Title

Taxpayer Payback Period Analysis of Rehabilitation Services Programs: A First Model and its Sensitivity Analysis

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TAXPAYER PAYBACK PERIOD ANALYSIS OF REHABILITATION SERVICES PROGRAMS: A FIRST MODEL AND ITS SENSITIVITY ANALYSIS

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Project for Cost Benefit Analysis and Evaluation of Rehabilitation Services

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PREFACE

In this paper, Mr. Stahl presents a refined version of a model for calculating the payback period of public program investments in vocational rehabilitation. He then performs a sensitivity analysis of the assumptions. The model and a preliminary sensitivity analysis were initially developed during the summer of 1971. The sensitivity analysis identified those key assumptions which most affected the overall results of the model. Subsequent research has focused on improving our estimation or collecting improved data on which to base the assumptions. Although our final benefit-cost analysis of rehabilitation programs will use a more detailed payback model yet than the one presented here, Mr. Stahl's model and sensitivity analysis should prove most useful to researchers in this field.

Frederick C. Collignon
October, 1972

I. INTRODUCTION

This paper is part of an effort to develop different types of cost-benefit estimates for rehabilitation services programs of the Rehabilitation Services Administration, U.S. Department of Health, Education and Welfare. The goals of this paper are:

(1) To formalize a disaggregated payback period model developed by Ronald W. Conley in his book, The Economics of Vocational Rehabilitation.[1] In this model the vocational rehabilitation effort is evaluated by the number of years needed to offset initial expenditures on rehabilitation by increased net returns to government due to rehabilitation, mostly through savings in welfare payments or increased tax payments.

(2) To estimate values for the payback period and check the sensitivity of our estimates to variations in the model's parameters. In order to do this, a special version of the model has been chosen. Several parameters considered crucial for the sensitivity of the payback period estimates have been varied. This effort is similar to our earlier sensitivity analysis for a GNP oriented model with an objective function maximizing the discounted net present value of investment.[2]

In the following, a conceptual framework for the payback period value is developed first (Part II); then the quantitative model developed

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[1] Baltimore: Johns Hopkins Press, 1964, Chapter 4, in particular pp. 84-93.

[2] See Averous, C. with Stahl, K. and Cole, C. "Cost Benefit Analysis of Rehabilitation Services Programs: A First Model and Its Sensitivity Analysis," Working Paper No. 163/RS001, Institute of Urban and Regional Development, University of California, Berkeley.

by Conley is formalized in Part III; a new model is developed for a sensitivity analysis (Part IV); the sensitivity analysis is presented and commented upon in Part V, and finally suggestions for the development of an extended payback period model are made in Part VI.

II. THE CONCEPTUAL FRAMEWORK FOR THE PAYBACK PERIOD MODEL

In this section, we develop a general framework for the taxpayer payback period model, based on the verbal description given by Conley.

II.1 General Comments

The payback period may be defined as "the number of years after the initial year it takes to accumulate sufficient benefits net of operation costs to cover the initial year's cost of the project."

Formally, the payback period P for the rehabilitation program is given by solving the inequality

$$\min_{P} \left[\sum_{i=1}^{P} \right] dTB_i \geq dTC_0$$

where dTB_i = total program benefits net of operating costs in year i due to expenditures on rehabilitation in year 0;

dTC_0 = initial program cost in year 0.

The following features characterize Conley's approach to this model:

(1) Benefits and costs are considered for taxpayers only, exclusive of the rehabilitants and their families. Furthermore, costs and benefits are considered only for the Federal government.

(2) Only economic benefits and costs are considered, and they are taken at face value. Thus, there is no attempt to evaluate differently benefits or costs accruing to the various segments of the population, for

example, to regional groups, or to groups distinguished according to income or race.

(3) The evaluation is based on a comparison of the situation of clients before and after rehabilitation, as opposed to an evaluation via control groups, or program variation.

Consequently, the objective function underlying the evaluation may be described as maximization of short run returns to a given level of public expenditures.[3] In this paper, we maintain all of the features introduced by Conley.

