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**Contributing Shares**

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## **CONTRIBUTING SHARES**

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## CONTRIBUTING SHARES

The contrast between the complexity and variety of financial structures and securities employed by the modern corporation and the spartan simplicity of the prescriptions of the Modigliani-Miller (1958) and Miller (1977) analyses has led researchers to relax the assumptions of the classical approach, in an effort to uncover the determinants of corporate financing decisions. In addition to taxation and considerations of corporate control and risk sharing, two issues have attracted particular attention; the managerial agency problem that is caused by the inability of shareholders to monitor completely the actions of managers, and the adverse selection problem faced by firms in issuing securities to the public.

Grossman and Hart (1982), Jensen (1986a), Hart and Moore (1990) and Stulz (1990) all assume that management has an incentive to undertake new investment projects even when they are unprofitable, and show how capital structure may be used to limit the problem of managerial agency. Jensen points out that debt reduces managerial discretion over the use of internally generated funds, and Stulz, assuming that shareholders may limit managerial access to external funds, shows how an optimal debt ratio may be obtained by balancing the benefits of unprofitable investments foregone against the costs of foregoing profitable investments for lack of internal finance. Hart and Moore extend the analysis to allow managerial access to external finance and show that debt may also be used to prevent management from raising external funds to finance investment projects with negative net present values. The efficiency of these debt solutions to the managerial agency problem will depend upon the correlation between the cash flows from existing assets and the funding required for profitable investment

opportunities.

Myers and Majluf (1985) and others are concerned with the adverse selection problem of a firm that must sell new equity in order to finance an investment project. In order to focus on this problem they ignore the possibility of agency problems between shareholders and managers. Brennan and Kraus (1987) have shown that, in the absence of a managerial agency problem, the Myers-Majluf problem may, under certain circumstances, be resolved by the appropriate choice of financing instrument. In this paper we are concerned with the resolution of the Myers-Majluf problem when there is also a managerial agency problem.

Following Jensen (1986a) and Hart and Moore (1990) we assume that management has a tendency to overinvest - that is, to undertake negative NPV projects even if this is contrary to the interests of current shareholders.<sup>1</sup> Such an assumption makes sense only if there are costs to managers of making low risk zero NPV investments such as purchases of marketable securities, or of returning funds to shareholders by way of stock repurchases or liquidation. Purchases of marketable securities will be negative NPV investments to the extent that the corporation must pay tax on the investment income; stock repurchases have been prohibited in many jurisdictions until recently. It seems likely that there are such costs. Most importantly, a manager may be unwilling to admit that he can find no profitable outlet for the funds at his

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<sup>1</sup> Myers and Majluf (1985) in contrast assume that an investment opportunity is discarded if it turns out to have a negative NPV.

command, since this may be an indication of his lack of entrepreneurial skills<sup>2</sup> and have adverse reputational consequences. This view suggests that there may be an important distinction between internally generated funds and funds raised externally, as implied by Jensen. The only reason for the manager not to invest the former is the lack of a profitable opportunity, whereas a manager may refrain from raising new external finance for the perfectly respectable reason that the share price is too low.<sup>3</sup> In this paper we assume that managers will invest internal funds in the best real investment opportunity available, but that they will raise external finance only if it is in the interest of the old shareholders to do so. However, we also assume, consistent with the reputational considerations outlined above, and contrary to Myers and Majluf, that new external finance will be invested in real investment projects even if those projects have negative NPVs. Thus, we take a view intermediate between those of Myers and Majluf on the one hand and Hart and Moore and Stulz on the other; we assume that managers act in the interests of shareholders to the extent that this is consistent with investing all available funds in real investment projects.

Given this setting, we show that it may be efficient for a firm to issue a class of shares that are partly paid, or assessable. Owners of such shares which, in the United States, have been issued primarily by mining companies, are liable to assessments as determined by the board of directors, for amounts up to the difference between the par value and the original purchase price.

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<sup>2</sup> See Trueman (1988) for a similar explanation of why mutual fund managers trade.

<sup>3</sup> The free cash flow model of Jensen (1986) implicitly assumes that the manager will not raise external finance to pursue unprofitable growth. Hart and Moore (1990) take the opposite view.

For example, the Delaware Statute states that:

Any corporation may issue the whole or any part of its shares as partly paid and subject to call for the remainder of the contribution to be paid therefor...Upon the declaration of any dividend on fully paid shares, the corporation shall declare a dividend on partly paid shares of the same class, but only upon the basis of the percentage of the consideration actually paid thereon.<sup>4</sup>

The discretion of the board concerning assessments may or may not be used to favor the interests of the partly paid shareholders, and may be abused, even though investors are protected to some extent by their right to refuse assessments and thereby forfeit their shares<sup>5</sup>. To understand the nature of the possible abuse consider the following scenario. An investor is given an assessable share whose pro-rata share of assets is \$2, and which is assessable for a further \$8. Having received such a gift an unsuspecting investor might well respond positively to a perceived bona fide assessment for a further \$2 since by meeting the assessment he would have an asset worth \$4, whereas by refusing the assessment he would have to forfeit the share. But then, faced with a further assessment for \$2, the investor must choose between losing the amount already contributed and meeting the assessment. It is apparent that an unscrupulous management might thus be able to extract funds from the investor well in excess of the value of the initial gift, and to syphon off those funds by way of perquisites and excessive salaries. Such a

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<sup>4</sup> Delaware Law Section 156. Prentice-Hall Corporation Statutes Vol. 3 (1986).

