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Authors
Antusch, Stefan
Drees, Manuel
Kersten, Jörn
et al.

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Neutrino mass operator renormalization in two Higgs doublet models and the MSSM

Stefan Antusch, Manuel Drees, Jörn Kersten, Manfred Lindner, Michael Ratz

Physik-Department T30, Technische Universität München, James-Franck-Straße, D-85748 Garching, Germany

Abstract

In a recent re-analysis of the Standard Model (SM) (β)-function for the effective neutrino mass operator, we found that the previous results were not entirely correct. Therefore, we consider the analogous dimension five operators in a class of two Higgs doublet models (2HDM's) and the minimal supersymmetric Standard Model (MSSM). Deriving the renormalization group equations for these effective operators, we confirm the existing result in the case of the MSSM. Some of our 2HDM results are new, while others differ from earlier calculations. This leads to modifications in the renormalization group evolution of leptonic mixing angles and CP phases in the 2HDM's.

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1. Introduction

The discovery of neutrino masses requires an extension of the Standard Model (SM). The most promising scenario for giving masses to neutrinos is the see-saw mechanism [1], which provides a convincing explanation for their smallness. It typically introduces heavy, gauge-singlet neutrinos and thereby gives small Majorana masses to the SM neutrinos. When the SM is viewed as an effective field theory, Majorana masses for the neutrinos can be introduced via higher-dimensional operators of SM fields. The lowest dimensional operator of this kind has dimension 5 and couples two lepton and two Higgs doublets. It ap-

pears, e.g., in the see-saw mechanism by integrating out the heavy singlets.

The experimental results in the neutrino sector provide an interesting new way for testing theories beyond the SM. In order to compare the experimental results with predictions from models beyond the SM, like unified theories, it is essential to evolve the masses, mixing angles and CP phases from high to low energies. This is accomplished with the renormalization group equations (RGE's) for the neutrino mass operators in the theory valid at intermediate energy scales. This theory may be the SM, but can also be an extension like a two Higgs doublet model (2HDM) or the minimal supersymmetric Standard Model (MSSM).

In a recent Letter [2] we discussed the derivation of the RGE for the dimension 5 neutrino mass operator in the SM. In this Letter we derive the RGE's for the

E-mail addresses: santusch@ph.tum.de (S. Antusch),
drees@ph.tum.de (M. Drees), jkersten@ph.tum.de (J. Kersten),
lindner@ph.tum.de (M. Lindner), mratz@ph.tum.de (M. Ratz).

corresponding operators in a class of 2HDM's and the MSSM.

2. Effective neutrino mass operators in 2HDM's

In many extensions of the SM, the Higgs sector is enlarged by introducing additional $SU(2)_L$ doublet scalar fields $\phi^{(i)}$ ($1 \leqslant i \leqslant N_H$). These can couple to the SM fermions via the Yukawa couplings

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}}^{(i)} = &-\left(Y_e^{(i)}\right)_{gf} \overline{e_R^g} \phi_a^{(i)\dagger} \delta^{ab} \ell_{Lb}^f \\
&-\left(Y_d^{(i)}\right)_{gf} \overline{d_R^g} \phi_a^{(i)\dagger} \delta^{ab} Q_{Lb}^f \\
&-\left(Y_u^{(i)}\right)_{gf} \overline{u_R^g} Q_{Lb}^f \varepsilon^{ba} \phi_a^{(i)}.
\end{aligned} \tag{1}
$$

ℓ_L^f and Q_L^f, $f \in \{1,2,3\}$ are the $SU(2)_L$ doublets of SM leptons and quarks, respectively. e_R^f, d_R^f and u_R^f denote the $SU(2)_L$-singlet (right-handed) charged leptons, down-type quarks and up-type quarks. ε is the totally antisymmetric tensor in 2 dimensions and $a, b, c, d \in \{1, 2\}$ are SU(2) indices. Summation over repeated indices is implied throughout this Letter. We have chosen the notation in Eq. (1) in such a way that all $\phi^{(i)}$ transform as $(\mathbf{2}, \frac{1}{2})$ under $SU(2)_L \otimes U(1)_Y$. In particular, for $N_H = 1$ we obtain the SM.