II.2 Disaggregation of Benefits

In general, the net benefits from the rehabilitation effort accruing in year i to taxpayers dTB_i are composed of the following terms:

$$dTB_i = dTER_i + dTEF_i + dTEO_i - dTAC_i - dW_i$$

where $dTER_i$ = incremental tax revenues from rehabilitants in year i due to rehabilitation efforts in year 0

$dTEF_i$ = incremental tax revenues from families of rehabilitants due to rehabilitation efforts in year 0

$dTEO_i$ = variations in tax revenues from other persons (excluding rehabilitants and their families) due to rehabilitation efforts in year 0

dAC_i = variations in associated program costs due to rehabilitation in year 0 borne by the public in year i

dW_i = variations in public assistance payments supporting rehabilitants and their families and borne by the public in year i, which are due to the rehabilitation effort in year 0.

[3] One limitation inherent in the payback period model may be mentioned here: Since benefits are considered only between time 0 and P, any net benefits beyond P attributable to the project are not taken into account. An advantage of this model lies in the fact that practically all returns and costs can be accounted for without conceptual difficulty.

The meaning of dTER and dTEF should be self evident. The interpretation and estimation of the term dTEO depends crucially on the state of the economy, in particular in the employment categories open to rehabilitants.[4]

The term dAC roughly comprises the change in all annual cost (medical, nursing and custodial care) borne by the public which is attributable to the rehabilitation effort in year 0. In contrast to the GNP oriented model, reductions in welfare payments dW can be counted as benefits, since reductions in welfare payments represent savings to the public sector.

II.3 Disaggregation of Costs

The conceptual equation for the incremental cost of year 0's program effort dTC_0 to the public includes the following terms:

$$dTC_0 = R - dRC + dRR + dRP + RR + RT + TEF$$

where R = total nominal program cost in year 0,

dRC = program costs not attributable to current year's closures (costs related to program expansion and increased number of carry overs)

dRR = costs due to repeaters of the rehabilitation program

dRP = program costs borne by other public agencies

RR = governmental expenditures for research in rehabilitation

RT = governmental expenditures for training of professionals in rehabilitation

TEF = tax revenues from rehabilitants foregone during the rehabilitation process.

In conclusion, let us comment on the differences between the taxpayer payback period approach and the GNP net present value oriented

[4] For a general discussion, see Averous, Stahl, Cole, op. cit., p. 5f.

approach. The model described here differs from the GNP oriented model in the following ways:

(1) All incremental costs borne by and incremental benefits accruing to the federal public agencies rather than to society as a whole have to be accounted for; in particular payments considered merely redistributive in the GNP approach and usually not introduced in this accounting scheme, such as public assistance payments, have to be included in our model.[5]

(2) Conversely, costs and benefits shifted to private persons or private charities should not be accounted for in a taxpayer payback model unless such costs and benefits have impacts on governmental revenues or expenditures, in particular taxes or savings in welfare payments.

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[5] Redistributive payments are only accounted for in a GNP oriented analysis if benefits and costs going to different income groups are weighed differently.

III. THE MODEL IN DETAIL

For several reasons which will not be discussed here, Conley restricts consideration in the payback period model to the following variables: dTER, the incremental tax revenues from the rehabilitated population, and dW, the variations in welfare payments due to rehabilitation on the benefit side; R, the program cost, on the cost side. Thus, his model reduces to

$$\min_{P} \left[\sum_{i=0}^{P} (dTER_i - dW_i) \right] \geqq R_0.$$

Note that since rehabilitated clients may earn income during the program year, calculation of the payback period includes the program year.

In this "first round" of the discussion of the payback period model Conley's discussion will be formalized only, without elaborating in detail upon the assumptions used and the specifications possible. Accordingly, the variables discussed in detail will be dTER, the incremental tax revenues from rehabilitants attributable to rehabilitation, and dW, the change in welfare payments due to rehabilitation.