<sup>5</sup> See Bloomenthal (1990).

possibility causes the Securities and Exchange Commission to regard assessable shares, which can be offered pursuant to Regulation A, with considerable suspicion; indeed it treats a **gift** of assessable shares as involving a **sale** of a security<sup>6</sup>, and under Rule 136 of the 1933 Securities Act an assessment is treated as an offer for sale which must be registered. This severe attitude on the part of the regulators has made assessable shares virtually unknown in the United States, and Section 23 of the Model Business Corporations prohibits their issuance. However, oil and gas partnerships sold under Rule 133, which are restricted to sophisticated investors, frequently do include provision for the assessment of partners. In Canada, the Canada Business Corporation Act prohibits partly paid shares because "the shareholder is put in a position where he can be forced to put up more money almost at the whim of the issuing corporation or suffer the loss of all or part of what he has already invested"<sup>7</sup>

In Australia in contrast, assessable shares are quite widely used and are known as contributing shares. As of February 1986 there were 56 companies listed on the Australian Associated Stock Exchanges that had contributing shares outstanding. Of these 11 were limited liability companies and 36 were 'no liability' companies<sup>8</sup> which, unlike limited liability companies, cannot enforce assessments, although the shares of shareholders who refuse

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<sup>6</sup> See Bloomenthal (1990). Shareholders generally have the right to refuse the assessment and forfeit their shares.

<sup>7</sup> Dickenson at al. (1971, p 38).

<sup>8</sup> The remaining 11 were trustee companies and trusts that invest in the shares of other companies.



assessments are subject to forfeit and auction to pay the assessment<sup>9</sup>. Interestingly, just as assessable shares were used in the US mainly by mining companies, the no liability corporate form in Australia is reserved for oil and mineral exploration firms. In this paper we show that contributing shares may be a useful device for ameliorating the Myers-Majluf (1985) underinvestment problem which, we argue, is likely to be particularly acute for mining exploration firms. On the other hand, their utility is limited by a moral hazard problem that exists on account of the directors' discretion over the call strategy. We show how the shares may be valued for given call strategies, and use data on Australian contributing shares to compare the call strategies anticipated by investors with those followed by the companies. We find some evidence of a conflict between investor anticipations and realized call strategies. However, the call strategies are generally consistent with what we would expect if contributing shares were issued to ameliorate the Myers-Majluf underinvestment problem.

In Section I we show how the right of a company to sell equity at a predetermined price may mitigate the Myers-Majluf problem. In Section II pricing restrictions for contributing shares are derived under different assumptions about the call policy to be followed by the firm. Section III develops a parametric model for valuing contributing shares. Section IV discusses the empirical evidence on the pricing and call strategies for contributing shares in Australia.

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<sup>9</sup> The defaulting shareholders receive any excess of the auction proceeds over the assessment amount.

## Contributing Shares and Investment Incentives

The adverse selection problem described by Myers and Majluf (1985), that may cause a firm to forego a profitable investment opportunity, may be particularly acute for mining and mineral exploration firms which wish to raise funds to complete an exploration project or to develop an existing property, for then the asymmetry of information between investors and management about the value of prospects is likely to be especially great.<sup>10</sup> In principle, the problem could be overcome by raising sufficient capital to cover any possible future investment outlays at the outset, before the information asymmetry arises; funds that cannot be invested in positive NPV projects could be parked in riskless marketable securities or returned to investors by share repurchase<sup>11</sup>. However, it is clear that such actions constitute an admission on the part of management that it has no positive NPV projects. Management may be reluctant to make such an admission for reputational reasons, and may therefore prefer to undertake a negative NPV real investment<sup>12</sup> project rather than to admit the absence of positive NPV projects<sup>13</sup>. In this

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<sup>10</sup> See Brennan (1990) for a discussion of information asymmetries in the context of natural resources. Exploration companies may be reluctant to communicate the true value of their prospects for competitive reasons - see Bhattacharya and Ritter (1983).

<sup>11</sup> Share repurchases were prohibited in Australia prior to 1990.

<sup>12</sup> Note that it may be difficult for investors to determine ex-post whether a particular risky investment project had a positive NPV.

case funds that are raised at the outset will be invested in new projects even if they have negative NPV's.<sup>14</sup> Then it becomes important that the incremental funds be provided only when the investment opportunity has a positive NPV, and the preparatory slack solution of Myers and Majluf is no longer costless<sup>15</sup>.

In this section we assume that any funds raised will be invested in real opportunities even if they have negative NPV's, and show that contributing shares may provide at least a partial solution to the Myers and Majluf problem. Thus consider a firm that is established at  $t_0$  to explore for a mineral deposit. The exploration program has two phases. The first phase costs  $I_1$  and results in the development, at a possibly random date  $t_1$ , of an asset whose value which becomes known only to the managers of the firm at  $t_1$  is denoted by  $\tilde{a}$ . The second phase of the program starts at  $t_1$  and costs  $I_2$ ; the net present value of the second phase as assessed at  $t_1$  is denoted by  $\tilde{b}$ .

