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BREAKING OF SCALE INVARIANCE IN THE PRESENCE OF INSTANTONS[†)]

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ABSTRACT

The instanton tunnelling amplitude induces a non-perturbative violation of scale invariance which can be understood in terms of the anomaly in the trace of the stress energy tensor. The scaling violation determined by the trace anomaly is compared with the explicitly constructed instanton amplitude in the one-loop approximation.

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In quantum chromodynamics (QCD) because of the chiral anomaly the instanton tunnelling amplitude induces a non-perturbative violation of the chiral U(1) symmetry[1]. The purpose of this note is to observe that because of the anomaly in the trace of the stress energy tensor[2] the instanton also induces a non-perturbative breaking of scale and conformal symmetry. The non-conservation of the dilation charge is just proportional to the gauge field action. In Euclidean space, where the tunneling amplitude is computed, the gauge field action is greater than or equal to the topologically determined quantity

$$\int d^4x\, G_{a\mu\nu} G_a^{\mu\nu} \geq \left| \int d^4x\, G_{a\mu\nu} \tilde{G}_a^{\mu\nu} \right| = \frac{32\pi^2 |q|}{g^2} \tag{1}$$

where g is the coupling constant, $\tilde{G}_a^{\mu\nu} = \frac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}\, G_{a\alpha\beta}$, and q is the winding number. Therefore all solutions with $q \neq 0$ induce non-perturbative breaking of scale (and conformal) symmetry.

Unlike the chiral anomaly, the trace anomaly is not free of higher order perturbative corrections, since the theory cannot be regulated in a scale invariant way. The lowest order trace anomaly occurs at the one-loop level -- the triangle, square, and pentagon diagrams familiar from the chiral anomaly. For the instanton with $q = 1$ we can compare the scale breaking computed from the lowest order trace anomaly with 't Hooft's explicit construction[3] of the one-loop quantum corrections to the instanton tunnelling amplitude. The results agree. In particular, the anomaly requires that the contribution of the non-zero eigenmodes be independent of the coupling constant subtraction point μ, as is verified by the explicit one-loop calculation of Ref. 3).

The simple details for the one-loop approximation are presented below. The note then concludes with two comments: on the breaking of conformal symmetry and on the possible experimental significance of the non-perturbative breaking of scale invariance.

The trace anomaly may be regarded as the scale invariance analogue of the more familiar anomaly of chiral symmetry. The trace of the tress tensor is the divergence of the dilatation current[4], $\theta \equiv \theta_\mu{}^\mu = \partial_\mu D^\mu$, whose charge $D = \int d^3x\, D^0$ generates scale transformations, $[D(t), \phi(x = \vec{x},t)] = -i\,(d_\phi + x\cdot\partial)\phi(x)$ (which defines the scale dimension d_ϕ of the field ϕ). In the $\langle \theta\, J^\mu J^\nu \rangle$ amplitude in spinor electrodynamics the lowest order triangle diagram requires that the naive trace $\theta = m\bar{\psi}\psi$ be modified by addition of an "anomalous" term[2]

$$\Theta = m\bar{\Psi}\Psi + \frac{\alpha}{8\pi} F_{\mu\nu} F^{\mu\nu} \tag{2}$$

just as the triangle diagram for $\langle \partial_\tau J_5^\tau J^\mu J^\nu \rangle$ induces the chiral anomaly

$$\partial_\mu J_5^\mu = 2im\bar{\Psi}\gamma_5\Psi + \frac{\alpha}{4\pi}\varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} \tag{3}$$

Equation (3) is valid to any finite order in perturbation theory, but Eq. (2) is modified by radiative corrections, which have recently been computed[5] to all orders in α in terms of functions defined by the renormalization group.