III.1 Incremental Tax Revenues from the Rehabilitated Population

Throughout his version of the taxpayer payback period model, Conley assumes that any changes in the program effort do not affect the tax rates upon which the estimation of tax return differentials depends. Furthermore, Conley assumes zero mortality of the rehabilitated population during the payback period. Only income tax returns to the Federal government are considered. In order to evaluate dTER for year i after rehabilitation, Conley disaggregates the population undergoing the rehabilitation effort into seven earnings brackets, and eight brackets according to number of dependents.

III.1.1 Definitions

For the disaggregation according to income to account for different tax rates let us define

PB = column vector (7x1) of number of rehabilitants by income group at acceptance (the first entry being the population segment with zero earnings)

PA = column vector (7x1) of rehabilitants by income group at closure

E = diagonal matrix (7x7) of mean yearly earnings, with e_{ii}, the typical element, showing the mean yearly earnings in the income bracket considered

In particular, Conley uses

$$E = \begin{bmatrix} 0 & & & & & & \bigcirc \\ & 250 & & & & & \\ & & 750 & & & & \\ & & & 1{,}500 & & & \\ & & & & 2{,}000 & & \\ & & & & & 3{,}500 & \\ \bigcirc & & & & & & 4{,}250 \end{bmatrix}$$

For disaggregation according to number of dependents to account for tax deductions, let us define

F = row vector (1x8) whose elements f_i show the percentage of the total number of rehabilitants in a program year having (i-1) dependents

TD = diagonal matrix (8x8) whose typical elements t_{ii} represent total tax deductions from yearly earnings applicable according to number of dependents

In particular, if $600.00 per dependent is tax deductible,

$$TD = \begin{bmatrix} 0 & & & & \bigcirc \\ & 600 & & & \\ & & 1{,}200 & & \\ & & & 1{,}800 & \\ \bigcirc & & & & \ddots \end{bmatrix}$$

For the determination of tax returns, let

TR = row vector (1x7) whose typical elements show the tax rate applying to each consecutive $1,000.00 of yearly income.

In particular, considering estimation of federal income tax returns only, Conley uses

$$TR = [.14, .15, .16, .17, \ldots\ldots .17]$$

TM = auxiliary matrix (7x7) disaggregating income brackets to consecutive $1,000.00 elements, the typical element tm_{ij} showing the fraction of mean yearly income j taxed at rate i.

Following the mean yearly earnings figures given in the diagonal matrix E, above, the matrix TM looks as follows:

$$TM = \begin{bmatrix} 1 & 1 & 1 & \frac{2}{3} & \frac{2}{5} & \frac{2}{7} & \frac{4}{17} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{2}{5} & \frac{2}{7} & \frac{4}{17} \\ 0 & 0 & 0 & 0 & \frac{1}{5} & \frac{2}{7} & \frac{4}{17} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{7} & \frac{4}{17} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{17} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, as example, $tm_{14} = 2/3$ means that 2/3 of the fourth income bracket ($e_{44} = 1,500$) are taxable at the first tax rate ($tr_1 = .14$).

Finally, let

K = auxiliary unit column vector (8x1)

III.1.2 Determination of Yearly Taxable Income Increments of the Rehabilitated Population

Since we are interested only in differential tax returns due to rehabilitation, let us compute first a matrix $A_{(7x8)}$ disaggregating the differences in number of rehabilitants per earnings bracket by number of dependents:

$$A_{(7x8)} = [PA - PB]_{(7x1)} \cdot F_{(1x8)}$$

The typical element a_{ij} shows the change in the number of rehabilitants in the i^{th} earning bracket having (j-1) dependents. Note that this matrix is established under the assumption that the distribution of dependents does not vary among income brackets. This assumption does not hold for the general population, say, of the U.S. It should be checked for the rehabilitated population in particular.

Next, we premultiply the matrix A by the mean earnings matrix, E, to obtain a matrix $B_{(7x8)}$ of total earnings differentials per income bracket and number of dependents:

$$B_{(7x8)} = E_{(7x8)} \cdot [PA - PB]_{(7x1)} \cdot F_{(1x8)}$$

The typical element b_{ij} of B shows the total earnings differential of rehabilitants in the i^{th} income bracket and with (j-1) number of dependents.

