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Authors

Srivastava, Nisheeth Schrater, Paul

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Classical conditioning via inference over observable situation contexts

Nisheeth Srivastava (nsrivastava@ucsd.edu)

Department of Psychology, University of California San Diego San Diego, CA 92093 USA

Paul R Schrater (schrater@umn.edu)

Department of Psychology, University of Minnesota Minneapolis, MN 55455 USA

Abstract

In this paper, we demonstrate that predicting stimulus cooccurrence patterns in a Bayes-optimal manner endogenously explains classical conditioning. Simulated experiments with a standard Bayesian implementation of this model show that it is capable of explaining a broader range of effects than any previous theory of classical conditioning. By simplifying the mathematical structure of statistical modelling of conditioning and demonstrating its ability to explain a large set of experimentally observed effects, our work advances Bayes-optimal inference about stimulus co-occurrence as a rational principle explaining classical conditioning.

Keywords: learning; Bayesian modelling; computer simulation; decision-making

Introduction

Classical conditioning is a form of associative learning wherein animals are trained to respond positively or negatively to some stimulus towards which they were previously neutrally disposed. The classic example of such conditioning arises from its accidental discovery by Ivan Pavlov, who discovered that dogs kept in his lab to be used as experiment subjects would salivate as if they were about to feed at the sound of a bell that typically heralded mealtime (Pavlov, 1927). Such transference of somatic response from a positive stimulus (food) to a prima facie neutral one (bell sound) through situational co-occurence has since been called classical (or Pavlovian) conditioning. In any classical conditioning experiment, a stimulus for which an animal is known to have a positive or negative response (called unconditioned stimulus, or US) is paired with a neutral one (called the conditioned stimulus, or CS) until the animal transfers the somatic response typically evinced when encountering the US to encounters with CS. When such a conditioned response (CR) is elicited, the animal is considered to have been conditioned.

Despite the importance and long lineage of research in classical conditioning, formal theories still find it difficult to explain a wide variety of conditioning-related behaviors seen in experiments. The Rescorla-Wagner model (Rescorla & Wagner, 1972) is the most influential formal model of conditioning, but cannot discriminate non-linear cue combinations and other interesting conditioning effects. The comparator class of theories has multiple formal instantiations, all of which share in the intellectual attempt to coopt environmental cues beyond the labeled CS to help predict the US. Attention-based models explain conditioning effects using information-theoretic measures of informativeness of various CS. Both these latter theories can explain some things

that Rescorla-Wagner can't, but are perceived to have their own limitations, particularly in that they remain formally underspecified. Thus, there is no theoretical consensus on modelling classical conditioning, and until recently, little hope of arriving at one.

Recently, sophisticated modelling by Courville et al (Courville, Daw, & Touretzky, 2006) and (Gershman & Niv, 2012) has shown that comparator-type theories formally instantiated as Bayesian inference about the latent structure of an animal's environment have substantial predictive power. These formal models statistically ground previously somewhat heuristic presentations of the comparator hypothesis (Stout & Miller, 2007) by using animals' known propensity to make situational frequency judgments (Hasher & Zacks, 1984) to inform conditional probability estimates of the form p(US|CS,US,S), where S are non-CS environmental factors that can help predict US. Simulation studies demonstrate that such Bayesian latent variable comparator models explain substantially more conditioning effects than earlier less formal comparator accounts. For instance, Courville et al. (2006) review a large number of studies documenting effects that have traditionally been explained using attentional models, and demonstrate that each one of them can be explained using a latent variable model that postulates that animals try to infer what cause could have led to their current stimulus set observation. Gershman and Niv (2012) present a different latent cause model and demonstrate its utility in explaining a number of conditioning effects, including ones that comparator theories have erstwhile been hard pressed to explain. These modelling advances have made a concrete case for investigating context-sensitive US prediction as a primary explanation for classical conditioning.

However, while latent cause modelling is a promising new direction, the theory is still immature, with a number of different formal specifications possible, each with a different set of potentially implausible statistical assumptions. For example, (Courville et al., 2006) assume that animals learn the rate of change of parameters for a fixed set of latent models. In contrast, (Gershman & Niv, 2012) use infinite mixture modelling to infer latent causes dynamically, but are forced to treat causes as clusters of observation vectors, assume a Dirichlet process prior on the set of causes, and impute only MAP estimation to the cause inference. It is not currently clear which assumptions in these models are essential to the

theory, which are simply modelling requirements, and which are entirely superfluous.

