk averseness parameter μ affects variables under the *OFS* strategy only. The seller's risk averseness makes all the freight rate, the base rate, and the seller's profit decreased

($\frac{\partial p_F^{SB,OFS}}{\partial \mu} < 0$, $\frac{\partial p_B^{SB,OFS}}{\partial \mu} < 0$, and $\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \mu} < 0$). If the seller's risk averseness is smaller than that

of the buyer, the base rate increases within $\overline{p}$ and $\overline{p} + \delta$, and if the seller's risk averseness is larger than that of the buyer, the base rate decreases within $\overline{p} - \delta$ and $\overline{p}$. It basically tells us that as the seller's risk averseness increases, the fuel surcharge amount will increases. This is a consistent with what we observed above that the seller is likely dependent more upon the fuel surcharges when she is highly risk-averse. At the same time, however, the seller is more likely reduce the chance of imposing a fuel surcharge when the buyer's risk averseness level is high enough. In addition to the movement of the base rate, an increased fuel surcharge leaves a room for the freight rate to be decreased. The seller profit will be impacted the seller's risk averseness. It should be away from the benchmark profit as the seller's risk averseness increases.

3.4 *Ex-Post* Analysis, Risk Sharing, and Burden of Risks

In this section, we study how much the seller can actually recover her profit loss from the volatile fuel price and how she can share the risk of uncertainty placed on the cost of operation. To investigate how to utilize the fuel surcharge mechanism, we compute and compare their *ex-post* profits when the actual fuel price falls into either $\overline{p} + \delta$ or $\overline{p} - \delta$.[15] Starting this section, we

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[15] Summary of ex-post analysis results is in the appendix (**Table 4**).

focus our analysis only on risk-averse seller cases leaving risk-neutral seller cases as a benchmark.

3.4.1 Risk-Averse Seller vs. Risk-Neutral Buyer

First, we observe the case where the seller is risk-averse while the buyer is risk-neutral, Under this scenario in this section, note that we do not consider the corner solution where the seller sets up the optimal base rate at the lower boundary of the fuel price fluctuation ($\bar{p}-\delta$) in order to make the comparison of the strategies clear. Thus, in this section, we focus only on the case where the seller uses the *AFS* strategy.

Derivations of *ex-post* profits, utilities, and social welfare values are as follows: *Ex-post* profits for the seller obtained in each of two realized fuel prices are the same and are given by

$$\pi\Big|_{P_A=\bar{p}+\delta}^{AFS} = \left(p_{F,SORA}^{SB,AFS} + \frac{P_A - p_{B,SORA}^{SB,AFS}}{\varepsilon} - \frac{P_A}{\varepsilon} \right)m$$

$$= \left(\theta - \frac{\delta+\rho_{AFS}}{\varepsilon} + \frac{\bar{P}+\delta-\left(\bar{P}-\delta-\rho_{AFS}\right)}{\varepsilon} - \frac{\bar{P}+\delta}{\varepsilon} \right)m = \left(\theta - \frac{\bar{p}}{\varepsilon} \right)m.$$

$\pi\Big|_{P_A=\bar{p}-\delta}^{AFS}$ derives the same profit of $\left(\theta - \dfrac{\bar{p}}{\varepsilon} \right)m$. Then, for each of realized prices, the risk-neutral buyer's utility is given by, respectively,

$$U\Big|_{P_A=\bar{p}+\delta}^{AFS} = \left(\theta - p_{F,SORA}^{SB,AFS} - \frac{P_A - p_{B,SORA}^{SB,AFS}}{\varepsilon} \right)m$$

$$= \left(\theta - \left(\theta - \frac{\delta+\rho_{AFS}}{\varepsilon} \right) - \frac{\bar{P}+\delta-\left(\bar{P}-\delta-\rho_{AFS}\right)}{\varepsilon} \right)m = -\frac{\delta m}{\varepsilon}$$

In a same fashion, we get, $U\Big|_{P_A=\bar{p}-\delta}^{AFS} = \dfrac{\delta m}{\varepsilon}$. Therefore, we have the social welfares as

$$SW\Big|_{P_A=\bar{p}+\delta}^{AFS} = \left(\theta - \frac{\bar{p}+\delta}{\varepsilon} \right)m \text{ and } SW\Big|_{P_A=\bar{p}-\delta}^{AFS} = \left(\theta - \frac{\bar{p}-\delta}{\varepsilon} \right)m.$$

Given that the seller is risk averse but the buyer is risk neutral, the *AFS* is chosen as the optimal strategy. If the actual fuel price is realized at the high expected value, the *ex-post* social welfare will be smaller than the *ex-ante* social welfare. Specifically, when the realized fuel price is unfavorable, only the buyer's utility will decrease having a negative utility while the seller keeps the same profit as before. On the contrary, the actual fuel price is realized at the low expected value, the *ex-post* social welfare will be larger than the *ex-ante* social welfare. In this case, still the seller's profit stays the same, but the buyer's utility increases from zero to a positive utility. We conclude the proposition below.

Proposition 2: *Assume that the seller is risk averse but the buyer is risk neutral. The seller transfers all the risks to the buyer. Therefore, when the actual fuel price is realized (i) at* $p_A = \bar{p} + \delta$, *the buyer bears all the burden of uncertain cost of operation, and (ii) at* $p_A = \bar{p} - \delta$, *the buyer carries all the benefits from the uncertainty.*

Proof. See the Appendix.

When the actual fuel price is unfavorable ($\bar{p} + \delta$), the social welfare reduces by $\dfrac{\delta m}{\varepsilon}$, which is the same as how much the buyer's ex-post utility decreases. The seller should spend this amount as an additional cost of operation. However, the seller passes through all of this amount to the buyer. Therefore, we can conclude that the buyer bears all the burden created due to the uncertain cost of fuel. On the other hand, when the actual fuel price is favorable ($\bar{p} - \delta$), all of additional social welfare will go to the buyer. The amount is $\dfrac{\delta m}{\varepsilon}$, and it is exactly same as how much the seller can save from the cost reduction. Thus, we can conclude that the buyer carries all the benefits from the uncertain cost of fuel in this case.

3.4.2 Risk-Averse Seller vs. Risk-Averse Buyer

Now, we study the case where both the seller and the buyer are risk averse, so that the seller chooses the *OFS* as the optimal strategy. When the actual fuel price is realized as $p_A = \bar{p} + \delta$, the seller's *ex-post* profits is given by

$$
\pi\big|_{P_A=\bar{p}+\delta}^{OFS} = \left(p_{F,RA}^{SB,OFS} + \frac{p_A - p_{B,RA}^{SB,OFS}}{\varepsilon} - \frac{p_A}{\varepsilon} \right) m = \left(\theta - \frac{\delta\mu}{\varepsilon} - \frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda+\mu)^2} \right.
$$
$$
\left. + \frac{1}{\varepsilon}\left(\bar{p}+\delta - \left(\bar{p} + \frac{\delta(\lambda-\mu)}{\lambda+\mu} \right) \right) - \frac{\bar{p}+\delta}{\varepsilon} \right) m = \left(\theta - \frac{\bar{p}}{\varepsilon} \right) m - \frac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} - \frac{\lambda\delta m}{\varepsilon(\lambda+\mu)}.
$$

Compared to the seller's *ex-ante* profit, this *ex-post* profit can be larger or smaller depending on the following conditions: (i) if $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, $\pi_{RA}^{SB,OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{OFS}$, and (ii) if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$,

$\pi_{RA}^{SB,OFS} < \pi\big|_{P_A=\bar{p}+\delta}^{OFS}$. Under the same situation, the buyer's *ex-post* utility is given by

$$
U\big|_{P_A=\bar{p}+\delta}^{OFS} = \left(\theta - p_{F,RA}^{SB,OFS} - \frac{p_A - p_{B,RA}^{SB,OFS}}{\varepsilon} \right) m = \left(\theta - \left(\theta - \frac{\delta\mu}{\varepsilon} - \frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda+\mu)^2} \right) \right.
$$
$$
\left. - \frac{1}{\varepsilon}\left(\bar{p}+\delta - \left(\bar{p} + \frac{\delta(\lambda-\mu)}{\lambda+\mu} \right) \right) \right) m = \left(\frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda+\mu)^2} - \frac{\mu\delta}{\varepsilon(\lambda+\mu)} \right) m.
$$

We observe that, compared to zero ex-ante utility, (i) if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, $U\big|_{P_A=\bar{p}+\delta}^{OFS} > 0$, and (ii) if

$\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, $U\big|_{P_A=\bar{p}+\delta}^{OFS} < 0$.

In a same fashion, we can find the *ex-post* profit and utility when the actual fuel price is realized as $p_A = \bar{p} - \delta$. Here, we note that there will be no fuel surcharges schedules since the realized fuel price is lower than the base price. Therefore, the seller's *ex-post* profit and the buyer's *ex-post* utility are, respectively, given by

$$\pi\big|^{OFS}_{p_A=\bar{p}-\delta} = \left(p^{SB,OFS}_{F,RA} - \frac{p_A}{\varepsilon} \right) m = \left(\theta - \frac{\delta\mu}{\varepsilon} - \frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2 (\lambda+\mu)^2} - \frac{\bar{p}-\delta}{\varepsilon} \right) m$$

$$= \left(\theta - \frac{\bar{p}}{\varepsilon} \right) m - \frac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2 (\lambda+\mu)^2} + \frac{\lambda\delta m}{\varepsilon(\lambda+\mu)}$$

and $U\big|^{OFS}_{p_A=\bar{p}-\delta} = \left(\theta - p^{SB,OFS}_{F,RA} \right) m = \left(\theta - \left(\theta - \frac{\delta\mu}{\varepsilon} - \frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2 (\lambda+\mu)^2} \right) \right) m = \left(\frac{\lambda\mu^2\delta^2 m}{2\varepsilon^2 (\lambda+\mu)^2} + \frac{\mu\delta}{\varepsilon(\lambda+\mu)} \right) m$.

In this case, the buyer's *ex-post* utility is strictly positive, which means that it is greater than his *ex-ante* utility of zero. After comparison, we also can find that this *ex-post* profit is always larger than the *ex-ante* profit for the seller.

Recall our definition of the social welfare (*SW*) as the sum of the seller's profit and the buyer's utility. Then, the social welfare becomes $SW\big|^{OFS}_{p_A=\bar{p}+\delta} = \left(\theta - \frac{\bar{p}+\delta}{\varepsilon} \right) m$ under that the realized fuel price is at the high end ($p_A = \bar{p}+\delta$), and $SW\big|^{OFS}_{p_A=\bar{p}-\delta} = \left(\theta - \frac{\bar{p}-\delta}{\varepsilon} \right) m$ under that the realized fuel price is at the low end ($p_A = \bar{p}-\delta$). We summarize the findings below.

Lemma 2: *Given that both the buyer and the seller are risk averse, and so the seller chooses the OFS as the optimal strategy. When the actual fuel price is realized:*

1. *At* $p_A = \bar{p}+\delta$, *(i) if* $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, *then* $\pi^{SB,OFS}_{RA} > \pi\big|^{OFS}_{p_A=\bar{p}+\delta}$, $U^{SB,OFS}_{RA} > U\big|^{OFS}_{p_A=\bar{p}+\delta}$, *and*

   $SW^{SB,OFS}_{RA} > SW\big|^{OFS}_{p_A=\bar{p}+\delta}$, *and*

   (ii) if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, $\pi^{SB,OFS}_{RA} < \pi\big|^{OFS}_{p_A=\bar{p}+\delta}$, $U^{SB,OFS}_{RA} < U\big|^{OFS}_{p_A=\bar{p}+\delta}$, *and* $SW^{SB,OFS}_{RA} < SW\big|^{OFS}_{p_A=\bar{p}+\delta}$.

2. *At* $p_A = \bar{p}-\delta$, $\pi^{SB,OFS}_{RA} < \pi\big|^{OFS}_{p_A=\bar{p}-\delta}$, $U^{SB,OFS}_{RA} < U\big|^{OFS}_{p_A=\bar{p}-\delta}$, *and* $SW^{SB,OFS}_{RA} < SW\big|^{OFS}_{p_A=\bar{p}-\delta}$.

Proof. See the Appendix.

When both the buyer and the seller are risk averse, the seller optimally chooses to practice the *OFS* strategy. In a case that the actual fuel price is realized high ($p_A = \bar{p} + \delta$), we may expect that one or both of players lose their utilities. However, we observe two different directions as follows: (i) First, as expected, *ex-post* profit, utility, and social welfare are all less than those of ex-ante outcomes. (ii) The second direction shows us thtat *ex-post* profit, utility, and social welfare can be larger than *ex-ante* social welfare when the condition specified above is met. This happens when both the buyer and the seller are sufficiently risk averse. It is because, given $0 \le \lambda$, $\mu \le 1$, and a fixed value of other parameters ($\frac{2\varepsilon}{\delta m}$), the ratio $\frac{\lambda \mu}{\lambda + \mu}$ is always high with both of λ and μ are high enough. Therefore, we can conclude that if both players are highly risk-averse at the moment of contracting, the *ex-post* profit, utility, and social welfare will become greater than those that would be obtained based on the contract values even if the seller spends more cost of fuel in the future.

Another case that the *ex-post* social welfare becomes greater than its *ex-ante* profit is when the actual fuel price is realized at the lower expected boundary ($p_A = \bar{p} - \delta$). Both players' profit/utility are higher in any case after the actual fuel price is realized, and thus it is obvious that the social welfare is higher *ex-post*. The seller spends less amount of the fuel cost, and its consequences benefit the whole market. This happens regardless of who, between two, is more risk averse.

With the nature of risk averseness, none of players will have ex-post profit/utility whose average is equivalent with their ex-ante profit/utility. In other words, for example, even if the buyer expects a zero utility at the moment of contracting, once he makes a contract decision, his decision involves a choice of risk. Thus, the buyer assumes that he already have a positive

expected return on average when things are realized in the future. Otherwise, he may reject the seller's offer. Since the seller is also risk-averse, same thing happens to the seller as well. Intuitively, it is natural for us to observe higher profit and utility even when the actual fuel price is realized unfavorably.

In sum, at the moment the contract is made, the social welfare is identical to the seller's expected profit given that that all of buyer's utility is assumed to be extracted. After the actual fuel price is realized and falls above the average fuel price, the seller's profit may decreases or increases and so does the *ex-post* welfare. Therefore, the buyer utility may also either decrease or increase depending on the levels of risk-averseness. However, when the realized fuel price falls below the average fuel price, both the seller profit and buyer utility increase and so does the *ex-post* social welfare. Therefore, we observe that the buyer and the seller share the burden of risk in some way.

Let us now find how the seller and the buyer share the risk form the uncertainty.

Proposition 3. *Assume that both of the seller and the buyer are risk-averse. For each unit of the fuel cost change, the seller carries* $\dfrac{\lambda}{\lambda + \mu}$ *proportion of the unit cost change, while the buyer carries* $\dfrac{\mu}{\lambda + \mu}$ *proportion of the unit cost change.*

Proof. See the Appendix.

Compared to the seller's optimal *ex-ante* profit function, her *ex-post* profit decreases by $\dfrac{\lambda \delta m}{(\lambda + \mu)\varepsilon}$. Assuming the actual fuel price is realized at $p_A = \bar{p} + \delta$, the seller spends an additional cost of $\dfrac{\delta}{\varepsilon}$ per mile. We observe that an amount of $\dfrac{\lambda \delta m}{(\lambda + \mu)\varepsilon}$ is exactly $\dfrac{\lambda}{\lambda + \mu}$ of

$\dfrac{\delta}{\varepsilon} m$, which is the total additional cost spending for the seller due to an unit of fuel cost increase.

Given that, the buyer will be responsible for $\dfrac{\mu}{\lambda + \mu}$ of $\dfrac{\delta}{\varepsilon} m$. Even when the actual fuel price

falls at $p_A = \bar{p} - \delta$, the seller will be benefited from the cost saving by the same amount of

$\dfrac{\lambda \delta m}{(\lambda + \mu)\varepsilon}$. On the other hand, it is clear that the buyer's benefit from the cost saving should be

$\dfrac{\mu \delta m}{(\lambda + \mu)\varepsilon}$. It is interestingly noted that either benefit or loss from the seller's cost changes is

dependent on the opponent player's risk averseness level. Therefore, from the seller's

perspective, for example, if the realized price is unfavorable so that there is a loss on her profit,

her loss will be relatively larger than what the buyer's loses if the buyer's risk averseness is

larger than the seller's. However, if there is an additional benefits, the seller's benefit will be

larger than the buyer's benefit when the buyer's risk averseness is large. It means that when the

seller need to spend an additional cost, the buyer's heavy risk averseness hurts the seller's profit

more. The flip side of this tells us that the seller can extract more profits from the buyer if the

buyer is lightly risk averse. How each of the players share the risk of uncertainty is dependent

upon their relative risk averseness.

3.4.2.1 In Comparison with the Case of No Fuel Surcharge

The analysis above shows us how much *ex-post* utilities are realized and how players share loss

or benefits from the uncertainty of such realization. Then, next we should study what role the

fuel surcharge mechanism plays. It is because the seller may choose not to provide any fuel

surcharge schedule if her loss is expected to be too large or her burden of risk is too big by

offering any types of surcharge schedule. In order to investigate it, we extend our analysis to *NFS* strategy cases as follows.

Suppose the seller does not choose to impose any fuel surcharges even if both the seller and the buyer are risk averse. Under this scenario, when the realized fuel price is $p_A = \bar{p} + \delta$, the seller's *ex-post* profit is given by

$$\pi\big|_{p_A=\bar{p}+\delta}^{NFS} = \left(p_{F,RA}^{SB,NFS} - \frac{p_A}{\varepsilon} \right)m = \left(\theta - \frac{\bar{p}+\delta}{\varepsilon} \right)m.^{16}$$

Then with the payment of $p_{F,RA}^{SB,NFS} = \theta$, the buyer's utility will be zero ($U\big|_{p_A=\bar{p}+\delta}^{NFS} = 0$). The *ex-post* social welfare just becomes identical to the seller's profit and is given by

$$SW\big|_{p_A=\bar{p}+\delta}^{NFS} = \left(\theta - \frac{\bar{p}+\delta}{\varepsilon} \right)m.$$

Similarly, we can find the *ex-post* profit, utility, and social welfare when the fuel price is realized as $p_A = \bar{p} - \delta$, and they are given by

$$\pi\big|_{p_A=\bar{p}-\delta}^{NFS} = \left(\theta - \frac{\bar{p}-\delta}{\varepsilon} \right)m, \; U\big|_{p_A=\bar{p}+\delta}^{NFS} = 0, \text{ and } SW\big|_{p_A=\bar{p}-\delta}^{NFS} = \left(\theta - \frac{\bar{p}-\delta}{\varepsilon} \right)m.$$

We observe that the *ex-post* social welfares under the *NFS* strategy are indifferent from those when the seller offers a fuel surcharge scheme with the *OFS* strategy regardless of the location of the realized fuel prices. Under this situation, the seller's decision of whether she offers a fuel surcharge schedule will depend on a pricing scheme that maximizes her profit. Thus, we compare the seller's profits and the buyer's utility under the *NFS* and the *OFS* schedules and summarize the findings as follows:

Lemma 3. *Suppose that both the seller and the buyer are risk-averse.*

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[16] Optimal values for *ex-post NFS* cases are summarized in **Table 4**.

1. At $p_A = \bar{p} + \delta$, (i) If $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, then $\pi\big|_{P_A=\bar{p}+\delta}^{OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$ and $U\big|_{P_A=\bar{p}+\delta}^{OFS} < U\big|_{P_A=\bar{p}+\delta}^{NFS}$, and

   (ii) if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, then $\pi\big|_{P_A=\bar{p}+\delta}^{OFS} < \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$ and $U\big|_{P_A=\bar{p}+\delta}^{OFS} > U\big|_{P_A=\bar{p}+\delta}^{NFS}$.

2. At $p_A = \bar{p} - \delta$, $\pi\big|_{P_A=\bar{p}-\delta}^{OFS} < \pi\big|_{P_A=\bar{p}-\delta}^{NFS}$ and $U\big|_{P_A=\bar{p}-\delta}^{OFS} > U\big|_{P_A=\bar{p}-\delta}^{NFS}$.

The ex-post social welfare between the OFS and the NFS strategies are always indifferent.

Proof. See the Appendix.

First, let us observe the scenario of a favorable fuel price realization ($p_A = \bar{p} - \delta$). From the comparison of the *OFS* strategy's profit and utility with those of the *NFS* strategy under the scenario of both risk-averse players, we observe that the seller always earns higher *ex-post* profit by offering the *NFS* strategy than offering the *OFS* strategy when the actual fuel price is realized at the lower value. For the buyer, however, offering the *OFS* pricing scheme always brings him a higher *ex-post* utility regardless of levels of both players' risk averseness. In other words, the seller's *OFS* offer decision benefits the buyer in terms of his utility, which means that the buyer is willing to accept the offer if he knew the location of the actual fuel price at $p_A = \bar{p} - \delta$.

The interesting circumstances come with the situation where the actual fuel price falls high unfavorably at $p_A = \bar{p} + \delta$. In this particular case, we observe two outcomes. (i) When $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, the seller has higher profit when she offers the *OFS* than the *NFS* strategy, even if her *ex-post* profit is realized below the *ex-ante* profit. The buyer is worse off with the *OFS* than the *NFS* strategy having a negative utility. (ii) However, when $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, the seller has higher profit when she offers the *NFS* than the *OFS* strategy, even though this *OFS* strategy *ex-post* profit is higher than its *ex-ante* profit. The buyer, on the contrary, is better off with the *OFS* and

the *NFS* strategy. Combining the profits and utilities comparisons shown in Lemma 2 and Lemma 3, we conclude the role of fuel surcharges in the following proposition.

Proposition 4: *The fuel surcharge is more effective for the seller when the actual fuel price is realized high at $p_A = \bar{p} + \delta$ than it is realized low at $p_A = \bar{p} - \delta$. In this case, the seller has a smaller profit variation under the OFS strategy than under the NFS strategy. However, the buyer has a larger utility variation under the OFS strategy than the NFS strategy.*

Proof. See the Appendix.

Figure 2 and 3 demonstrate the profit and utility comparisons, respectively, of the *OFS* strategy with the *NFS* strategy *ex-ante* and *ex-post*. First, we examine the actual fuel price is realized favorably ($p_A = \bar{p} - \delta$). Under the *OFS* strategy, even though the seller earns a higher *ex-post* profit than she would have under the *ex-ante* strategy, she will have even higher profit under the *NFS* strategy. On the contrary, the buyer earns a higher *ex-post* utility with the *OFS* all the time. In terms of variation for uncertainty, however, the seller and the buyer have zero variation for their respective profits and utilities regardless of which type of strategy they play. Therefore, in this particular case, no one carries any risks.

When the actual fuel price is realized unfavorably at $p_A = \bar{p} + \delta$, the seller will have a higher profit under the *NFS* than under the *OFS* strategy if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, and will have a lower profit under the *NFS* than under the *OFS* strategy if $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$. Thus, we observe that the profits under the *NFS* is lower than the low value of the *OFS* strategy and is higher than the high value of the *OFS* strategy. It means that the profit variation that the seller faces with under the *OFS* strategy is smaller. Alternatively speaking, we find that the fuel surcharge mechanism reduces the variation created due to the cost uncertainty. For the buyer, on the other hand, the
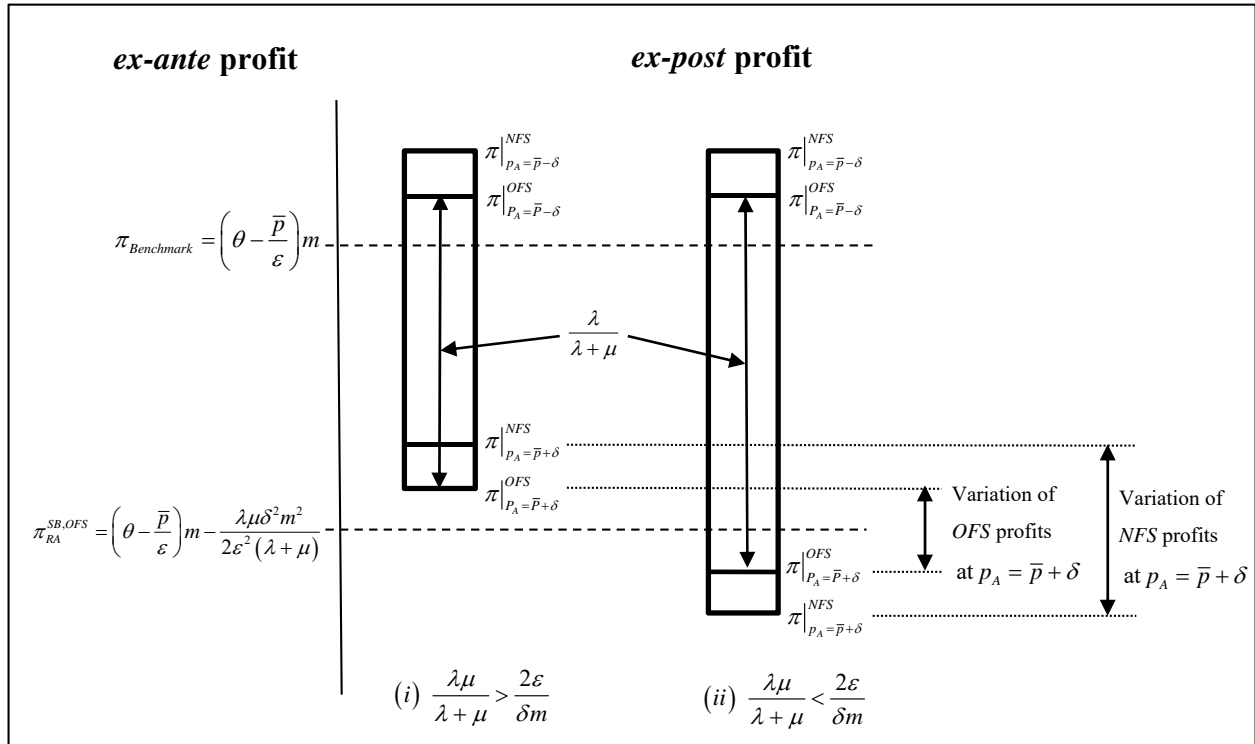
Figure 2. Comparison of the seller's *ex-ante* and *ex-post* profits
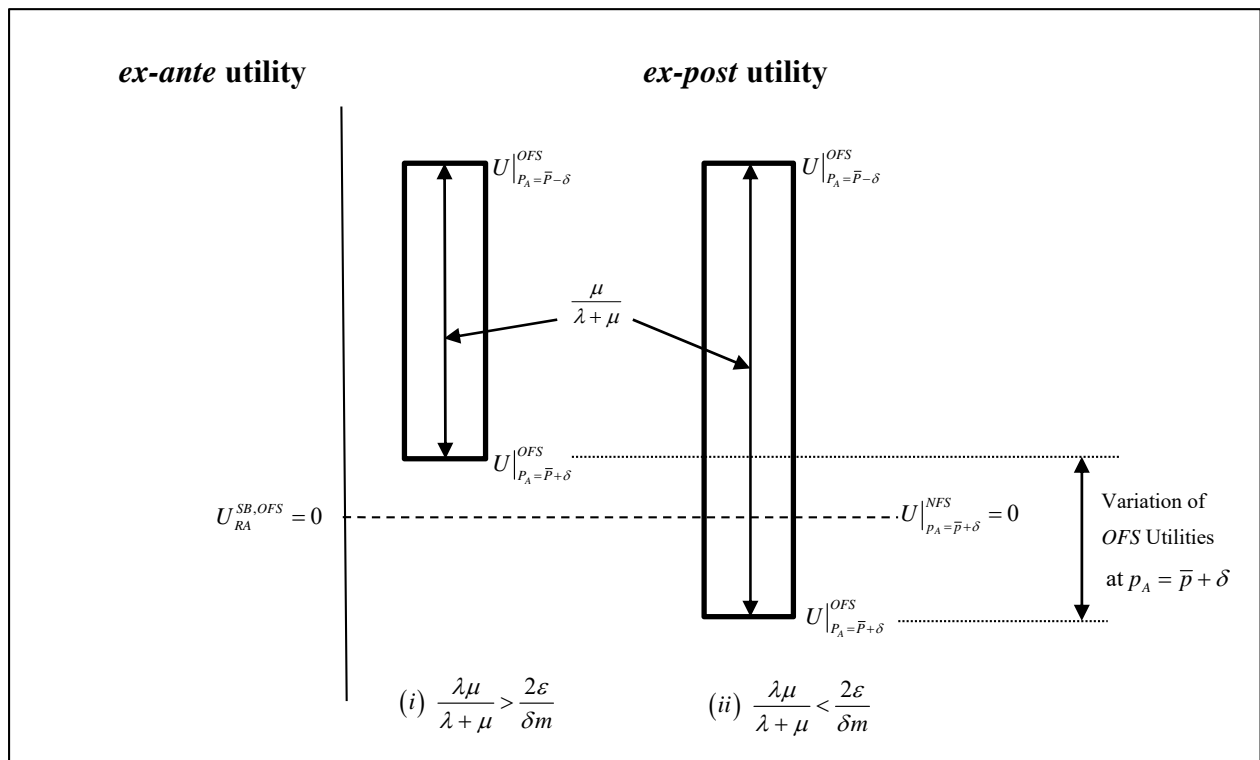
Figure 3. Comparison of the buyer's *ex-ante* and *ex-post* utilities

utility gap between the one with $\frac{\lambda\mu}{\lambda+\mu} > \frac{2\varepsilon}{\delta m}$ and the one with $\frac{\lambda\mu}{\lambda+\mu} < \frac{2\varepsilon}{\delta m}$ becomes greater

under the *OFS* strategy than under the *NFS*. It means that the seller's fuel surcharge mechanism

makes the buyer have larger variation for the uncertain risk. Therefore, the buyer is expected to

be worse off under the *OFS* when the actual fuel price is realized unfavorably. In sum, we

conclude that a fuel surcharge pricing mechanism can be utilized effectively with the *OFS*

strategy at actual fuel price is realized unfavorably: i.e., at $p_A = \bar{p} + \delta$, the seller is better off, and

at $p_A = \bar{p} - \delta$, the buyer is better off.