In order to determine the total tax deductible earnings differential, we postmultiply the matrix A by the matrix TD of tax deductions by number of dependents. In addition, since some fraction η of gross earnings is tax deductible (Conley uses $\eta = .10$), the matrix $C_{(7x8)}$ of total tax deductible earnings by income brackets and number of dependents is given by

$$C_{(7x8)} = A_{(7x8)} \cdot TD_{(8x8)} + \eta \cdot B_{(7x8)}$$

$$= [(PA - PB) \cdot F \cdot TD] + \eta \cdot [E(PA - PB) \cdot F]$$

where η is a scalar.

Consequently, the matrix of taxable income increments, TE, is given by

$$TE_{(7x8)} = B_{(7x8)} - C_{(7x8)}$$

$$= E \cdot [PA - PB] \cdot F - (PA - PB) \cdot F \cdot TD - \eta E(PA - PB) \cdot F$$

or

$$\boxed{TE = (1-\eta) \cdot E \cdot [PA-PB] \cdot F - [PA - PB] \cdot F \cdot TD}$$

Using the auxiliary unit column vector K, we may aggregate the taxable income into column vector $TER_{(7x1)}$ showing the total taxable increment in earnings of rehabilitated population by income brackets:

$$\boxed{TER_{(7x1)} = TE_{(7x8)} \cdot K_{(8x1)}}$$

III.1.3 Determination of Yearly Incremental Tax Returns from the Rehabilitated Population

Using the tax rate vector TR, and the auxiliary matrix TM defined above, we now can compute the total incremental tax gain per year from the rehabilitated population, dTER:

$$\boxed{dTER_{(1x1)} = TR_{(1x7)} \cdot TM_{(7x7)} \cdot TER_{(7x1)}}$$

Since zero mortality of the rehabilitated population is assumed for the payback period, dTER is constant for all relevant years i.

III.2 Variations in Welfare Payments Due to Rehabilitation

In order to estimate the variations in public assistance payments, dW, Conley uses the following simple formula:

$$dW = (WA - WB) \cdot 12$$

where WA = aggregate monthly public assistance payments to the rehabilitated population after, and

WB = before rehabilitation

Since $dW < 0$, $(-dW)$ positively adds to benefits. Conley assumes the difference in welfare payments to be constant for the time period in question, so dW_i is constant for all relevant years i.

IV. MODEL FOR A PRELIMINARY SENSITIVITY ANALYSIS

In order to examine the sensitivity of the taxpayers' payback period estimates to some assumptions made by Conley in deriving his estimates, a first model has been developed. In this model, some of these assumptions have been made explicit in the form of new variables and parameters introduced; estimates derived by systematically varying some of the parameters will be presented and discussed in part V.

In order to facilitate the computation which has been done by hand, the model is simplified in some respects as compared to Conley's model discussed in part III:

(1) There is no disaggregation by earnings levels to account for the progressivity of tax rates. Instead, an aggregate tax rate has been used.

(2) There is no disaggregation of the rehabilitated population according to number of dependents to account for tax deductions.

Expansions in comparison to Conley's model include the following features:

(3) Benefits are discounted according to a discount rate τ.[6]

(4) Incremental tax returns from the rehabilitated population dTER, have been adjusted for

- mortality of rehabilitated population
- changes in productivity with and without rehabilitation over time

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[6] There is no conceptual rationale for evaluating benefits equally, no matter in which year they are occurring.

(5) Several variables not considered in Conley's estimate have been included in the model:

On the benefit side:

- dTEO, variations in tax revenues from other persons.

On the cost side:

- dRC, program cost not attributable to current year's closures
- dRR, costs due to repeaters in the rehabilitation program
- RR, research expenditures
- RT, expenditures for training of professionals in rehabilitation
- TEF, tax revenues from earnings foregone due to the rehabilitation process.

Thus, except for dTEF, dAC and dRP which are excluded because of lack of data, all variables of the conceptual framework developed in part II are included in our model.