In this paper, we adopt a simpler third interpretation for these latent variables, viz. that they are not latent but rather simply index stimuli co-occurrences the animal has observed in the past. We show that this interpretation leads to a simple formal specification of the comparator account of classical conditioning that expands upon the explanatory power of latent cause-based models, while additionally being observable and directly testable. As evidence for the predictive ability of our model, we present simulated replications of ten classic conditioning effects by agents using our model. Its success further strengthens the case for normative Bayesian inference as a rational basis for animal behavior (Knill & Pouget, 2004).

The situation prediction model

The fundamental novelty of our approach to modelling classical conditioning is that we don't try to model it. Instead, we try to model what we believe is a superordinate goal for animals - predicting situations and their corresponding task-sets. We consider it more ecologically natural to think of animals being driven by the need to orient themselves to their environments by constantly making adaptive predictions. If we assume that animals do partition the environment as discrete situations, trying to predict which one they are in becomes a natural and essential goal. It is this essential process that we try to model.

We model this basic orienting process as sequential Bayesian updating, where animals use evidence about stimulus co-occurrences observed in the past $o_{1:t-1}$ to predict the likelihood of being in situation context *c* and having to respond to it at time *t*. The standard Bayesian formulation of this inference is,

$$p(c|o_{1:t}) \propto p(o_t|c) \times p(c|o_{1:t-1}).$$
 (1)

To specialize this model to the classical conditioning paradigm, we simply define observations in a specific way. We assume that, in addition to generic environmental stimuli *S*, the animal can observe CS and US in the conditioning setup. We therefore model an observation at time *t* as $o_t = \{US, CS, S\}(t)$, where each stimulus in turn is a binary attribute which is either present or not present in the observation vector o_t . We interpret *c* as the observation-indexed *situation context* within which an animal has to respond to stimuli that are biologically relevant (US). Formally, we simply index *c* using *o*, thereby instantiating *C* as the set of all previous observations unique with respect to observation set membership.

With these additional assumptions constraining our interpretation of the model defined in (1), we obtain a specialization calculating $p(c|\{US, CS, S\}(1:t))$ as,

$$p(c|\{US, CS, S\}(1:t)) \propto p(\{US, CS, S\}(t)|c)$$
(2)

$$\times p(c|\{US, CS, S\}(1:t-1)).$$

The key quantity of interest is the likelihood term p(US(t), CS(t), S(t)|c). Both within the temporal constraints of conditioning experiments and in the real world, animals will typically encounter situations where biologically irrelevant stimuli S and CS are observed, and biologically important US is to be predicted, making the separate computation of p(US|CS,S) ecologically natural. It is easy to derive the probabilistic strength of association p(US|CS) from this computation, and we believe that the stimulus specific CRs measured in conditioning experiments are simply behavioral measurements of this quantity. Thus, we see that our likelihood term matches the interpretation of CR in the conditioning responses as playing a facilitative role in the animal's deeper goal of situational context prediction.

In our representation, each context has an associated set of stimuli $c^j \in c$, and the probability that an incoming observation vector o is generated from this context simply requires an assessment of set membership mismatches. Let $p(o^i|c^j)$ be the probability that the i^{th} element of the observation vector o matches the j^{th} element of c. Then, the probability that stimulus $x = o^i$ is generated by an object set c is,

$$p(x|c) = 1 - \frac{\beta}{|c|} \sum_{j}^{|c|} (1 - \delta(x - c^{j}))$$
(3)

where β is a parameter controlling the magnitude of the penalty imposed for each mismatch observed, with p(x|o) similarly defined.