We assume that all funds raised are invested in real projects, that the interest rate is zero, and that investors are risk neutral. Then, if the funds for the whole investment program are

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<sup>13</sup> Note that the situation is different when internal finance is not available. Management has the good excuse for not investing that finance is not available on suitable terms - because of the Myers-Majluf problem.

<sup>14</sup> Some anecdotal evidence to this effect is to be found in Jensen (1986b). McConnell and Muscarella (1985) present evidence that oil companies in the 1970's were overinvesting in exploration and development.

<sup>15</sup> Note that the debt solutions of Hart and Moore (1990) and Stulz (1990) to the managerial agency problem of overinvestment ignore the Myers-Majluf problem, in the one case by prohibiting any external finance and in the other by assuming away any information asymmetry.

raised at  $t_0$ , both phases of the program will be undertaken for sure and  $V_0$ , the value of the firm at  $t_0$ , before any funds are raised, is given by

$$V_0^* = E_0[\tilde{a} + \tilde{b}] - I_1 \quad (1)$$

where  $E_0[\ ]$  denotes expectations conditional on the information available at time 0. Note that this policy is likely to be inefficient since the second stage of the project will proceed even if  $\tilde{b}$  is negative. The first best financing solution would ensure that the second phase of the project was undertaken only if  $\tilde{b} > 0$ . Thus,  $V_0^*$ , the value of the firm under the first best policy, is given by:

$$V_0^* = E_0[\tilde{a} + \max(\tilde{b}, 0)] - I_1 \quad (2)$$

Now consider an alternative financing policy under which the equity funds to finance the firm are raised in two stages, and suppose for simplicity that the firm considers an issue of equity at time  $t_1$  when it has exhausted its first tranche of investment funds,  $I_1$ , so that it has no financial slack. As in Myers and Majluf<sup>16</sup>, the second stage issue decision is assumed to be taken in order to maximize the wealth of the first stage shareholders.

It is reasonable to assume that investors have some information about the first phase of the investment program, and we shall assume that at time  $t_1$  investors receive a noisy signal  $\tilde{y}$

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<sup>16</sup> Dybvig and Zender (1990) discuss managerial contracts that will induce managers to maximize the value of the firm without regard to the interests of old shareholders.

of the sum of the value of the asset in place and the net present value of the investment opportunity:

$$\tilde{y} = \tilde{a} + \tilde{b} + \tilde{\epsilon} \quad (3)$$

where  $\tilde{\epsilon}$  is a random noise term.

Let  $P'(y)$  denote the value of the original equity shares if a new issue of equity is made at time  $t_1$ . Then, since the managers are assumed to make the equity issuance decision to maximize the value of the original equity, it follows from the analysis of Myers and Majluf that a new stock issue will be made to fund the required second phase investment  $I_2$  if and only if:

$$\frac{b}{I_2} > \frac{a}{P'(y)} - 1 \quad (4)$$

where  $P'(y)$  is defined by:

$$P'(y) = E_0[ a + b | y, b > I_2(\frac{a}{P'(y)} - 1) ] \quad (5)$$

Condition (4) defines the acceptance region, the values of (a,b) for which it will be optimal from the point of view of the original equityholders to raise capital and invest; for values of (a,b) that fall outside the acceptance region the second phase of the project will be rejected. In Figure 1 the acceptance region for the signal  $y_i$  ( $i=1,2$ ) is the area lying above the line  $OC_i$ . Since the acceptance regions include areas for which  $b < 0$  and exclude areas for which  $b > 0$ , it is apparent that the value of the firm under the two stage financing policy is in general less

than  $V_0^*$ , the firm value under the first best policy<sup>17</sup> - it may even be less than under the preparatory slack policy of raising the whole amount of the potential investment budget,  $I \equiv I_1 + I_2$ , at  $t_0$ , depending on the joint distribution of  $a$  and  $b$ .

Note that a prior issue of debt as suggested by Stulz (1990) and Hart and Moore (1990) will not alleviate the problem of over- and underinvestment. Thus, supposing for simplicity that riskless debt with a promised payment of  $D$  at  $t_2$  has been issued at  $t_0$ , the acceptance region would be changed as follows: the line  $OC_i$  would rotate in an anti-clockwise direction about its intercept with the  $a$ -axis. The effect of this is perverse in that it reduces the size of the acceptance region for  $b > 0$ , while increasing it for  $b < 0$ .

To this point we have followed Myers and Majluf in leaving the joint distribution of  $a$  and  $b$  unspecified. However, in the case of a mineral exploration firm, it is likely that  $a$  and  $b$  are positively correlated: if the first phase of exploration has been unsuccessful then it is unlikely to be worth investing further funds in exploration or development in the same area. On the other hand, if the first phase has been successful then it is likely that the returns to further investment will be high. To illustrate the potential role of contributing shares in this context we shall consider the extreme case in which the net present value of the new investment project is an exact linear function of the asset in place:

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<sup>17</sup> Note that this is true even if management follows the Myers-Majluf convention and never invests in negative NPV projects.

$$b = \alpha + \beta a \quad (6)$$

where  $\alpha < -I_2$ <sup>18</sup>.