4   Two Buyers with a Risk-Averse Seller

We now consider two buyers who are differentiated in terms of their (i) service valuations, (ii)

service mileages, and (iii) risk averseness. We consider them to be either a 'high' or a 'low' type.

A high type (or low type) buyer is defined as a buyer whose service valuation is high (or low).

Thus, high and low type valuation buyers have their respective reservation values of θ_H and θ_L,

where $\theta_H > \theta_L$. In terms of their mileages, we consider that each buyer brings their own mileages

to be served; i.e., m_H and m_L, for high and low type, respectively. However, the high type

buyer's mileage is not necessarily assumed to be higher than that of the low type buyer.

Regarding their risk averseness levels, we assume that the high type buyer is less risk-averse than

the low type buyer. That is, $\lambda_H < \lambda_L$, where $0 \le \lambda_L, \lambda_H \le 1$. It is a plausible assumption because a

big shipper has less risk averseness in general. Misra et al. (2005) state that risk aversion is likely

to vary across firms, and that a key driver of this heterogeneity is firm size and that firm size is

related to a firm's degree of risk aversion. They also add that evidence suggests that there is a

strong relation between a firm size and its attitude towards risk. There is an empirical evidence

that smaller firms will tend to be risk averse (Smith, 1995). In addition, Mayers and Smith

(1982) prove that larger firms tend to re-insure less and thus show less risk-averse attitude.

4.1 Risk Averse Buyers

Since we study the most generalized market, in this extension of multiple buyers, let us assume

that the seller is always risk-averse hereafter. Therefore, we investigate a market where a risk-

averse seller faces with two risk-averse buyers. Both buyers enter the market simultaneously.

(i) The Case of *NFS* Strategy ($\bar{p} + \delta \leq p_B$)

Each individual buyer's expected utility function with no surcharge is as follows:

$$EU_i = \left(\theta_i - p_{Fi} \right) m_i , \ i = H, L.$$

The seller's mean-variance profit function is given by:[17]

$$\pi = \sum_{i=H,L} \left(p_{Fi} - \frac{\bar{p}}{\varepsilon} \right) m_i - \frac{\mu}{2} \frac{\delta^2}{\varepsilon^2} \left(\sum_{i=H,L} m_i \right)^2 .$$

The first term of right-hand side of the above equation represents the seller's mean profit and

the second term represents the variance due to its risk averseness. By solving this problem

with the constraint of the expected utility, we find the optimal freight rate and the profit,

which are given, respectively, by[18]

$$p_{Fi,RA}^{MB,NFS} = \theta_i , \ i = H, L \ \text{ and } \ \pi_{RA}^{MB,NFS} = \left(\theta_H - \frac{\bar{p}}{\varepsilon} \right) m_H + \left(\theta_L - \frac{\bar{p}}{\varepsilon} \right) m_L - \frac{\mu \delta^2}{2\varepsilon^2} \left(m_H^2 + m_L^2 \right),$$

where '*MB*' stands for 'Multiple Buyers' and '*RA*' stands for '(both players are) Risk-

Averse.'

(ii) The Case of *OFS* Strategy ($\bar{p} - \delta \leq p_B < \bar{p} + \delta$)

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[17] Derivations of Mean-Variance Functions of utilities and profits for all cases are in Appendix B2.
[18] Derivations of the optimal values for all cases are shown in Appendix B2.

Each buyer's expected utility function is given by

$$EU_i = \left(\theta_i - p_{Fi} - \frac{\overline{p} + \delta - p_B}{2\varepsilon} \right) m_i - \frac{\lambda_i}{2} \left(\frac{(\overline{p} + \delta - p_B) m_i}{2\varepsilon} \right)^2.$$

Solving this function for p_F, we find $\hat{p}_{Fi} = \theta_i - \frac{\overline{p} + \delta - p_B}{2\varepsilon} - \frac{\lambda_i (\overline{p} + \delta - p_B)^2 m_i}{8\varepsilon^2}$. Given this

value, the seller solves her expected profit, and it is given by

$$E\pi\big|_{\hat{p}_{Fi}} = \sum_{i=H,L} \left(p_{Fi} + \left(\frac{\overline{p} + \delta - p_B}{2\varepsilon} \right) - \frac{\overline{p}}{\varepsilon} \right) m_i - \frac{\mu}{2} \left(\frac{\overline{p} - \delta - p_B}{2\varepsilon} \right)^2 \left(\sum_{i=H,L} m_i \right)^2.$$

Solving a simple optimization problem, we have the optimal prices and profit as

$$p_{B,RA}^{MB,OFS} = \overline{p} - \delta + \frac{2\delta \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2},$$

$$p_{FH,RA}^{MB,OFS} = \theta_H - \frac{\mu\delta \left(m_H + m_L \right)^2}{\varepsilon \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)} - \frac{\lambda_H \mu^2 \delta^2 m_H \left(m_H + m_L \right)^4}{2\varepsilon^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)^2},$$

$$p_{FL,RA}^{MB,OFS} = \theta_L - \frac{\mu\delta \left(m_H + m_L \right)^2}{\varepsilon \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)} - \frac{\lambda_L \mu^2 \delta^2 m_L \left(m_H + m_L \right)^4}{2\varepsilon^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)^2},$$

and $\pi_{RA}^{MB,OFS} = \left(\theta_H - \frac{\overline{p}}{\varepsilon} \right) m_H + \left(\theta_L - \frac{\overline{p}}{\varepsilon} \right) m_L - \frac{\mu\delta^2 \left(m_H + m_L \right)^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{2\varepsilon^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)}.$

Feasibility: $\overline{p} - \delta \leq p_{B,RA}^{MB,OFS} < \overline{p} + \delta$.

The optimal base rate under this case should be greater than or equal to $\overline{p} - \delta$ and should be

less than $\overline{p} + \delta$. Otherwise, it is infeasible. Therefore, the following analysis follows:

(1) First, $\overline{p} - \delta \leq p_{B,RA}^{SB,OFS}$: $\overline{p} - \delta \leq \overline{p} - \delta + \frac{2\delta \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2}$. This is always true

since the third term of the right-hand side of the inequality is always positive.

(2) Second, let us check $p_{B,RA}^{MB,OFS} < \bar{p} + \delta$: $\bar{p} - \delta + \dfrac{2\delta\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2} < \bar{p} + \delta$

$$\Leftrightarrow \dfrac{2\delta\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2} < 2\delta \Leftrightarrow \lambda_H m_H^2 + \lambda_L m_L^2 < \lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2$$

$\Leftrightarrow 1 < \mu\left(m_H + m_L\right)^2$ (or $\dfrac{1}{\mu} < \left(m_H + m_L\right)^2$). Therefore, in order to keep this strategy feasible,

either the seller's risk-averseness should be low enough compared to buyers' small mileages

or one and/or both of buyers' mileages should be big enough compared to the seller's high-

level of risk averseness. Therefore, we confirm that $p_{B,RA}^{SB,OFS}$ is the optimal base rate as far as

the condition in (2) holds.

Condition 3 (Feasibility Condition). $1/\mu < \left(m_H + m_L\right)^2$.

Given a seller's risk-averseness, there should be a minimum total mileages that buyers ask to be

served. When both buyer's mileages are the same, the condition becomes $1/4\mu < m^2$.

However, we also should note the followings. Like Corollary 2 mentions in the single

buyer case, we can easily predict that there should be an upper feasible limit of service mileages

that two buyers together request to be served even if the seller makes a set of contracts at the

same time. Having an assumption of a non-negative profit ($\pi_{RA}^{MB,OFS} \geq 0$) and setting the service

valuation and the buyers' risk averseness are the same across the buyers for computational

convenience; i.e., $\theta_H = \theta_L = \theta$ and $\lambda_H = \lambda_L = \lambda$, we derive the upper limit of the mileage sum as

$\left(m_H + m_L\right) \leq \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)\dfrac{2\varepsilon^2\left(\lambda + \mu\right)}{\lambda\mu\delta^2}$. Along with the Condition 3 above, which is $\dfrac{1}{\mu} < \left(m_H + m_L\right)^2$

for the base rate to be placed within the upper and lower limits, $p_{B,RA}^{MB,OFS} \in [\bar{p} - \delta, \bar{p} + \delta)$, we find

the feasible range of the both buyers' mileage sum as $\dfrac{1}{\sqrt{\mu}} < \left(m_H + m_L\right) \le \left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)\dfrac{2\varepsilon^2\left(\lambda + \mu\right)}{\lambda\mu\delta^2}$.

The *OFS* strategy will be feasible as long as this relationship holds. Therefore, we find that there was no minimum boundary for the mileage under the single buyer case, but now under the two-buyer case, there is a minimum required mileage for the seller to serve as a sum of two buyers' mileages. Doubled risk averseness from two buyers has the seller to setup more insurance to earn a positive profit.

(iii) The Case of *AFS* Strategy ($0 \le p_B < \overline{p} - \delta$)

Each buyer's expected utility function is given by

$$EU_i = \left(\theta_i - p_{Fi} - \dfrac{\overline{p} - p_B}{\varepsilon}\right)m_i - \dfrac{\lambda_i}{2}\left(\dfrac{\delta m_i}{\varepsilon}\right)^2.$$

First, we solve the buyer's utility function for the freight rate. It is given by

$\hat{p}_{Fi} = \theta_i - \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\lambda_i\delta^2 m_i}{2\varepsilon^2}$. Then given this value, we solve the seller's profit maximization problem as described below to get the optimal base rate.

$$E\pi\big|_{\hat{p}_F} = \sum_{i=H,L}\left(p_{Fi} + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon}\right)m_i$$

Same as the single buyer case, the base rate is not determined by the first order condition of the seller's problem, which means that the any base rate within the feasible region can be chosen, we can define it as $p_B = \overline{p} - \delta - \rho_{AFS}$, where ρ_{AFS} ($0 < \rho_{AFS} \le \overline{p} - \delta$). Given this, we have the optimal freight rates and profit as

$$p_{B,RA}^{MB,AFS} = \bar{p} - \delta - \rho_{AFS}, \quad p_{FH,RA}^{MB,AFS} = \theta_H - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda_H \delta^2 m_H}{2\varepsilon^2},$$

$$p_{FL,RA}^{MB,AFS} = \theta_L - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda_L \delta^2 m_L}{2\varepsilon^2}, \text{ and}$$

$$\pi_{RA}^{MB,AFS} = \left(\theta_H - \frac{\bar{p}}{\varepsilon} \right) m_H + \left(\theta_L - \frac{\bar{p}}{\varepsilon} \right) m_L - \frac{\delta^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{2\varepsilon^2}.$$

We summarize the optimal solutions in the following proposition as an equilibrium.

Proposition 5: *Assume Condition 3 holds. In a single-seller-multiple-buyers contract where all seller and buyers are risk averse, the optimal contract $\left(p_{Fi}^*, p_B^* \right)$ specifies solutions to the seller's problem such that the seller offers the OFS strategy with $\left\{ p_{Fi,RA}^{MB,OFS}, p_{B,RA}^{MB,OFS} \right\}$ as specified above in Section 4.1.*

Proof. See the Appendix.

In fact, if both buyers' risk levels are assumed to be the same as being risk-averse, contracting with two buyers do not seem to be much different for the seller from making two individual contracts with each of two buyers and then summing them up. At least, the optimal strategy chosen by the seller should be the same as the *OFS* strategy regardless of the number of buyers, even though, there is a condition for the mileage selection for this strategy to be optimal as we already observed above. However, the seller's profits are not just a sum of profits from two individual risk-averse buyers.

Corollary 3. *Comparing the optimal profit and the optimal base rate from the multiple buyers' analysis with those from the single buyer cases, we have the following results:*

(i)   The total expected profit of the seller who faces two types of risk averse buyers are always smaller than the expected profit sum of that the seller would earn from making contracts with the same two buyers individually.

(ii)   *The base rate under the two buyer case is lower than that under the single buyer case.*

Proof. See the Appendix.

When the seller decides an optimal strategy and the base rate for fuel surcharge, the location of the base rate is determined depending on how serious the seller considers the uncertainty. As we observed in section 3, when a risk-averse seller faces with a risk-averse buyer, she tries to be dependent more on the fuel surcharge if her risk averseness level is relatively high given the buyer's risk averseness level. Higher the risk averse the seller is, lower the base rate the seller sets up, so that she could have higher chance to charge a fuel surcharge.[19] However, if the number of risk-averse buyer increases for the seller to face with at the same time, her dependency on the fuel surcharge also increases as well. Thus, the seller tries to increase the chance of charging the fuel surcharge, and this is how the base rate ends up with being lowered. In other words, facing with more number of risk-averse buyers has the seller feel riskier than facing with a single buyer.

4.1.1. Comparative Statics

We discuss comparative statics in this section. We continue to focus only on the case where all players are risk-averse. Comparative statics under this scenario are summarized in Table 6.[20]

Mileages (m_i). Given that the *OFS* is chosen as an optimal strategy, we study how two buyers' service mileages together or individually affect the seller's optimal variable decisions.

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[19] We also observe that even in an extreme case of the buyer's risk averseness neutrality, the seller is likely to take the *AFS* strategy.

[20] Table 6 is in the Appendix A.

Regarding the optimal base rate, we observe $\frac{\partial p_{B,RA}^{MB,OFS}}{\partial m_H} < 0$ and $\frac{\partial p_{B,RA}^{MB,OFS}}{\partial m_L} > 0$.[21] First of all, the

biggest difference from the single buyer case is that the optimal base rate under the two-buyer

case is affected by the length of mileages. The first order conditions of the base rate with respect

to mileages of each type buyers go to different directions depending on which type's mileage

changes. For example, given a fixed low type's mileage, if the mileage of the hype type buyer

increases, the seller decreases the base rate. However, from the seller's perspective, who between

the low and high type buyers makes the total mileage increased is not important because the cost

associated with the service per mile will be indifferent. If the mileages from both type of buyers

either increase or decrease at the same time, since one type makes the base rate high while the

other type makes it low, the optimal base rate will be balanced out.

However, if only one type buyer's mileage increases, then the base rate can be adjusted

as the way we found out. The sign of the first order conditions of the base rate with respect to

mileages, in fact, depends on the difference between the two type buyer's risk averseness; i.e.,

$(\lambda_H - \lambda_L) < 0$ since we assumed that $\lambda_H < \lambda_L$. Therefore, we argue that if we want to explain the

effect of mileage on the base rate, we should also incorporate the risk averseness parameters on

the explanation. If the low type buyer is the only one or a majority who increases the total

mileage, the service itself possesses a higher risk averseness from the buyers overall compared to

the seller's given risk averseness. This leads the seller to adjust the base rate high. That is, if

$(m_H + m_L)$ increases due to an increase in m_L, $p_{B,RA}^{MB,OFS}$ increases. This is a consistent result as

what we observed with the single buyer case in section 3.1.2 and 3.2.2. We found that the seller

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[21] $\dfrac{\partial p_{B,RA}^{MB,OFS}}{\partial m_H} = \dfrac{4\delta\mu(\lambda_H - \lambda_L)m_H m_L^2}{\left((\lambda_H + \mu)m_H^2 + (\lambda_L + \mu)m_L^2\right)^2} < 0$ and $\dfrac{\partial p_{B,RA}^{MB,OFS}}{\partial m_L} = \dfrac{4\delta\mu(\lambda_L - \lambda_H)m_H^2 m_L}{\left((\lambda_H + \mu)m_H^2 + (\lambda_L + \mu)m_L^2\right)^2} > 0$.

tries to increase the base rate relying on a lower fuel surcharges when the buyer becomes more

risk averse. In sum, a change in mileage does not directly affect the base rate under two buyers.

However, according to which type of buyer's mileage changes, the seller will adjust the level of

the base rate depending on the level of the focal buyer's risk averseness.

Another interesting observations can be found from the relationship between the mileages

and the freight rates. However, they are consistent findings as we observe above for the base rate

changes. We find that $\dfrac{\partial p_{FH,RA}^{MB,OFS}}{\partial m_H} < 0$, $\dfrac{\partial p_{FL,RA}^{MB,OFS}}{\partial m_H} < 0$, $\dfrac{\partial p_{FH,RA}^{MB,OFS}}{\partial m_L} > 0$, and $\dfrac{\partial p_{FL,RA}^{MB,OFS}}{\partial m_L} > 0$ and then < 0.

First two inequalities show that if the high type buyer's mileage increases, the seller decreases

the freight rates that will assign to both the high type and the low type buyers. Since an increase

in the high type's mileage makes the seller decreases the base rate as explained above. This

drives a higher fuel surcharge, and thus a decreasing in freight rates seems reasonable.

Similar explanation applies for the low type buyer's mileage changes at first place. The

third and fourth inequalities shows the relationships between the low type buyer's mileage

changes and the freight rates. When there is an increase in mileage of the low type buyer, it leads

the seller to increase the base rate, and thus to decrease the fuel surcharge. Therefore, it seems

reasonable for the seller to charge both buyers a higher freight rate overall. However, what we

see from the last inequality is that if the mileage of the risk averse low type buyer is high enough,

the seller starts to decrease the freight rate for the buyer. This is an interesting observation and

gives us a motivation to look at the case in the forthcoming section 4.2.[22] It is because we realize

that the way the seller treats two buyers are different even under the equivalent situation. If the

risk averse low type buyer's mileage is sufficiently high, the seller is willing to decrease the

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[22] In section 4.2, we study a market with two buyers where one is a risk-neutral buyer and the other is a risk-averse buyer.

lower type's freight rate even with a low fuel surcharge. It means that the impact of increased mileage is weaker with the low type buyer case. Therefore, we interpret this phenomenon as the fuel surcharge works weakly to coordinate with the low type buyers. Therefore, there is still a room for them to be charged a lower fuel surcharge along with a lower freight rate.

The observation of the relationship between mileages and the seller's profit is very similar to that of the single buyer case under the *OFS* strategy. For both types of buyers, the seller's profit increases initially as the buyers' mileages increase ($\frac{\partial \pi_{RA}^{MB,OFS}}{\partial m_H} > 0$), but the profit decreases once the mileage goes over a certain threshold ($\frac{\partial \pi_{RA}^{MB,OFS}}{\partial m_H} < 0$). This is because an increase in disutility due to a fuel price uncertainty exceeds the amount of an increase in profit. Therefore, the total profit will start decreasing beyond the threshold. Eventually the seller's profit goes down to be negative if the service mileage is over a certain point. We already discussed this in section 4.1. There will be a maximum sum mileage that creates a positive seller's profit. If the seller is sufficiently risk-averse, dealing with higher mileage is too risk since she needs to spend fuel costs under a high uncertainty. Therefore, a high level of risk averseness can give the seller a chance that she does not even make a contract with buyers.

Service Valuation (θ_i). We study how each of two buyers' service valuations affects the seller's decisions. Even with two types of buyers, the service valuation parameter θ_i works in a similar way as we observed under a single buyer scenario. Higher value of this parameter always brings higher seller's profit in all cases ($\frac{\partial \pi_{RA}^{MB,OFS}}{\partial \theta_i} > 0$). Since the seller's transportation service that the buyer values is in fact the maximum amount what the seller can extracts, theoretically the optimal freight rate starts with θ_i when there is no risk averseness at all, and then it starts

decreasing from θ_i depending on how high the risk averseness is. The optimal choice under this case where all players are risk averse is the *OFS* strategy. Therefore, there must be some parts that are deducted from θ_i for the freight rate. Still, the freight rate always increases as the service valuation increases ($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \theta_i} > 0$). However, it should be noted that only one type buyer's service value affects that type of buyer's freight rate. For example, the low type buyer's freight rate charged will not be affected by his own value for the seller's service ($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \theta_{-i}} = 0$).

Buyer Risk Averseness (λ_i): Each buyer's risk averseness raises the base rate when it increases ($\frac{\partial p_{B,RA}^{MB,OFS}}{\partial \lambda_i} > 0$). This has already been proven earlier in sections 4.2.2 and 3.3.2 when a risk averse seller faces with a single risk averse buyer. Given the seller's risk averseness level, as the buyer's risk averseness increases, the base rate will increase as well. An increase in the base rate generates a smaller fuel surcharge if it is imposed. That is, when the buyer is more risk averse, the seller tries to impose less fuel surcharges. The seller can reduce a chance of imposing a fuel surcharge. In addition to that, even within this optimal strategy of the *OFS*, the seller reduces the magnitude of fuel surcharges by increasing the level of the base rate along with the level of the buyer's risk averseness.

Given this change on the base rate, the freight rate should be increased at the same time to compensate the seller's fuel cost. For example, the high type buyer's risk averseness increases, the freight rates charged to both the high type buyer and the low type buyer increases.

($\frac{\partial p_{FH,RA}^{MB,OFS}}{\partial \lambda_H} > 0$ and $\frac{\partial p_{FL,RA}^{MB,OFS}}{\partial \lambda_H} > 0$). However, we also observe that this increase in risk averseness

can make the high type buyer's freight rate lower ($\frac{\partial p_{FH,RA}^{MB,OFS}}{\partial \lambda_H} < 0$) when the value of the seller's

risk averseness (μ) is large enough. Likewise, we observe that $\frac{\partial p_{FL,RA}^{MB,OFS}}{\partial \lambda_L} > 0$ for small values of

μ, but for larger values of μ, $\frac{\partial p_{FL,RA}^{MB,OFS}}{\partial \lambda_L} < 0$. As we discussed earlier under the single buyer

cases, it is because, even with the optimal strategy is the *OFS*, the seller is likely to act like she

offers the *AFS* when her own risk averseness is strong. The seller's dependency on the fuel

surcharge is large when the buyer's risk averseness is relatively small enough. However, we note

that it is a temporary reaction when the seller is highly risk averse while the buyer's risk

averseness is minimal. As the buyer's risk averseness increases, a decrease in the freight rate

turns out to an increase, and this increase in the fuel freight rate can be substituted with a

decrease in the fuel surcharge. In order to do that, the base rate should be increased. However,

the seller's this behavior applies to the high type buyer only, for example, when only the high

type buyer's risk averseness increases. In the same fashion, if the low type buyer's risk

averseness increases, (i) only the low type buyer's freight rate will be increased with relatively

low level of the seller's risk averseness, but it will be decreased as the seller's risk averseness

goes to high enough, and (ii) the high type buyer's freight rate will be increased monotonically.

Finally, the seller's profit decreases as the buyers' risk averseness levels increase

($\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \lambda_i} < 0$). An increase in λ_i makes the seller's profit being away from the benchmark profit

level. When this happens, the seller tries to keep the base rate high, so that she could charge a

lower fuel surcharge to the buyer. Instead, the seller tends to raise the freight rate in general. It is

a way for the seller not to drop her profit too low.

Seller Risk Averseness (μ): Above in section 4.1, we found the condition for feasible

mileage sum, which is given by $\frac{1}{\sqrt{\mu}} < (m_H + m_L) \leq \left(\theta - \frac{\bar{p}}{\varepsilon}\right) \frac{2\varepsilon^2 (\lambda + \mu)}{\lambda \mu \delta^2}$. Given this condition,

one interesting thing we observe is that the larger the seller's risk averseness is, the larger the

range of mileage sum is guaranteed. In other words, as the seller's risk averseness increases, the

seller will have a larger feasible range of mileages she can accept and serve to the buyer. This is

a bit counterintuitive. However, this seems how the seller responds the market changes according

to her risk averseness level. As we discussed in the single buyer case in section 3.3.2, as the

seller's risk averseness increases, her dependency on fuel surcharges increases; for example, the

seller lowers the base rate ($\frac{\partial p_{B,RA}^{MB,OFS}}{\partial \mu} < 0$). Since her insurance is activated in this way, the seller

now can serve higher mileages than before. Same phenomenon occurs here under the two-buyer

case. With an increase in her own risk averseness, the seller does not seem to intend to serve

more mileages. However, she ends up with being able to serve more mileages by reducing her

uncertainty by charging fuel surcharges.

However, to cooperate all of these risks, it seems inevitable for the seller to have a less

profit. We observe that all decision variables are decreasing as her own risk averseness increases

($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \mu} < 0$, $\frac{\partial p_{B,RA}^{MB,OFS}}{\partial \mu} < 0$, and $\frac{\partial \pi_{RA}^{MB,OFS}}{\partial \mu} < 0$). The base rate moves to the opposite direction as

the seller's risk averseness moves. It basically tells us that as the seller's risk averseness

increases, the fuel surcharge amount will increases as well in one direction. This is a consistent

with what we have observed under the single buyer cases that the seller is likely dependent more

upon the fuel surcharges when she is highly risk-averse. At the same time, however, the seller is

more likely reduce the chance of imposing a fuel surcharge when the buyer's risk averseness

level is high enough. In addition to the movement of the base rate, an increased fuel surcharge leaves a room for the freight rate to be decreased. The seller profit will be impacted the seller's risk averseness. It should be away from the benchmark profit as the seller's risk averseness increases.

Fuel Efficiency (ε): We find no special impact of the fuel efficiency parameter under the two-buyer case, different from what we found in the single buyer case. The freight rates assigned to both types of buyers and the seller's profit increase as the fuel efficiency parameter ε increases ($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \varepsilon} > 0$ and $\frac{\partial \pi_{RA}^{MB,OFS}}{\partial \varepsilon} > 0$). Both the fuel surcharge and fuel cost decrease with the fuel efficiency. Hence, in the case that buyers pay a fuel surcharge, they can save their total utility since an increase in fuel efficiency makes the buyers to have less fuel surcharge and less disabilities from the uncertain payment risks. It also leads the seller to be able to have more profit with less fuel costs. An increase in fuel efficiency directly decreases the seller's fuel cost consumption and her disutility from the fuel price fluctuation. Therefore, the seller has an incentive to increase the fuel efficiency if the long term profit that she can earn exceeds the short term investment for an improvement of fuel efficiencies. Overall, an increase in this parameter ε independently makes the whole system more efficient, and thus it brings higher social welfare.