Here we are interested in the trace anomaly of QCD, which has also recently been expressed[6] to all orders in the renormalized coupling constant g:

$$\Theta = (1+\gamma_m)\, N(\bar{\Psi} m_R \Psi) + \frac{\beta}{2g} N(G_{a\mu\nu} G_a^{\mu\nu}) \tag{4}$$

We use the notation of Collins et al.: $\gamma_m(g)$ and $\beta(g)$ are defined in the context of the renormalization group, N denotes the normal product definition of the operators, and $G_a^{\mu\nu}$ is the normal gauge covariant field tensor. We will use Eq. (4) in Green's functions with only gauge field or current external legs, in which case the one-loop approximation to Eq. (4) is[*]

$$\Theta = \bar{\Psi} m \Psi - \frac{b}{2} g^2 G_{a\mu\nu} G_a^{\mu\nu} \tag{5}$$

where

$$\beta = -bg^3 + \ldots \tag{6}$$

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*) In such Green's functions the factor γ_m in Eq. (4) only appears in two-loop order. I thank John Ellis for dispelling my confusion on this point.

To compare with Ref. 3) we choose SU(2) to be the gauge group, so

$$b = \frac{1}{16\pi^2}\left(\frac{22}{3} - \frac{2}{3}N_F\right) \tag{7}$$

where N_F is the number of quark flavours.

The breaking of the dilatation charge is characterized by

$$\Delta D \equiv \int d^4x \; \partial_\mu D^\mu = \int d^4x \; \Theta \tag{8}$$

We want to compute in the one-loop approximation the contribution to ΔD due to the anomalous term in Eq. (5) in the presence of the $q = 1$ instanton background field. The leading order anomaly in Eq. (5) is already a one-loop effect, so it suffices to evaluate $\langle G^{\mu\nu}_a \, G_{a\mu\nu}\rangle_{instanton}$ in tree approximation. The amplitude in this approximation is just the classical instanton field[7] and we compute

$$\begin{aligned} \langle \Delta D_{anomaly}\rangle_{instanton} &= -\frac{b}{2}g^2\int d^4x \; G^{cl}_{a\mu\nu} \, G^{\mu\nu\,cl}_a \\ &= -16\pi^2 b i \end{aligned} \tag{9}$$

Here G^{cl} denotes the classical solution with $q = 1$, for which the inequality of Eq. (1) is saturated. The factor "i" in Eq. (9) is due to the continuation of Eq. (1) back to Minkowski space.

Now we compare this result with the explicit one-loop calculation of Ref. 3). To this order the contribution to the $q = 1$ tunnelling amplitude from the instantons within Δz of the space-time point z and with size parameter within $\Delta\rho$ of size ρ is[3]

$$\begin{aligned} \langle \tilde{0} | 0 \rangle = {} & 2^{14}\pi^6 g^{-8} m^{N_F} \rho^{N_F-5} (\Delta z)^4 \Delta\rho \\ & \cdot \exp\left\{-\frac{8\pi^2}{g^2(\mu)} + 16\pi^2 b \ln \mu\rho + \text{const.}\right\} \end{aligned} \tag{10}$$

We want to compute the change in D in such a transition due to the anomalous contribution to θ. Under a dilatation ρ, $\Delta\rho$, and Δz are rescaled, while μ and m are held fixed, so the generator is

$$D = -i\left(\rho\frac{\partial}{\partial\rho} + \Delta\rho\frac{\partial}{\partial(\Delta\rho)} + \Delta z\frac{\partial}{\partial(\Delta z)}\right) \tag{11}$$

with the phase fixed by the quantum equal time commutation relation $[D,\phi] = -i\,(d_\phi + x\cdot\partial)\phi$.

However, we want to compute only the contribution to ΔD from the anomaly, $\theta - \bar{\psi}m\psi$ in Eq. (5), so we subtract the scaling violation generated by the factor m^{N_F} in Eq. (10). For our purpose the relevant differential operator is then

$$D_{anomaly} = -i\left(\rho\frac{\partial}{\partial\rho} + \Delta\rho\frac{\partial}{\partial(\Delta\rho)} + \Delta z\frac{\partial}{\partial(\Delta z)} - m\frac{\partial}{\partial m}\right) \tag{12}$$