This linear relation is represented by the dotted line AB in Figure 1. It is apparent now that for  $y = y_1$  no new capital will be raised, even if  $b > 0$ , whereas for  $y = y_2$  the new investment will often be made even if  $b < 0$ . Thus, as in the models of Stulz and Hart and Moore, inefficiencies caused by both underinvestment and overinvestment may occur. Now suppose that the firm is able to sell new equity at a price which corresponds to a valuation of  $P^*$  for the original equity<sup>19</sup>, where  $P^*$  is chosen so that the ray through O with slope  $I_2/P^*$  passes through the point X. Inspection of Figure 1 reveals that  $b > 0$  whenever  $(a,b)$  falls within the acceptance region defined by the price corresponding to  $P^*$ , so that no positive NPV investments are rejected. It is still possible however that if  $P'(y) > P^*$  for some  $y$  then the second phase of the investment program will be carried out, even when the net present value of doing so is negative. Nevertheless, this will only occur when high signal values are accompanied by low asset values - in other words, only for extreme negative realizations of the signal error. Moreover, under the Myers-Majluf assumption that negative NPV projects are not undertaken, the ability to sell equity at a predetermined price would achieve the first best solution.

Thus the ability of the firm to sell equity at a predetermined price corresponding to  $P^*$

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<sup>18</sup> If  $\alpha > -I_2$ , then the proposed solution is not available.

<sup>19</sup> For example, if there was only one old share the new shares would be issued at a price of  $P^*$ .

may significantly ameliorate the Myers Majluf problem in those cases where investors are not able to evaluate well the value of assets in place and investment opportunities, but where the net present value is highly correlated with the value of assets in place. We have argued that this may not be unrepresentative for many mineral exploration companies which are established to explore a small number of prospects. We shall see in the following section that contributing shares allow the firm to sell equity at a predetermined price and may therefore be a rational contract in such circumstances<sup>20</sup>.

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<sup>20</sup> It has been suggested to us that warrants play much the same role as do contributing shares in allowing the company to raise equity at a predetermined price. However, warrants allow the firm to sell equity at a predetermined price only when that price is below the current market price, whereas it is evident from Figure 1 that the efficiency gains come from being able to raise equity at a price  $P^*$  that is above the current market price  $P'(y)$ .



## II

### Restrictions on Contributing Share Prices under Alternative Call Policies

A contributing share is a contingent claim whose payoffs and value depend on the value of the underlying firm and its stochastic process, as well as the policy that is followed by the issuing firm in making a call for contributions. Thus define:

**P** the price of a contributing share when a fraction  $(1 - f)$  of the issue price has been paid<sup>21</sup>

**S** the price of a fully paid share

**$n_1$**  the number of contributing shares outstanding

**$n_2$**  the number of fully paid shares outstanding

**M** the per share par value for both classes of shares

The rules of the Australian Associated Stock Exchange, like the Delaware Statute, require that the dividend paid on the contributing share,  $d_1$ , be equal to the product of the dividend paid on a fully paid share and the fraction of the issue price paid to date:

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<sup>21</sup> The issue price is typically the same as the par value of the share. If the issue price is above the par value then  $f$  is defined as the fraction of the issue price that has been paid.

$$d_1 = (1-f)d_2 \quad (7)$$

This implies that each contributing share is entitled to a fraction  $q$  of the aggregate dividend, where

$$q = \frac{(1-f)}{(1-f)n_1 + n_2} \quad (8)$$

Define  $V$  as the total value of the firm's equity<sup>22</sup>. Then, assuming that the firm is entirely equity financed,

$$V = n_1P + n_2S \quad (9)$$

In the event of a call for contributions, whose timing is at the discretion of the directors of the company, each contributing share will pay the outstanding balance  $fM$ <sup>23</sup>, and will then own a fraction  $1/(n_1 + n_2)$  of the total value of the firm,  $V$ , augmented by the new capital raised,  $K \equiv n_1fM$ . Therefore the value of a contributing share at the time of call is given by

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<sup>22</sup> We shall assume that the firm is financed entirely by equity in the form of contributing and fully paid shares. The issuers of contributing shares considered in this paper make very little use of debt. However, they do issue warrants. For an extension of the analysis to incorporate warrants see Dunlop (1989).

<sup>23</sup> For simplicity we consider only 100% calls; in reality, the firm may issue a call for less than the outstanding balance.

$$P = \frac{V + K}{n_1 + n_2} - fM \quad (10)$$

$$= hV - k \quad (11)$$

where  $h \equiv 1/(n_1 + n_2)$  and  $k \equiv n_2 fM / (n_1 + n_2)$ . Expressions (10) and (11) implicitly assume that the contributing shareholder has an unlimited liability to make contributions if called upon to do so. This is the case with so-called limited liability companies, which are mainly industrial firms. However, owners of contributing shares in so-called "no-liability" companies<sup>24</sup> have the option of not paying the call and thereby forfeiting their shares. In this paper we are concerned exclusively with no-liability companies. Then, taking account of the default option of the shareholder, the value of a contributing share at call is given by:

$$P = \max[hV - k, 0] \quad (12)$$

Substituting for  $V$  from equation (10), and using the definition of  $q$ , it may be verified that expression (13) is equivalent to:

$$P = \max[S - fM, 0] \quad (13)$$

Arbitrage considerations dictate that the value of a contributing share satisfy an upper and a lower bound:

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<sup>24</sup> This corporate form is reserved for oil and mineral exploration firms.