Average Fuel Price ($\overline{p}$): An increase in the average fuel price raises the level of the base rate ($\frac{\partial p_{B,RA}^{MB,OFS}}{\partial \overline{p}} > 0$). Under the *OFS* strategy, if the average fuel price increases, the base rate also should increase accordingly since it is determined within a range for the strategy chosen. However, the seller's profit decreases when the average fuel price increases ($\frac{\partial \pi_{RA}^{MB,OFS}}{\partial \overline{p}} < 0$). Since the fuel price directly affects the seller's cost of operation and thus her profit, it is obvious

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that more fuel consumption with per unit fuel price brings the seller less profit. Lastly, since the optimal freight rates for both types of buyers are not functions of the average fuel price, and thus, this particular parameter does not affect the freight rates.

  Fuel Price Fluctuation (δ): Again, we observe the similar effects of the fuel price variation on decision variables in two buyers scenario. Given an average fuel price, if the level of fuel price fluctuation increases, the base rate may decreases or may increases ($\frac{\partial p_{B,RA}^{MB,OFS}}{\partial \delta} < 0$ or > 0). The sign changes depending on which player's risk averseness level is larger. First, given mileages from the buyers, if the sum of two buyers' risk averseness is smaller than the twice of the seller's risk averseness ($\lambda_H + \lambda_L < 2\mu$), the base rate decreases, and so the seller tends to depend more on the fuel surcharge pricing. In this situation, the freight rates for both buyers will decrease as well when the fuel price uncertainty increases ($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \delta} < 0$). It is a plausible pricing scheme since the seller needs to bake in the decreased freight rate with an increasing fuel surcharge. However, even with this reaction, an increase in δ hurts the seller's profit ($\frac{\partial \pi_{RA}^{MB,OFS}}{\partial \delta} < 0$). Even if the seller uses a fuel surcharge pricing to compensate her in some way, her profit cannot reach the benchmark profit level. Along with other parameters, the fuel price uncertainty level creates a disutility on the seller's profit and thus on the social welfare.

  Second, under the same situation such that the uncertainty parameter (δ) increases, however, if the buyers' risk averseness sum is higher than the twice of the seller's risk averseness ($\lambda_H + \lambda_L > 2\mu$), the base rate increases as the fuel price fluctuates more. It means that the seller tries to depend less on the fuel surcharge. Since either both buyer's risk averseness are relatively large or one buyer's risk averseness is sufficiently large in this case, it is reasonable for

the seller to raise the level of the base rate to accommodate the buyer's risk averseness tendency

into the contract. Interestingly, the seller still reduces the freight rates ($\frac{\partial p_{Fi,RA}^{MB,OFS}}{\partial \delta} < 0$) charging

the buyers lower fuel surcharges. It means that the seller's total charge to the buyers is not

sufficiently compensated from the high volatility of the fuel price, especially when the buyers are

highly risk averse. Since the total payment decreases, the seller's profit decreases as well as δ

increases ($\frac{\partial \pi_{RA}^{SB,OFS}}{\partial \delta} < 0$). Therefore, we find that the seller is not always able to respond

positively against the changes in fuel price uncertainty.

4.2 Asymmetric Buyer Differentiation: Buyers with Different Types

So far, we studied the case where the seller faces with two independent risk-averse buyers. Even

though we could find some specific changes in term of the optimal values chosen and have some

insights from the parameter changes, the overall optimal policies still stay the same under the

similar scenarios as we observed in the single buyer cases. In an extreme case where those

parameters that differentiate two buyers each other are the same ($m_H = m_L = m$, $\theta_H = \theta_L = \theta$ and

$\lambda_H = \lambda_L = \lambda$), the seller's profit will just become a simple sum of two profits from the contracts

with each of the buyers. Therefore, we can conclude that multiple contracts do not change the

seller's decision about the optimal choices of pricing strategy in general if they are made

simultaneously. However, what if two sellers are differentiated asymmetrically?

In this section, let us suppose that the seller faces with two buyers whose levels of their

risk averseness are different; that is, one buyer is risk-averse and the other one is risk-neutral. Let

us define the risk-averse buyer as a low type buyer and the risk-neutral buyer as a high type

buyer because we defined earlier that the high type buyer is assumed to be less risk-averse. For

example, a large shipper can be a high type buyer since large shippers are typically insensitive to

the risk of uncertainty. At its extreme case, we can consider that this high type buyer has no risk averseness at all, which he is assumed to be a risk-neutral buyer. For mathematical simplicity without loss of generality, let us assume, for the further analysis, that the buyers' mileages and service values are identical across the buyers; i.e., $m_H = m_L = m$ and $\theta_H = \theta_L = \theta$.

(1) When the *OFS* strategy is chosen:

Suppose the seller practices the *OFS* strategy. The risk-averse (low-type) buyer's mean-variance utility function and the risk-neutral (high-type) buyer's utility are given by, respectively,

$$U_{L,Mixed}^{MB,OFS} = \left(\theta - p_{FL} - \frac{\overline{p} + \delta - p_B}{2\varepsilon} \right) m - \frac{\lambda_L}{2} \left(\left(\frac{\overline{p} + \delta - p_B}{2\varepsilon} \right) m \right)^2 \quad \text{and}$$

$$U_{H,Mixed}^{MB,OFS} = \left(\theta - p_{FH} - \frac{\overline{p} + \delta - p_B}{2\varepsilon} \right) m.$$

Note that only the low type buyer is risk-averse, so he follows a mean-variance utility function having a risk averseness parameter; that is, $\lambda_L > 0$ while $\lambda_H = 0$. The risk-averse seller solves her mean-variance profit function, which is given by

$$\pi_{MB,Mixed}^{OFS} = \left(p_{FH} + p_{FL} - \frac{\overline{p} - \delta + p_B}{2\varepsilon} \right) m - \frac{\mu}{2} \left(\left(\frac{-\overline{p} + \delta + p_B}{\varepsilon} \right) m \right)^2.$$

Then, optimal solution are given by

$$p_{B,Mixd}^{MB,OFS} = \overline{p} + \frac{\delta(\lambda_L - 4\mu)}{\lambda_L + 4\mu}, \quad p_{FH,Mixd}^{MB,OFS} = \theta - \frac{4\mu\delta}{\varepsilon(\lambda_L + 4\mu)}, \quad p_{FL,Mixed}^{MB,OFS} = \theta - \frac{4\mu\delta}{\varepsilon} - \frac{8\lambda_L \mu^2 \delta^2 m}{\varepsilon^2 (\lambda_L + 4\mu)^2},$$

$$\text{and} \quad \pi_{Mixed}^{MB,OFS} = 2\left(\theta - \frac{\overline{p}}{\varepsilon} \right) m - \frac{2\lambda_L \mu \delta^2 m^2}{\varepsilon^2 (\lambda_L + 4\mu)}.$$

(2) When the *AFS* strategy is chosen:

Suppose now the seller practices the *AFS* strategy. The problems for the risk-averse low-type buyer and the risk-neutral high-type buyer are respectively given by

$$U_{L,Mixed}^{MB,AFS} = \left(\theta - p_{FL} - \frac{\bar{p} - p_B}{\varepsilon}\right)m - \frac{\lambda_L}{2}\left(\frac{\delta}{\varepsilon}m\right)^2 \text{ and } U_{H,Mixed}^{MB,AFS} = \left(\theta - p_{FH} - \frac{\bar{p} - p_B}{\varepsilon}\right)m.$$

The seller solves her profit function, which is given by

$$\pi_{MB,Mixed}^{AFS} = \left(p_{FH} + p_{FL} - \frac{2p_B}{\varepsilon}\right)m.$$

Like the other single buyer *OFS* cases, we find $p_{B,Mixed}^{MB,AFS} = \bar{p} - \delta - \rho_{AFS}$ by defining ρ_{AFS} as

$\rho_{AFS} \in (0, \bar{p} - \delta]$. Then, optimal solution are given by

$$p_{B,Mixed}^{MB,AFS} = \bar{p} - \delta - \rho_{AFS}, \quad p_{FH,Mixd}^{MB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon}, \quad p_{F,Mixed}^{MB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda_L \delta^2 m}{2\varepsilon^2},$$

$$\text{and } \pi_{Mixed}^{MB,AFS} = 2\left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\lambda_L \delta^2 m^2}{2\varepsilon^2}.$$

Proposition 6. *Suppose that a risk-averse seller make contracts with two types of buyers. One is a risk-averse low type buyer and the other one is a risk-neutral high type buyer. Then, the seller will chooses the OFS strategy as the optimal pricing schedule by setting the base rate within the range of $\bar{p} - \delta$ and $\bar{p} + \delta$.*

Proof. Let us compare two profits above. $\pi_{Mixed}^{MB,OFS}$ and $\pi_{Mixed}^{MB,AFS}$. If we suppose $\pi_{Mixed}^{MB,OFS} > \pi_{Mixed}^{MB,AFS}$,

the following is true: $\pi_{Mixed}^{MB,OFS} > \pi_{Mixed}^{MB,AFS} \Leftrightarrow 2\left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{2\lambda_L \mu \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)} > 2\left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\lambda_L \delta^2 m^2}{2\varepsilon^2}$

$\Leftrightarrow \frac{2\lambda_L \mu \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)} < \frac{\lambda_L \delta^2 m^2}{2\varepsilon^2} \Leftrightarrow \lambda_L > 0$. This inequality always holds. $\hspace{2cm}$ *Q.E.D.*

When a contract can be made individually, the seller would choose the *AFS* strategy for the risk-neutral high type buyer and the *OFS* for the risk-averse low type buyer as we discussed in section 3. However, when the seller meets two different types of buyers simultaneously, since the seller can pick only one base rate, we find that the *OFS* schedule is still the optimal for the

seller. We examine how this seller's decision affect the both types of buyers. To do that, we perform the *ex-post* analysis and compare the profits and utilities under the two possible actual fuel prices ($p_A = \bar{p} - \delta$ and $p_A = \bar{p} + \delta$).[23] Then, we draw the following proposition.

Proposition 7. *Suppose that the risk-averse seller deals with two types of buyers simultaneously. One is risk-averse and the other one is risk-neutral.*

(i) *For each unit of fuel cost change, the seller, the risk-neutral buyer, and the risk-averse buyer carry* $\dfrac{\lambda_L}{\lambda_L + 4\mu}$, $\dfrac{2\mu}{\lambda_L + 4\mu}$, *and* $\dfrac{2\mu}{\lambda_L + 4\mu}$ *proportions of the unit cost change, respectively.*

(ii) *The low type (risk-averse) buyer carries relatively higher burden than anyone of either the seller or the high type (risk-neutral) buyer.*

Proof. See the Appendix.

We found that even though one player's risk sharing is still dependent upon the opponent player's risk averseness level, the seller does not equally share her risks with the buyers anymore when there are more than a single buyer. Earlier under the single buyer case, the seller's proportion was $\dfrac{\lambda}{\lambda + \mu}$, compared to that of the buyer, $\dfrac{\mu}{\lambda + \mu}$. It means that whoever has a smaller risk averseness deals with a burden of risk a bit more at the optimal level of contract. In this particular case, however, the seller carries a $\dfrac{\lambda_L}{\lambda_L + 4\mu}$ portion of the change in the cost of operation while the set of buyers will carry a $\dfrac{4\mu}{\lambda_L + 4\mu}$ portion of it. Therefore, comparing these two and given $0 < \lambda_L < 1$ and $0 < \mu < 1$, a risk sharing of the seller is a lot lighter than the sum of

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[23] Summary of the ex-post analysis results under the two buyers cases are in the Appendix A (Table 7).

each buyers' portion. That is, as the number of buyer increases, there will be more room for the seller to transfer her burden of risk to buyers given levels of risk averseness. For example, even if the levels of risk averseness of the seller and the low type buyer are the same, the seller has only one-fourth of the amount that a single risk-averse buyer should carry.[24]

We argue that the low type buyer bears more burden of risk due to an uncertain fuel cost. The high type buyer is risk-neutral. Thus, if the high type buyer make a contract with the seller individually, he needs to carry all of the seller's risks with the *AFS* contract as we discussed in section 3. Even with a case that this buyer could make an *OFS* contract, the he would carry

$\dfrac{\mu}{\lambda_H + \mu}$ portion of any additional cost occurred. Compared this proportion with what he carries

under the two buyers case ($\dfrac{2\mu}{\lambda_L + 4\mu}$) and an assumption of any non-risk-neutral behavior, the

chance this high type buyer bears a higher risk burden is minimal as far as $\lambda_H < \lambda_L$.[25]

On the contrary, when the seller makes two contract simultaneously, a risk burden of the low type buyer is likely to be higher than he would have under the single buyer case. This happens especially when the seller tries to protect herself heavily. Given a set of risk averseness values (λ_L and μ), on one hand, the absolute amount of risk burden that the low type buyer should carry may be smaller since he can still share the buyers' portion of an additional unit cost with the high type buyer. On the other hand, however, since the low type buyer is risk averse, and his risk averseness level is relatively high, if the low type buyer wants to carry a smaller

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[24] $\dfrac{\lambda_L}{\lambda_L + 4\mu} < \dfrac{2\mu}{\lambda_L + 4\mu} + \dfrac{2\mu}{\lambda_L + 4\mu} \Leftrightarrow \dfrac{\mu}{\lambda_L} > \dfrac{1}{4}.$

[25] For a chance of $\dfrac{2\mu}{\lambda_L + 4\mu} > \dfrac{\mu}{\lambda_H + \mu}$, an inequality of $2\lambda_H - \lambda_L > 2\mu$ should be held. However, given $\lambda_H < \lambda_L$ this is possible only when $\lambda_H \simeq \lambda_L$ and μ is nearly zero.

portion of burden than before, the seller's risk averseness level should be at least less than a half

of his own risk averseness level.[26] This is somewhat a rare case in practice. Therefore, we

conclude that the one who bears the most risks due to uncertain fuel cost is the low type buyer.

This argument supports the claim we addressed in the beginning of the current study.

In sum, the seller is likely to share any additional risk driven by an uncertain cost of fuel

with the buyers. However, if the low type buyer's risk averseness is relatively high, the seller

choose the *OFS* strategy as an optimal policy even though there is another buyer who is risk-

neutral, whose desirable policy would be the *AFS* strategy. If there are more number of buyer

than a single one, the seller can have a higher chance to reduce the risk burden from an uncertain

cost of operation, and transfer this burden to the buyers. Even though these two types of buyers

share the same amount of risk proportions, under the *OFS* strategy, the high type buyer's relative

burden of risk is actually reduced from what he would have under the *AFS* strategy, and this

reduction on risk burden is transferred to the low type buyer. Moreover, if the seller's risk

averseness level increases, the low type buyer's risk burden will increase.

5   Conclusion

There were two claims that mentioned earlier as we started with this project. The first one was

how the seller can be indifferent with the total rate unchanged if she offers the buyer a lower fuel

surcharge with a slight adjustment of the freight rate. The way we approached this question was

to investigate the players' risk-averse behavior. We showed that the risk-averse seller is likely to

offer the buyer the *OFS* contract charging the buyer reduced fuel surcharge keeping the total

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[26] This condition follows $\dfrac{2\mu}{\lambda_L + 4\mu} < \dfrac{\mu}{\lambda_L + \mu} \Leftrightarrow \dfrac{\mu}{\lambda_L} < \dfrac{1}{2}$.

payment balanced if the buyer is risk-averse as well. Thus, we showed that a fuel surcharge scheme can be used as a medium for the seller to mitigate the risk averseness and to reduce the uncertainty burden between the trade members.

The second claim argues that the small shipper bears the higher burden of the cost of fuel than the large shipper does. To investigate who between the small and large buyers would primarily carry the risk burden, we setup a market with two buyers who are differentiated with their risk averseness levels, and finally have seen that a fuel surcharge policy can also coordinate the market where multiple players exist. Our finding actually supports the claim if the small shipper is assumed to be a risk-averse buyer, compared to the risk-neutral large shipper. In this particular case, first, the seller's risk sharing burden will be lighter with multiple buyers, and second, each buyer will share their heavier portion of uncertain cost of operation. However, the chosen fuel surcharge mechanism will coordinate the risk sharing in a way that the risk-neutral large buyer can transfer its own burden of such risk to the risk-averse small buyer.

In addition, our investigation showed many other interesting results. For example, first, we found conditions for the case where imposing fuel surcharges is not always an outperforming strategy over a strategy without it. Second, along with the pass-through of burden of risk, buyers can also have a chance to benefit from the uncertain cost of fuel and fuel surcharges imposed to them. This happens when the uncertain cost of fuel turns out to be realized as a favorable actual cost. Third, even with an unfavorably realized fuel cost, the fuel surcharge pricing mechanism plays an important role to reduce the players' variation over uncertain cost of fuel.

Major contributions of this study is twofold: First one is its theoretical application. There are largely two types of fuel surcharge formulas that are widely used in industry; one is the linehaul-based fuel surcharge, and the other one is the mileage-based fuel surcharge. We adopt

the latter one, which is most commonly used in industry in practice. There has been studies on fuel surcharges, but with only few in academia, especially in business areas. However, they all use some existing additive pricing theory or other non-practical theory-based pricing models to describe the fuel surcharge mechanism. With our best knowledge, our current work is the first paper that uses the real-world formula. In addition to this, we found most of existing studies use the typical linear demand to model this specific market. However, there are some issues found on demand models they use in order to reflect the theories on the practice. Demand does not depend on the mileages. We take this into account to our model setup. Second contribution of this study is that, since we adopt the real-world formula, use the demand model that reflects the real world, and setup assumptions in ways to be highly plausible, the outcomes of the analysis can be easily applicable to the practice. This is important in term of its managerial application of the paper.

As one of the future research, we suggest an examination of the fuel efficiency. In the current study, fuel efficiency is treated as an exogenous parameter. However, fuel efficiency can be improved based on the carrier's investment. From the formula we introduced in section 2.1, raising the escalator for fuel efficiency can lower total costs. Industry research shows that when sellers use a larger escalator number for fuel efficiency, they tend to spend less in total transportation costs. However, sellers estimate about the cost of fuel when they quote the total payment, and then determine how much of that cost is covered by the seller's fuel pricing schedule compared to their actual fuel costs. If costs stay in a relatively close range, it is usually close enough for a buyer. For example, when a seller uses a six cent escalator instead of a five cent escalator, the seller is unlikely to drop the customer, since the difference is only a penny per unit. This may be another useful instrument that the seller can play with to make their decisions to modify the payments.

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APPENDIX A. Tables and Figures

Table 1. Summary of Optimal Values in a Single-Seller-Single-Buyer Market

3.1.1 Single buyer case with no risk-averse

	$p_{F,NoRA}^{SB}$	$p_{B,NoRA}^{SB}$	π_{NoRA}^{SB}
Case 1: *NFS* ($\bar{p}+\delta \le p_B$)	θ	n/a	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$
Case 2: *OFS* ($\bar{p}-\delta \le p_B < \bar{p}+\delta$)	$\theta - \dfrac{\rho_{OFS}}{2\varepsilon}$	$\bar{p}+\delta-\rho_{OFS}$, where $0 < \rho_{OFS} \le 2\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$
Case 3: *AFS* ($0 \le p_B < \bar{p}-\delta$)	$\theta - \dfrac{\delta+\rho_{AFS}}{\varepsilon}$	$\bar{p}-\delta-\rho_{AFS}$, where $0 < \rho_{AFS} \le \bar{p}-\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$

3.1.2 Single buyer case when only the buyer is risk-averse

	$p_{F,BORA}^{SB}$	$p_{B,BORA}^{SB}$	π_{BORA}^{SB}
Case 1: *NFS* ($\bar{p}+\delta \le p_B$)	θ	n/a	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$
Case 2: *OFS* ($\bar{p}-\delta \le p_B < \bar{p}+\delta$)	$\theta - \dfrac{\rho_{OFS}}{2\varepsilon} - \dfrac{\lambda m \rho_{OFS}^2}{8\varepsilon^2}$	$\bar{p}+\delta-\rho_{OFS}$, where $0 < \rho_{OFS} \le 2\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\lambda m^2 \rho_{OFS}^2}{8\varepsilon^2}$
Case 3: *AFS* ($0 \le p_B < \bar{p}-\delta$)	$\theta - \dfrac{\delta+\rho_{AFS}}{\varepsilon} - \dfrac{\lambda\delta^2 m}{2\varepsilon^2}$	$\bar{p}-\delta-\rho_{AFS}$, where $0 < \rho_{AFS} \le \bar{p}-\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\lambda\delta^2 m^2}{2\varepsilon^2}$

3.2.1 Single buyer case when only the seller is risk-averse

	$p_{F,SORA}^{SB}$	$p_{B,SORA}^{SB}$	π_{SORA}^{SB}
Case 1: *NFS* ($\bar{p}+\delta \le p_B$)	θ	n/a	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\mu\delta^2 m^2}{2\varepsilon^2}$
Case 2: *OFS* ($\bar{p}-\delta \le p_B < \bar{p}+\delta$)	$\theta - \dfrac{\delta}{\varepsilon}$	$\bar{p}-\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$
Case 3: *AFS* ($0 \le p_B < \bar{p}-\delta$)	$\theta - \dfrac{\delta+\rho_{AFS}}{\varepsilon}$	$\bar{p}-\delta-\rho_{AFS}$, where $0 < \rho_{AFS} \le \bar{p}-\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)$

3.2.2 Single buyer case when both the seller and the buyer are risk-averse

	$p_{F,RA}^{SB}$	$p_{B,RA}^{SB}$	π_{RA}^{SB}
Case 1: *NFS* ($\bar{p}+\delta \le p_B$)	θ	n/a	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\mu\delta^2 m^2}{2\varepsilon^2}$
Case 2: *OFS* ($\bar{p}-\delta \le p_B < \bar{p}+\delta$)	$\theta - \dfrac{\delta\mu}{\varepsilon} - \dfrac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda+\mu)^2}$	$\bar{p}+\dfrac{\delta(\lambda-\mu)}{\lambda+\mu}$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$
Case 3: *AFS* ($0 \le p_B < \bar{p}-\delta$)	$\theta - \dfrac{\delta+\rho_{AFS}}{\varepsilon} - \dfrac{\lambda\delta^2 m}{2\varepsilon^2}$	$\bar{p}-\delta-\rho_{AFS}$, where $0 < \rho_{AFS} \le \bar{p}-\delta$	$m\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right) - \dfrac{\lambda\delta^2 m^2}{2\varepsilon^2}$

Table 3. Comparative statics for optimal strategies (Single buyer case)

(1) When only buyer is risk-averse								
NFS	m	ε	θ	$\bar{p}$	δ	λ	μ	ρ
$p_{B,BORA}^{SB,NFS}$	0	0	0	0	0	0	0	0
$p_{F,BORA}^{SB,NFS}$	0	0	+	0	0	0	0	0
$\pi_{BORA}^{SB,NFS}$	+	+	+	−	0	0	0	0
(2) When only seller is risk-averse								
AFS	m	ε	θ	$\bar{p}$	δ	λ	μ	ρ
$p_{B,SORA}^{SB,AFS}$	0	0	0	+	−	0	0	−
$p_{F,SORA}^{SB,AFS}$	0	+	+	0	−	0	0	−
$\pi_{RA}^{SB,AFS}$	+	+	+	−	0	0	0	0
(3) When both players are risk-averse								
OFS	m	ε	θ	$\bar{p}$	δ	λ	μ	ρ
$p_{B,RA}^{SB,OFS}$	0	0	0	+	$+/-^{\dagger}$	+	−	0
$p_{F,RA}^{SB,OFS}$	−	+	+	0	−	$+/\mp^{\dagger\dagger}$	−	0
$\pi_{RA}^{SB,OFS}$	±	+	+	−	−	−	−	0

('+' increase; '−' decrease; '±(or ∓)' increase, then decrease (vice versa); '+/−' may increase or may decrease; '0' not dependent)

† $\dfrac{\partial p_{B,RA}^{SB,OFS}}{\partial \delta}$ for $\lambda \neq \mu$.

†† $\dfrac{\partial p_{F,RA}^{SB,OFS}}{\partial \lambda} > 0$ for small values of μ, but for larger values of μ, $\dfrac{\partial p_{F,RA}^{SB,OFS}}{\partial \lambda}$ changes its sign from negative to positive.