Note that there are tight phenomenological constraints on Yukawa couplings. As pointed out in [3–5], it is very hard to construct viable models in which one type of SM fermions e, d and u couples to two or more Higgs bosons, since this in general leads to tree-level flavor-changing neutral currents (FCNC's). Therefore, we will only consider models in which the fermions couple to at most one Higgs. As a consequence, the suffix "(i)" on the Yukawa couplings in Eq. (1) is redundant and will be omitted in the following.

2.1. Classification of 2HDM's

We concentrate on models with two Higgs doublets for simplicity, i.e., $N_H = 2$, and consider only schemes in which each of the right-handed SM fermions couples to exactly one Higgs boson. All inequivalent possibilities are classified in Table 1. By convention, the scalar which couples to e is defined to be $\phi^{(1)}$. In order to avoid FCNC's, we impose the $\mathbb{Z}_2$ symmetry

$$
\phi^{(1)} \to \phi^{(1)}, \qquad \phi^{(2)} \to -\phi^{(2)}, \tag{2}
$$

Table 1

Classification of the 2HDM's with natural suppression of FCNC's and tree-level mass terms for all SM fermions except neutrinos. Note that model (i) is usually referred to as "type I" and (ii) as "type II" in the literature

Coupling scheme				
Model	(i)	(ii)	(iii)	(iv)

and corresponding transformations in the fermion sector. For example, in scheme (ii) all fields transform trivially except for

$$
\begin{pmatrix} \phi^{(2)} \\ u \end{pmatrix} \to -\begin{pmatrix} \phi^{(2)} \\ u \end{pmatrix}. \tag{3}
$$

The most general Higgs self-interaction Lagrangian is then

$$
\begin{aligned}
\mathcal{L}_{\text{2Higgs}} = &-\frac{\lambda_1}{4}\left(\phi^{(1)\dagger}\phi^{(1)}\right)^2 - \frac{\lambda_2}{4}\left(\phi^{(2)\dagger}\phi^{(2)}\right)^2 \\
&- \lambda_3\left(\phi^{(1)\dagger}\phi^{(1)}\right)\left(\phi^{(2)\dagger}\phi^{(2)}\right) \\
&- \lambda_4\left(\phi^{(1)\dagger}\phi^{(2)}\right)\left(\phi^{(2)\dagger}\phi^{(1)}\right) \\
&- \left[\frac{\lambda_5}{4}\left(\phi^{(1)\dagger}\phi^{(2)}\right)^2 + \text{h.c.}\right].
\end{aligned} \tag{4}
$$

2.2. Effective neutrino mass operators

The lowest-dimensional effective neutrino mass operators compatible with the symmetry (2) are given by

$$
\mathcal{L}_\kappa = \mathcal{L}_\kappa^{(11)} + \mathcal{L}_\kappa^{(22)}, \tag{5}
$$

where

$$
\begin{aligned}
\mathcal{L}_\kappa^{(ii)} = &\frac{1}{4}\kappa_{gf}^{(ii)} \overline{\ell_{Lc}^C}{}^g \varepsilon^{cd} \phi_d^{(i)} \ell_{Lb}^f \varepsilon^{ba} \phi_a^{(i)} \\
&+ \text{h.c.} \quad (i = 1, 2).
\end{aligned} \tag{6}
$$

ℓ_L^C is the charge conjugate of the lepton doublet, and $\kappa_{gf}^{(ii)}$ are symmetric matrices with respect to the generation indices g and f. Note that it is possible that only one of these operators, e.g., $\mathcal{L}_\kappa^{(22)}$, arises from integrating out heavy degrees of freedom in a specific model. However, as we shall see, both mix due to the

renormalization group evolution and therefore have to be taken into account simultaneously.

As long as the symmetry (2) is valid, $\mathcal{L}_\kappa^{(11)}$ and $\mathcal{L}_\kappa^{(22)}$ represent the only possible dimension 5 operators containing two ℓ_L fields. If this symmetry was broken, further couplings would appear in the Higgs interaction Lagrangian (4).