**Condition 1**

$$P \leq S \quad (14)$$

This upper bound follows immediately from the fact that the contributing share is never entitled to a payoff larger than that of a fully paid share. The lower bound is determined by the consideration that the contributing share cannot sell for less than its pro-rata share of the value of the fully paid share less the value of the contributions remaining to be made. Taking account of limited liability, this gives rise to:

**Condition 2**

$$P \geq \max[S(1 - f) - fM, 0] \quad (15)$$

In order to derive further restrictions on the pricing of contributing shares it is necessary to make some assumptions about the call policy followed by the issuer. We consider initially three extreme policies - the no-call policy, the value maximizing policy, and the value minimizing policy.

**Proposition 1:** Under a no-call policy in which no calls are ever made on the contributing shareholders, the value of a contributing share is given by:

$$P = (1 - f)S \quad (16)$$

This follows immediately from the observation that the voting and dividend rights of the contributing shareholders are proportioned to the fraction of the issue price that has been paid.

**Proposition 2:** Under a value-maximizing call policy, in which the call decision is made to maximize the value of the contributing shares,

(i) the value of a contributing share satisfies:

$$P \leq \max[S(1 - f), S - fM] \quad (17)$$

(ii) a call is never made if  $S < M$ .

The first part of the proposition follows from the observation that under a value maximizing policy the value of the contributing share cannot be less than under the no-call policy, or less than if the call were made immediately. The second part follows from the observation that under a value-maximizing strategy the contributing share is equivalent to  $(1 - f)$  fully paid shares, plus a perpetual warrant to purchase  $f$  shares at an exercise price  $M$ : it never pays to exercise the warrant when  $S < M$ .

**Proposition 3:** Under a value-minimizing call policy:

$$(i) \quad P \leq \min[S(1 - f), \max[S - fM, 0]] \quad (18)$$

$$- \max[S - fM, 0] - f \max[M - S, 0]$$

$$(ii) \quad \text{No call is made when } S > M \quad (19)$$

The first part of the proposition states that the value under the value minimizing policy can never be more than under the no-call policy, or if a call is made immediately. The second part follows from the observation that under the value-minimizing strategy the contributing share is equivalent to  $(1 - f)$  fully paid shares less  $f$  put options with exercise price  $M$ : it is never optimal to exercise the put when it is out of the money.

The pricing restrictions we have derived for the value maximizing and minimizing policies implicitly assume that the management is able to employ the funds raised by a call in a zero NPV project such as a share repurchase or an investment in other marketable securities. We suggested in the previous section that management may be reluctant to do this for reputational reasons. If the investment of funds is restricted to real projects and if, as assumed in the previous section, decisions are made in the interest of the old fully paid shareholders, then a call will be made only if  $S < M$  and the asset-investment opportunity pair falls into the acceptance region of Figure 1. We shall call this a Myers Majluf ("MM") call policy. As the following proposition states, the value of the contributing shares under a MM policy is always less than under the no-call policy.

**Proposition 4:** Under an MM call policy:

(i)  $P \leq S(1 - f)$

(ii) calls are only made when  $S < M$ .

The second part of the proposition follows from the previous discussion, and the first part is an immediate consequence of the fact that calls are unprofitable for the contributing shareholder.

The pricing restrictions under the different call policies are summarized in Figure 2. In this figure regions 1 and 4 are prohibited by arbitrage considerations; the line OE is the rational price schedule under the no-call policy; only region 2 is consistent with a value maximizing strategy, and only region 3 is consistent with a value minimizing strategy. Under a MM call policy the price must always lie below the line OE.

### III

#### An Explicit Pricing Model

If the call policy of the issuing firm may be written as at most a function of the value of the firm and time, the contributing share may be treated as a simple contingent claim on the value of the firm and valued by the classic techniques of Black and Scholes (1973). Thus, assume that the issuing firm pays no dividends<sup>25</sup>, that the interest rate is a constant  $r$ , that the firm is financed exclusively by equity, and that its value follows a stochastic process:

$$\frac{dV}{V} = \mu dt + \sigma dz \quad (20)$$

where  $dz$  is the increment to a Gauss-Wiener process. Then the value of a contributing share may be written as a function of the current value of the firm,  $P(V)$ . Let  $\lambda(V)$  denote the instantaneous probability rate of a call when the firm value is  $V$ , and let  $\kappa(V)$  denote the value of a contributing share if it is called when the firm value is  $V$ . Then, assuming that there is no risk premium associated with the event of a call, standard arbitrage arguments may be used to

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<sup>25</sup> This is not a very restrictive assumption in the current context. Out of 381 Mining and Oil companies listed on the Australian Stock Exchanges as at February 1986, only 43 had paid dividends in the previous 5 years. Two of these companies were included in the sample used in the empirical study below - they had each paid only a single dividend in the previous five years.



show that the value of a contributing share satisfies the ordinary differential equation:

$$\frac{1}{2}P''V^2\sigma^2 + P'rV + \lambda(V)[\kappa(V) - P] - rP = 0 \quad (21)$$

If  $\kappa(V) = hV - k$  and  $\lambda(V) = \lambda$ , then the complete solution to equation (21) may be written as

$$P(V) = C_1V^{\alpha_1} + C_2V^{\alpha_2} + aV - \frac{\lambda b}{(r + \lambda)} \quad (22)$$

where

$$\alpha_1 = \left\{ \frac{1}{2}\sigma^2 - r + \left[ \left( r - \frac{1}{2}\sigma^2 \right)^2 + 2\sigma(r + \lambda) \right]^{\frac{1}{2}} \right\} / \sigma^2 \quad (23)$$

and

$$\alpha_2 = \left\{ \frac{1}{2}\sigma^2 - r - \left[ \left( r - \frac{1}{2}\sigma^2 \right)^2 + 2\sigma(r + \lambda) \right]^{\frac{1}{2}} \right\} / \sigma^2 \quad (24)$$

and  $C_1$  and  $C_2$  are constants chosen to satisfy the boundary conditions. Note that  $\alpha_1 > 1$  and  $\alpha_2 < 0$ .