Predictive evidence for US associated with each context p(US|x,c) is imputed individually. As a rule of thumb, if a context does not contain US, it is unlikely to predict future US and vice versa. This evidence is compiled across all previously inferred $c \in C$ to predict strength of association with US for any x, conditional on it being present in the current observation vector o_t . While only the CS is relevant for modelling conditioning, our model is designed to predict strength of association with all possible stimuli. In settings where the environmental stimulus set S is fixed, our model predicts that the conditioning response CR will be proportional to,

$$p(US(t)|CS(t)) = \frac{\sum_{c}^{C} p(US(t)|CS(t),c)p(CS(t)|c)p(c|o_t)}{\sum_{c}^{C} p(CS(t)|c)p(c|o_t)},$$
(4)

where $p(c|o_t)$ is the posterior distribution across contexts given we have observed o_t , in turn computed as,

$$p(c|o_t) = \frac{\sum_x p(x|o_t) p(x|c) p(c)}{\sum_c \sum_x p(x|o_t) p(x|c) p(c)}.$$
(5)

Changes in the environmental context can be easily incorporated by marginalizing over *S*. Additionally, we can also compute cumulative conditioning responses arising from multiple stimuli, e.g. $\{CS_1, CS_2, S\}$ in cue combination settings by marginalizing across the subset X of the cues in ques-

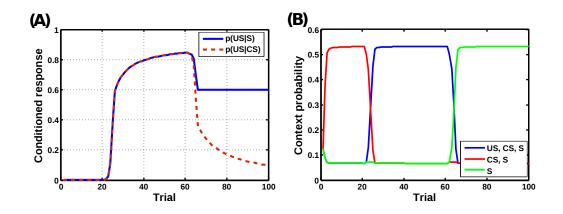


Figure 1: **Simulation illustrating the basic operation of our model.** We assume that agents can estimate the likelihood of seeing a stimulus worth responding to conditioned on the probability of being in particular situations. (A) Our Bayesian model estimates conditional probabilities of US with respect to multiple environmental cues, giving us the traditional CR as a special case. (B) The principal component of our model is situation inference, which gates the accumulation of reinforcement evidence.

tion,

$$CR = \sum_{x}^{X} p(US(t)|x)p(x) = \sum_{x}^{X} p(US(t)|x) \sum_{c}^{C} p(x|c)p(c|o_{t}).$$
(6)

Figure 1 shows the basic operation of our model as a simulation, with the measured conditioning response associated with both CS and S plotted across an acquisition-extinction episode in panel A and the corresponding latent context probabilities plotted in panel B. For the first 20 trials in this experiment, the agent is exposed to the unreinforced CS in the environmental condition S, resulting in a high context probability for {CS, S}. Then, during acquisition between trials 20 and 60, the context {US, CS, S} expectedly becomes prominent. Finally, the agent is exposed to the unreinforced CS in a different environmental condition, resulting in the dominance of the context indexed by {CS}. Our simulation of CR (dotted line) shows the correct acquisition and extinction behavior expected of a model of classical conditioning.

Results

Our theory makes accurate predictions for a large number of conditioning-related effects. In the interests of succinctness, we restrict ourselves to reporting here 10 simulated effect replications, selected for their prominence across multiple theoretical accounts of conditioning. No existing model of conditioning can account for all 10 of the effects that we explain below. In all the following figures, trial numbers are counted from the last reinforced trial, i.e., the last reinforced presentation occurs at trial 0. For all subsequent trials, the simulated agent sees unreinforced situation contexts in configurations determined by the corresponding test scenarios. Individual training and pre-exposure blocks, where specified, consisted of 5 trials in each of our simulations. Finally, since US is a binary random variable, p(US|CS) = 0.5 will indicate absence of conditioning if the training sequence is zero contingency; all values lower than this can reflect absence of CR depending on training conditions. Values higher than 0.5 will always imply the existence of a CR.

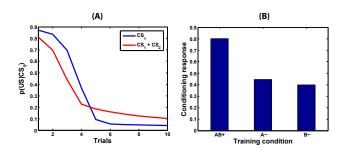


Figure 2: **Stimulus generalization.** Replicating (**A**) external inhibition, where the trained CS is accompanied by a novel stimulus during testing and (B) non-linear (positive) patterning, where individual elements A and B are presented unreinforced, while the combined cue AB is reinforced during training.