IV.1 The Model

The model used for the sensitivity analysis is of the following form:

$$\min_{P} \left[\sum_{i=0}^{P} \frac{dTER_i - dTEO_i - dW_i}{(1+r)^i} \right] \geq R - dRC + dRR + RR + RT + TEF \qquad (1)$$

where the variables are as above. In the following, we consider some terms in detail.

IV.1.1 Tax Revenues from the Rehabilitated Population

The tax revenues from the rehabilitated population $dTER_i$ are determined as follows:

$$dTER_0 = (1 - \pi) \cdot \tau \cdot dER_0 \qquad i=0$$

$$dTER_i = \tau \cdot dER_i \qquad i>0$$

where π = fraction of year 0 during which earnings are foregone due to rehabilitation

τ = an aggregate tax rate

dER_i = change in total earnings of the rehabilitated population in year i due to rehabilitation.

(a) Adjustments of $dTER_i$ for mortality

Since the mortality tables used for the estimates[7] provide mortality rates for ten 5-year age groups, an aggregate index giving the mortality rate m_i between year i and year i+1 has been developed as follows:

$$m_i = \frac{1}{10\, P_0} \sum_{j=1}^{10} \frac{1}{5} M_j P_j$$

where P_0 = total population rehabilitated in year 0,

P_j = population rehabilitated in year 0 of the j^{th} 5-year age group

M_j = mortality rate of the j^{th} 5-year age group

Thus, the index developed is a linear approximation for yearly mortality rates weighed by the actual rehabilitated population.

The total population rehabilitated in year 0 surviving to year i, P_i, then is given by iteration:

$$P_i = (1-m_i)^i \cdot P_0$$

(b) Adjustments for productivity differentials with and without rehabilitation

Assuming remuneration of workers according to the value of marginal product, the increment in earnings due to rehabilitation over time should be adjusted for changes in the productivity of the individual workers. Differences in productivity of workers with and without rehabilitation may develop in different ways over time.

[7] Cf. Conley, op. cit., p. 77.

In order to introduce such variations explicitly, consider the following expression:

$$dER_i = (ERA\,[1+\alpha]^i \cdot WIA_1 \cdot P_i) - (ERB\,[1+\beta]^i \cdot WIB_1 \cdot P_i)$$

where dER_i = change in total earnings in year i due to the rehabilitation effort, as above

ERB, ERA = level of mean yearly earnings of remuneratively employed rehabilitants at acceptance, B (closure A)

β, α = annual rate of change of yearly earnings without ,β (with, α) rehabilitation through time reflecting variations in productivity

WIB_1, WIA_1 = proportion of rehabilitants remuneratively employed at acceptance, B (closure A)

P_i = rehabilitated population surviving to year i, as above.

Substituting for P_i,

$$dER_i = [ERA\,(1+\alpha)^i \cdot WIA_1 \cdot (1-m_i)^i \cdot P_0] - [ERB\,(1+\beta)^i\, WIB_1 \cdot (1-m_i)^i P_0]$$

$$= [ERA\,(1+\alpha)^i\, WIA_1 - ERB\,(1+\beta)^i\, WIB_1] \cdot (1-m_i)^i P_0$$

In consequence, tax revenues from rehabilitants in year i are determined as follows:

$$dTER_i = (1-\pi) \cdot \tau \cdot [ERA \cdot WIA_1 - ERB \cdot WIB_1] \cdot P_0 \qquad i=0$$

$$dTER_i = \tau \cdot [ERA\,(1+\alpha)^i\, WIA_1 - ERB\,(1+\beta)^i\, WIB_1] \cdot [P_0 - \frac{1}{50} \sum_{j=1}^{10} M_j P_j] \qquad i=1,2,..p \qquad (2)$$

IV.1.2 Other Variables

(a) Variations in tax revenues from other persons

The determination of such variations depends upon the state of the economy, in particular that segment of employment open to rehabilitated workers.

$$dTEO_i = \kappa \cdot dTER_i \qquad (3)$$

In this expression, the parameter κ defines the proportion of tax revenues (or earnings since tax revenues are linear in earnings) from rehabilitated workers which are merely due to displacement of other workers. Thus, as unemployment in the relevant segments of the economy is perceived to be increasing, the magnitude of κ should be shifted upwards.