2.3. Calculation of the RGE

We work in the MS renormalization scheme at the one-loop level. The wavefunction renormalization constants $Z_i = 1 + \delta Z_i$ are defined in the usual way. For the Higgs fields $\phi^{(i)}$ we obtain

$$
\begin{aligned}
\delta Z_{\phi^{(i)}} = -\frac{1}{16\pi^2}\Big[& \delta_{i1} 2 \operatorname{Tr}(Y_e^\dagger Y_e) + z_u^{(i)} 6 \operatorname{Tr}(Y_u^\dagger Y_u) \\
& + z_d^{(i)} 6 \operatorname{Tr}(Y_d^\dagger Y_d) \\
& + \tfrac{1}{2}(\xi_B - 3)g_1^2 + \tfrac{3}{2}(\xi_W - 3)g_2^2 \Big]\frac{1}{\epsilon},
\end{aligned}
$$
(7)

where $\epsilon := 4 - d$ is the deviation from 4 dimensions in dimensional regularization, and ξ_B and ξ_W are the gauge fixing parameters used in R_ξ gauge. g_1 and g_2 are the U(1)$_Y$ and SU(2)$_L$ gauge coupling constants, respectively. The coefficients $z_f^{(i)}$ are defined to be 1 if the fermion f couples to the Higgs boson $\phi^{(i)}$ and 0 otherwise. For the models classified in Table 1 they are given by Table 2.

The wavefunction renormalization for ℓ_L is given by

$$
\delta Z_{\ell_L} = -\frac{1}{16\pi^2}\Big[Y_e^\dagger Y_e + \tfrac{1}{2}\xi_B g_1^2 + \tfrac{3}{2}\xi_W g_2^2 \Big]\frac{1}{\epsilon}.
$$
(8)

Table 2
The coefficients $z_f^{(i)}$ for the two Higgs doublet models classified in Table 1

	(i)	(ii)	(iii)	(iv)
$z_u^{(1)}$	1	0	1	0
$z_u^{(2)}$	0	1	0	1
$z_d^{(1)}$	1	1	0	0
$z_d^{(2)}$	0	0	1	1

With the counterterms for the effective vertices defined by

$$
\begin{aligned}
\mathcal{C}_\kappa = &\; \frac{1}{4}\delta\kappa_{gf}^{(11)} \overline{\ell_{Lc}^C}{}^g \varepsilon^{cd}\phi_d^{(1)}\ell_{Lb}^f \varepsilon^{ba}\phi_a^{(1)} \\
& + \frac{1}{4}\delta\kappa_{gf}^{(22)} \overline{\ell_{Lc}^C}{}^g \varepsilon^{cd}\phi_d^{(2)}\ell_{Lb}^f \varepsilon^{ba}\phi_a^{(2)} + \text{h.c.},
\end{aligned}
$$
(9)

we find for the vertex correction

$$
\begin{aligned}
\delta\kappa^{(ii)} = -\frac{1}{16\pi^2}\Big[& \delta_{i1} 2\kappa^{(ii)}\big(Y_e^\dagger Y_e\big) + \delta_{i1} 2\big(Y_e^\dagger Y_e\big)^T \kappa^{(ii)} \\
& - \lambda_i \kappa^{(ii)} - \delta_{i1}\lambda_5^*\kappa^{(22)} - \delta_{i2}\lambda_5\kappa^{(11)} \\
& + \big(\xi_B - \tfrac{3}{2}\big)g_1^2 \kappa^{(ii)} \\
& + \big(3\xi_W - \tfrac{3}{2}\big)g_2^2 \kappa^{(ii)} \Big]\frac{1}{\epsilon}.
\end{aligned}
$$
(10)

The relevant Feynman diagrams are shown in Fig. 1. Using the technique described in [2], the β-functions $\beta_{\kappa^{(ii)}} = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\kappa^{(ii)}$ with μ denoting the renormalization scale, can be obtained from the counterterms,

$$
\begin{aligned}
16\pi^2\beta_{\kappa^{(ii)}} = &\; \big(\tfrac{1}{2} - 2\delta_{i1}\big)\big[\kappa^{(ii)}\big(Y_e^\dagger Y_e\big) + \big(Y_e^\dagger Y_e\big)^T \kappa^{(ii)}\big] \\
& + \big[\delta_{i1} 2\operatorname{Tr}\big(Y_e^\dagger Y_e\big) + z_u^{(i)} 6\operatorname{Tr}\big(Y_u^\dagger Y_u\big) \\
& + z_d^{(i)} 6\operatorname{Tr}\big(Y_d^\dagger Y_d\big)\big]\kappa^{(ii)} \\
& + \lambda_i \kappa^{(ii)} + \delta_{i1}\lambda_5^*\kappa^{(22)} \\
& + \delta_{i2}\lambda_5\kappa^{(11)} - 3g_2^2\kappa^{(ii)}.
\end{aligned}
$$
(11)

The terms proportional to λ_5 are responsible for the mixing of the effective operators mentioned before. Our result for $\beta_{\kappa^{(11)}}$ differs from the one in [7] by a factor of 3 because of the term $\tfrac{1}{2} - 2\delta_{i1}$ in the first line. We had earlier found [2] an analogous discrepancy in β_κ for the SM, which has recently been confirmed [8].