The first pair of effects, illustrated in Figure 2, can both be seen as instances of animals generalizing US-CS contingency information in interesting ways that simple Rescorla-Wagner (RW) models find hard to replicate. For example, the example of non-linear patterning seen in 2(B) poses a major challenge for RW, since positive contingency of the US with AB must necessarily transfer to both A and B by its predictions. Instead, real animals, comparator-based accounts and our simulation all show CR in the presence of AB, but not A or B, as shown in 2(A). Our model explains patterning through separate evidence accumulation for the situation contexts triggered by the presence respectively of A,B and AB. External inhibition occurs when the presence of a novel stimulus during testing reduces CR, as shown in 2(A). This effect is, in fact, known since the earliest conditioning experiments, when Pavlov noticed the effects of extraneous noises on his dogs' CRs during testing. Our model explains this effect via a reduction in the likelihood p(x|c) for an observation containing novel stimuli.

Another simple manipulation of the basic conditioning experiment involves pre-exposing a test group of subjects to the US or CS before training begins. Learned irrelevance is a prominent effect that arises in such settings, often invoked by attention-based modelers, wherein prior exposure to US intermittently paired with CS retards subsequent acquisition of CR for CS. Figure 3(A) illustrates our model's replication of this effect using two pre-exposure blocks containing together five reinforced and five unreinforced presentations of CS in random order, with the reduced sensitivity to training emerging from a flattened prior distribution p(c|o) of possible situations where US is expected. In other words, the model anticipates that US will be present in a large range of situations, the presence of CS being one of them. This dilutes the probabilistic binding p(US|CS), leading to lower CR, and slower acquisition until p(c|o) becomes sufficiently peaked through repeated exposure to CS.

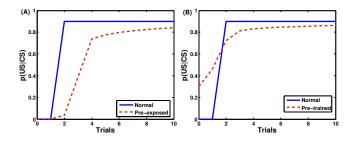


Figure 3: **Pre-exposure effects.** Replicating pre-exposure effects like, (**A**) Learned irrelevance through random prior presentation of US and CS leads to no initial CR (see trial 2) and slower acquisition of CR compared to normal conditioning without pre-exposure to the random trials. (**B**) The Hall-Pearce effect, where pre-training with a weaker version of the US retards subsequent acquisition with normal US.

Note that our explanation does not invoke notions of surprise and salience, like (Mackintosh, 1975) and other attention-based accounts of this effect. The explanation for retarded CR acquisition arises entirely from our straightforward Bayesian formalism. A similar resolution occurs for the well-known Hall-Pearce effect, wherein pre-training with a weaker version of the US retards subsequent CR acquisition with a stronger version. While our model does not permit explicit representations of stimulus strength, we model this effect by modifying our set membership computation in Equation 3 to permit partial matches. For our purposes, this modification is restricted to defining p(x = weak US|c) = $0.5, \forall c \text{ s.t.} US \in c \text{ and } p(x = US|c) = 0.5, \forall c \text{ s.t.} \text{weak } US \in c$ c, following the intuition that a weak version of the same US will be judged similar to the US. With this assumption, our model replicates the Hall-Pearce effect using one preexposure block presenting CS reinforced with weak US as defined above, as illustrated in 3. As with learned irrelevance, no attention or surprise-related concepts are invoked.

When multiple conditioning stimuli are used, it turns out that presenting one stimulus reinforced alone and unreinforced when paired with another stimulus causes the second stimulus to become a conditioned 'inhibitor', viz. its presence suggests that US will likely not appear. Presenting both stimulus configurations for one trial block each leads to a replication of conditioned inhibition in our simulation, as shown in 4(A). Bayesian inference provides an extremely natural explanation for this effect, since negative evidence for US availability accumulates transparently in the $\{CS_1, CS_2\}$ situation context in our model. Since CR is computed by marginalizing across all such contexts, incorporating evidence from this context reduces $p(US|\{CS_1, CS_2\})$, which, by the logic of our model is the same as measuring CR in a summation test.

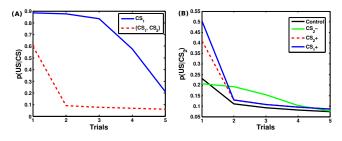


Figure 4: **Conditioned inhibition.** Replicating (A) the basic conditioned inhibition effect, where interleaved presentation of reinforced CS_1 and an unreinforced combination of CS_1 and CS_2 causes CS_2 to be inhibited, as measured by summation tests, and (B) variations thereof testing the effects of subsequent inflation and deflation of the inhibited CS.