The changes in welfare payments dW_i, are treated as in Conley's model, thus, the actual difference in welfare payments before, and after rehabilitation is assumed to remain constant over time. Hence

$$dW_i = dW \quad \forall i$$

(b) Tax revenue foregone due to rehabilitation

The tax revenue foregone due to rehabilitation, TEF in year 0, is determined as follows:

$$TEF = \pi \cdot \tau \cdot ERB \cdot P_0 \qquad (4)$$

where the terms are as above.

V. SENSITIVITY ANALYSIS

A sensitivity analysis has been performed for the years 1960, 1963, 1966, 1969, and 1970.[8]

The sensitivity analysis is presented as follows: First, the variables to be sensitized are tabulated in order of costs and benefits together with the values assumed. Second, the data used for calculation are tabulated; and third, the results of the sensitivity analysis are presented and evaluated.

V.1 Sensitized Variables and Parameters

Variations have been performed *ceteris paribus*. In the notes below the tables the basic values for each sensitized variable are indicated.

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[8] For a general description of the approach to the sensitivity analysis, see Averous, Stahl and Cole, *op. cit.*, p. 17ff.

Table 1: Sensitivity Adjustments for Cost Side Parameters and Variables

Variable	Sensitivity Adjustment?	Values	Data Sources
R	known	See Table 3	See Table 3
dRC a/	yes	.05R, .02R, .005R	Conley's estimate[9]
dRR	no	0 b/	
RR	known	See Table 3	See Table 3
RT	known	See Table 3	See Table 3
TEF	yes	Sensitized by ERB. See equation (4)[9]	See Table 3
π	no	.85	Conley's estimate[10]
τ	no	.25	

Notes: a/ Basic values: ERB: "reported" values given in Table 3;
dRC: .02 R.

b/ dRR is considered zero since the problem of repetition is taken up by assumptions about productivity changes introduced explicitly.

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[9] Cf. Conley, R. "A Benefit Cost Analysis of the Vocational Rehabilitation Program," Journal of Human Resources, Vol. IV/2 (1969) pp. 226-252.

[10] Cf. Conley, R. "The Economics of Vocational Rehabilitation," *op. cit.*, p. 64.

Table 2: Sensitivity Adjustments for Benefit Side Parameters and Variables

Variable	Sensitivity Adjustment?	Values	Data Sources
dTER	yes	Sensitized by ERB and M_j. See equations (2)	
π	no	.85	Conley's estimate[10]
τ	no	.25	
M_j	yes	Railroad Disabled, U.S. pop., average between railroad disabled and U.S. population	Conley[11]
ERB	yes	See Table 3	See Table 3
α, β	yes	α = -.015 β= .015 = .0 = .015 = .0 = .0 = .025 = .025	
dTEO	no		
κ	no	κ = 0.1	
dW	known	See Table 3	See Table 3
r	yes	0.0, .08, .12	

Notes: Basic values: M_j: average mortality rates; ERB: reported values given in Table 3; α = .0; β = .0; r = .08.

[10] Cf. Conley, R., "The Economics of Vocational Rehabilitation," *op. cit.*, p. 64.

[11] Cf. Conley, R., "The Economics of Vocational Rehabilitation," *op. cit.*, p. 77.

Values for the dated variables in the model are as follows in Table 3.

Table 3: Values for Dated Variables in in the Sensitivity Analysis

Variable	1960[a/]	1963[a/]	1966[b/]	1969[c/]	1970[c/]
R (in $1,000)	80,500	117,050	216,800	458,000	561,300
RR	5,513	10,338	55,000 (RR and RT combined)	63,402 (RR and RT combined)	58,275 (RR and RT combined)
RT	6,117	12,108			
ERB[d/], reported:	967	1,435	2,171	2,991	3,099
variations ($\pm$ $500.00)	450	900	1,700	2,500	2,500
	1,380	1,800	2,600	3,500	3,500
ERA	2,443	2,532	2,801	3,666	3,823
dW (in millions)	11,000	10,900	17,800	17,250	17,250
WIA_1	.83	.83	.83	.83	.83
WJB_1	.78 (years)	.27 (qrtly)	.18 (wk)	.220	.227