In Eq. (9) we computed the violation of $D_{anomaly}$ in the presence of the instanton background field. That is, we assumed the instanton to be present with probability one. We now compute the analogous quantity by applying $D_{anomaly}$, Eq. (12), to the transition amplitude and normalizing to the value of the amplitude. So we have

$$\langle \Delta D_{anomaly}\rangle_{instanton} = D_{anomaly}\left[\ln\langle\tilde{0}|0\rangle\right] \tag{13}$$

$D_{anomaly}$ vanishes when applied to the factor $g^{-8}\, m^{N_F}\, \rho^{N_F-5}\, \Delta\rho(\Delta z)^4$, which is the contribution of the zero eigenmodes. The entire contribution to Eq. (13) is from the exponent in Eq. (10), which is due to the non-zero eigenmodes. We have

$$\begin{aligned}\langle \Delta D_{anomaly}\rangle_{instanton} &= -i\rho\frac{\partial}{\partial\rho}\left\{-\frac{8\pi^2}{g^2(\mu)} + 16\pi^2 b\ln\mu\rho\right\} \\ &= -i\,16\pi^2 b i\end{aligned} \tag{14}$$

in agreement with Eq. (9). Turning the argument around we can say that the anomaly requires the one quantum loop correction of the non-zero eigenmodes to have just that μ dependence which cancels the leading (one loop) μ dependence of the classical action, $\exp\left[-8\pi^2/g^2(\mu)\right]$.

If we attempt to compute $\langle \Delta D_{anomaly} \rangle_{instanton}$ to all orders in quantum fluctuations, we encounter a second way in which the trace anomaly differs from the chiral anomaly. To compute the breaking of the chiral U(1) charge, $\Delta Q_5 = \int d^4x\, \partial_\mu J_5^\mu$, in the presence of the instanton we must compute $\int d^4x \langle \tilde{G}_a^{\mu\nu} G_{a\mu\nu} \rangle_{instanton}$. Because this volume integral can be written as an integral over the surface at infinity where the integrand is determined by the boundary conditions[7], it is given to all orders in quantum loops by the classical value of Eq. (1)*). This fact, together with the Adler-Bardeen theorem on the absence of higher order corrections for the chiral anomaly itself, means that $\langle \Delta Q_{5,anomaly} \rangle_{instanton}$ is equal to $2N_F q$ to all orders in the loop expansion.

For the trace anomaly, not only are there higher order corrections given by the factor $\beta/2g$, but in addition the integral $\int d^4x \langle G_{a\mu\nu} G_a^{\mu\nu} \rangle_{instanton}$ is just the gauge field action which is also corrected by quantum fluctuations [which must be positive in Euclidean space because of Eq. (1)].

A second difference: in QCD the chiral U(1) symmetry is broken (as far as we know) only by the instanton, while scale symmetry is broken by the well-known perturbative effects and in addition by the non-perturbative effects due to the instanton.

I conclude with two comments:

i) The trace anomaly is also responsible for an anomaly in the divergence of the conformal current[8],

$$\partial_\mu K^{\alpha\mu}(x) = 2x^\alpha \theta(x) \tag{15}$$

where for QCD we substitute Eq. (4) for θ. The instanton therefore non-perturbatively breaks the conformal charge, by

$$\Delta K^\alpha = \int d^4x \;\partial_\mu K^{\alpha\mu}$$

*) I am grateful to David Olive for an explanation of this point.

ii) The instanton suggests an interesting possible solution of the chiral U(1) problem in QCD[1]. But lacking even an order of magnitude estimate of its effect on the pseudoscalar mass matrix, it would be premature to claim that the problem is already solved. In general, there is no contact with experiment: what is calculable is exponentially small. If the instanton really does solve the chiral U(1) problem, then it must have a large though presently incalculable effect on the pseudoscalar masses. Then, because of the trace anomaly, it would also have a large effect on the scalar meson masses. The instanton should also induce non-perturbative scaling violations in deep-inelastic phenomena. Whether these are big enough to observe will depend on whether in the amplitude under consideration the integration over the instanton size ρ has important contributions from sizes much larger or smaller than the short distance scale being probed.

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