As Gershman and Niv (2012) point out, there are multiple variations of the basic conditioned inhibition result. For instance, once a stimulus has been trained as an inhibitor, subsequently reinforcing it attenuates the inhibition, while extinguishing it has little impact on the conditioning response. This pattern is borne out in our simulations using one posttraining block, as illustrated in 4(B). The y-intercept is the key observation in this figure, and shows clearly that the CS_2 condition yields a similar CR to the baseline conditioned inhibition setting, while CS_2 + leads to increased CR at test trial 1, though still inhibitory with respect to CS_2 . However, our theory fails, like Gershman's model, to explain an observation (Amundson, Wheeler, & Miller, 2005) that post-training inflation of the conditioned excitor (CS_1+) enhances conditioned inhibition of CS_2 . Our theory predicts the opposite result, using positive evidence for $US - CS_1$ binding to increase the overall conditioning response, without reducing the relative share of CS₂.

Latent variable models also find it difficult to explain other cue competition based conditioning effects. The observable nature of our context variables renders our theory somewhat more successful in explaining such data. Overshadowing is a classic conditioning effect, and has heretofore been one of the most compelling pieces of evidence supporting value-based models of conditioning. In overshadowing, the CR for a stimulus trained jointly with another stimulus is weaker than the CR obtained when training with it alone. Rescorla-Wagner and other value-based models find it easy to explain this result as a sharing of value across multiple sites in the compound conditioning case. Our model explains this effect, as illustrated in 5(A) via a simple conditionalization argument. Given the same number of trials in each case, evidence for CS_1 in the compound case has to be weighted by the observation likelihood of encountering CS_1 alone. Since this quantity is sub-unity by the design of the experiment, overshadowing emerges naturally. Note that it is our model's use of observation-indexed context variables that permits likelihood computation. Latent variable models using more abstract context variables cannot compute such quantities, which makes it harder for them to explain cue competition effects.

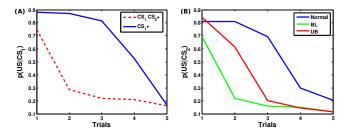


Figure 5: **Cue competition.** Replicating effects like (**A**) overshadowing, where compound conditioning with CS_1 and CS_2 results in a weaker conditioning response than training with either of the individual CS, and (**B**) the classic blocking effect, where reinforcement with CS_1 followed by reinforcement with { CS_1, CS_2 } elicits no trained response for CS_2 as well as an unblocking effect, where blocking is attenuated by testing in a new environmental context.

Figure 5(B) illustrates our model's replication of Kamin's classic blocking effect, wherein it is found that reinforcing the compound $\{CS_1, CS_2\}$ after reinforcing CS_1 yields no training response for CS_2 . The existence of this effect motivated the use of a maximum (saturation) level of training associated with any US in the Rescorla-Wagner framework. It is also explained in attention-based frameworks by positing that since CS_1 already predicts US, the animal doesn't need to learn a new predictor CS_2 . Our model provides a novel inductive explanation. The observation likelihood $p(x = CS_2 | c = \{CS_1, CS_2\})$ reduces the positive evidence for $p(US|CS_2)$, the prior encounters with the context containing just CS_1 lower the prior context probability $p(c|o^{1:t})$ of inferring the context $\{CS_1, CS_2\}$ itself. These sequential dilutions multiplicatively reduce the $p(US|CS_2)$ computation to the value we see in 5(B). Also, our model replicates the unblocking effect described by (Courville et al., 2006), wherein blocking is attentuated by testing in a new environmental context. Here, adding a novel environmental stimulus reduces the relevance of the original CS_1 trials in the context probability calculation, resulting in a relatively higher value for contexts containing CS_2 than the blocking case.

The last pair of effects we consider have been been difficult to explain in the past, particularly for latent variable models. The first - the partial reinforcement extinction effect (PREE) refers to the observation that CRs trained using intermittent and variable schedules are larger in magnitude and take longer to extinguish. As we show in 6(A), our model predicts the second aspect of this effect, but not the first. In doing so, it joins the model of (Courville et al., 2006), who make precisely the same prediction. Our model explains this aspect of PREE following much the same outline as (Gershman & Niv, 2012) and (Courville et al., 2006). Intermittent observation of the reinforcing context makes it harder for the inductive algorithm to judge when it is no longer in a reinforcing trial block, thereby taking longer to extinguish the CR.