Table 4. Summary of *Ex-Post* Analysis Results under Single Buyer Cases

3.4.1. Risk-averse seller and risk-neutral buyer (optimal strategy: *AFS*)

Ex-ante	*Ex-post* ($p_A = \bar{p} + \delta$)	
$\pi_{RA}^{SB,AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m$ $U_{RA}^{SB,OFS} = 0$ $SW_{RA}^{SB,AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m$	$\pi\big|_{P_A=\bar{P}+\delta}^{AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m$ $U\big|_{P_A=\bar{P}+\delta}^{AFS} = -\dfrac{\delta}{\varepsilon}m$ $SW\big|_{P_A=\bar{P}+\delta}^{AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$	$\pi\big|_{P_A=\bar{P}+\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$ $U\big|_{P_A=\bar{P}+\delta}^{NFS} = 0$ $SW\big|_{P_A=\bar{P}+\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$
	Ex-post ($p_A = \bar{p} - \delta$)	
	$\pi\big|_{P_A=\bar{P}-\delta}^{AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m$ $U\big|_{P_A=\bar{P}-\delta}^{AFS} = \dfrac{\delta}{\varepsilon}m$ $SW\big|_{P_A=\bar{P}-\delta}^{AFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$	$\pi\big|_{P_A=\bar{P}-\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$ $U\big|_{P_A=\bar{P}-\delta}^{NFS} = 0$ $SW\big|_{P_A=\bar{P}-\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$

3.4.2. Risk-averse seller and risk-averse buyer (optimal strategy: *OFS*)

Ex-ante	*Ex-post* ($p_A = \bar{p} + \delta$)	
$\pi_{RA}^{SB,OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$ $U_{RA}^{SB,OFS} = 0$ $SW_{RA}^{SB,OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$	$\pi\big|_{P_A=\bar{P}+\delta}^{OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} - \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)}$ $U\big|_{P_A=\bar{P}+\delta}^{OFS} = \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} - \dfrac{\mu\delta m}{\varepsilon(\lambda+\mu)}$ $SW\big|_{P_A=\bar{P}+\delta}^{OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$	$\pi\big|_{P_A=\bar{P}+\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$ $U\big|_{P_A=\bar{P}+\delta}^{NFS} = 0$ $SW\big|_{P_A=\bar{P}+\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m$
	Ex-post ($p_A = \bar{p} - \delta$)	
	$\pi\big|_{P_A=\bar{P}-\delta}^{OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} + \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)}$ $U\big|_{P_A=\bar{P}-\delta}^{OFS} = \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} + \dfrac{\mu\delta m}{\varepsilon(\lambda+\mu)}$ $SW\big|_{P_A=\bar{P}-\delta}^{OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$	$\pi\big|_{P_A=\bar{P}-\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$ $U\big|_{P_A=\bar{P}-\delta}^{NFS} = 0$ $SW\big|_{P_A=\bar{P}-\delta}^{NFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m$

Table 5. Summary of Optimal Values in a Single-Seller-Multiple-Buyer Market

4.1 Two buyers case when all the seller and the buyers are risk-averse ($i = H, L$ & $i \neq -i$)			
	p_{Fi}^{MB}	p_B^{SB}	π^{MB}
Case 1: NFS ($\bar{p} + \delta \leq p_B$)	θ_i	Any $p_B \geq \bar{p} + \Delta$	$\left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L$ $- \dfrac{\mu\delta^2\left(m_H^2 + m_L^2\right)}{2\varepsilon^2}$
Case 2: OFS ($\bar{p} - \delta \leq p_B < \bar{p} + \delta$)	$\theta_i - \dfrac{\mu\delta\left(m_i + m_{-i}\right)^2}{\varepsilon\left(\begin{array}{c}\lambda_i m_i^2 + \lambda_{-i}m_{-i}^2 \\ +\mu\left(m_i + m_{-i}\right)^2\end{array}\right)}$ $- \dfrac{\lambda_i \mu^2 \delta^2 m_i \left(m_i + m_{-i}\right)^4}{2\varepsilon^2\left(\begin{array}{c}\lambda_i m_i^2 + \lambda_{-i}m_{-i}^2 \\ +\mu\left(m_i + m_{-i}\right)^2\end{array}\right)^2}$	$\bar{p} - \delta$ $+ \dfrac{2\delta\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{\left(\begin{array}{c}\lambda_H m_H^2 + \lambda_L m_L^2 \\ +\mu\left(m_H + m_L\right)^2\end{array}\right)}$	$\left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L$ $- \dfrac{\mu\delta^2\left(m_H + m_L\right)^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2\right)}$
Case 3: AFS ($0 \leq p_B < \bar{p} - \delta$)	$\theta_i - \dfrac{\delta + \rho_{AFS}}{\varepsilon} - \dfrac{\lambda_i \delta^2 m_i}{2\varepsilon^2}$	$\bar{p} - \delta - \rho_{AFS}$, where $0 < \rho_{AFS} \leq \bar{p} - \delta$	$\left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L$ $- \dfrac{\delta^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2}$

Table 6. Comparative statics for the optimal *OFS* strategy (Two buyers case)

All seller and two buyers are risk-averse										
OFS	m_H	m_L	θ_H	θ_L	λ_H	λ_L	μ	$\bar{p}$	δ	ε
$p_{B,RA}^{MB,OFS}$	$-$	$+$	0	0	$+$	$+$	$-$	$+$	$+/-$	0
$p_{FH,RA}^{MB,OFS}$	$-$	$+$	$+$	0	$+/-$	$+$	$-$	0	$-$	$+$
$p_{FL,RA}^{MB,OFS}$	$-$	$\pm$	0	$+$	$+$	$+/-$	$-$	0	$-$	$+$
$\pi_{RA}^{MB,OFS}$	$\pm$	$\pm$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$

('+' increase; '−' decrease; '±(or ∓)' increase, then decrease (vice versa); '+/−' may increase or may decrease; '0' not dependent)

Table 7. Summary of *Ex-Post* Analysis Results under Multiple Buyers Cases

4.2 Risk-averse seller with a risk-neutral buyer and a risk-averse buyer (optimal: *OFS*)	
Ex-ante	*Ex-post* ($p_A = \overline{p} + \delta$)
	$\pi\big|_{p_A=\overline{p}+\delta}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{2\lambda_L \delta m}{\varepsilon(\lambda_L + 4\mu)} - \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)^2}$
	$U\big|_{H,p_A=\overline{p}+\delta}^{MB,OFS} = -\dfrac{4\mu\delta m}{\varepsilon(\lambda_L + 4\mu)}$
	$U\big|_{L,p_A=\overline{p}+\delta}^{MB,OFS} = -\dfrac{4\mu\delta m}{\varepsilon(\lambda_L + 4\mu)} + \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)^2}$
$\pi_{Mixed}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{2\lambda_L \mu \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)}$	$SW\big|_{p_A=\overline{p}+\delta}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{2\delta m}{\varepsilon}$
$U_{H,Mixed}^{MB,OFS} = U_{L,Mixed}^{MB,OFS} = 0$	*Ex-post* ($p_A = \overline{p} - \delta$)
$SW_{Mixed}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{2\lambda_L \mu \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)}$	$\pi\big|_{p_A=\overline{p}-\delta}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m + \dfrac{2\lambda_L \delta m}{\varepsilon(\lambda_L + 4\mu)} - \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)^2}$
	$U\big|_{H,p_A=\overline{p}-\delta}^{MB,OFS} = \dfrac{4\mu\delta m}{\varepsilon(\lambda_L + 4\mu)}$
	$U\big|_{L,p_A=\overline{p}-\delta}^{MB,OFS} = \dfrac{4\mu\delta m}{\varepsilon(\lambda_L + 4\mu)} + \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2(\lambda_L + 4\mu)^2}$
	$SW\big|_{p_A=\overline{p}-\delta}^{MB,OFS} = 2\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)m + \dfrac{2\delta m}{\varepsilon}$

APPENDIX B. Proofs and Derivations

B1. Proofs.

Proof of Proposition 1.

Since all other combinations of risk-averse and risk-neutral players' cases can reach the seller's

benchmark profit with zero buyer utility, we only prove the case of that both the buyer and the

seller are risk-averse. Thus, given a risk-averseness assumption, we compare profits of three

different cases that were driven above. First we compare (i) $\pi_{RA}^{SB,NFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\mu m^2 \delta^2}{2\varepsilon^2}$ and

$\pi_{RA}^{SB,OFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$. Suppose $\pi_{RA}^{SB,NFS} < \pi_{RA}^{SB,OFS}$. Then, $m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\mu m^2 \delta^2}{2\varepsilon^2}$

$< m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} \Leftrightarrow \dfrac{\mu m^2 \delta^2}{2\varepsilon^2} > \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} \Leftrightarrow 1 > \dfrac{\lambda}{\lambda+\mu}$. This inequality always holds

given $0 < \lambda, \mu \le 1$. (ii) Now we compare $\pi_{RA}^{SB,AFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\delta^2 m^2}{2\varepsilon^2}$ and $\pi_{RA}^{SB,OFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right)$

$- \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$. Suppose $\pi_{RA}^{SB,AFS} < \pi_{RA}^{SB,OFS}$. Then, $m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\delta^2 m^2}{2\varepsilon^2} < m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$

$\Leftrightarrow \dfrac{\lambda\delta^2 m^2}{2\varepsilon^2} > \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} \Leftrightarrow 1 > \dfrac{\mu}{\lambda+\mu}$. Again, this inequality always holds. Therefore, based on

(i) and (ii), we conclude that the seller will choose the *OFS* strategy when both the buyer and the

seller are risk averse. *Q.E.D.*

Proof of Proposition 2.

(i) $\dfrac{\pi_{SORA}^{SB,AFS}}{SW_{SORA}^{SB,AFS}} = 1 < \dfrac{\pi\big|_{P_A=\overline{p}+\delta}^{AFS}}{SW\big|_{P_A=\overline{p}+\delta}^{AFS}}$ since $\pi_{SORA}^{SB,AFS} = \pi\big|_{P_A=\overline{p}+\delta}^{AFS}$ and $SW_{SORA}^{SB,AFS} > SW\big|_{P_A=\overline{p}+\delta}^{AFS}$, and

$$\frac{U_{SORA}^{SB,AFS}}{SW_{SORA}^{SB,AFS}} = 0 > \frac{U\big|_{p_A=\bar{p}+\delta}^{AFS}}{SW\big|_{p_A=\bar{p}+\delta}^{AFS}} \text{ since } U\big|_{p_A=\bar{p}+\delta}^{AFS} < 0. \text{ From } \frac{\pi\big|_{p_A=\bar{p}+\delta}^{AFS} + U\big|_{p_A=\bar{p}+\delta}^{AFS}}{SW\big|_{p_A=\bar{p}+\delta}^{AFS}} = 1,$$

$$SW\big|_{p_A=\bar{p}+\delta}^{AFS} - \pi\big|_{p_A=\bar{p}+\delta}^{AFS} = -\frac{\delta}{\varepsilon}m, \text{ which is identical to } U\big|_{p_A=\bar{p}+\delta}^{AFS}. \text{ Therefore, all decrease in the utility}$$

is the same as the amount of decrease in the social welfare.

(ii) A similar analysis applies to the case of favorable actual fuel price, and we find the similar

results in the opposite direction. That is, the social welfare increases as much as the utility

increases after the realization. For both cases, an amount of $\dfrac{\delta}{\varepsilon}m$ is exactly how much more (or

less) the total cost of operation that the seller needs to spend. Q.E.D.

Proof of Lemma 2.

Under the *OFS* strategy, the social welfare is $SW_{RA}^{SB,OFS} = \left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)}$. We simply

compare this with *ex-post* social welfares given the realized fuel price:

1. The realized actual fuel price is $p_A = \bar{p} + \delta$, (i) assume $SW_{RA}^{SB,OFS} > SW\big|_{p_A=\bar{p}+\delta}^{OFS}$. Then,

$$\left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} > \left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\delta}{\varepsilon}m \Leftrightarrow \frac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} < \frac{\delta}{\varepsilon}m \Leftrightarrow \frac{\lambda\mu}{\lambda+\mu} < \frac{2\varepsilon}{\delta m}. \text{ (ii) Then, the}$$

condition of $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$ gives us $SW_{RA}^{SB,OFS} < SW\big|_{p_A=\bar{p}+\delta}^{OFS}$.

2. The actual price is realized at $p_A = \bar{p} - \delta$. We find $SW_{RA}^{SB,OFS} < SW\big|_{p_A=\bar{p}-\delta}^{OFS}$ since

$$\left(\theta - \frac{\bar{p}}{\varepsilon}\right)m - \frac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} < \left(\theta - \frac{\bar{p}}{\varepsilon}\right)m + \frac{\delta}{\varepsilon}m \Leftrightarrow \frac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)} > -\frac{\delta}{\varepsilon}m \Leftrightarrow \frac{\lambda\mu}{\lambda+\mu} > -\frac{2\varepsilon}{\delta m}, \text{ and this}$$

inequality always holds. Q.E.D.

Proof of Proposition 3.

To find how much portion of an additional unit cost of fuel the seller and the buyer share, we can find the ratio of the difference between the *ex-ante* and *ex-post* profit/utility to the difference between the *ex-ante* and *ex-post* social welfare as follows:

$$\frac{\pi\big|_{P_A=\bar{p}+\delta}^{OFS}-\pi_{RA}^{SB,OFS}}{SW\big|_{P_A=\bar{p}+\delta}^{OFS}-SW_{RA}^{SB,OFS}}=\frac{\left(\theta-\dfrac{\bar{p}}{\varepsilon}-\dfrac{\lambda\mu^2\delta^2 m}{2\varepsilon^2\left(\lambda+\mu\right)^2}-\dfrac{\lambda\delta}{\varepsilon\left(\lambda+\mu\right)}\right)m-\left(m\left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)-\dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2\left(\lambda+\mu\right)}\right)}{m\left(\theta-\dfrac{\bar{p}+\delta}{\varepsilon}\right)-\left(m\left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)-\dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2\left(\lambda+\mu\right)}\right)}$$

$$=\frac{\lambda}{\lambda+\mu},\text{ and }\frac{U\big|_{P_A=\bar{p}+\delta}^{OFS}-0}{SW\big|_{P_A=\bar{p}+\delta}^{OFS}-SW_{RA}^{SB,OFS}}=\frac{\left(\dfrac{\lambda\mu^2\delta^2 m}{2\varepsilon^2\left(\lambda+\mu\right)^2}-\dfrac{\mu\delta}{\varepsilon\left(\lambda+\mu\right)}\right)m}{m\left(\theta-\dfrac{\bar{p}+\delta}{\varepsilon}\right)-\left(m\left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)-\dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2\left(\lambda+\mu\right)}\right)}=\frac{\mu}{\lambda+\mu}.$$

This concludes that the seller is responsible for $\dfrac{\lambda}{\lambda+\mu}$ of an additional unit of cost, and the buyer

is responsible for $\dfrac{\mu}{\lambda+\mu}$ of an additional unit of cost. $\hspace{3cm}$ Q.E.D.

Proof of Lemma 3.

1. (i) Now, assume $\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon}<1$. Given this inequality, we suppose $\pi\big|_{P_A=\bar{p}+\delta}^{OFS}>\pi\big|_{P_A=\bar{p}+\delta}^{NFS}$. Then,

$$\pi\big|_{P_A=\bar{p}+\delta}^{OFS}=\left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m-\dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2\left(\lambda+\mu\right)^2}-\dfrac{\lambda\delta m}{\varepsilon\left(\lambda+\mu\right)}>\pi\big|_{P_A=\bar{p}+\delta}^{NFS}=\left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m-\dfrac{\delta}{\varepsilon}m.\text{ Further,}$$

$$\dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2\left(\lambda+\mu\right)^2}+\dfrac{\lambda\delta m}{\varepsilon\left(\lambda+\mu\right)}<\dfrac{\delta}{\varepsilon}m\Leftrightarrow\dfrac{\lambda\mu^2\delta m}{2\varepsilon\left(\lambda+\mu\right)^2}+\dfrac{\lambda}{\left(\lambda+\mu\right)}<1\Leftrightarrow\dfrac{\lambda}{\left(\lambda+\mu\right)}\left[\dfrac{\mu^2\delta m}{2\varepsilon\left(\lambda+\mu\right)}+1\right]<1.$$

Since $\dfrac{\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon}<\dfrac{1}{\lambda}$, $\dfrac{\lambda}{\left(\lambda+\mu\right)}\left[\dfrac{\mu^2\delta m}{2\varepsilon\left(\lambda+\mu\right)}+1\right]<\dfrac{\lambda}{\left(\lambda+\mu\right)}\left[\dfrac{\mu}{\lambda}+1\right]=1$. Therefore, the inequality

between profits holds: i.e., $\pi\big|_{P_A=\bar{p}+\delta}^{OFS}>\pi\big|_{P_A=\bar{p}+\delta}^{NFS}$.

(ii) Following Lemma 2, we found that to satisfy the inequality $SW\big|^{OFS}_{P_A=\bar{p}+\delta} > SW^{SB,OFS}_{RA}$, the

following condition should be met: $\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > 1$. Now, suppose $\pi\big|^{OFS}_{P_A=\bar{p}+\delta} < \pi\big|^{NFS}_{P_A=\bar{p}+\delta}$. Then,

$$\pi\big|^{OFS}_{P_A=\bar{p}+\delta} = \left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} - \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)} < \pi\big|^{NFS}_{P_A=\bar{p}+\delta} = \left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\delta}{\varepsilon}m . \text{ Further,}$$

$$\dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} + \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)} > \dfrac{\delta}{\varepsilon}m \Leftrightarrow \dfrac{\lambda\mu^2\delta m}{2\varepsilon(\lambda+\mu)^2} + \dfrac{\lambda}{(\lambda+\mu)} > 1 \Leftrightarrow \dfrac{\lambda}{(\lambda+\mu)}\left[\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right] > 1.$$

Since $\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > 1$, $\dfrac{\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > \dfrac{1}{\lambda}$. By using this, $\dfrac{\lambda}{(\lambda+\mu)}\left[\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right] > \dfrac{\lambda}{(\lambda+\mu)}\left[\dfrac{\mu}{\lambda} + 1\right]$

$$> \dfrac{\lambda}{(\lambda+\mu)}\cdot\dfrac{\mu}{\lambda} + \dfrac{\lambda}{(\lambda+\mu)} = 1 . \text{ It proves that } \dfrac{\lambda}{(\lambda+\mu)}\left[\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right] > 1 . \text{ Therefore,}$$

$$\pi\big|^{OFS}_{P_A=\bar{p}+\delta} < \pi\big|^{NFS}_{P_A=\bar{p}+\delta} .$$

2. (i) We analyze when the actual fuel price is at $p_A = \bar{p}-\delta$. First, given $\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > 1$,

Suppose $\pi\big|^{OFS}_{P_A=\bar{p}-\delta} = \left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m - \dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} + \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)} < \pi\big|^{NFS}_{P_A=\bar{p}-\delta} = \left(\theta-\dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{\delta}{\varepsilon}m. \text{ Then,}$

$$-\dfrac{\lambda\mu^2\delta^2 m^2}{2\varepsilon^2(\lambda+\mu)^2} + \dfrac{\lambda\delta m}{\varepsilon(\lambda+\mu)} < \dfrac{\delta}{\varepsilon}m \Leftrightarrow -\dfrac{\lambda\mu^2\delta m}{2\varepsilon(\lambda+\mu)^2} + \dfrac{\lambda}{(\lambda+\mu)} < 1 \Leftrightarrow \dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right] < 1.$$

Since $\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > 1$, $\dfrac{\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} > \dfrac{1}{\lambda}$. By using this, we can create the following inequality

relationships: $\dfrac{\lambda-\mu}{(\lambda+\mu)} = \dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu}{\lambda} + 1\right] > \dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right]$. However, since

$\dfrac{\lambda-\mu}{(\lambda+\mu)} < 1$, $\dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)} + 1\right] < 1$. we find that the supposition is true. It means that

$$\pi\big|^{OFS}_{P_A=\bar{p}-\delta} < \pi\big|^{NFS}_{P_A=\bar{p}-\delta} .$$

(ii) Now, we start the analysis with an supposition of $\pi\Big|_{p_A=\bar{p}-\delta}^{OFS} > \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$ given that

$\dfrac{\lambda\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} < 1$. Given a similar analysis in (i) above, we check whether the following inequality

holds: $\dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)}+1\right] > 1$. Since $\dfrac{\mu}{\lambda+\mu}\cdot\dfrac{\delta m}{2\varepsilon} < \dfrac{1}{\lambda}$, $\dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)}+1\right]$

$> \dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu}{\lambda}+1\right]$. This leads $\dfrac{\lambda-\mu}{(\lambda+\mu)} < 1 < \dfrac{\lambda}{(\lambda+\mu)}\left[-\dfrac{\mu^2\delta m}{2\varepsilon(\lambda+\mu)}+1\right]$. This is not sufficient to

prove $\pi\Big|_{p_A=\bar{p}-\delta}^{OFS} > \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$. However, from the equation of $SW\Big|_{p_A=\bar{p}-\delta}^{OFS} = SW\Big|_{p_A=\bar{p}-\delta}^{NFS}$ and

$U\Big|_{p_A=\bar{p}-\delta}^{OFS} > U\Big|_{p_A=\bar{p}-\delta}^{NFS}$, we observe that the inequality, $\pi\Big|_{p_A=\bar{p}-\delta}^{OFS} > \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$, is not possible.

Therefore, we conclude that $\pi\Big|_{p_A=\bar{p}-\delta}^{OFS} < \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$. $\qquad$ Q.E.D.

Proof of Proposition 4.

By using what we derived in Lemma 2 and Lemma 3, we show the following:

1. At $p_A = \bar{p}-\delta$, the seller's profits were compared as $\pi_{RA}^{SB,OFS} < \pi\Big|_{p_A=\bar{p}-\delta}^{OFS}$ and

$\pi\Big|_{p_A=\bar{p}-\delta}^{OFS} < \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$. Thus, we conclude that $\pi_{RA}^{SB,OFS} < \pi\Big|_{p_A=\bar{p}-\delta}^{OFS} < \pi\Big|_{p_A=\bar{p}-\delta}^{NFS}$. For the buyer's utility,

we found $U_{RA}^{SB,OFS} = 0 < U\Big|_{p_A=\bar{p}-\delta}^{OFS}$, but $U\Big|_{p_A=\bar{p}-\delta}^{NFS} = 0 < U\Big|_{p_A=\bar{p}-\delta}^{OFS}$. Therefore, we conclude that

$U_{RA}^{SB,OFS} = U\Big|_{p_A=\bar{p}-\delta}^{NFS} < U\Big|_{p_A=\bar{p}-\delta}^{OFS}$.

2. At $p_A = \bar{p}+\delta$, (i) if $\dfrac{\lambda\mu}{\lambda+\mu} > \dfrac{2\varepsilon}{\delta m}$, then the seller's profits were found as $\pi_{RA}^{SB,OFS} < \pi\Big|_{p_A=\bar{p}+\delta}^{OFS}$

and $\pi\Big|_{p_A=\bar{p}+\delta}^{OFS} < \pi\Big|_{p_A=\bar{p}+\delta}^{NFS}$, which derive $\pi_{RA}^{SB,OFS} < \pi\Big|_{p_A=\bar{p}+\delta}^{OFS} < \pi\Big|_{p_A=\bar{p}+\delta}^{NFS}$. For the buyer's utilities, the

previous findings of $U_{RA}^{SB,OFS} = 0 < U\Big|_{p_A=\bar{p}+\delta}^{OFS}$ and $U\Big|_{p_A=\bar{p}+\delta}^{NFS} = 0 < U\Big|_{p_A=\bar{p}+\delta}^{OFS}$ are summarized as

$U_{RA}^{SB,OFS} = U\big|_{P_A=\bar{p}+\delta}^{NFS} = 0 < U\big|_{P_A=\bar{p}+\delta}^{OFS}$. (ii) if $\dfrac{\lambda\mu}{\lambda+\mu} < \dfrac{2\varepsilon}{\delta m}$, then the seller's profit inequalities were

$\pi_{RA}^{SB,OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{OFS}$ and $\pi\big|_{P_A=\bar{p}+\delta}^{OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$, and it concludes that $\pi_{RA}^{SB,OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$. The

buyer's utilities were found as $U\big|_{P_A=\bar{p}+\delta}^{OFS} < U\big|_{P_A=\bar{p}+\delta}^{NFS} = U_{RA}^{SB,OFS} = 0$.

From $\pi_{RA}^{SB,OFS} < \pi\big|_{P_A=\bar{p}+\delta}^{OFS} < \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$ and $\pi_{RA}^{SB,OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{OFS} > \pi\big|_{P_A=\bar{p}+\delta}^{NFS}$, we find that the *ex-*

post profit change under the *OFS* strategy is smaller than the *ex-post* change under the *NFS*

strategy. Also, from $U_{RA}^{SB,OFS} = U\big|_{P_A=\bar{p}+\delta}^{NFS} = 0 < U\big|_{P_A=\bar{p}+\delta}^{OFS}$ and $U\big|_{P_A=\bar{p}+\delta}^{OFS} < U\big|_{P_A=\bar{p}+\delta}^{NFS} = U_{RA}^{SB,OFS} = 0$, we

conclude that the change of the buyer's utility under the *ex-post OFS* is larger than that under the

ex-post NFS strategy.   *Q.E.D.*

Proof of Proposition 5.

We compare profits of three different cases that were driven in section 4.1. (i) First, let us

suppose that $\pi_{RA}^{MB,NFS} < \pi_{RA}^{MB,OFS}$. Then, $\left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L - \dfrac{\mu\delta^2}{2\varepsilon^2}\left(m_H^2 + m_L^2\right)$

$< \left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L - \dfrac{\mu\delta^2\left(m_H + m_L\right)^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2\right)}$. Then, $\dfrac{\mu\delta^2}{2\varepsilon^2}\left(m_H^2 + m_L^2\right)$

$> \dfrac{\mu\delta^2\left(m_H + m_L\right)^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2\right)} \Leftrightarrow 1 > \dfrac{\lambda_H m_H^2 + \lambda_L m_L^2}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2} \Leftrightarrow \mu\left(m_H + m_L\right)^2 > 0$.

This inequality always holds. (ii) Now, we compare $\pi_{RA}^{MB,OFS}$ with $\pi_{RA}^{MB,AFS}$. Suppose that

$\pi_{RA}^{MB,OFS} < \pi_{RA}^{MB,AFS}$. Then, $\left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L - \dfrac{\mu\delta^2\left(m_H + m_L\right)^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2\right)}$

$< \left(\theta_H - \dfrac{\bar{p}}{\varepsilon}\right)m_H + \left(\theta_L - \dfrac{\bar{p}}{\varepsilon}\right)m_L - \dfrac{\delta^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2} \Leftrightarrow \dfrac{\mu\delta^2\left(m_H + m_L\right)^2\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{2\varepsilon^2\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2\right)}$

$$> \frac{\delta^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{2\varepsilon^2} \Leftrightarrow \frac{\mu \left(m_H + m_L \right)^2}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2} > 1 \Leftrightarrow 0 > \lambda_H m_H^2 + \lambda_L m_L^2,$$ which is not true.

Therefore, based on (i) and (ii), we conclude that the risk-averse seller will choose the *OFS*

strategy when both the buyer and the seller are risk averse. *Q.E.D.*

Proof of Corollary 3.

(i) We can simply find the difference between $\pi_{RA}^{MB,OFS}$ and the sum of $\pi_{H,RA}^{SB,OFS}$ and $\pi_{L,RA}^{SB,OFS}$. We

can find $\pi_{H,RA}^{SB,OFS}$ from the profit derived in the section 3.2.2 as $m_H \left(\theta_H - \dfrac{\overline{p}}{\varepsilon} \right) - \dfrac{\lambda_H \mu \delta^2 m_H^2}{2\varepsilon^2 \left(\lambda_H + \mu \right)}$.

Also, $\pi_{L,RA}^{SB,OFS} = m_L \left(\theta_L - \dfrac{\overline{p}}{\varepsilon} \right) - \dfrac{\lambda_L \mu \delta^2 m_L^2}{2\varepsilon^2 \left(\lambda_L + \mu \right)}$. The sum of these two will be

$$\pi_{H,RA}^{SB,OFS} + \pi_{L,RA}^{SB,OFS} = m_H \left(\theta_H - \frac{\overline{p}}{\varepsilon} \right) - \frac{\lambda_H \mu \delta^2 m_H^2}{2\varepsilon^2 \left(\lambda_H + \mu \right)} + m_L \left(\theta_L - \frac{\overline{p}}{\varepsilon} \right) - \frac{\lambda_L \mu \delta^2 m_L^2}{2\varepsilon^2 \left(\lambda_L + \mu \right)}.$$ Therefore,

compared to the expression of $\pi_{RA}^{MB,OFS}$, we conclude that $\pi_{RA}^{MB,OFS} < \pi_{H,RA}^{SB,OFS} + \pi_{L,RA}^{SB,OFS}$ only when

$$\frac{\mu \delta^2 \left(m_H + m_L \right)^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{2\varepsilon^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)} > \frac{\lambda_H \mu \delta^2 m_H^2}{2\varepsilon^2 \left(\lambda_H + \mu \right)} + \frac{\lambda_L \mu \delta^2 m_L^2}{2\varepsilon^2 \left(\lambda_L + \mu \right)}.$$ It leads that

$$\frac{\left(m_H + m_L \right)^2 \left(\lambda_H m_H^2 + \lambda_L m_L^2 \right)}{\left(\lambda_H m_H^2 + \lambda_L m_L^2 + \mu \left(m_H + m_L \right)^2 \right)} > \frac{\left(\lambda_H m_H^2 \left(\lambda_L + \mu \right) + \lambda_L m_L^2 \left(\lambda_H + \mu \right) \right)}{\left(\lambda_H + \mu \right) \left(\lambda_L + \mu \right)}.$$ This is mathematically too

complicated to compare at a glance. However, since all expressions for both sides are positive,

we can simplify them by minimizing two types to be only one type without loss of generality;

that is, $m_H = m_L = m$ and $\lambda_H = \lambda_L = \lambda$. Then, above inequality simply becomes as follows:

$$\frac{\left(2m \right)^2 \left(2\lambda m^2 \right)}{2\lambda m^2 + \mu \left(2m \right)^2} > \frac{2\lambda m^2 \left(\lambda + \mu \right)}{2 \left(\lambda + \mu \right)} \Leftrightarrow \frac{2 \left(\lambda + \mu \right)}{\left(\lambda + 2\mu \right)} > 1 \Leftrightarrow 2\lambda + 2\mu > \lambda + 2\mu.$$ Then, we prove that this

is always true with given λ and μ . Therefore, we conclude that $\pi_{RA}^{MB,OFS} < \pi_{H,RA}^{SB,OFS} + \pi_{L,RA}^{SB,OFS}$.