Under a value minimizing strategy  $\kappa(V) = \lambda(V) \equiv 0$ , for  $V > k/h$ , so that  $C_1 = 0$  and the value of a contributing share is given by:

$$P(V) = C_2V^{\frac{-2r}{\sigma^2}} \quad (25)$$

where  $C_2$  is a constant chosen to satisfy the boundary condition that the value of the share be given by expression (12), when the call is made at  $V = v$ :

$$hv - k - C_2 v^{\frac{-2r}{\sigma^2}} \quad (26)$$

Since the call policy is chosen to minimize the value of the share, it is immediately seen that the optimal policy is to call when  $V = k/h$ , and that this policy implies that the value of the assessable shares is equal to zero.

Under a value-maximizing strategy a call is never made<sup>26</sup> and  $\kappa(V) - \lambda(V) = 0, C_2 = 0$ <sup>27</sup>, so that

$$P(V) = C_1 V \quad (27)$$

The condition that  $S \geq P \geq \text{Max}[S - fM, 0]$  then implies that  $C_1 = h$ , so that  $P = S$ , and the value of a contributing share is equal to the value of a fully paid share.

We defined a Myers-Majluf call policy as one in which a call is made only if  $S < M$  and

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<sup>26</sup> Since no dividends are paid the contributing shareholder loses nothing by postponing the date of his contribution indefinitely.

<sup>27</sup> Otherwise the conditions that  $0 \leq P \leq V$  will be violated for sufficiently small values of  $V$ .

the asset-investment pair falls into the acceptance region. The condition that  $S < M$  following a call is equivalent to the condition that  $V < [n_1(1 - f) + n_2]M \equiv \hat{V}$  immediately prior to the call. If the probability that the asset investment pair falls into the acceptance region is independent of  $V$ , then the probability of a call will be constant for  $V > \hat{V}$ . Therefore define the following generalization of a Myers-Majluf call policy:

$$\begin{aligned} \lambda(V) &= \lambda_M \text{ for } V < V^* \\ \lambda(V) &= \lambda_0 \text{ for } V > V^* \end{aligned} \quad (28)$$

Under the generalized Myers-Majluf call policy, the value of a contributing share is given by the solution to equation (21) where  $\lambda(V) = \max[hV - k, 0]$ , and  $\lambda(V)$  is given by (28):

$$P_1(V) = C_{11} V^{\alpha_{1M}} + C_{12} V^{\alpha_{2M}}, \quad V < k/h \quad (29)$$

$$P_2(V) = C_{21} V^{\alpha_{1M}} + C_{22} V^{\alpha_{2M}} + hV - \frac{\lambda_M k}{r + \lambda_M} \quad k/h \leq V \leq \hat{V} \quad (30)$$

$$P_3(V) = C_{31} V^{\alpha_{10}} + C_{32} V^{\alpha_{20}} + hV - \frac{\lambda_0 k}{r + \lambda_0} \quad \hat{V} < V \quad (31)$$

where  $\alpha_{iM}$  and  $\alpha_{i0}$  ( $i=1,2$ ) are defined by equations (22) and (23) with  $\lambda = \lambda_M$  and  $\lambda_0$

respectively and the  $C_{ij}$  are constants determined as follows. Since  $P < V$  and  $\alpha_{2M} < 0$   $C_{12} = 0$ ; similarly, since  $\alpha_{10} > 1$ ,  $C_{31} = 0$ . The remaining four constants are determined by the set of four linear equations implied by the conditions that the valuation function be smooth and continuous at  $V = k/h$  and  $V = \hat{V}$  :

$$P_1(k/h) - P_2(k/h) \tag{32}$$

$$P'_1(k/h) - P'_2(k/h) \tag{33}$$

$$P_2(\hat{V}) - P_3(\hat{V}) \tag{34}$$

$$P'_2(\hat{V}) - P'_3(\hat{V}) \tag{35}$$

## IV

### **Empirical Evidence on Contributing Shares**

The original sample consists of 30 No Liability firms whose fully paid and contributing shares were traded simultaneously on the Associated Australian Stock Exchanges for a period of at least one year between July 1981 and March 1986; three firms were dropped because they were very thinly traded. For each class of shares the last sale price of the day was taken as the closing price. Details of the issues were taken from Personal Investment and its predecessor Australian Stock Exchange Journal and were verified against the company files held at the Sydney Stock Exchange.