Notes: a/ The 1960 and 1963 data are taken from Conley (1965).

b/ The 1966 data are those used by Conley (1969) except RR and RT which have been provided by R.S.A.

c/ The 1969 and 1970 data are from an information memorandum (R.S.A.-IM-72-4).

d/ Variations in ERB are taken quite heavy (subtraction and addition of $500 to reported annual income) to allow for reporting errors and errors due to reporting time.[12]

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[12] See Conley's discussion, Conley (1965), p. 66 ff.

V.2 Results of the Sensitivity Analysis

The outcomes of the sensitivity tests upon the payback period appear in Table 4.

Table 4: Results of Sensitivity Tests on Payback Periods for Selected Years

Variations in Parameters and Variables			Estimated Payback Period[a/] 1960	1963	1966	1969	1970
	α	β					
α,β	-.015	-.015	3.85	3.55	4.5	5.25	5.535
	.0	-.015	3.76	3.49	4.384	5.056	5.324
	.0	.0	3.80	3.52	4.395	5.125	5.420
	.025	.025	3.524	3.45	4.21	4.825	4.900
discount rate	r = .00		2.75	2.42	3.01	3.49	3.65
	r = .08		3.80	3.52	4.395	5.125	5.420
	r = .12		4.03	3.70	4.785	5.696	5.99
mortality rate	railroad disabled		3.845	3.54	4.488	5.235	5.52
	average between RR disabled and U.S. population		3.80	3.52	4.395	5.125	5.420
	U.S. population		3.76	3.505	4.302	5.018	5.285
dRC	.05R		3.60	3.44	4.20	5.02	5.30
	.02R		3.80	3.52	4.395	5.125	5.420
	.005R		3.85	3.57	4.42	5.14	5.45
ERB	reported		3.80	3.52	4.395	5.125	5.420
	reported -500		2.95	3.30	4.01	4.78	4.95
	reported +500		4.70	3.80	4.805	5.57	5.905

Notes: a/ The payback period has been computed up to decimal fractions of years. Note that the payback estimates given are calculated including the program year.

In the interpretation of the results, let us consider variations in the estimation of benefits first. Recalling the bias inherent to the payback model, we could expect, in comparison to a discounted net present value estimate, a high sensitivity to variables affecting short term returns. It is therefore not too surprising that variations in α and β, the productivity changes with and without rehabilitation, do not violently change the numerical value of the estimates derived. The same argument holds for the variations in the mortality rate computed.

Significant changes in the results occur where the discount rate or the earnings before rehabilitation are subject to variations. In the case of the discount rate, variations in the results increase over time, i.e., the 1970 variation is both absolutely and proportionately larger than the 1960 variation. The converse holds for the variation of earnings before rehabilitation.

The former effect may be attributable to the increase in the magnitude of earnings and cost variables over time, which implies an increasing weighting effect of the discount rate. Conversely, the latter effect is due to the constant absolute variation of ±500.00 considered, implying an increasing relative variation of ERB as we move backwards in time: In 1970, the relative variation considered is 20% about the reported annual earnings; in 1960, however, it is more than 46% about reported annual earnings.

The only variation on the cost side that we have introduced in our analyses considers cost adjustments for carry overs. As we might expect, relatively large variations occur due to the bias of the payback approach toward early years after rehabilitation.

In general, the sensitivity analysis shows that imperfect knowledge about the future does not as heavily influence the outcome of payback period estimates as it does estimates based on lifetime benefit streams, such as discounted net present value estimates presented in earlier papers.[13] Conversely, imperfections in the data base may have a substantial impact. Both estimates -- the payback period as well as the discounted present value type of estimate -- are highly sensitive to the choice of the discount rate. Nevertheless, the aggregate variations in values for the payback period over the ten year period, appear to exceed variations due to large changes in parameters that simulate potential errors. Although the conclusion is not statistically testable, we feel justified in arguing (1) that the payback period for the program will range from three to six years, and (2) that it has been increasing.