(ii) We compare $p_{B,RA}^{MB,OFS} = \bar{p} - \delta + \dfrac{2\delta\left(\lambda_H m_H^2 + \lambda_L m_L^2\right)}{\lambda_H m_H^2 + \lambda_L m_L^2 + \mu\left(m_H + m_L\right)^2}$ and $p_{B,RA}^{SB,OFS} = \bar{p} + \dfrac{\delta\left(\lambda - \mu\right)}{\lambda + \mu}$

(from section 3.2.2). From the expression of $p_{B,RA}^{SB,OFS}$, we know that, given a level of the seller's

risk averseness, lower the buyer's risk averseness level is, lower the base price is. Therefore, as

we did in (i) above, we can focus on comparing a single 'low' type buyer and two 'low' type

buyers' cases. That is, $p_{B,RA}^{MB,OFS}$ becomes $\bar{p} - \delta + \dfrac{4\delta\lambda m^2}{2\lambda m^2 + 4\mu m^2}$ having $m_H = m_L = m$ and

$\lambda_H = \lambda_L = \lambda$. Now, assume that $p_{B,RA}^{MB,OFS} < p_{B,RA}^{SB,OFS}$. It is equivalent with $\bar{p} - \delta + \dfrac{4\delta\lambda m^2}{2\lambda m^2 + 4\mu m^2}$

$< \bar{p} + \dfrac{\delta\left(\lambda - \mu\right)}{\lambda + \mu}$. It leads $-\delta + \dfrac{2\delta\lambda}{\lambda + 2\mu} < \dfrac{\delta\left(\lambda - \mu\right)}{\lambda + \mu} \Leftrightarrow -1 + \dfrac{2\lambda}{\lambda + 2\mu} < \dfrac{\lambda - \mu}{\lambda + \mu} \Leftrightarrow -\dfrac{2\mu}{\lambda + 2\mu} < \dfrac{\lambda - \mu}{\lambda + \mu}$

$\Leftrightarrow \dfrac{\lambda - 2\mu}{\lambda + 2\mu} < \dfrac{\lambda - \mu}{\lambda + \mu}$. This inequality is always true since the numerator in the left-hand side is

smaller than that in the right-hand side and the denominator in the left-hand side is bigger that

that in the right-hand side. Hence, we conclude that $p_{B,RA}^{MB,OFS} < p_{B,RA}^{SB,OFS}$. $\qquad$ Q.E.D.

Proof of Proposition 7.

For each unit of additional cost change, how much proportions of this unit cost change are shared

by the seller, the high type buyer, and the low type buyer are as follows: (i) The seller's portion

will be the difference between two actual *ex-post* profits under $p_A = \bar{p} - \delta$ and $p_A = \bar{p} + \delta$; that

is, $\left.\pi\right|_{p_A = \bar{p} - \delta}^{MB,OFS} - \left.\pi\right|_{p_A = \bar{p} + \delta}^{MB,OFS} = 2\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{2\lambda_L \delta m}{\varepsilon\left(\lambda_L + 4\mu\right)} - \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2\left(\lambda_L + 4\mu\right)^2} - 2\left(\theta - \dfrac{\bar{p}}{\varepsilon}\right)m + \dfrac{2\lambda_L \delta m}{\varepsilon\left(\lambda_L + 4\mu\right)}$

$+ \dfrac{8\lambda_L \mu^2 \delta^2 m^2}{\varepsilon^2\left(\lambda_L + 4\mu\right)^2} = \dfrac{4\lambda_L \delta m}{\varepsilon\left(\lambda_L + 4\mu\right)}$. (ii) Each high type buyer's proportion will be derived in the same

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way; that is, $U\Big|_{H,p_A=\overline{p}-\delta}^{MB,OFS} - U\Big|_{H,p_A=\overline{p}+\delta}^{MB,OFS} = \dfrac{4\mu\delta m}{\varepsilon(\lambda_L+4\mu)} + \dfrac{4\mu\delta m}{\varepsilon(\lambda_L+4\mu)} = \dfrac{8\mu\delta m}{\varepsilon(\lambda_L+4\mu)}$. (iii) The low type

buyer's proportion should be derived in the same way: that is, $U\Big|_{L,p_A=\overline{p}-\delta}^{MB,OFS} - U\Big|_{L,p_A=\overline{p}+\delta}^{MB,OFS} =$

$\dfrac{4\mu\delta m}{\varepsilon(\lambda_L+4\mu)} + \dfrac{8\lambda_L\mu^2\delta^2 m^2}{\varepsilon^2(\lambda_L+4\mu)^2} + \dfrac{4\mu\delta m}{\varepsilon(\lambda_L+4\mu)} - \dfrac{8\lambda_L\mu^2\delta^2 m^2}{\varepsilon^2(\lambda_L+4\mu)^2} = \dfrac{8\mu\delta m}{\varepsilon(\lambda_L+4\mu)}$. Summarizing these, the

proportions are given by $\dfrac{4\lambda_L\delta m}{\varepsilon(\lambda_L+4\mu)} : \dfrac{8\mu\delta m}{\varepsilon(\lambda_L+4\mu)} : \dfrac{8\mu\delta m}{\varepsilon(\lambda_L+4\mu)} = \dfrac{\lambda_L}{\lambda_L+4\mu} : \dfrac{2\mu}{\lambda_L+4\mu} : \dfrac{2\mu}{\lambda_L+4\mu}$ for

the seller, the high type buyer, and the low type buyer, respectively. $\qquad$ *Q.E.D.*

B2. Derivations.

Derivations of Mean-Variance Functions for the Case of Single Risk-Averse Seller and

Single Risk-Neutral Buyer (Section 3.2.1)

The Case of *NFS* Strategy

Since $U=(\theta-p_F)m$ with a probability of one-half, the expected utility is given by

$EU=(\theta-p_F)m$. Hence, its mean and variance are $E[EU]=U_{SB}^{NFS}=(\theta-p_F)m$.

With a probability of one-half, $\pi=\left(p_F-\dfrac{\overline{p}+\delta}{\varepsilon}\right)m$ or $\pi=\left(p_F-\dfrac{\overline{p}-\delta}{\varepsilon}\right)m$. We have

$E\pi=\dfrac{1}{2}\left(p_F-\dfrac{\overline{p}+\delta}{\varepsilon}\right)m+\dfrac{1}{2}\left(p_F-\dfrac{\overline{p}-\delta}{\varepsilon}\right)m$. The mean and variance are, respectively,

$E[E\pi]=\left(p_F-\dfrac{\overline{p}}{\varepsilon}\right)m$ and $V[E\pi]=\dfrac{1}{2}\left(\left(p_F-\dfrac{\overline{p}+\delta}{\varepsilon}\right)m-\left(p_F-\dfrac{\overline{p}}{\varepsilon}\right)m\right)^2$

$$+\frac{1}{2}\left(\left(p_F - \frac{\overline{p}-\delta}{\varepsilon}\right)m - \left(p_F - \frac{\overline{p}}{\varepsilon}\right)m\right)^2 = \left(\frac{\delta m}{\varepsilon}\right)^2.$$ Therefore, the mean-variance function of the

seller's profit is given by $\pi_{SB}^{NFS} = \left(p_F - \frac{\overline{p}}{\varepsilon}\right)m - \frac{\mu}{2}\left(\frac{\delta m}{\varepsilon}\right)^2.$

The Case of *OFS* Strategy

With one-half chance, $U = (\theta - p_F)m$, and with one-half, $U = \left(\theta - p_F - \frac{\overline{p}+\delta-p_B}{\varepsilon}\right)m$. It leads

$EU = \frac{1}{2}\left(\theta - p_F - \frac{\overline{p}+\delta-p_B}{\varepsilon}\right)m + \frac{1}{2}(\theta - p_F)m$. The expected utility is given by

$E[EU] = U_{SB}^{OFS} = \left(\theta - p_F - \frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m.$

With a probability of one-half, $\pi = \left(p_F + \frac{\overline{p}+\delta-p_B}{\varepsilon} - \frac{\overline{p}+\delta}{\varepsilon}\right)m$, and with the other one-half

probability, $\pi = \left(p_F - \frac{\overline{p}-\delta}{\varepsilon}\right)m$. Thus, $E\pi = \frac{1}{2}\left(p_F + \frac{\overline{p}+\delta-p_B}{\varepsilon} - \frac{\overline{p}+\delta}{\varepsilon}\right)m + \frac{1}{2}\left(p_F - \frac{\overline{p}-\delta}{\varepsilon}\right)m.$

The expected profit is $E[E\pi] = \left(p_F + \frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m$, and the variance is

$$V[E\pi] = \frac{1}{2}\left(\left(p_F + \frac{\overline{p}+\delta-p_B}{\varepsilon} - \frac{\overline{p}+\delta}{\varepsilon}\right)m - \left(p_F + \frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m\right)^2$$

$$+\frac{1}{2}\left(\left(p_F - \frac{\overline{p}-\delta}{\varepsilon}\right)m - \left(p_F + \frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m\right)^2$$

$$=\frac{1}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\delta}{\varepsilon}\right)m\right)^2 + \frac{1}{2}\left(\left(-\frac{\overline{p}+\delta-p_B}{2\varepsilon} + \frac{\delta}{\varepsilon}\right)m\right)^2 = \left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\delta}{\varepsilon}\right)m\right)^2.$$

Therefore, the seller's profit mean-variance function becomes

$$\pi_{SB}^{OFS} = \left(p_F + \frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m - \frac{\mu}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon} - \frac{\delta}{\varepsilon}\right)m\right)^2.$$

The Case of *AFS* Strategy

With a probability of one-half, $U = \left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{\varepsilon} \right) m$, and with the other probability of

one-half, $U = \left(\theta - p_F - \dfrac{\overline{p} - \delta - p_B}{\varepsilon} \right) m$. We have the expected utility of

$EU = \dfrac{1}{2} \left(\theta - p_F - \dfrac{\overline{p} - \delta - p_B}{\varepsilon} \right) m + \dfrac{1}{2} \left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{\varepsilon} \right)$. The mean function of the expected

utility is given by $E[EU] = U_{SB}^{AFS} = \left(\theta - p_F - \dfrac{\overline{p} - p_B}{\varepsilon} \right) m$.

The seller's profit is $\pi = \left(p_F + \dfrac{\overline{p} - \delta - p_B}{\varepsilon} - \dfrac{\overline{p} - \delta}{\varepsilon} \right) m$ with a one-half chance, and

$\pi = \left(p_F + \dfrac{\overline{p} + \delta - p_B}{\varepsilon} - \dfrac{\overline{p} - \delta}{\varepsilon} \right) m$ with another one-half chance. Thus, we have

$E\pi = \dfrac{1}{2} \left(p_F + \dfrac{\overline{p} - \delta - p_B}{\varepsilon} - \dfrac{\overline{p} - \delta}{\varepsilon} \right) m + \dfrac{1}{2} \left(p_F + \dfrac{\overline{p} + \delta - p_B}{\varepsilon} - \dfrac{\overline{p} + \delta}{\varepsilon} \right) m$. The mean and variance of

the expected profits are, respectively, $E[E\pi] = \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon} \right) m$, and

$V[E\pi] = \dfrac{1}{2} \left(\left(p_F + \dfrac{\overline{p} - \delta - p_B}{\varepsilon} - \dfrac{\overline{p} - \delta}{\varepsilon} \right) m - \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon} \right) m \right)^2$

$+ \dfrac{1}{2} \left(\left(p_F + \dfrac{\overline{p} + \delta - p_B}{\varepsilon} - \dfrac{\overline{p} + \delta}{\varepsilon} \right) m - \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon} \right) m \right)^2 = 0$. Therefore, the mean-variance

function of the seller's profit is given by $\pi_{SB}^{AFS} = \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon} \right) m$.

Derivations of Mean-Variance Functions for the Case of Single Risk-Averse Seller and Single Risk-Averse Buyer (Section 3.2.2)

The Case of *NFS* Strategy

Since $U = (\theta - p_F)m$ with a probability of one-half, the expected utility is given by

$EU = (\theta - p_F)m$. Hence, its mean and variance are $E[EU] = (\theta - p_F)m$ and

$V[EU] = \frac{1}{2}((\theta - p_F)m - (\theta - p_F)m)^2 + \frac{1}{2}((\theta - p_F)m - (\theta - p_F)m)^2 = 0$, respectively.

Therefore, the mean-variance utility of the buyer is given by $U_{SB}^{NFS} = (\theta - p_F)m$.

With a probability of one-half, $\pi = \left(p_F - \dfrac{\overline{p} + \delta}{\varepsilon}\right)m$ or $\pi = \left(p_F - \dfrac{\overline{p} - \delta}{\varepsilon}\right)m$. We have

$E\pi = \dfrac{1}{2}\left(p_F - \dfrac{\overline{p} + \delta}{\varepsilon}\right)m + \dfrac{1}{2}\left(p_F - \dfrac{\overline{p} - \delta}{\varepsilon}\right)m$. The mean and variance are, respectively,

$E[E\pi] = \left(p_F - \dfrac{\overline{p}}{\varepsilon}\right)m$ and $V[E\pi] = \dfrac{1}{2}\left(\left(p_F - \dfrac{\overline{p} + \delta}{\varepsilon}\right)m - \left(p_F - \dfrac{\overline{p}}{\varepsilon}\right)m\right)^2$

$+ \dfrac{1}{2}\left(\left(p_F - \dfrac{\overline{p} - \delta}{\varepsilon}\right)m - \left(p_F - \dfrac{\overline{p}}{\varepsilon}\right)m\right)^2 = \left(\dfrac{\delta m}{\varepsilon}\right)^2$. Therefore, the mean-variance function of the

seller's profit is given by $\pi_{SB}^{NFS} = \left(p_F - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{\mu}{2}\left(\dfrac{\delta m}{\varepsilon}\right)^2$.

The Case of *OFS* Strategy

With one-half chance, $U = (\theta - p_F)m$, and with one-half, $U = \left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{\varepsilon}\right)m$. It leads

$EU = \dfrac{1}{2}\left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{\varepsilon}\right)m + \dfrac{1}{2}(\theta - p_F)m$. The expected utility is given by

$E[EU] = \left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{2\varepsilon}\right)m$. The variance of the utility is given by

$V[EU] = \dfrac{1}{2}\left((\theta - p_F)m - \left(\theta - p_F - \dfrac{\overline{p} + \delta - p_B}{2\varepsilon}\right)m\right)^2$

$$+\frac{1}{2}\left(\left(\theta-p_F-\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)m-\left(\theta-p_F-\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m\right)^2=\frac{1}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m\right)^2$$

$$+\frac{1}{2}\left(-\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m\right)^2=\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m\right)^2.$$ The mean-variance function of the buyer's utility

is given by $U_{SB}^{OFS}=\left(\theta-p_F-\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m-\frac{\lambda}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)m\right)^2.$

With a probability of one-half, $\pi=\left(p_F+\frac{\overline{p}+\delta-p_B}{\varepsilon}-\frac{\overline{p}+\delta}{\varepsilon}\right)m$, and with the other one-half

probability, $\pi=\left(p_F-\frac{\overline{p}-\delta}{\varepsilon}\right)m$. Thus, $E\pi=\frac{1}{2}\left(p_F+\frac{\overline{p}+\delta-p_B}{\varepsilon}-\frac{\overline{p}+\delta}{\varepsilon}\right)m+\frac{1}{2}\left(p_F-\frac{\overline{p}-\delta}{\varepsilon}\right)m.$

The expected profit is $E[E\pi]=\left(p_F+\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\overline{p}}{\varepsilon}\right)m$, and the variance is

$$V[E\pi]=\frac{1}{2}\left(\left(p_F+\frac{\overline{p}+\delta-p_B}{\varepsilon}-\frac{\overline{p}+\delta}{\varepsilon}\right)m-\left(p_F+\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\overline{p}}{\varepsilon}\right)m\right)^2$$

$$+\frac{1}{2}\left(\left(p_F-\frac{\overline{p}-\delta}{\varepsilon}\right)m-\left(p_F+\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\overline{p}}{\varepsilon}\right)m\right)^2$$

$$=\frac{1}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\delta}{\varepsilon}\right)m\right)^2+\frac{1}{2}\left(\left(-\frac{\overline{p}+\delta-p_B}{2\varepsilon}+\frac{\delta}{\varepsilon}\right)m\right)^2=\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\delta}{\varepsilon}\right)m\right)^2.$$

Therefore, the seller's profit mean-variance function becomes

$$\pi_{SB}^{OFS}=\left(p_F+\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\overline{p}}{\varepsilon}\right)m-\frac{\mu}{2}\left(\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}-\frac{\delta}{\varepsilon}\right)m\right)^2.$$

The Case of *AFS* Strategy

With a probability of one-half, $U=\left(\theta-p_F-\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)m$, and with the other probability of

one-half, $U = \left(\theta - p_F - \dfrac{\overline{p} - \delta - p_B}{\varepsilon} \right) m$. We have the expected utility of

$$EU = \frac{1}{2} \left(\theta - p_F - \frac{\overline{p} - \delta - p_B}{\varepsilon} \right) m + \frac{1}{2} \left(\theta - p_F - \frac{\overline{p} + \delta - p_B}{\varepsilon} \right). \text{ The mean function of the expected}$$

utility is given by $E[EU] = \left(\theta - p_F - \dfrac{\overline{p} - p_B}{\varepsilon} \right) m$, and its variance is given by

$$V[EU] = \frac{1}{2} \left(\left(\theta - p_F - \frac{\overline{p} - \delta - p_B}{\varepsilon} \right) m - \left(\theta - p_F - \frac{\overline{p} - p_B}{\varepsilon} \right) m \right)^2$$

$$+ \frac{1}{2} \left(\left(\theta - p_F - \frac{\overline{p} + \delta - p_B}{\varepsilon} \right) m - \left(\theta - p_F - \frac{\overline{p} - p_B}{\varepsilon} \right) m \right)^2 = \frac{1}{2} \left(\frac{\delta m}{\varepsilon} \right)^2 + \frac{1}{2} \left(-\frac{\delta m}{\varepsilon} \right)^2 = \left(\frac{\delta m}{\varepsilon} \right)^2.$$

Therefore, the buyer's mean-variance utility is given by

$$U_{SB}^{AFS} = \left(\theta - p_F - \frac{\overline{p} - p_B}{\varepsilon} \right) m - \frac{\lambda}{2} \left(\frac{\delta m}{\varepsilon} \right)^2.$$

The seller's profit is $\pi = \left(p_F + \dfrac{\overline{p} - \delta - p_B}{\varepsilon} - \dfrac{\overline{p} - \delta}{\varepsilon} \right) m$ with a one-half chance, and

$$\pi = \left(p_F + \frac{\overline{p} + \delta - p_B}{\varepsilon} - \frac{\overline{p} - \delta}{\varepsilon} \right) m \text{ with another one-half chance. Thus, we have}$$

$$E\pi = \frac{1}{2} \left(p_F + \frac{\overline{p} - \delta - p_B}{\varepsilon} - \frac{\overline{p} - \delta}{\varepsilon} \right) m + \frac{1}{2} \left(p_F + \frac{\overline{p} + \delta - p_B}{\varepsilon} - \frac{\overline{p} + \delta}{\varepsilon} \right) m. \text{ The mean and variance of}$$

the expected profits are, respectively, $E[E\pi] = \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon} \right) m$, and

$$V[E\pi] = \frac{1}{2} \left(\left(p_F + \frac{\overline{p} - \delta - p_B}{\varepsilon} - \frac{\overline{p} - \delta}{\varepsilon} \right) m - \left(p_F + \frac{\overline{p} - p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon} \right) m \right)^2$$

$$+ \frac{1}{2} \left(\left(p_F + \frac{\overline{p} + \delta - p_B}{\varepsilon} - \frac{\overline{p} + \delta}{\varepsilon} \right) m - \left(p_F + \frac{\overline{p} - p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon} \right) m \right)^2 = 0. \text{ Therefore, the mean-variance}$$

function of the seller's profit is given by $\pi_{SB}^{AFS} = \left(p_F + \dfrac{\overline{p} - p_B}{\varepsilon} - \dfrac{\overline{p}}{\varepsilon}\right)m$.

Derivations of Optimal Solutions (Section 3.2.2)

The case of *NFS* strategy: The seller's problem is $\max\limits_{p_F} E\pi = \left(p_F - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{\mu \delta^2 m^2}{2\varepsilon^2}$ subject to

$EU = (\theta - p_F)m$. First, the seller simply can choose the freight rate that leaves the buyer with

zero utility. Thus, from $EU = 0$, $p_{F,RA}^{SB,NFS} = \theta$. We substitute this into the profit function to have

the optimal profit as $\pi_{RA}^{SB,NFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\mu m^2 \delta^2}{2\varepsilon^2}$.

The case of *OFS* strategy: From the buyer's problem, $EU = \left(\theta - p_F - \left(\dfrac{\overline{p} + \delta - p_B}{2\varepsilon}\right)\right)m$

$-\dfrac{\lambda}{2}\left(\dfrac{m\left(\overline{p} + \delta - p_B\right)}{2\varepsilon}\right)^2$, we solve it for the best responding freight rate as

$\hat{p}_F = \theta - \dfrac{\overline{p} + \delta - p_B}{2\varepsilon} - \dfrac{\left(\overline{p} + \delta - p_B\right)^2 \lambda m}{8\varepsilon^2}$. Substitute this into the seller's profit function, we

solve $\max\limits_{p_B} E\pi\big|_{\hat{p}_F} = \left(\hat{p}_F + \left(\dfrac{\overline{p} + \delta - p_B}{2\varepsilon}\right) - \dfrac{\overline{p}}{\varepsilon}\right)m - \dfrac{\mu}{2}\left(\left(\dfrac{\overline{p} + \delta - p_B}{2\varepsilon} - \dfrac{\delta}{\varepsilon}\right)m\right)^2$. Differentiating it with

respect to p_B and making it equal to zero ($\dfrac{\partial E\pi\big|_{\hat{p}_F}}{\partial p_B} = 0$), we find the optimal p_B as

$p_{B,RA}^{SB,OFS} = \overline{p} + \dfrac{\delta(\lambda - \mu)}{\lambda + \mu}$, followed by the optimal freight rate of $p_{F,RA}^{SB,OFS} = \theta - \dfrac{\delta\mu}{\varepsilon} - \dfrac{\lambda\mu^2\delta^2 m}{2\varepsilon^2(\lambda + \mu)^2}$,

and the optimal profit of $\pi_{RA}^{SB,OFS} = m\left(\theta - \dfrac{\overline{p}}{\varepsilon}\right) - \dfrac{\lambda\mu\delta^2 m^2}{2\varepsilon^2(\lambda + \mu)}$.

The case of *AFS* strategy: Again, the buyer's mean-variance utility function,

$$EU = \left(\theta - p_F - \frac{(\bar{p} - p_B)}{\varepsilon}\right)m - \frac{\lambda}{2}\left(\frac{\delta m}{\varepsilon}\right)^2,$$ gives the zero-utility extracting freight rate as

$$\hat{p}_F = \theta - \frac{(\bar{p} - p_B)}{\varepsilon} - \frac{\lambda m \delta^2}{2\varepsilon^2}.$$ Substituting it into the seller's problem, we solve an maximization

objective function of $\max\limits_{p_B} E\pi\big|_{\hat{p}_F} = \left(\hat{p}_F + \frac{\bar{p} - p_B}{\varepsilon} - \frac{\bar{p}}{\varepsilon}\right)m$. However, since its F.O.C. is not a

function of p_B, we don't get a single optimal p_B. Thus, we define $p_{B3}^{SB} = \bar{p} - \delta - \rho_{AFS}$, where

$0 < \rho_{AFS} \leq \bar{p} - \delta$. Substituting this rate, we have the optimal freight rate as

$$p_{F,RA}^{SB,AFS} = \theta - \frac{\delta + \rho_{AFS}}{\varepsilon} - \frac{\lambda \delta^2 m}{2\varepsilon^2}.$$ Plugging p_{F2}^{SB} and p_{B2}^{SB} back into the profit function, we get the

optimal expected profit as $\pi_{RA}^{SB,AFS} = m\left(\theta - \frac{\bar{p}}{\varepsilon}\right) - \frac{\lambda \delta^2 m^2}{2\varepsilon^2}$.

Derivations of Mean-Variance Functions for the Case of Single Risk-Averse Seller and

Multiple Risk-Averse Buyers (Section 4.1)

Since buyers are independent entities, their individual mean-variance utility functions are the

same as the ones derived under the single buyer case (Section 3). Only the seller has a new

mean-variance profit function as follows:

The case of *NFS* strategy

With a probability of one-half, $\pi = \sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p} + \delta}{\varepsilon}\right)m_i$ or $\pi = \sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p} - \delta}{\varepsilon}\right)m_i$. We have

$$E\pi = \frac{1}{2}\sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p} + \delta}{\varepsilon}\right)m_i + \frac{1}{2}\sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p} - \delta}{\varepsilon}\right)m_i.$$ The mean and variance are, respectively,

$$E[E\pi] = \sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p}}{\varepsilon}\right)m_i \text{ and } V[E\pi] = \frac{1}{2}\left(\sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p} + \delta}{\varepsilon}\right)m_i - \sum\limits_{i=H,L}\left(p_{Fi} - \frac{\bar{p}}{\varepsilon}\right)m_i\right)^2$$

$$+\frac{1}{2}\left(\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}+\delta}{\varepsilon}\right)m_i-\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}}{\varepsilon}\right)m_i\right)^2=\left(\frac{\delta}{\varepsilon}\left(m_H+m_L\right)\right)^2.$$ Therefore, the mean-variance

function of the seller's profit is given by $\pi_{RA}^{SB,NFS}=\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}}{\varepsilon}\right)m_i-\frac{\mu}{2}\frac{\delta^2}{\varepsilon^2}\left(\sum_{i=H,L}m_i\right)^2.$

<u>The Case of *OFS* Strategy</u>

$$\pi=\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)-\frac{\overline{p}+\delta}{\varepsilon}\right)m_i \text{ and } \pi=\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}-\delta}{\varepsilon}\right)m_i\text{ , with each of one-half}$$

probability. Thus, $E\pi=\frac{1}{2}\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)-\frac{\overline{p}+\delta}{\varepsilon}\right)m_i+\frac{1}{2}\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}-\delta}{\varepsilon}\right)m_i.$ The

expected profit is $E[E\pi]=\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)-\frac{\overline{p}}{\varepsilon}\right)m_i$, and the variance is

$$V[E\pi]=\frac{1}{2}\left(\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)-\frac{\overline{p}+\delta}{\varepsilon}\right)m_i-\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)-\frac{\overline{p}}{\varepsilon}\right)m_i\right)^2$$

$$+\frac{1}{2}\left(\sum_{i=H,L}\left(p_{Fi}-\frac{\overline{p}-\delta}{\varepsilon}\right)m_i-\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)-\frac{\overline{p}}{\varepsilon}\right)m_i\right)^2$$

$$=\frac{1}{2}\left(\frac{\overline{p}-\delta-p_B}{2\varepsilon}\right)^2\left(\sum_{i=H,L}m_i\right)^2+\frac{1}{2}\left(\frac{\overline{p}-\delta-p_B}{2\varepsilon}\right)^2\left(\sum_{i=H,L}m_i\right)^2=\left(\frac{\overline{p}-\delta-p_B}{2\varepsilon}\right)^2\left(\sum_{i=H,L}m_i\right)^2.$$

Therefore, the seller's profit mean-variance function becomes

$$\pi_{RA}^{SB,OFS}=\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{2\varepsilon}\right)-\frac{\overline{p}}{\varepsilon}\right)m_i-\frac{\mu}{2}\left(\frac{\overline{p}-\delta-p_B}{2\varepsilon}\right)^2\left(\sum_{i=H,L}m_i\right)^2.$$

<u>The Case of *AFS* Strategy</u>

The seller's profit is $\pi=\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}-\delta-p_B}{\varepsilon}\right)-\frac{\overline{p}-\delta}{\varepsilon}\right)m_i$ with an one-half chance, and

$$\pi=\sum_{i=H,L}\left(p_{Fi}+\left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right)-\frac{\overline{p}+\delta}{\varepsilon}\right)m_i\text{ with another one-half chance. Thus, we have}$$

$$E\pi = \frac{1}{2}\sum_{i=H,L}\left(p_{Fi} + \left(\frac{\overline{p}-\delta-p_B}{\varepsilon}\right) - \frac{\overline{p}-\delta}{\varepsilon}\right)m_i + \frac{1}{2}\sum_{i=H,L}\left(p_{Fi} + \left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right) - \frac{\overline{p}+\delta}{\varepsilon}\right)m_i .$$ The mean

and variance of the expected profits are, respectively, $E[E\pi] = \sum_{i=H,L}\left(p_{Fi} + \frac{\overline{p}-p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m_i$, and

$$V[E\pi] = \frac{1}{2}\left(\sum_{i=H,L}\left(p_{Fi} + \left(\frac{\overline{p}-\delta-p_B}{\varepsilon}\right) - \frac{\overline{p}-\delta}{\varepsilon}\right)m_i - \sum_{i=H,L}\left(p_{Fi} + \frac{\overline{p}-p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m_i\right)^2$$

$$+ \frac{1}{2}\left(\sum_{i=H,L}\left(p_{Fi} + \left(\frac{\overline{p}+\delta-p_B}{\varepsilon}\right) - \frac{\overline{p}+\delta}{\varepsilon}\right)m_i - \sum_{i=H,L}\left(p_{Fi} + \frac{\overline{p}-p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m_i\right)^2 = 0.$$ Therefore, the

mean-variance function of the seller's profit is given by $\pi_{RA}^{SB,AFS} = \sum_{i=H,L}\left(p_{Fi} + \frac{\overline{p}-p_B}{\varepsilon} - \frac{\overline{p}}{\varepsilon}\right)m_i .$

CHAPTER 2: STORE BRAND COMPETITION AND STRATEGIC

QUALITY DECISIONS

"20% of grocery sales are private brands, up to 15% prior to the recession."