Note that in 2HDM's running effects are in general larger than in the SM due to the fact that the Yukawa couplings are enhanced, e.g., $(Y_e)_{2\text{HDM}} = (Y_e)_{\text{SM}}/\cos\beta$, where $\tan\beta = v^{(1)}/v^{(2)}$ with $v^{(i)}$ being the vacuum expectation value of the Higgs field $\phi^{(i)}$.

3. The effective neutrino mass operator in the MSSM

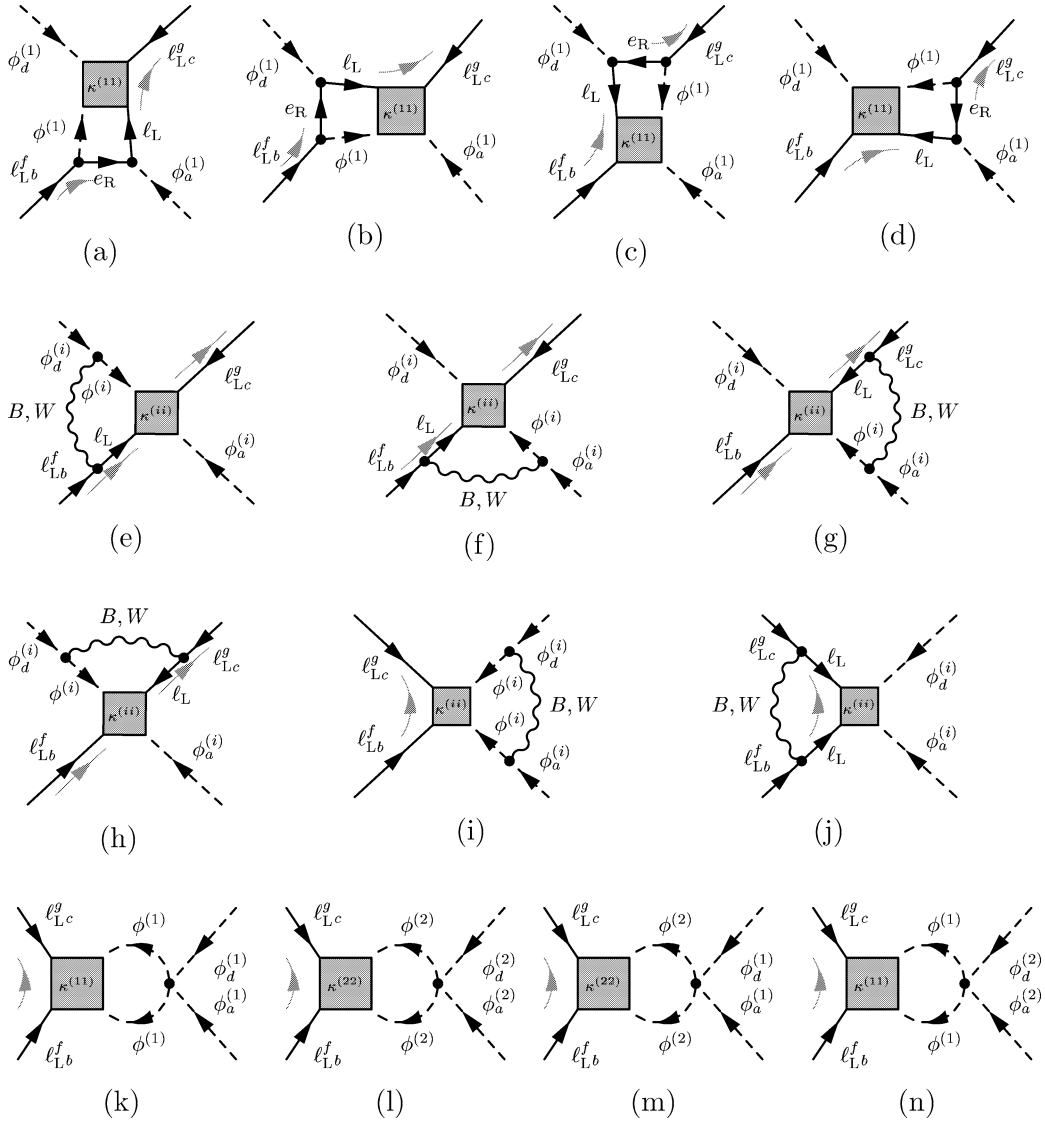In the MSSM, the effective dimension 5 operator that gives Majorana masses to the SM neutrinos is