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[13] See Averous, Stahl and Cole, *op. cit.*

VI. SUGGESTIONS FOR FURTHER DEVELOPMENT OF THE PAYBACK PERIOD MODEL

(1) This analysis is incomplete insofar as disaggregations that account for variations in the tax rate and tax deductions are not included in the sensitized model. Most probably, the model will be highly sensitive to these variations. Furthermore, the assumptions made by Conley in his model could be relaxed.[14]

(2) So far, changes in aggregate welfare payments due to rehabilitation are assumed to stay constant in time. If changes in welfare payments are expected to vary substantially during the years directly following the rehabilitation process, the sensitivity of the payback estimates to such variations could be substantial. In this case, the sensitivity of the estimates could be analyzed in a way similar to that pursued for changes in earnings through time in the GNP model.[15] Thus, the term dW_i should be disaggregated to take account explicitly for

- changes over time in the number of clients on welfare (with, and without rehabilitation)
- changes over time and client population in the level of welfare receipts.

Eventually, the sensitivity of the estimates to mortality rates of welfare recipients at variance to the productive (rehabilitated) population could be investigated.

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[14] See in particular, p. 10, this paper.

[15] See Averous, Stahl and Cole, op. cit., p. 11f.

(3) No analysis of the effects of different assumptions about the state of the economy is included. Most probably they will have a significant influence on the payback period results.

(4) Any payback period results should be presented with various discount rates to allow for best possible control of this parameter.

(5) The analysis suggests a careful consideration of employment at acceptance, and through time, with or without the program, to yield more precise estimates for tax returns for the time periods shortly after rehabilitation. Control group analysis could be of great value in this respect.

(6) Finally, an analysis of the program cost borne by other public agencies may have a significant impact on the payback period estimate, as the variation due to changes in dRC suggests.

GLOSSARY

Note: "d" refers to the variation of a quantity with/without the program.

dTB_i	benefits net of associated program costs to the taxpayer in year i
dTC_0	program cost in year 0
$dTER_i$	incremental tax revenues from rehabilitated population in year i
$dTEF_i$	incremental tax revenues from family of rehabilitants in year i
$dTEO_i$	variations in tax revenues from other persons in year i
dAC_i	variations in associated program costs due to rehabilitation effort borne by the public in year i
dW_i	variations in public assistance payments supporting rehabilitants and other families in year i
R	total nominal program cost in year 0
dRC	program cost not attributable to current year's closures (cost related to program expansion and increased numbers of carry overs)
dRR	special service (repeaters) costs
dRP	program costs borne by other public agencies
RR	governmental expenditures for research in rehabilitation
RT	governmental expenditures for training of professionals in rehabilitation
TEF	tax revenues from rehabilitants foregone during the rehabilitation process
PB	vector as on page 8
PA	vector as on page 8
E	matrix as on page 8
F	vector as on page 8

TD	matrix as on page 8
TR	vector as on page 9
TM	matrix as on page 9
K	vector as on page 9
A	matrix as on page 10
B	matrix as on page 10
C	matrix as on page 10
TE	matrix as on page 11
TER	vector as on page 11
π	fraction of year 0 during which earnings are foregone due to rehabilitation
τ	an aggregate tax rate
dER_i	variation in total earnings of the rehabilitated population in year i
P_0	population rehabilitated, year 0
P_j	population rehabilitated, year 0, in the j^{th} 5 year age group
M_j	mortality rate in the j^{th} 5 year age group
P_i	total population rehabilitated in year 0 surviving to year i
ERB, ERA	level of mean yearly earnings of remuneratively employed rehabilitants at acceptance, B (closure A)
α,β	annual rate of change of yearly earnings with (without) rehabilitation through time reflecting variations in productivity
WIB_1, WIA_1	proportion of rehabilitants remuneratively employed at acceptance, B (closure A)
κ	proportion of total tax revenues originating from displacement of formerly employed workers through rehabilitants
r	discount rate