(The Wall Street Journal, Oct. 2, 2011)

1. INTRODCUTION

Research on store brands has found a retailer's introduction of a store brand can be used as either a tool for store differentiation, store loyalty, or store profitability. More recently, various studies show that many retailers who offer store brands position them very close to leading national brands in terms of their product's characteristics (e.g., Sayman et al. 2002, Morton and Zettelmeyer 2004); in other words, store brand products benefit from their relationship with the national brand. We argue that retailers do not position their store brand very close to the national brands, not only because of retailer's competitive reactions to the national brands, but also because of the strategic decisions of other store-brand retailers. To our knowledge, not many studies have been investigated such retail-level competitive structures. The only studies that consider retail competition are Corstjens and Lal (2000) and Geylani et al. (2009). Their studies consider that retailers introduce quality-equivalent store brands and thus focus on competitive strategies such as brands advertising or whether or not to introduce a store brand.

Past studies also pay little attention to consumers who are generally non-brand seeking and thus are more favorable to purchasing store brand products. Thus, we assume that a retailer's actual competitor who competes on the common consumer segment should be the other retailers who provide their own store brands, rather than the national brand that the retailer carries on stock. There is usually a separate segment of consumers who are loyal to a national brand, and they will not easily switch between their loyal brands and store brands only because of product

prices. Therefore, in this study we focus on investigating how a retailer can design its store brand under a competitive setting. Specifically, the main research question we will answer in the current study is how a retailer positions its own store brand when it competes with both another retailer's store brand and a national brand when two segments of consumers exist; those who are loyal to the national brand and those who are "price shoppers" (i.e., less brand-sensitive, but more price-sensitive consumers).

The purpose of this paper is to apply economic theory to analyze the relationship between retailer-level competition and the quality of store brand products. We first investigate why retailers offer store brand products and examine whether and how this competition affects the quality levels of store brand products. Specifically, we would like to address the following research questions: (i) How does store brand competition affect retailers' and manufacturer's pricing decisions? (ii) How does the intensity of retailer-level competition affect the quality gap between the manufacturer and the retailers?

The above questions seem to be a set of traditional questions that have already been answered in previous literatures, especially in those papers we mentioned earlier. However, through the current study we try to observe the market even when we relax two major assumptions that previous studies have made. First, Corstjens and Lal (2000) assume that the manufacturer does not behave strategically. However, this assumption restricts explaining fluctuations of each player's profit margins, i.e. it is hard to observe explicitly the effects of retailer-level competition on pricing decisions (both wholesale and retail). Second, Geylani, Jerath, and Zhang (2009) assume that the quality of the store brand is the same as that of the national brand. Thus, even though they explain how retail competition affects profit margins, they do not capture the effect of product quality on player strategies.

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Therefore, even if this study focuses only on a symmetric equilibrium, with the logit model demand, we can make a generalization to observe how firms behave by relaxing restrictions made in previous studies. Furthermore, we can later extend the model to exclusive manufacturer-retailer competition settings, which have not yet been addressed. Then, we will be able to observe how firms will make strategic decisions accordingly.

Our research is also related to topics of product line rivalry and market segmentation. These are the topics that have been popular, especially in marketing area and have been variously studied with different setups and dimensions. Desai (2001) investigates multiproduct firms facing with the cannibalization problem in designing their product lines. In terms of the market characteristics and the direction of the study, there are some partial similarities between Desai's and our current work. For example, Desai (2001) develops a model of a market characterized by both quality and taste differentiation just like we do, even though he uses Hotelling (1929) model while the current study uses a logit model to describe two levels of market segmentations. Also, the main part of his paper suggests how intense competition in the low-valuation segment affects the market, which is also one of our main questions. His study shows that the more intense competition in the low-valuation segment makes it more attractive for the high-valuation consumers to buy the products meant for the low-valuation segment, and such phenomenon worsens the cannibalization problem. That being said, we can find an insight from our findings in terms of cannibalization. Since our result shows that more intense competition in low-valuation segment (i.e., store brand segment) makes the low quality even lowered, such widening gap in qualities can worsen the cannibalization problem.

However, the biggest difference between Desai (2001) and other similar product line studies and our current research is as follows: The two levels of product qualities cannot be

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differentiated and considered as a product line products in our study. Even though these two quality products are being sold at one retailer competing with other retailer with a similar product layers and customer segments, the high-valuation products are not produced by the retailer. The manufacturer determines the quality and wholesale price of the high-valuation product. It means that the manufacturer is an individual entity that behaves strategically just like the retailers. Even though the price of this high-valuation product is determined by the retailer, it is still hard for us to see that the retailer has a product line products just like what we used to use the term 'a product line' in practice and in academia.

The rest of the paper is organized as follows. Section 2 introduces the model. We illustrate the market where a single retailer makes a product quality decision with a single manufacturer in Section 3. In Section 4, we analyze the market of two retailers with one manufacturer and compare the players' interactions on decision making. We extend our duopoly analysis to a study of the monopolistic competition in Section 5. Finally, we conclude and discuss future research in Section 6. Derivations and proofs are given in the Appendix.

2. MODEL

We examine competitive price and quality discrimination with horizontal and vertical taste differences. A manufacturer produces a national brand product (N) and supplies it through two retailers, A and B. These symmetric retailers also carry their own store brands (S). We assume that the manufacturer produces the national brand with zero marginal cost, without loss of generality, and also strategically decides on wholesale prices to maximize its own profit. The retailer sells a store brand of quality q_S and a national brand of quality q_N, at marginal costs of c_S and w, respectively, where c_S denotes the unit production cost of a store brand product and

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w denotes the wholesale price. We make two additional assumptions. (i) First, we plausibly assume that the quality of the national brand product is greater than that of the store brand; i.e., $q_N > q_S$.[27] (ii) Second, we also assume that the unit-production cost for the store brand is less than the wholesale price; i.e., $c_S \leq w$. The wholesale prices (per-unit production cost) at which the retailers procure store brands are, in general, lower than the wholesale prices at which they procure national brands (Geylani, Jerath and Zhang, 2009).

Given the initial setup and assumptions described above, a consumer's utility function is given by

$$U_{ij} = V + \theta q_{ij} - p_{ij} - \varepsilon_i, \; i = A, B, \; j = N, S,$$

where V is the reservation value of a product, θ is a consumer's willingness to pay for quality, q_{ij} is seller i's quality level for product j, p_{ij} is seller i's retail price for product j, and ε_i captures retailer brand preferences of consumers.[28] Therefore, the random variable of this expression denotes a consumer's willingness to pay for a specific brand j. Also, note that the reservation value V is assumed to be sufficiently large so that all consumers always buy a differentiated good.

We assume that there is a unit mass of consumers, which is normalized to one. The parameter θ is distributed uniformly on $[0,1]$. By defining θ^* as the indifferent consumer's taste parameter, it can be derived as follows: For any consumer, if she is indifferent between buying a national brand and buying a store brand, her indifferent taste parameter is set by $U_N = U_S$, which is equivalent as $V + \theta q_N - p_N - \varepsilon = V + \theta q_S - p_S - \varepsilon$. Solving this equation for

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[27] We exclude the case of equal quality levels between the national brand and the store brand because we want to make the mathematical results articulate and moreover, are not interested in any corner solution whose importance is minimal in terms of its insights.

[28] ε_i captures the factors that affect utility but are not included in $V + \theta q_{ij} - p_{ij}$.

θ, we find that $\theta^* \equiv \dfrac{p_N - p_S}{q_N - q_S}$. Consumers whose willingness to pay is less than this indifferent

taste parameter will prefer the low quality brand ($\theta < \theta^*$), and consumers whose willingness to

pay is larger than this parameter will prefer the high quality brand ($\theta > \theta^*$). We assume, as in the

popular logit model, that ε_i is distributed identically and independently according to the type I

extreme value distribution.[29] The mean of ε_i is zero and the standard deviation is $SD(\sigma)$,[30]

where the parameter σ captures the intensity of retailer rivalry stemming from heterogeneity in

retailer brand preference.[31]

We analyze two market structures. As a benchmark, we first examine a monopoly market

that consists of one national brand manufacturer and a retailer. Second, we extend our analysis to

a duopoly market, where two retailers provide their own store brands but also sell the same

national brand. Consumers choose a retail store and a quality variant simultaneously.

3. MONOPOLY MARKET

We investigate the single manufacturer-single retailer market first. The retailer's profit function

is given by

$$\pi_R^M = (p_N - w)D_N + (p_S - c_S)D_S.\text{[32]}$$

The retailer's profit consists of a profit from selling national brand product and a profit from

selling its own store brand product. Assuming all consumers buy one of either national or store

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[29] The type I extreme value distribution is also referred to as the Gumbel distribution.

[30] This is equal to $(\pi/\sqrt{6})\sigma$ for the logit model. See Anderson, de Palma and Thisse (1992).

[31] $\sigma \in (0.1]$. The parameter σ is interpreted as preference intensity in the literatures; a higher value of σ means that price becomes a less important factor in determining which variant a consumer will buy.

[32] Note that the superscript M represents 'Monopoly' and the subscript M represents 'Manufacturer,' just like the subscript R represents 'Retailer.'

brands in equilibrium, market demand is given by $D_S = \theta^*$ and $D_N = 1 - \theta^*$. The optimal national

brand price can be found from the first-order condition with respect to p_N and is given by

$$\hat{p}_N = p_S + \frac{1}{2}(q_N - q_S) + \frac{1}{2}(w - c_S).$$

Note that we assume that $c_S \leq w$. The optimal store brand price is defined as a price such that the

consumer with the lowest marginal willingness to pay for quality is still willing to buy, which is

basically the same as the reservation price. Thus, it is given by

$$p_S = V.$$

By plugging optimal prices into the manufacturer's profit function, $\pi_M^M = wD_N$, the wholesale

price is given by

$$\hat{w} = \frac{1}{2}(c_S + q_N - q_S).$$

Lemma 1. *For a monopolistic retailer, the price-cost margin of the national brand product,*

$p_N - w$, *is greater than the price-cost margin of the store brand product,* $p_S - c_S$.

Proof. Using the optimal price of the national brand, $p_N = p_S + \frac{1}{2}(q_N - q_S) + \frac{1}{2}(w - c_S)$, and the

requirement that the demand for the national brand product should be positive $(\theta^* < 1)$ at

equilibrium prices: $p_N = p_S + \frac{1}{2}(q_N - q_S) + \frac{1}{2}(w - c_S) > p_S + \frac{1}{2}(p_N - p_S) + \frac{1}{2}(w - c_S) \Leftrightarrow 2p_N >$

$2p_S + (p_N - p_S) - (w - c_S) \Leftrightarrow p_N - w > p_S - c_S.$ *Q.E.D.*

This is consistent with what the conventional wisdom says; i.e., if a monopoly retailer

offers two quality levels of products, it can induce a self-selection of consumers and extract

higher unit profit margins from the consumer with a higher willingness to pay for quality (e.g.,

Mussa and Rosen (1978) and Verboven (1999)).

Condition 1 (for a monopoly retailer): *Let* $\Delta q = q_N - q_S$. *(i) Then, since* $\theta^* \equiv \dfrac{p_N - p_S}{q_N - q_S} \in [0,1]$,

$p_N - p_S \le \Delta q$. *(ii) Also,* $w = \dfrac{1}{2}(c_S + \Delta q) \ge c_S$. *Therefore, combining (i) and (ii), we setup a*

condition of $c_S \le \Delta q$.

Now, let's find the optimal quality levels of products. By substituting the best responding

prices obtained above, the retailer and the manufacturer's profit functions are, respectively, given

by

$$\hat{\pi}_R = V + \frac{1}{16}\Delta q - \frac{7}{8}c_S + \frac{c_S^2}{16\Delta q} \text{ and } \hat{\pi}_M = \frac{(c_S + \Delta q)^2}{8\Delta q}.$$

Solving these functions now with respect to Δq, I find that the optimal gap between the quality

levels of the national brand and store brand products should be the same as the retailer's unit

production cost of store brand as follows:

$$\Delta q^M = q_N^M - q_S^M = c_S.$$

Along with it, the optimal values and profits are given by

$$p_N^M = \frac{1}{2}c_S + V, \ p_S^M = V, \text{ and } \pi_R^M = V - \frac{3}{4}c_S, \text{ (for the retailer)}$$

$$w^M = c_S \text{ and } \pi_M^M = \frac{1}{2}c_S. \text{ (for the manufacturer)}$$

Retailer perspective: Only parameter plays a role in these results is the unit cost of

production for the store brand product. An increase in this cost makes the price of the national

brand product increased while it makes the retailer's profit decreased ($\dfrac{\partial p_N^M}{\partial c_S} > 0, \dfrac{\partial \pi_R}{\partial c_S} < 0$). As

we found above, the optimal unit cost of store brand production is equivalent to the gap between

the national and store brands' qualities. Therefore, even though product quality levels are choice

variables, we can obtain comparative static insights from the relationship of this quality difference and the cost of store brand production. The retailer's profit can decrease in two different ways in terms of product quality levels: (i) an increase in the national brand product quality and/or (ii) a decrease in the store brand product quality. Given everything else is fixed, if the quality of the national brand product is set high, it will give the retailer a negative impact in the first place. On the other hand, at a certain unit production cost and the quality of the national brand, the retailer is likely to improve its own product quality not to have any slack for the condition $c_S \leq \Delta q$, which was defined above. That is, when $c_S = \Delta q$, the retailer earns a highest profit. In terms of the retailer's price decision on the national brand product, an increase in the unit cost of the store brand production due to an increase in the national brand product quality leave the room for the retailer increase the price of the national brand product.

$\quad$ *Manufacturer perspective*: An increase in the unit production cost of the store brand makes both the wholesale price of the national brand product and the manufacturer's profit increased ($\frac{\partial w}{\partial c_S} > 0$, $\frac{\partial \pi_M}{\partial c_S} > 0$). Again, given that an exogenous unit production cost for the store brand product and an expected quality level of the store brand are fixed, the manufacturer is likely to increase the quality of the national brand until it reaches the optimal. It will affect its profit in a positive way. Another insight we may argue is that a decrease in the store brand's quality generates more room for the unit cost of store brand production, and thus this increase in the production cost may lead for the manufacturer to increase the wholesale price of its own product.

4. DUOPOLY MARKET

In this section we consider a market where two retailers compete with each other. However, still there is a single manufacturer that produces a national brand product and supplies it to consumers via those two competing symmetric retailers. Consumer demands are assumed to follow the logistic distribution.

4.1. Time line of the game

The price and quality decisions are sequenced in the following time line: (1) National brand quality choice – The manufacturer decides on the level of national brand product quality (if it is considered to be endogenous); (2) Store brand quality choice – The retailers simultaneously decide the quality levels of store brand products; (3) Wholesale price choice – The manufacturer of a national brand decides on the wholesale price that will be charged to the retailer; and (4) Retail price choice – The retailers simultaneously decide the retail prices of all national and store brand products they offer at stores.

4.2. Consumers' choices

Even though a consumer decides a location and a variant simultaneously, her decision process can be seen as a sequential process in which a retailer is first chosen, and then a particular product is selected.

(1) For each retailer, the consumer decides what quality variant is preferred. From the utility function we described above, we define θ_i^* as the indifferent consumer's marginal

willingness to pay for the variant of retailer i, i.e., $\theta_i^* \equiv \dfrac{p_{iN} - p_{iS}}{q_{iN} - q_{iS}}$.

(2) The consumer compares the preferred variants of the two different retailers. Figure 1 shows what comparisons the consumer makes. For example, if consumers' marginal willingness to pay to the retailer A is smaller than that to the retailer B, i.e., $\theta_A^* < \theta_B^*$, then a consumer with

$\theta \in \left[0, \theta_A^*\right]$ prefers the store brand of retailer A to the national brand, and prefers the store

brand of retailer B to the national brand, so that she compares retailer A's store brand with

retailer B's store brand. If there is another consumer with $\theta \in \left[\theta_A^*, \theta_B^*\right]$ under $\theta_A^* < \theta_B^*$, she

will prefer the national brand of retailer A to the store brand, and will prefer the store brand

of retailer B to the national brand, so that she compares retailer A's national brand with

retailer B's store brand. If there is also a consumer with $\theta \in \left[\theta_B^*, 1\right]$ under $\theta_A^* < \theta_B^*$, he will

prefer the national brand of retailer A to the store brand, and will prefer the national brand of

retailer B to the store brand, so that he compares retailer A's national brand with retailer B's

national brand. Therefore, as shown in Figure 1 below, three consumer segments are

considered for the case of $\theta_A^* < \theta_B^*$, and also three consumer segments are considered for the

case of $\theta_A^* < \theta_B^*$.

Figure 1. Consumer Comparisons of Variants across Retailer Brands
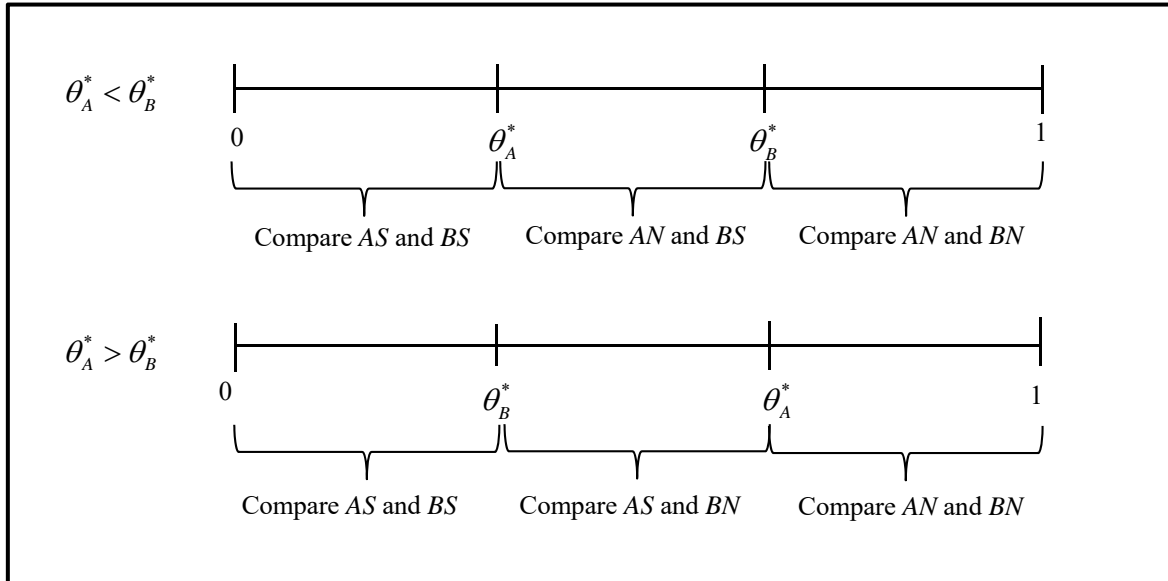
The outcome of the cross-retailer brand comparisons depends on the extreme value

distributed random variable ε_i, $i = A, B$. The probability that the consumer will choose retailer i

over retailer $-i$, where $-i \neq i$, given the quality taste parameter θ, is given by the standard logit formula:

$$\Pr\left(U_{ij}(\theta) \geq U_{-ij}(\theta)\right) = \frac{\exp\left(\left(\theta q_j - p_{ij}\right)/\sigma\right)}{\exp\left(\left(\theta q_j - p_{ij}\right)/\sigma\right) + \exp\left(\left(\theta q_j - p_{-ij}\right)/\sigma\right)} \ , \ i = A, B, \ j = N, S.$$

In context of the current study, this formula represents the probability that a consumer's utility obtained from choosing retailer i to buy a product variant j is greater than or equal to her utility obtained from choosing retailer $-i$ to buy a product variant j.[33]

Using this probability and consumer segments described earlier, we can derive consumer demand functions for each segmentations. First, the aggregate demand for the store brand product of retailer A is given by [34]

$$D_{AS} = \begin{cases} \displaystyle\int_0^{\theta_A^*} \frac{\exp\left(-p_{AS}/\sigma\right)}{\exp\left(-p_{AS}/\sigma\right) + \exp\left(-p_{BS}/\sigma\right)} d\theta & \text{for } 0 \leq \theta_A^* < \theta_B^* \\[4mm] \displaystyle\int_0^{\theta_B^*} \frac{\exp\left(-p_{AS}/\sigma\right)}{\exp\left(-p_{AS}/\sigma\right) + \exp\left(-p_{BS}/\sigma\right)} d\theta + \int_{\theta_B^*}^{\theta_A^*} \frac{\exp\left(-p_{AS}/\sigma\right)}{\exp\left(-p_{AS}/\sigma\right) + \exp\left(\theta\left(q_N - q_S\right) - p_{BN}/\sigma\right)} d\theta \\[4mm] \hspace{9cm} \text{for } \theta_B^* < \theta_A^* \leq 1. \end{cases}$$

The first function represents the demand of consumers who buy the store brand product at the retailer A against the retailer B when $\theta_A^* < \theta_B^*$. The second function expresses the demand as a sum of those who choose a store brand product from the retailer A rather than the retailer B (the first term) and those who choose a store brand product at the retailer A rather than the national brand product at the retailer B (the second term).

---

[33] For example, $\Pr\left(U_{AN}(\theta) \geq U_{BS}(\theta)\right) = \dfrac{\exp\left(\left(\theta q_N - p_{AN}\right)/\sigma\right)}{\exp\left(\left(\theta q_N - p_{AN}\right)/\sigma\right) + \exp\left(\left(\theta q_N - p_{BS}\right)/\sigma\right)}$ represents the probability that the utility from having the national product at Store A is greater than or equal to the utility from having the store brand product at Store B.

[34] Examples of demand derivations are in the Appendix.

The aggregate demand for the national brand sold by retailer A is given by

$$
D_{AN} = \begin{cases}
\displaystyle\int_{\theta_A^*}^{\theta_B^*} \frac{\exp(-p_{AN}/\sigma)}{\exp(-p_{AN}/\sigma) + \exp(-\theta(q_N - q_S) - p_{BS}/\sigma)}\, d\theta + \int_{\theta_B^*}^{1} \frac{\exp(-p_{AN}/\sigma)}{\exp(-p_{AN}/\sigma) + \exp(-p_{BN}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } 0 \le \theta_A^* < \theta_B^* \\[4pt]
\displaystyle\int_{\theta_A^*}^{1} \frac{\exp(-p_{AN}/\sigma)}{\exp(-p_{AN}/\sigma) + \exp(-p_{BN}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } \theta_B^* < \theta_A^* \le 1.
\end{cases}
$$

Analogously, the aggregate demand for the store brand and national brand provided by retailer B are given by, respectively

$$
D_{BS} = \begin{cases}
\displaystyle\int_{0}^{\theta_A^*} \frac{\exp(-p_{BS}/\sigma)}{\exp(-p_{BS}/\sigma) + \exp(-p_{AS}/\sigma)}\, d\theta + \int_{\theta_A^*}^{\theta_B^*} \frac{\exp(-p_{BS}/\sigma)}{\exp(-p_{BS}/\sigma) + \exp(\theta(q_N - q_S) - p_{AN}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } 0 \le \theta_A^* < \theta_B^* \\[4pt]
\displaystyle\int_{0}^{\theta_B^*} \frac{\exp(-p_{BS}/\sigma)}{\exp(-p_{BS}/\sigma) + \exp(-p_{AS}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } \theta_B^* < \theta_A^* \le 1,
\end{cases}
$$

and

$$
D_{BN} = \begin{cases}
\displaystyle\int_{\theta_B^*}^{1} \frac{\exp(-p_{BN}/\sigma)}{\exp(-p_{BN}/\sigma) + \exp(-p_{AN}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } 0 \le \theta_A^* < \theta_B^* \\[4pt]
\displaystyle\int_{\theta_B^*}^{\theta_A^*} \frac{\exp(-p_{BN}/\sigma)}{\exp(-p_{BN}/\sigma) + \exp(-\theta(q_N - q_S) - p_{AS}/\sigma)}\, d\theta + \int_{\theta_A^*}^{1} \frac{\exp(-p_{BN}/\sigma)}{\exp(-p_{BN}/\sigma) + \exp(-p_{AN}/\sigma)}\, d\theta \\
\hspace{9cm} \text{for } \theta_B^* < \theta_A^* \le 1.
\end{cases}
$$

4.3. Analysis

Retailers set retail prices for the national brand and their own private label products to maximize profits. Their profit functions are given by

$$
\pi_{Ri} = (p_{iN} - w_i) D_{iN} + (p_{iS} - c_{iS}) D_{iS}, \quad i = A, B.
$$

The manufacturer chooses the wholesale prices at which they will supply product to retailers. Assuming zero cost of production, the profit function is given by

$$
\pi_M = w_A D_{AN} + w_B D_{BN}.
$$

4.3.1 Retail Prices Decisions

We now solve this game backwards, starting from the retail prices. Let us first consider that

$0 \leq \theta_A^* < \theta_B^*$.[35] Retailer A's first-order conditions for profit maximization, with respect to p_{AS} and

p_{AN}, are given by

$$\frac{\partial \pi_{RA}}{\partial p_{AS}} = \left(p_{AN} - w_A \right) \frac{\partial D_{AN}}{\partial p_{AS}} + \left(p_{AS} - c_{AS} \right) \frac{\partial D_{AS}}{\partial p_{AS}} + D_{AS} \text{, and}$$

$$\frac{\partial \pi_{RA}}{\partial p_{AN}} = \left(p_{AN} - w_A \right) \frac{\partial D_{AN}}{\partial p_{AN}} + D_{AN} + \left(p_{AS} - c_{AS} \right) \frac{\partial D_{AS}}{\partial p_{AN}}.$$

We also can get retailer B's first-order conditions ($\frac{\partial \pi_{RB}}{\partial p_{BS}}$, $\frac{\partial \pi_{RB}}{\partial p_{BN}}$) in the same fashion as retailer

A's problems. Note that we focus only on finding a symmetric equilibrium. Therefore, we

impose $p_{AS} = p_{BS} = p_S$ and $p_{AN} = p_{BN} = p_N$ for further analysis hereafter.

Substituting all the demands and the demand derivatives above, the first order conditions

can be rewritten as:

$$\frac{\partial \pi_{RA}}{\partial p_{AS}} = \left(p_{AN} - w_A \right) \left(\frac{1}{\Delta q \Omega} \right) + \left(p_{AS} - c_{AS} \right) \left(-\frac{\theta^*}{4\sigma} \right) + \frac{\theta^*}{2} = 0 \text{, and}$$

$$\frac{\partial \pi_{RA}}{\partial p_{AN}} = \left(p_{AN} - w_A \right) \left(-\frac{1}{\Delta q \Omega} - \frac{1-\theta^*}{4\sigma} \right) + \frac{1-\theta^*}{2} + \left(p_{AS} - c_{AS} \right) \left(\frac{1}{2\Delta q} \right) = 0,$$

where $\Omega = 1 + \exp\left[\theta \Delta q \left(\frac{1-\sigma}{\sigma} \right) \right]$.