In order to determine whether the pricing of contributing shares violated the arbitrage conditions (14) and (15) the closing prices of contributing and non-contributing shares were compared for all days on which the trading volume in each class was at least 10,000 shares. This minimum volume requirement was imposed in order to minimize the problems caused by non-synchronicity of the closing prices for the two classes of shares. Violations were observed on 49 out of 2004 firm-days. All the violations related to the same firm, and all were violations of the

lower bound condition (15). No explanation is known for these violations.<sup>28</sup>

Table 1 reports the frequency of violations of the boundary conditions imposed by the different call policies described in Section II for 10,000 share days. Approximately 80% of the price observations are inconsistent with the boundary condition imposed by a value-maximizing call policy. Moreover, the higher is the number of available observations for an individual firm, the higher is the proportion that are inconsistent with value maximization. Only firm 27 had no observations that were inconsistent with value maximization<sup>29</sup>; however, firm 17 had only 9.5% of its observations inconsistent with value maximization, and this fell to 7.1% when all 168 matching daily trades were included; firm 12 had only 12% of its observations inconsistent with value maximization, and this falls to 4.4% of all matching daily trades. It should be noted of course that even those observations which are consistent with a value-maximizing policy do not imply that the shares are priced consistent with such a policy. Thus there is strong evidence that, except for three firms, investors did not believe that the issuers of contributing shares were following call policies designed to maximize the value of the contributing shares.

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<sup>28</sup> The low price of the contributing shares of Energy Oil and Gas was commented on by the firm's directors who, in a telex to the Perth Stock Exchange dated February 21, 1983, stated that "The directors of Energy Oil and Gas NL have noted that a discrepancy exists between the market price of fully paid shares and the market price of contributing shares and options. Whilst the directors are unable to substantiate any reason for the market preference for fully paid shares they wish to clarify the position with regard to options and payments of calls on contributing shares".

<sup>29</sup> While this firm had only 42 trading days that met our minimum trading volume requirement, all 285 days on which matching trades were found were consistent with value maximization (and therefore inconsistent with value minimization or a MM call policy).

The other extreme possibility is that firms should pursue call policies to maximize the value of the fully paid shares or to minimize the value of the contributing shares. No firm has fewer than 60% of its observations inconsistent with value-minimization. Thus it seems that investors do not expect firms to pursue policies that will minimize the value of the contributing shares. Since a value minimizing policy would generally mean making a call when the prospects for the company are bad, it is possible that the absence of a value minimizing policy can be attributed to a management reluctance to invest in riskless zero NPV projects.

Under a Myers-Majluf call strategy, calls are made only if  $S < M$  and the asset investment opportunity pair falls within the acceptance region. Thus calls are always unprofitable for the contributing shareholders, and the contributing shares are worth less than the prorated value of a fully paid share as described in Proposition 4. For five of the firms, more than 90% of the price observations are consistent with a Myers-Majluf call policy; on the other hand, nine of the firms have over 90% of their observations inconsistent with a Myers-Majluf call policy.

Table 2 provides further detail on the pricing of contributing shares along with information about the calls that actually were made<sup>30</sup>. Column 3 of the table reports the fraction of price observations for which the fully paid shares traded below par. Columns 4-7 of the table show the proportion of price observations for which the fully paid shares were above

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<sup>30</sup> We attempted to estimate the parameters of the valuation model developed under a MM call policy, equations (28)-(30), but for most firms were unsuccessful, either because the call probability parameters were unstable or because there were insufficient observations for  $S > M$  and  $S < M$  to permit identification of the parameters.

(below) par and the contributing shares were above (below) the no call value of  $S(1 - f)$ . If calls are expected to be made when the fully paid shares are above (below) par then we should expect the contributing shares to trade above (below) their no call value. There is a strong tendency for the contributing shares to trade above (below) the no call value when the fully paid shares are trading above (below) par - note the preponderance of observations in columns 3 and 6 - suggesting that investors expect calls to be made both above and below par. In particular it is quite rare for the contributing shares to trade below the no call value when the fully paid are above par. For nine firms over 90% of the contributing share prices are below the no call value and four of these firms actually made calls below par. Five firms had over 90% of their contributing share prices above the no call value - only one of these firms made a call on the contributing shares (above par). The average ratio of the contributing share price to its no call value given in column 7 was 126%; for firms that made calls that were unprofitable for the contributing shareholders the average ratio was 78%, while for firms that made no calls the average ratio was 140%. For the two firms that made profitable calls the average price ratio was 202%.

Ten firms actually made calls during our sample period, three firms making two calls. Columns 9 and 10 show the fraction of par value remaining unpaid before and after the calls. Calls were typically for 10-20% of the par value, and in no case was a call made for the whole of the outstanding unpaid balance. This is inconsistent with a policy of maximizing or minimizing the value of the contributing shares, but is consistent with what we have described as a Myers-Majluf policy of calling only when there is a profitable use for the funds. Column



12 reports the loss realized by the contributing shareholders for each call, as a fraction of the no call value. This loss is calculated as the difference between the cash subscribed and the value of the increased ownership share as measured by the corresponding fraction of a fully paid share at the first price observation after the contribution. Of the fourteen calls, eleven were unprofitable for the contributing shareholders while the remaining three were profitable. One of these (firm 8) raised a negligible amount of capital, and one yielded only a 4% profit to contributing shareholders. However the third (firm 24) made a call for contributions when its shares were trading at a 40% premium to par. This is clearly inconsistent with what we have called a Myers-Majluf call strategy.