Fig. 1. One-loop diagrams contributing to the vertex renormalization of $\kappa^{(ii)}$. The one-loop diagrams (a)–(d) that arise due to the Yukawa coupling Y_e affect only the renormalization of $\kappa^{(11)}$. The one-loop gauge diagrams (e)–(j) are the same as in the SM. B and $W \in \{W^1, W^2, W^3\}$ are the gauge bosons of SU(1)$_Y$ and SU(2)$_L$. Diagrams (k)–(n) come from the Higgs interaction Lagrangian. While the diagrams (k) and (l) have a counterpart in the SM, the diagrams (m) and (n) appear only in the 2HDM's and lead to a mixing between the operators $\mathcal{L}_\kappa^{(11)}$ and $\mathcal{L}_\kappa^{(22)}$. The gray arrow indicates the fermion flow as defined in [6].

contained in the F-term of the superpotential

$$\mathscr{w}_\kappa^{\mathrm{MSSM}} = -\frac{1}{4}\kappa_{gf}\boldsymbol{L}_c^g \varepsilon^{cd}\boldsymbol{H}_d^{(2)}\boldsymbol{L}_b^f \varepsilon^{ba}\boldsymbol{H}_a^{(2)} + \text{h.c.} \quad (12)$$

$\boldsymbol{L}$ and $\boldsymbol{H}^{(2)}$ are the chiral superfields that contain the SU(2)$_L$ doublets, the Higgs doublet with weak hypercharge $+\frac{1}{2}$ and the corresponding superpartners. The part of the superpotential describing the Yukawa inter-

actions is given by

$$
\begin{aligned}
\mathcal{W}_{\text{Yuk}}^{\text{MSSM}} = (Y_e)_{gf}\, \boldsymbol{E}^{\text{C}g}\, \boldsymbol{H}_a^{(1)} \varepsilon^{ab} \boldsymbol{L}_b^{f} \\
+ (Y_d)_{gf}\, \boldsymbol{D}^{\text{C}g}\, \boldsymbol{H}_a^{(1)} \varepsilon^{ab} \boldsymbol{Q}_b^{f} \\
+ (Y_u)_{gf}\, \boldsymbol{U}^{\text{C}g}\, \boldsymbol{H}_a^{(2)} (\varepsilon^T)^{ab} \boldsymbol{Q}_b^{f}.
\end{aligned} \tag{13}
$$

The superfields $\boldsymbol{E}^{\text{C}}$, $\boldsymbol{D}^{\text{C}}$ and $\boldsymbol{U}^{\text{C}}$ contain the SU(2)$_{\text{L}}$-singlet charged leptons, down-type quarks and up-type quarks, respectively, and $\boldsymbol{Q}$ contains the SU(2)$_{\text{L}}$ quark doublets. The Higgs superfield $\boldsymbol{H}^{(1)}$ has weak hypercharge $-\frac{1}{2}$. Calculating the RGE in the MSSM yields

$$
\begin{aligned}
16\pi^2 \beta_\kappa^{\text{MSSM}} = \left(Y_e^\dagger Y_e\right)^T \kappa + \kappa \left(Y_e^\dagger Y_e\right) \\
+ 6\,\text{Tr}\left(Y_u^\dagger Y_u\right)\kappa - 2g_1^2 \kappa - 6g_2^2 \kappa,
\end{aligned} \tag{14}
$$

confirming the existing MSSM result [7,9].

4. Conclusion

A recent check of the SM β-function [8] showed that previously published results were not quite correct. Therefore, in this Letter we have derived the RGE's for the effective dimension 5 operators which yield a Majorana mass for neutrinos after the electroweak symmetry breaking in four types of 2HDM's and in the MSSM. For the MSSM, we confirmed the earlier result [7,9]. However, when we applied our general result (11) to the 2HDM discussed in [7], we found that the non-diagonal part of one of the β-functions, relevant for the evolution of the mixing angles, is enhanced by a factor of 3.

Acknowledgements

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