---

[35] An analysis of $\theta_B^* < \theta_A^*$ will be actually redundant. Given an assumption of symmetric retailers, the existing analysis ($0 < \theta_A^* < \theta_B^*$) will give us the same insight.

Definition 1: $\Omega = 1 + \exp\left[\theta\Delta q\left(\frac{1-\sigma}{\sigma}\right)\right]$. This is the ratio of one retailer's output to the other

retailer's output.

In a duopoly market, what we are interested in is two extreme cases: either a case of (i) a highly competitive intensity between retailers or (ii) locally-monopolized retailers. Since the main purpose of our study is to observe how price and quality of store brand products are determined in a competitive setting, from now on throughout the end of this particular section (Section 4), we will focus our study only on a highly intensified competition market.[36]

Recall $\Omega = 1 + \exp\left[\theta\Delta q\left(\frac{1-\sigma}{\sigma}\right)\right]$. The parameter σ captures the intensity of retailer

rivalry; i.e., a higher value of σ means that the price becomes a less important factor in determining which variant a consumer will buy. Thus, a lower value of σ refers to a stronger intensity of competition. Hence, let us consider a situation of high enough competition where σ approaches zero. Then, we observe that $\lim_{\sigma\to 0}\Omega = \infty$, which means that there will be sufficiently large value of Ω, and thus $1/\lim_{\sigma\to 0}\Omega = 0$.

Note that there is another terms that have σ in each equation of the first order conditions above; they are $-\frac{\theta^*}{4\sigma}$ and $-\frac{1-\theta^*}{4\sigma}$. These terms are decreasing in σ as well and eventually approach to zeros when high competitive intensity is represented. However, they will not completely disappear while $1/\lim_{\sigma\to 0}\Omega = 0$ since the decreasing rate of these two terms are a lot slower than the decreasing rate of the exponential term of Ω. Therefore, we will still be able to

---

[36] We extend our analysis to the case of locally monopolized retailers in Section 5. Please see Section 5 for a separate analysis of another duopoly market in detail.

keep both terms of $-\dfrac{\theta^*}{4\sigma}$ and $-\dfrac{1-\theta^*}{4\sigma}$ in our expressions by defining the *upper boundary* of σ as follows:

Definition 2a: (Upper Boundary σ) There is $0 < \sigma < \tilde{\sigma}$ such that $\dfrac{1}{1+\exp\left[\theta\Delta q\left(\dfrac{1-\tilde{\sigma}}{\tilde{\sigma}}\right)\right]} \approx 0$.[37]

This definition brings us a sufficiently small value of σ, which guarantees for us to observe a highly competitive retailers' market. Now, we only consider values of σ as given the definition above through the rest of the section. Then, the best responding prices at a unique symmetric equilibrium are given by

$$\hat{p}_N = \frac{1}{2}\left(c_S - w_A + \Delta q + 4\sigma + \sqrt{(c_S - w_A + \Delta q)^2 - 16\sigma^2}\right) \text{ and } \hat{p}_S = c_S + 2\sigma.$$

Condition 2 (for a duopoly retailer) *Given the price of the national brand product derived above, the following condition should hold:* $(c_S - w_A + \Delta q)^2 - 16\sigma^2 \geq 0$. *This leads to*

$c_S - w_A + \Delta q \geq 4\sigma \Leftrightarrow \Delta q \geq 4\sigma + w_A - c_S$.

Lemma 2. *For a duopolistic retailer, the price-margin of the national brand product is greater than the price-margin of the store brand product when there is an intense competition between retailers.*

---

[37] We can easily check a value of σ with a numerical example as follows: In the F.O.C. expression of $\dfrac{\partial \pi_A^R}{\partial p_{AS}}$ above, let's suppose $\sigma = 0.01$. Then, $e^{\left(\frac{1-\sigma}{\sigma}\right)} = 9.88903\times10^{42}$ while $\dfrac{1}{4\sigma} = 25$. Further, let $p_N = 0.4$, $p_S = 0.2$, $\Delta q = 0.4$, $c_S = 0.18$, and $w_A = 0.2$. Then the term with Ω becomes, $(p_{AN} - w_A)\left(\dfrac{1}{\Delta q \Omega}\right) = 1.25875\times10^{-9} \approx 0$ while the other terms with σ become $(p_{AS} - c_{AS})\left(-\dfrac{\theta^*}{4\sigma}\right) = -0.25$ and $(p_{AN} - w_A)\left(-\dfrac{1-\theta^*}{4\sigma}\right) = -12.5$. Compared to these last two terms, $(p_{AN} - w_A)\left(\dfrac{1}{\Delta q \Omega}\right)$ is almost zero and negligible. Therefore, we set $\tilde{\sigma} = 0.01$ in this example.

Proof. The gap of profit margins between the national brand and the store brand is given by,

$$\left(\hat{p}_N - w_N\right) - \left(p_S - c_S\right) = \frac{1}{2}\left(c_S + w_A + \Delta q + 4\sigma + \sqrt{\left(c_S - w_A + \Delta q\right)^2 - 16\sigma^2}\right) - w_N - 2\sigma$$

$$= \frac{1}{2}\left(c_S - w_A + \Delta q + \sqrt{\left(c_S - w_A + \Delta q\right)^2 - 16\sigma^2}\right) > \frac{1}{2}\left(4\sigma + \sqrt{\left(c_S - w_A + \Delta q\right)^2 - 16\sigma^2}\right) > 0,$$

along with the condition defined above.   *Q.E.D.*

When competition gets more intense, retailers has no room to make higher margins from selling store brand products. This is somewhat different from what Geylani, Jerath and Zhang (2009) argue; that is, retailers introduce store brands only when the competition intensity is small enough and if there is no margin advantage from store brands. However, our result shows that the per-unit margin of the store brand does not have to be larger than that of the national brand to be introduced, even under a high competition intensity context.

4.3.2 Wholesale Price Decision

We substitute best response prices obtained above ($\hat{p}_N$ and $\hat{p}_S$) into the manufacturer's problem and differentiate the objective profit function with respect to the wholesale price. Note that, in the context of a single manufacturer and the Antitrust Law, we can simplify the subscripts omitting '*A*' and '*B*' for the wholesale prices. The first order condition is given by

$$\frac{\partial \pi_M}{\partial w} = D_{AN} + w\frac{\partial D_{AN}}{\partial w} + D_{BN} + w\frac{\partial D_{BN}}{\partial w},$$

where $D_{AN}\big|_{\hat{p}_N, p_S} = \frac{1}{2\Delta q}\left(c_S + \Delta q + 2\sigma + \frac{1}{2}\left(-c_S - w - \Delta q - 4\sigma - \sqrt{\left(c_S - w + \Delta q\right)^2 - 16\sigma^2}\right)\right),$

$$D_{BN}\big|_{\hat{p}_N, p_S} = \frac{1}{4\Delta q}\left(c_S - 2\Delta q + w + \Delta q + \sqrt{\left(c_S - w + \Delta q\right)^2 - 16\sigma^2}\right), \quad w\frac{\partial D_{AN}}{\partial w}\bigg|_{\hat{p}_N, p_S} =$$

$$\frac{w}{4\Delta q}\left(1 - \frac{c_S - w + \Delta q}{\sqrt{\left(c_S - w + \Delta q\right)^2 - 16\sigma^2}}\right), \text{ and } w\frac{\partial D_{BN}}{\partial w}\bigg|_{\hat{p}_N, p_S} = \frac{w}{4\Delta q}\left(1 - \frac{c_S - w + \Delta q}{\sqrt{\left(c_S - w + \Delta q\right)^2 - 16\sigma^2}}\right).$$

Having this equal to zero, we find the optimal wholesale price, which is given by

$$\hat{w}^{HC} = \frac{1}{2}c_S + \frac{1}{2}\Delta q - \frac{8\sigma^2}{c_S + \Delta q},$$

where the superscript *HC* represent 'High Competition.'

Regardless of the competition level, the wholesale price under the duopoly with an intense competition is lower than the wholesale price under the monopoly; i.e., $\hat{w}^{HC} < w^M = \frac{1}{2}(c_S + q_N - q_S)$. Moreover, as competition intensity increases, the degree of decrease in wholesale price gets even smaller; i.e., $\frac{\partial \hat{w}^{HC}}{\partial \sigma} < 0$. After plugging this optimal wholesale price back into the best responding national and store brand prices, we have:

$$\hat{p}_N^{HC} = \frac{1}{2}(c_S + \Delta q + 4\sigma) \text{ and } \hat{p}_S^{HC} = c_S + 2\sigma.$$

4.3.3 Store Brand Quality Decisions

Given the best responding retail and wholesale prices, we find the optimal store brand product quality in the section. Retailer *i* solves its maximization problem given the optimal prices obtained in Section 4.3.1 and 4.3.2 as follows:

$$\pi_{Ri}^{HC} = \left(\hat{p}_{iN}^{HC} - \hat{w}^{HC}\right)D_{iN}\left(\hat{p}_{iN}^{HC}, \hat{p}_{iS}^{HC}, \hat{w}^{HC}\right) + \left(\hat{p}_{iS}^{HC} - c_{iS}\right)D_{iS}\left(\hat{p}_{iN}^{HC}, \hat{p}_{iS}^{HC}, \hat{w}^{HC}\right), \ i = A, B.$$

Given the retailer's symmetric status, the retailer *A*'s first-order condition for profit maximization, with respect to q_S, is given by

$$\frac{\partial \pi_{RA}}{\partial q_S} = \left(\frac{\partial \hat{p}_N^{HC}}{\partial q_S} - \frac{\partial \hat{w}_A^{HC}}{\partial q_S}\right)D_N^{HC} + \left(\hat{p}_N^{HC} - w_A^{HC}\right)\frac{\partial D_N^{HC}}{\partial q_S} + \left(\hat{p}_S^{HC} - c_S\right)\frac{\partial D_S^{HC}}{\partial q_S}.$$

Substituting the demands, the demand derivatives, and the prices derived above, the first order conditions of this differentiation can be rewritten as:

$$\frac{\partial \pi_{RA}}{\partial q_S} = \left(-\frac{1}{2} + \frac{1}{2} + \frac{8\sigma^2}{\left(c_S + q_N - q_S\right)^2} \right) \left(\frac{c_S + q_N - q_S}{4\left(q_N - q_S\right)} \right) + \left(\frac{2\sigma\left(c_S + q_N - q_S + 4\sigma\right)}{c_S + q_N - q_S} \right) \left(\frac{c_S}{4\left(q_N - q_S\right)^2} \right)$$
$$+ \left(c_S + 2\sigma - c_S \right) \left(-\frac{c_S - q_N + q_S}{4\left(q_N - q_S\right)^2} \right)$$

It can actually be simplified as $\dfrac{\partial \pi_{RA}}{\partial q_S} = \dfrac{\sigma\left(-2c_S^2 - \left(c_S - \left(q_N - q_S\right)\right)\left(q_N - q_S + 4\sigma\right)\right)}{2\left(q_N - q_S\right)^2\left(c_S + q_N - q_S\right)}$. Then, having

$\dfrac{\partial \pi_{RA}}{\partial q_S} = 0$, we solve the optimal quality of the store brand, which is given by

$$q_S^{HC*} = q_N + \frac{1}{2}\left(-c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} \right).$$

As is straightforwardly shown, the optimal quality level of the store brand product is

linear and increases with the quality level of the national brand. For any quality level of the

national brand over the threshold of $\bar{q}_N = \dfrac{1}{2}\left(c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} \right)$, the quality of the

store brand is positive. In addition, the optimal quality level of the store brand product decreases

with competition intensity; i.e., q_S^{HC*} concave-decreases as σ decreases ($\dfrac{\partial q_S^{HC*}}{\partial \sigma} > 0$).

4.3.4 National Brand Quality Decision

4.3.4.1 Exogenous National Brand Quality

When the quality of the national brand is assumed to be exogenously given, it can be defined

directly from the optimal level of the store brand product quality, which is shown in Section

4.3.3. It is given by

$$q_N = q_S^{HC*} - \frac{1}{2}\left(-c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} \right).$$

Therefore, we conclude that given a fixed level of the national brand, the quality gap between its

product and the store brand product is given by

$$\Delta q^{HC*} = \frac{1}{2}\left(c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right).$$

Proposition 1. *There exists a unique symmetric equilibrium where retailers choose*

$\left\{p_S^{HC*}, p_N^{HC*}, q_S^{HC*}\right\}$ *and the manufacturer chooses* $\left\{w^{HC*}\right\}$.

The corresponding optimal retail prices for the national and the store brand, quality for

the store brand, and wholesale price are given by, respectively

$$p_N^{HC*} = \frac{1}{4}\left(3c_S + 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right),$$

$$p_S^{HC*} = c_S + 2\sigma,$$

$$q_S^{HC*} = q_N + \frac{1}{2}\left(-c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right) \text{ and}$$

$$w^{HC*} = \frac{1}{4}\left(3c_S + 2\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} - \frac{2\sigma\left(4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}{c_S}\right).$$

Comparative Statics: For optimal prices, both the national and the sore brand product

prices decrease as the competition intensity increases ($\frac{\partial p_N^{HC*}}{\partial \sigma} > 0$, $\frac{\partial p_S^{HC*}}{\partial \sigma} > 0$). However, the

wholesale price increases as the competition gets more intense ($\frac{\partial w^{HC*}}{\partial \sigma} < 0$). For the profit

margins, both the national brand manufacturer's profit margin and the store brand retailers'

margins decreases as the competition intensity increases ($\frac{\partial\left(p_N^{HC*} - w^{HC*}\right)}{\partial \sigma} > 0$ and

$\frac{\partial\left(p_S^{HC*} - c_S\right)}{\partial \sigma} > 0$). As the competition intensity parameter σ increases, both the national brand

product price and the store brand product price increase. With its meaning of σ, this indicates

that retailers decrease prices for both national and store brands when the competition intensity

gets strong. In reality, prices for the national brand approaches the wholesale price, and the store

brand products get close to the marginal production cost under a strong competition; that is,

$\lim\limits_{\sigma \to 0} p_N^{HC*} = w^{HC*}$ and $\lim\limits_{\sigma \to 0} p_S^{HC*} = c_S$. The retailer's profit margins decrease for both product

variants. Thus, the retailer will have more restrictions on deciding prices since it becomes hard

for a retailer to compensate for a loss from one profit margin due to competition against the other

retailer. These will interact with firms' quality decisions. We summarize these insights in the

following proposition.

Proposition 2. *In a highly competitive market at the retailer level, as the intensity of competition*

grows, the quality level of the store brand product for retailers will diverge from the quality level

of the national brand product.

Proof. See the Appendix.

The retailers face with their marginal profit lowered due to a strong competition. This

brings the manufacturer a motivation to make its product quality higher by charging a higher

wholesale price:[38] First, we observe that the wholesale price decreases with the intense

competition ($\dfrac{\partial w^{HC*}}{\partial \sigma} < 0$), then observe that this lowered wholesale price leads the national brand

product's quality down ($\dfrac{\partial q_N^{HC}}{\partial w^{HC*}} > 0$), since $\dfrac{\partial q_N^{HC}}{\partial \sigma} < 0$ and $\dfrac{\partial w^{HC*}}{\partial \sigma} < 0$.[39] To increase the

wholesale price, the manufacturer is more likely to increase the quality level of the national

brand product. On the other hand, the retailers are more likely to decrease the product quality of

---

[38] We do not include quality-dependent production costs in the model. However, it is not implausible to assume that the marginal benefit from an increase in wholesale price is higher than a marginal cost increase due to an increase in quality level of the product.

[39] $\dfrac{\partial q_N^{HC}}{\partial \sigma} = \dfrac{\partial q_N^{HC}}{\partial w^{HC*}} \dfrac{\partial w^{HC*}}{\partial \sigma} \Leftrightarrow \dfrac{\partial q_N^{HC}}{\partial w^{HC*}} = \dfrac{\partial q_N^{HC}}{\partial \sigma} / \dfrac{\partial w^{HC*}}{\partial \sigma}$.

the store brands. The profit margin from the store brand approaches zero at a certain cost, which

is exogenously given in our model. Thus, price will decrease until it reaches the marginal cost. A

decrease in price leads the retailer to drop the product quality of the store brand or vice versa. It

can also be proved by observing that $\frac{\partial q_S^{HC*}}{\partial p_S^{HC*}} > 0$ if $\frac{\partial q_S^{HC*}}{\partial \sigma} > 0$ and $\frac{\partial p_S^{HC*}}{\partial \sigma} > 0$.[40] Therefore, we

conclude that the quality levels for the national and store brand products move in the opposite

direction against the competition intensity (See Figure 2).

Figure 2. Quality Levels of National and Store Brand Products
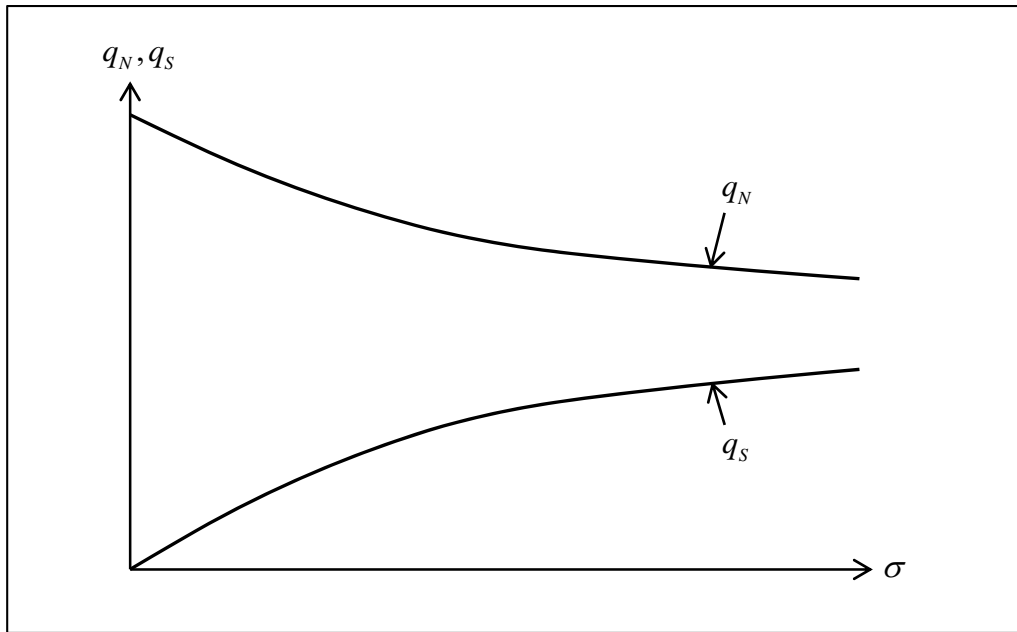
Let us summarize our analysis intuitively. When competition is intensified, retailers

primarily compete with prices. Then, as the intensity grows, the prices tend to decrease so that

the profit margins from both product variants will be lowered. Therefore, the price of the

national brand approaches the wholesale price and the price of the store brand product

---

[40] $\frac{\partial q_S^{HC*}}{\partial \sigma} = \frac{\partial q_S^{HC*}}{\partial p_S^{HC*}} \frac{\partial p_S^{HC*}}{\partial \sigma} \Leftrightarrow \frac{\partial q_S^{HC*}}{\partial p_S^{HC*}} = \frac{\partial q_S^{HC*}}{\partial \sigma} / \frac{\partial p_S^{HC*}}{\partial \sigma}$.

approaches the marginal production cost. This incentivizes the manufacturer to increase the wholesale price. Therefore, the quality level of the national brand product will be increased accordingly. However, given a fixed quality level of the national brand, the quality level of the store brand product will decrease due to its deterministic relationship with its price. Therefore, we conclude that, as the intensity of competition increases, the gap between the qualities of two product variants becomes larger.

Corollary 1. *Retailers under duopoly will not position the quality level of their store brand products as close to the quality of the national brand product as much as a monopolistic retailer would do.*

Proof. Recall the optimal quality level from the monopoly case: $q_S^M = q_N - c_S$. Therefore,

$\Delta q^M = c_S$. Then, $\Delta q^{HC*} - \Delta q^M = \dfrac{1}{2}\left(c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right) - c_S$

$= \dfrac{1}{2}\left(-c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right) > 0$. This is always positive, which means that the quality

gap between the national and the store brand is always greater under duopoly than under

monopoly. *Q.E.D.*

4.3.4.2 Endogenous National Brand Quality

Given the optimal values obtained from Section 4.3.1 through 4.3.3, we solve the manufacturer's problem by finding the first-order condition as follows:

$$\frac{\partial \pi_M^{HC}}{\partial q_N} = w^{HC*}\frac{\partial D_{AN}\left(p_N^{HC*}, p_S^{HC*}, q_S^{HC*}, w^{HC*}\right)}{\partial q_N} + w^{HC*}\frac{D_{BN}\left(p_N^{HC*}, p_S^{HC*}, q_S^{HC*}, w^{HC*}\right)}{\partial q_N}.$$

However, once we substitute all the optimal values into the equation, the above equation becomes only a function of parameters c_S and σ due to the deterministic relationship between

the optimal quality level of the store brand product and that of the national brand product.[41] Even though our goal in the present study is to observe how qualities of the store brand products are decided and positioned, and thus it seems sufficient to set the quality level of the national brand exogenously[42], it is also worth observing how the quality of the national brand product is characterized.

Let us first plug the optimal values of p_N^{HC*}, p_S^{HC*}, and w^{HC*} into the first order condition of the manufacturer's profit function. Then, the above equation turns to and is given by

$$\frac{\partial \pi_M^{HC}}{\partial q_N} = w^{HC*} \frac{\partial D_{AN}\left(p_N^{HC*}, p_S^{HC*}, w^{HC*}\right)}{\partial q_N} + w^{HC*} \frac{D_{BN}\left(p_N^{HC*}, p_S^{HC*}, w^{HC*}\right)}{\partial q_N}$$

$$= \frac{1}{2}\left(3c_S + 2\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} - \frac{2\sigma\left(4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}{c_S} \right)$$

$$\times \left(\frac{c_S + 4q_N - 4q_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}}{8(q_N - q_S)} \right).$$

We first perturb this first order condition by approximating it by using the Taylor Expansion, then we substitute q_S^{HC*} into the equation, and then finally solve the equation for q_N. Hence, we find the optimal quality level of the national brand q_N^{HC*}, which is given by

$$q_N^{HC*} = \frac{4\left(-c_S^2 - \sigma\left(c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)\right)}{3\left(c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}.$$

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$$\frac{\partial \pi_M^{HC}}{\partial q_N} = w^{HC*} \frac{\partial D_{AN}\left(p_N^{HC*}, p_S^{HC*}, q_S^{HC*}, w^{HC*}\right)}{\partial q_N} + w^{HC*} \frac{D_{BN}\left(p_N^{HC*}, p_S^{HC*}, q_S^{HC*}, w^{HC*}\right)}{\partial q_N} = \frac{c_S\left(3c_S - 12\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}{2\left(c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}.$$

[42] Setting up the quality of the national brand product exogenously is even more practical because the national manufacturers in general do not seriously consider the store brands when they design their own products at the very initial stage of product development.

If we observe how the national brand product quality (q_N^{HC*}) changes over the

competition parameter (σ), we find that the optimal quality increases as the competition

intensity between retailers increases; that is, $\dfrac{\partial q_N^{HC*}}{\partial \sigma} = -\dfrac{2\left(-c_S - 4\sigma + \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right)}{3\sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}} < 0$.[43]

This shows a consistent support with the result that we obtained from the case of the exogenous

national brand quality (q_N) assumption since, given a value of the store brand product quality

(q_S), the gap between the two quality levels becomes larger with the stronger competition

intensity.

5. Local Monopolistic Competition

In this section, we extend our analysis to the case where two retailers behave an individual

monopolist. The way we solve the problem is similar to Section 4, where the retailers compete

strongly each other. Thus, we start with solving retail and wholesale prices. Let us recall the ratio

of the retailers' output from one to the other; i.e., $\Omega = 1 + \exp\left[\theta\Delta q\left(\dfrac{1-\sigma}{\sigma}\right)\right]$ (Definition 1). When

the competition between two retailers is least intense, the competition parameter (σ) approaches

to its maximum boundary value of one, and thus we can find that $\lim\limits_{\sigma \to 1} \Omega = 2$.

Definition 2b: (Lower Boundary σ) There is $\underset{\sim}{\sigma} < \sigma < 1$ such that $\dfrac{1}{1 + \exp\left[\theta\Delta q\left(\dfrac{1-\tilde{\sigma}}{\tilde{\sigma}}\right)\right]} \approx \dfrac{1}{2}$.

---

[43] This condition always holds: Suppose $\dfrac{\partial q_N^{HC*}}{\partial \sigma} < 0$. Then, it means $2\left(c_S + 4\sigma - \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}\right) < 0 \Leftrightarrow$

$c_S + 4\sigma < \sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2} \Leftrightarrow (c_S + 4\sigma)^2 < 9c_S^2 + 8c_S\sigma + 16\sigma^2 \Leftrightarrow c_S^2 < 9c_S^2$, which is always true.

Retail Prices: We also use the Taylor expansion to approximate the first-order conditions. This can be done to mitigate their mathematical complexities without loss of generality. Therefore, the first-order conditions can be rewritten as

$$\frac{\partial \pi_{RA}}{\partial p_{AS}} = \frac{c_S p_N + 4\sigma p_N - 2\sigma w_A}{4\sigma \Delta q} - \frac{(c_S + p_N - 2\sigma) p_S}{4\sigma \Delta q}, \text{ and}$$

$$\frac{\partial \pi_{RA}}{\partial p_{AN}} = \frac{w_A (p_S + \Delta q) + 2\sigma (-c_S + 2p_S + w_A + \Delta q)}{4\sigma \Delta q} - \frac{(p_S + w_A + \Delta q + 4\sigma) p_N}{4\sigma \Delta q}.$$

By solving these equations simultaneously, the best responding retail prices are given by

$$\hat{p}_N = \frac{1}{2(c_S + w_A + \Delta q + 8\sigma)} \left(-c_S \Delta q + w_A \Delta q - 2c_S \sigma - 6\sigma w_A + 8\sigma^2 - \Theta \right), \text{ and}$$

$$\hat{p}_S = \frac{1}{2(c_S + w_A + 6\sigma)} \left(-c_S \Delta q - w_A \Delta q + 2\sigma c_S - 2\sigma w_A - 4\sigma \Delta q + 8\sigma^2 - \Theta \right),$$

where $\Theta = \sqrt{\begin{array}{l} \left(8\sigma^2 - c_S (\Delta q + 2\sigma) + w_A (\Delta q + 6\sigma) \right)^2 - 4(c_S + w_A + \Delta q + 8\sigma) \\ \times \left(2\sigma c_S^2 + 2\sigma (w_A - \Delta q)(w_A + 2\sigma) - c_S (w_A \Delta q + 2\sigma (w_A + \Delta q) - 4\sigma^2) \right) \end{array}}$.

Lemma 3. *If retailers are local monopolists, the price cost margin of the national brand product is not always greater than the price-cost margin of the store brand product.*

Proof. See the Appendix.

When competition gets more intense, retailers has no room to make higher margins from selling store brand products. However, the margin from the store brand product becomes larger than that from the national brand product when retailers are locally monopolized. This happens only if the quality gap between the store brand and the national brand products is sufficiently small. In other words, for a locally monopolized retailer, it has an incentive to make a high quality good which can be positioned very close to the national brand to make higher profit margins from the store brand product if it is beneficial to its total profits.