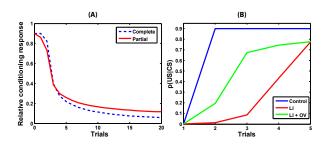


Figure 6: **Tricky effects.** The situational context model makes interesting predictions for effects that have proved difficult to explain such as (A) the partial reinforcement extinction effect, where intermittent and variable reinforcement schedules retard future cue extinction. and (B) pre-exposure overshadowing, where unreinforced pre-exposure to CS_1 appears to eliminate the over-shadowing effect of subsequent combined training with CS_1 and CS_2 .

Courville et al. (2006) also describe preexposure overshadowing (Experiment 3 in their paper), where unreinforced preexposure to CS_1 eliminates the overshadowing effect of subsequent compound training with $\{CS_1, CS_2\}$. This is, on the surface, a surprising result, since it is known that (i) pre-exposure to a stimulus tends to retard future acquisition (latent inhibition), and (ii) pairing two stimuli also leads to slower learning (overshadowing). Blaisdell and colleagues (Blaisdell, Bristol, Gunther, & Miller, 1998) demonstrated that combining both these conditions actually attenuates the learning deficit. In fact, as a mark of its intricacy, (Blaisdell et al., 1998) set it up as a challenge for noncomparator accounts of conditioning. Our model replicates this effect, as shown in 6(B), and explains it thus: prior exposure to CS_1 tilts the context probability p(c|o) in favor of contexts containing only CS_1 as opposed to the pair $\{CS_1, CS_2\}$. This causes evidence for the compound to disproportionately raise $p(US|CS_1)$ above the overshadowing baseline.

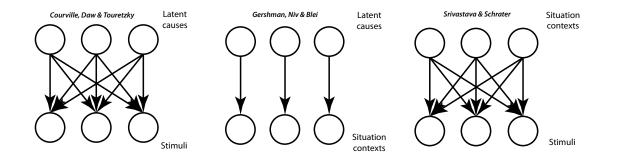


Figure 7: A comparison of the generative models underpinning our theory and its closest relatives - Courville's latent cause model and Gershman's infinite capacity latent cause model. The key difference is the observability of the context variables in our theory which permits more granular likelihood computations and a richer set of effect descriptions.

Discussion

The principal source of variation between the models in Figure 7 is in the interpretation of the context variables. Courville et al. (2006) assume that the latent variables are generative models that the animal thinks could explain the observations it sees. Gershman and Niv (2012) assume that latent variables are clusters of observations, imputing a specific generative model (Dirichlet process mixtures) to the clustering process. Our interpretation indexes contexts using observation co-occurence directly. The consequent transparency of representation allows our model to reproduce the predictions of its competitors, and predict some effects they cannot. The main contribution of this work is a streamlining of the assumptions underlying the inferential view of conditioning, thereby improving its explanatory power and testability.

An important limitation of our model is that it is eventbased, rather than time-based. While the time elapsed between events is certainly important for many kinds of learning, including reinforcement tasks that have explicit time dependence on the delivery and availability food or reinforcers, our methods have focused on another ubiquitous type of decision task, where the options selected are freely available within a context, and the context determines what should be selected. Such tasks are likely to be well-described by context-dependent conditioning, wherein ventromedial prefrontal cortex and hippocampus loops have been neurophysiologically implicated in context-specific retrieval of value information (Kalisch et al., 2006).

In a broad sense, value-based theories of conditioning describe a world where understanding is cheap and decisions are difficult. Such a description is apt for artificial control systems, the study of which, not coincidentally, gives us much of the mathematics that governs value-based learning theories. It is not, however, a fitting description of the way welladapted animals engage with their natural environments. The inference-based view of conditioning gives a better description of such a world - where understanding is difficult, and action selection, once the situation context is understood, is easy. The success of our simple model in explaining a variety of conditioning effects further strengthens this belief.

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