Wholesale Price: We plug the best responding national and store brand prices obtained above into the manufacturer's problem and differentiate it with respect to the wholesale price. The optimal wholesale price is given by

$$w_A^{LM} = -\frac{\Delta q\left(c_S\left(\Delta q + 2c_S\right) + \Lambda_1\right) + 2\sigma\left(2c_S^2 + 12c_S\Delta q + 4\Delta q^2 + \Lambda_1\right) + 4\sigma^2\left(7c_S + 18\Delta q\right) + 16\sigma^3}{\left(c_S + 6\sigma\right)\left(c_S + \Delta q + 8\sigma\right)\left[\begin{array}{c} \dfrac{2\left(-\Delta q - 2\sigma + \Lambda_2\right)}{c_S + 6\sigma} - \dfrac{2\left(\Lambda_1 - \left(\Delta q - 2\sigma\right)\left(c_S + 4\sigma\right)\right)}{\left(c_S + 6\sigma\right)^2} \\ -\dfrac{2\left(\Delta q + 6\sigma + \Lambda_2\right)}{c_S + \Delta q + 8\sigma} + \dfrac{2\left(\Lambda_1 + 8\sigma^2 - c_S\left(\Delta q + 2\sigma\right)\right)}{\left(c_S + \Delta q + 8\sigma\right)^2} \end{array}\right]}.$$

where $\Lambda_1 = \sqrt{\left(c_S + 4\sigma\right)\left(-8\sigma c_S^2 + c_S\left(\Delta q^2 + 4\sigma\Delta q - 44\sigma^2\right) + 4\sigma\left(\Delta q^2 + 8\sigma\Delta q + 4\sigma^2\right)\right)}$ and

$$\Lambda_2 = \frac{\left(c_S + 4\sigma\right)\left(2c_S^2\Delta q + c_S\left(\Delta q^2 + 20\sigma\Delta q + 4\sigma^2\right) + 4\sigma\left(\Delta q^2 + 10\sigma\Delta q - 4\sigma^2\right)\right)}{\sqrt{\Lambda_1}}.$$ Substituting this

best responding wholesale price into the best responding national and store brand prices obtained above, the optimal retail prices will be found as

$$\hat{p}_N^{LM} = \frac{1}{2\left(c_S + w_A^{LM} + \Delta q + 8\sigma\right)}\left(-c_S\Delta q + w_A^{LM}\Delta q - 2c_S\sigma - 6\sigma w_A^{LM} + 8\sigma^2 - \Theta\left(w_A^{LM}\right)\right), \text{ and}$$

$$\hat{p}_S^{LM} = \frac{1}{2\left(c_S + w_A^{LM} + 6\sigma\right)}\left(-c_S\Delta q - w_A^{LM}\Delta q + 2\sigma c_S - 2\sigma w_A^{LM} - 4\sigma\Delta q + 8\sigma^2 - \Theta\left(w_A^{LM}\right)\right).$$

Store Brand and National Brand Quality Decisions: Given the best responding retail and wholesale prices, the retailer i solves its maximization problem as follows:

$$\pi_{Ri}^{LM} = \left(\hat{p}_{iN}^{LM} - w^{LM}\right) D_{iN}\left(\hat{p}_{iN}^{LM}, p_{iS}^{LM}, \hat{w}^{LM}\right) + \left(p_{iS}^{LM} - c_{iS}\right) D_{iS}\left(\hat{p}_{iN}^{LM}, p_{iS}^{LM}, \hat{w}^{LM}\right), \ i = A, B,$$

where the superscript *LM* represents 'Local Monopoly.' However, because of its mathematical complexity, we will perform a numerical analysis to show our findings. In addition, instead of finding an individual quality level of the national and the store brand products, let us find the

change of retailer's profit according to the gap between two product qualities. That is, $\dfrac{\partial \pi_{Ri}^{LM}}{\partial \Delta q}$.

Therefore, the retailer A's first-order condition for profit maximization, with respect to Δq, is given by

$$\frac{\partial \pi_{RA}^{LM}}{\partial \Delta q} = \left(\frac{\partial \hat{p}_N^{LM}}{\partial \Delta q} - \frac{\partial \hat{w}_A^{LM}}{\partial \Delta q} \right) D_N^{LM} + \left(\hat{p}_N^{LM} - w_A^{LM} \right) \frac{\partial D_N^{LM}}{\partial \Delta q} + \left(\hat{p}_S^{LM} - c_S \right) \frac{\partial D_S^{LM}}{\partial \Delta q} \, .$$

Substituting all the demands, the demand derivatives, and the prices derived above along with some numerical values into this equation, our findings are depicted below:

Figure 3. The Locally Monopolized Retailer's Profit given Product Quality Gap ($\sigma \in [0.7,1]$)

(a) $c_S = 0.15,\ \Delta q = 0.005$

(b) $c_S = 0.15,\ \Delta q = 0.01$

(c) $c_S = 0.2,\ \Delta q = 0.005$

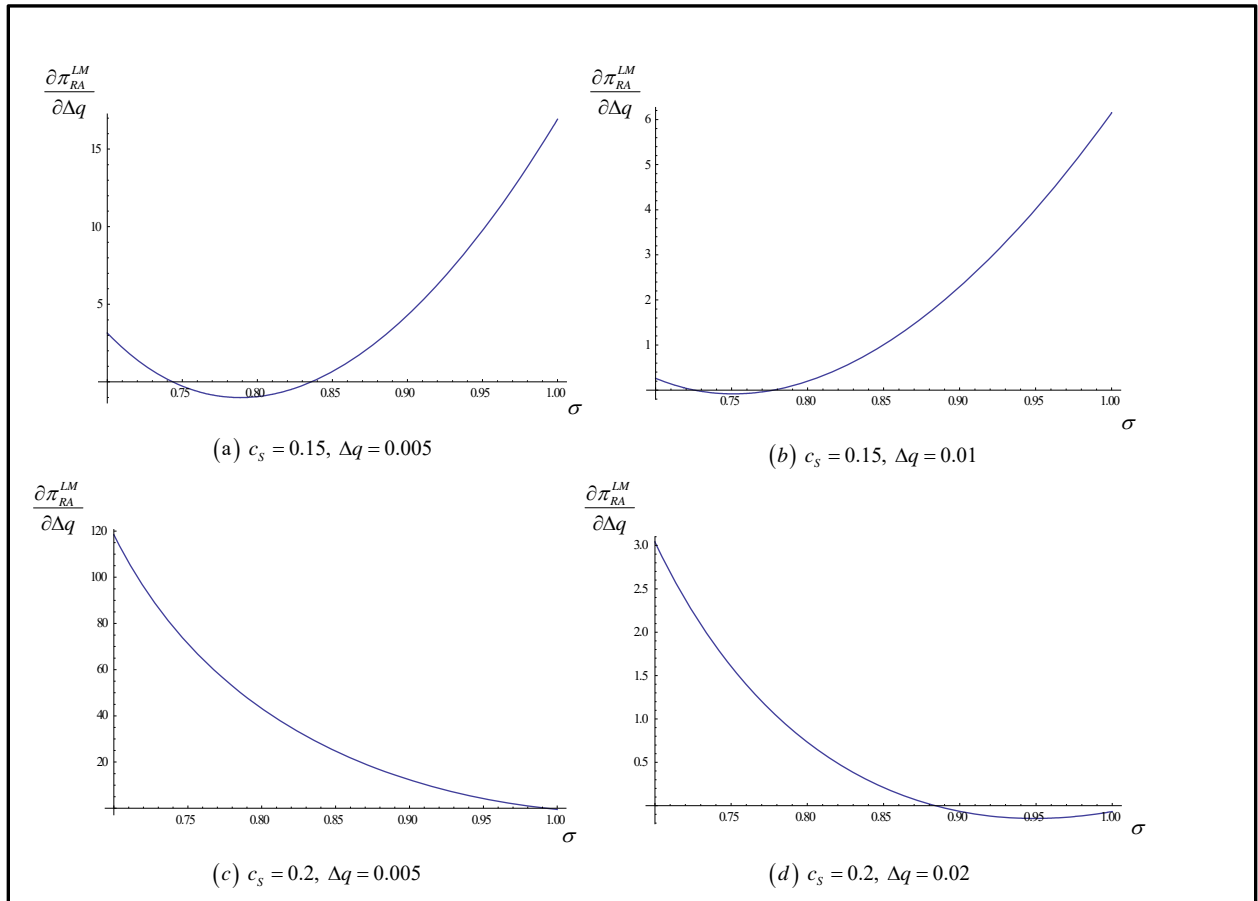(d) $c_S = 0.2,\ \Delta q = 0.02$

Figure 3 above shows us various scenarios of the effect of the competition intensity on the retailer profit changes over the quality gap between the national and store brands under the

assumption of locally monopolized retailers' market. First, as we know from Definition 2b, there should be a lower boundary of the competition intensity parameter (σ). We set it as $\underline{\sigma} = 0.7$ even though it could be lowered depending on other parameter values. Note that we parameterize the quality gap between the national and the store brand products (Δq) to observe the effect of σ. Given a value of Δq, if the competition intensity parameter σ increases, the retailer's profit may increases or decreases; that is, defining $\bar{\sigma}$ as such a threshold, $\dfrac{\partial \pi_{Ri}^{LM}}{\partial \Delta q} < 0$ until σ increases up to $\bar{\sigma}$, and then $\dfrac{\partial \pi_{Ri}^{LM}}{\partial \Delta q} > 0$ as σ increases after $\bar{\sigma}$.

Proposition 3. *When the retailer market is locally monopolized, as the quality gap between the national and the store brand products decreases, the retailer's profit (i) increases with relatively lower levels of competition intensity, but (ii) may decrease if the market becomes almost perfectly monopolized.*

First, for relatively high values of σ, which means that the retailers are less competitive, and thus become locally monopolized, each of them has no incentive to position her store brand quality far away from that of the national brand because it makes her profit lowered. *Ceteris paribus*, the retailer will strategically produce more similar product to the national brand in terms of its quality when the competition intensity becomes more loosen. In addition, when the cost of store brand production gets larger, this phenomenon becomes even more stable (Figure 3 (c) & (d)).

However, what we also observe is that the retailers seem to strategically position their product away from the national brand quality when they become a near-perfect monopolists (Figure 3 (a) & (b)). In fact, in this extreme case, the national manufacturer tries to be away from the retailers in terms of the product quality. As we discuss in Lemma 3, unlike other cases in

current study, the price-cost margin of the national brand may not necessarily be greater than the price-cost margin of the store brand product. An increase in the quality of their product requires the retailer to spend a higher production cost of the store brand product. However, a smaller gap between the national and the store brand product qualities also have the national brand manufacturer lower the wholesale price ($\frac{\partial w_i^{LM}}{\partial \Delta q} > 0$). Thus, if the retailer keeps trying to increase her product quality so that the quality gap becomes minimal, the manufacturer also increase the quality of the national brand determining a higher wholesale price. Therefore, the national brand manufacturer is likely to increase his product quality as well. Therefore, we can conclude that a minimal level of retailer competition may be able to show us a larger quality gap between two brands in its extreme while most of the case, this gap becomes close to each other as the retailers competition gets less intense in general.

6. CONCLUSION

In reality, we can observe various quality levels of similar kind of a store brand product in different retailer or wholesaler shops. These are the relative qualities when we compare those store brand products to the national brand products. This paper studies the characteristics of market situations that make the retailers develop their own strategies when they decide the quality of their own home brands. Given a conventional argument from the literature, which says that the retailer tends to increase its own home brand quality as close to the national brand product quality as possible, we observe how the retailers when making the quality decisions will be affected under a competition at the retailer level. Our result shows that the highly competing retailers position their store brands not too close to the quality of the national brand product, especially when the competition between the store brands becomes sufficiently intensified. On

the contrary, they can position their store brand very close to the national brand product in terms of the quality level of their product when they become locally monopolized. We also found some conditions that make these generalized results exceptional at an extreme case.

Our analysis can be extended to the following directions of future inquiry. First, it would be interesting to observe a duopoly market where consumers either sequentially decide what retailer they visit and what brand variant they purchase, or they sequentially decide the brand first and then the store to visit. Use of the nested logit model will help with such an analysis. Second, one of the restrictions of the current paper is to limit symmetric retailers. It would be very challenging to relax this limitation to asymmetric retailer competition model due to its methodological difficulties. However, an analysis of asymmetric retailers would even further generalize this studies. Finally, we can extend our model to look at upstream level competition. It would also be interesting to study how retailers' strategy changes depending on the national brand manufacturers' competition intensity.

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APPENDIX. Derivations and Proofs

Derivation of Demand Functions from the Choice Probabilities

For example, the demand for q_{AS}, if $0 \le \theta_A^* < \theta_B^*$, I compare AS and BS with the choice probability

$$\Pr\big(U_{AS}(\theta) \ge U_{BS}(\theta)\big) = \frac{\exp\big((\theta q_S - p_{AS})/\sigma\big)}{\exp\big((\theta q_S - p_{AS})/\sigma\big) + \exp\big((\theta q_S - p_{BS})/\sigma\big)}.$$

The aggregate demand for the store brand product of retailer A is given by

$$\int_0^{\theta_A^*} \frac{\exp(\theta q_S - p_{AS}/\sigma)}{\exp(\theta q_S - p_{AS}/\sigma) + \exp(\theta q_S - p_{BS}/\sigma)} d\theta = \int_0^{\theta_A^*} \frac{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma)}{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma) + \exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma)} d\theta$$

$$= \int_0^{\theta_A^*} \frac{\exp(-p_{AS}/\sigma)}{\exp(-p_{AS}/\sigma) + \exp(-p_{AS}/\sigma)} d\theta$$

If $\theta_B^* < \theta_A^* \le 1$, I compare AS and BS for $\theta \in \big[0, \theta_B^*\big]$ and AS and BN for $\theta \in \big[\theta_B^*, \theta_A^*\big]$. Therefore, the aggregate demand for the store brand product of retailer A is given by

$$\int_0^{\theta_B^*} \frac{\exp(\theta q_S - p_{AS}/\sigma)}{\exp(\theta q_S - p_{AS}/\sigma) + \exp(\theta q_S - p_{BS}/\sigma)} d\theta + \int_{\theta_B^*}^{\theta_A^*} \frac{\exp(\theta q_S - p_{AS}/\sigma)}{\exp(\theta q_S - p_{AS}/\sigma) + \exp(\theta q_N - p_{BN}/\sigma)} d\theta$$

$$= \int_0^{\theta_B^*} \frac{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma)}{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma) + \exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma)} d\theta$$

$$+ \int_{\theta_B^*}^{\theta_A^*} \frac{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma)}{\exp(\theta q_S/\sigma)\exp(-p_{AS}/\sigma) + \exp(\theta q_N/\sigma)\exp(-p_{BN}/\sigma)} d\theta$$

$$= \int_0^{\theta_B^*} \frac{\exp(-p_{AS}/\sigma)}{\exp(-p_{AS}/\sigma) + \exp(-p_{AS}/\sigma)} d\theta + \int_{\theta_B^*}^{\theta_A^*} \frac{\exp(-p_{AS}/\sigma)}{\exp(-p_{AS}/\sigma) + \dfrac{\exp(\theta q_N/\sigma)}{\exp(\theta q_S/\sigma)}\exp(-p_{BN}/\sigma)} d\theta$$

$$= \int_0^{\theta_B^*} \frac{\exp(-p_{AS}/\sigma)}{\exp(-p_{AS}/\sigma) + \exp(-p_{AS}/\sigma)} d\theta + \int_{\theta_B^*}^{\theta_A^*} \frac{\exp(-p_{AS}/\sigma)}{\exp(-p_{AS}/\sigma) + \exp(\theta(q_N - q_S)/\sigma)\exp(-p_{BN}/\sigma)} d\theta.$$

Derivations of the Differentiation of the Demand w.r.t. prices in Duopoly Case

A closed form solution for the integrals in the demand functions can be found as follows:

(1) $\dfrac{\partial D_{AS}}{\partial p_{AS}}$:

(i) For $0 \le \theta_1^* < \theta_2^*$, $D_{AS} = \displaystyle\int_0^{\theta_2^*} \dfrac{e^{-\frac{p_{AS}}{\sigma}}}{e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}} d\theta = \dfrac{\theta_2^* e^{-\frac{p_{AS}}{\sigma}}}{e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}}$. Then,

$$\dfrac{\partial D_{AS}}{\partial p_{AS}} = \dfrac{\partial}{\partial p_{AS}}\left(\dfrac{\theta_2^* e^{-\frac{p_{AS}}{\sigma}}}{e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}} \right) = \dfrac{\theta_2^*\left(e^{-\frac{p_{AS}}{\sigma}}\right)'\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right) - \left(e^{-\frac{p_{AS}}{\sigma}}\right)\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right)'}{\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right)^2}$$

$$= \dfrac{\theta_2^*\left(\left(-\frac{1}{\sigma}\right)e^{-\frac{p_{AS}}{\sigma}}\right)\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right) - \left(e^{-\frac{p_{AS}}{\sigma}}\right)\left(\left(-\frac{1}{\sigma}\right)e^{-\frac{p_{AS}}{\sigma}}\right)}{\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right)^2} = \dfrac{\theta_2^*\left(\left(-\frac{1}{\sigma}\right)e^{-\frac{p_{AS}}{\sigma}-\frac{p_{BS}}{\sigma}}\right)}{\left(e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}}\right)^2}$$

(By imposing symmetry)

$$= \dfrac{\theta_2^*\left(\left(-\frac{1}{\sigma}\right)e^{-\frac{p_S}{\sigma}-\frac{p_S}{\sigma}}\right)}{\left(e^{-\frac{p_S}{\sigma}} + e^{-\frac{p_S}{\sigma}}\right)^2} = \dfrac{\theta_2^*\left(\left(-\frac{1}{\sigma}\right)e^{-2\frac{p_S}{\sigma}}\right)}{\left(2e^{-\frac{p_S}{\sigma}}\right)^2} = \dfrac{\theta_2^*\left(\left(-\frac{1}{\sigma}\right)e^{-2\frac{p_S}{\sigma}}\right)}{4e^{-2\frac{p_S}{\sigma}}} = -\dfrac{1}{4\sigma}\theta_2^* .$$

(ii) For $\theta_2^* \le \theta_1^* \le 1$,

$$D_{AS} = \int_{\theta_2^*}^{\theta_1^*} \dfrac{e^{-\frac{p_{AS}}{\sigma}}}{e^{-\frac{p_{AS}}{\sigma}} + e^{\left[\theta(q_N - q_S) - \frac{p_{BN}}{\sigma}\right]}} d\theta$$

$$= \theta_1^* - \dfrac{1}{(q_N - q_S)}\ln\left[e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}}e^{\theta_1^*(q_N - q_S)}\right] - \theta_2^* + \dfrac{1}{(q_N - q_S)}\ln\left[e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}}e^{\theta_2^*(q_N - q_S)}\right]$$

$$= \left(\theta_1^* - \theta_2^*\right) - \dfrac{1}{(q_N - q_S)}\left\{\ln\left[e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}}e^{\theta_1^*(q_N - q_S)}\right] - \ln\left[e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}}e^{\theta_2^*(q_N - q_S)}\right]\right\}$$

$$\frac{\partial D_{AS}}{\partial p_{AS}} = -\frac{1}{(q_N - q_S)} - \frac{1}{(q_N - q_S)} \left\{ \frac{-\frac{1}{\sigma} e^{-\frac{p_{AS}}{\sigma}} - e^{-\frac{p_{BN}}{\sigma}} e^{(p_{AN} - p_{AS})}}{e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}} e^{(p_{AN} - p_{AS})}} - \frac{-\frac{1}{\sigma} e^{-\frac{p_{AS}}{\sigma}}}{e^{-\frac{p_{AS}}{\sigma}} + e^{-\frac{p_{BN}}{\sigma}} e^{(p_{AN} - p_{AS})}} \right\}$$

(By imposing symmetry)

$$= -\frac{1}{(q_N - q_S)} + \frac{1}{(q_N - q_S)} \frac{e^{-\frac{p_N}{\sigma}} e^{(p_N - p_S)}}{e^{-\frac{p_S}{\sigma}} + e^{-\frac{p_N}{\sigma}} e^{(p_N - p_S)}} = -\frac{1}{(q_N - q_S)} + \frac{1}{(q_N - q_S)} \frac{1}{e^{\frac{1-\sigma}{\sigma}(p_N - p_S)} + 1}.$$

(2) $\dfrac{\partial D_{AN}}{\partial p_{AS}}$:

(i) For $0 \le \theta_1^* < \theta_2^*$,

$$D_{AN} = \int_{\theta_1^*}^{\theta_2^*} \frac{e^{-\frac{p_{AN}}{\sigma}}}{e^{-\frac{p_{AN}}{\sigma}} + e^{\left[-\theta(q_N - q_S) - \frac{p_{BS}}{\sigma} \right]}} d\theta$$

$$= \theta_2^* - \frac{1}{(q_N - q_S)} \ln \left[e^{-\frac{p_{AN}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}} e^{-\theta_2^*(q_N - q_S)} \right] - \theta_1^* + \frac{1}{(q_N - q_S)} \ln \left[e^{-\frac{p_{AN}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}} e^{-\theta_1^*(q_N - q_S)} \right]$$

$$= (\theta_2^* - \theta_1^*) - \frac{1}{(q_N - q_S)} \left\{ \ln \left[e^{-\frac{p_{AN}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}} e^{-\theta_2^*(q_N - q_S)} \right] - \ln \left[e^{-\frac{p_{AN}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}} e^{-\theta_1^*(q_N - q_S)} \right] \right\}$$

$$\frac{\partial D_{AN}}{\partial p_{AS}} = \frac{1}{(q_N - q_S)} - \frac{1}{(q_N - q_S)} \left\{ -\frac{e^{-\frac{p_{BS}}{\sigma}} e^{-(p_{AN} - p_{AS})}}{e^{-\frac{p_{AN}}{\sigma}} + e^{-\frac{p_{BS}}{\sigma}} e^{-(p_{AN} - p_{AS})}} \right\}$$

(By imposing symmetry)

$$= \frac{1}{(q_N - q_S)} + \frac{1}{(q_N - q_S)} \frac{e^{-\frac{p_S}{\sigma}} e^{-(p_N - p_S)}}{e^{-\frac{p_N}{\sigma}} + e^{-\frac{p_S}{\sigma}} e^{-(p_N - p_S)}} = \frac{1}{(q_N - q_S)} + \frac{1}{(q_N - q_S)} \frac{1}{e^{-\left(\frac{1-\sigma}{\sigma}\right)(p_N - p_S)} + 1}.$$

(ii) For $\theta_2^* \le \theta_1^* \le 1$, it becomes zero.

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For the other derivatives, we derived them in a similar fashion as derivations in (1) and (2). For

example, (3) $\frac{\partial D_{AS}}{\partial p_{AN}} = \frac{1}{2(q_N - q_S)}$ for $0 \leq \theta_1^* < \theta_2^*$ and (4) $\frac{\partial D_{AN}}{\partial p_{AN}} = -\frac{1}{\Delta q \left(e^{-\left(\frac{1-\sigma}{\sigma}\right)(p_N - p_S)} + 1 \right)} - \frac{1 - \theta^*}{4\sigma}$

for $0 \leq \theta_1^* < \theta_2^*$.

Proof of Proposition 2.

First, note that lower σ represents higher competition intensity.

(i) $\frac{\partial q_S^{HC*}}{\partial \sigma} > 0$; As competition gets more intense, retailers lower the quality of their store brand

products (concave-decreasing).

$\frac{\partial q_S^{HC*}}{\partial \sigma} = \frac{1}{2}\left(4 - \frac{4(c_S + 4\sigma)}{\sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}} \right) = 2\left(1 - \frac{(c_S + 4\sigma)}{\sqrt{8c_S^2 + (c_S + 4\sigma)^2}} \right)$. Let's first assume that the last

expression is positive. Then, $\sqrt{8c_S^2 + (c_S + 4\sigma)^2} > (c_S + 4\sigma) \Leftrightarrow 8c_S^2 > 0$, which is always

satisfied. Therefore, $\frac{\partial q_S^{HC*}}{\partial \sigma} > 0$.

(ii) $\frac{\partial q_N^{HC}}{\partial \sigma} < 0$; As competition gets more intense, the manufacturer increases the quality level

of their national brand products (convex-increasing).

$\frac{\partial q_N^{HC*}}{\partial \sigma} = \frac{1}{2}\left(-4 + \frac{4(c_S + 4\sigma)}{\sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}} \right)$. Straightforwardly from the above proof, we find that

$\frac{\partial q_N^{HC}}{\partial \sigma} < 0$.

(iii) $\dfrac{\partial \Delta q^{HC*}}{\partial \sigma} < 0$; As competition gets more intense, the quality gap between the national brand

and the store brand increases (convex-increasing).

$$\frac{\partial \Delta q^{HC*}}{\partial \sigma} = \frac{1}{2}\left(-4 + \frac{4\left(c_S + 4\sigma\right)}{\sqrt{9c_S^2 + 8c_S\sigma + 16\sigma^2}}\right) < 0. \text{ It is straightforward from the proof in (ii).} \quad Q.E.D.$$

Proof of Lemma 2.

The gap of profit margins between the national brand and the store brand is given by

$$\left(p_N^{LM} - w_N\right) - \left(p_S^{LM} - c_S\right) = \frac{1}{2\left(c_S + w_A + \Delta q + 8\sigma\right)}\left(w_A \Delta q + 6\sigma w_A + 8\sigma^2 - c_S\left(\Delta q + 2\sigma\right) + \Theta\right) - w_A$$

$$- \frac{1}{2\left(c_S + w_A + 6\sigma\right)}\left(-c_S \Delta q - w_A \Delta q + 2\sigma c_S - 2\sigma w_A - 4\sigma \Delta q + 8\sigma^2 + \Theta\right) + c_S$$

$$= c_S - w_A + \frac{-\Theta + c_S\left(\Delta q - 2\sigma\right) + 4\sigma\left(\Delta q - 2\sigma\right) + w_A\left(\Delta q + 2\sigma\right)}{2\left(c_S + w_A + 6\sigma\right)} + \frac{\Theta + 8\sigma^2 - c_S\left(\Delta q + 2\sigma\right) + w_A\left(\Delta q + 6\sigma\right)}{2\left(c_S + w_A + \Delta q + 8\sigma\right)},$$

where $\Theta = \sqrt{\begin{pmatrix}\left(8\sigma^2 - c_S\left(\Delta q + 2\sigma\right) + w_A\left(\Delta q + 6\sigma\right)\right)^2 \\ -4\left(c_S + w_A + \Delta q + 8\sigma\right)\begin{pmatrix}2\sigma c_S^2 + 2\sigma\left(w_A - \Delta q\right)\left(w_A + 2\sigma\right) \\ +c_S\left(-w_A \Delta q - 2\sigma\left(w_A + \Delta q\right) + 4\sigma^2\right)\end{pmatrix}\end{pmatrix}}.$

Let us define that

$$\Delta \bar{q} = \frac{1}{2\left(4 + c_S + w_A\right)}$$

$$\times \left(\begin{matrix}-24 - 16c_S - 2c_S^2 - 8w_A - 2c_S w_A + \Theta \\ + \sqrt{\begin{matrix}\left(24 + 2c_S^2 + 8w_A + 2c_S\left(8 + w_A\right) - \Theta\right)^2 \\ -8\left(4 + c_S + w_A\right)\left(-8 + c_S^3 - 22w_A - 10w_A^2 - w_A^3 + c_S^2\left(12 + w^A\right) + c_S\left(34 + 2w_A - w_A^2\right) - \Theta\right)\end{matrix}} + \Theta\end{matrix}\right).$$

Then, assuming $\Theta > 0$, the margin gap becomes positive only when the quality gap is

sufficiently large; i.e., $\Delta q > \Delta \bar{q}$. Otherwise, $\left(p_N^{LM} - w_N\right) < \left(p_S^{LM} - c_S\right)$. $\qquad Q.E.D.$