Figure 3 plots the average price to par value ratio for the fully paid shares around the time of a call for contributions. In this figure trading days refer to the days on which both classes of shares traded. The figure is drawn both with and without firm 8 which has a disproportionate effect on the data since at the time of its call it was trading at over ten times the par value, yet the call raised only a small amount of money. Excluding this firm, there is no pronounced evidence of a price drop prior to the call; the average price ratio was 0.72 at the time of the call which is consistent with the unprofitability of the calls. Figure 4 plots the average ratio of the contributing share price to its no-call value around the date of a contribution. This ratio is well above unity 100 trading days before the call, implying that investors are expecting a call above par. However, by the time of the call this price ratio has dropped to around 0.70 implying that investors are expecting subsequent calls to be unprofitable to contributing shareholders.

In summary, most calls took place at a loss to contributing shareholders, because the fully paid shares were trading below par; in only one case did our (noisy) measure of the realized loss from contributing exceed 100%, implying that the firms were not attempting to minimize the value of the contributing shares. This is as predicted by what we have called a Myers-Majluf call policy. On the other hand, for 45% of the price observations the contributing share price exceeds its no call value, implying that investors expect future calls to be profitable on average, which is inconsistent with a Myers-Majluf call strategy.

The variation in the market's assessment of the profitability of future calls is puzzling in light of our model. One possibility is that corporate insiders change their relative ownership of contributing and fully paid shares and that this acts as a signal of the profitability of future calls. If this is so, then it seems that contributing shares, while offering a solution for one type of managerial agency problem, give rise to another. A second possibility is that there is an implicit contract that contributing share calls will be made (eventually) if the firm does well, so that management is not maximizing the value of the fully paid shares as our Myers-Majluf call policy assumes. In any event, the findings are of interest in light of the frequent assumption in financial models that management maximizes the interest of one class of investors (stockholders) at the expense of the others (bondholders). That is clearly not the case here, and similar considerations may offer an alternative to the signalling explanation of why convertible bonds are called 'late'.

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TABLE 1

Firm	Numbers of Observations	Violations of		
		Value Maximization	Value Minimization	Myers-Majluf
1	505	97.4%	82.0%	5.3%
2	38	29.0	97.4	97.4
3	176	97.7	81.0	2.3
4	134	91.8	73.1	8.2
5	36	86.1	61.1	58.3
6	138	80.6	92.7	92.0
8	21	100.00	100.00	0.0
10	19	84.2	100.00	100.00
11	18	66.7	83.3	33.3
12	10	10.0	100.0	100.0
13	126	100.00	70.6	0.0
15	27	96.3	96.3	11.1
16	107	77.6	66.1	34.6
17	21	9.5	100.0	100.0
18	108	69.4	81.5	69.5
19	53	41.5	79.2	67.9
20	14	28.6	100.0	100.0
22	14	50.0	100.0	50.0
23	139	99.3	100.0	99.3
26	168	28.6	97.0	92.2
27	42	0.0	100.0	100.0
28	73	93.2	85.0	78.1
WEIGHTED AVERAGE		79.8	84.4	42.6

Proportion of Matching Daily Trades Violating Boundary Conditions imposed by Different Call Policies on days when Volume exceeds 10,000 shares in both Classes of Share.

TABLE 2  
Summary of Pricing of Contributing Shares and Calls made.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Firm No.	No. Obs.	P < M % Obs	Proportion of Observations				Avg P/S(1-f)	f		Contrib share pr after call	Realized loss/S(1-f)	Cash Raised	
			FP > M P > S(1-f)	FP > M P < S(1-f)	FP < M P > S(1-f)	FP < M P < S(1-f)		Before Call	After Call			Per Share	/Firm
		%	%	%	%	%	%	%	%	\$	%	\$	%
1	709	94	2	2	3	93	75	40	30	0.23	4	0.05	9.8
2	179	5	89	2	6	3	129	30	20	0.10	20	0.05	10.2
3	318	100	0	0	1	99	50	56	38	0.15	14	0.09	39.1
4	301	100	0	0	6	94	68	60	40	0.17	19	0.10	18.4
5	128	23	38	20	16	26	107	60	50	0.20	4	0.05	10.3
6	359	3	90	6	1	4	110	40	30		5	0.05	4.0
7	57	100	0	0	0	100	25						
8	210	0	100	0	0	0	133	30	20	3.10	(13)	0.02	0.0
9	113	96	0	1	12	87	85						
10	18	23	33	0	67	0	337						
11	266	77	0	22	0	78	32	50	40	0.10	5	0.05	9.4

1	2	3	4	5	6	7	8	9	10	11	12	13	14
12	30	20	40	40	0	20	88						
13	89	43	38	16	8	38	93	40	38	0.01	108	0.02	15.6
14	249	79	10	4	14	72	90	30	21	0.15	10	0.05	14.4
15	152	51	42	0	58	0	209						
16	345	31	50	12	8	31	102						
17	46	78	2	0	57	41	104						
18	83	31	100	0	0	0	102						
19	78	0	100	0	0	0	191						
20	37	97	0	0	100	0	212						
21	178	100	0	0	1	99	40						
22	37	89	0	8	3	89	57						
23	24	92	0	0	75	25	115						
24	291	26	54	1	30	15	271	90	60	0.09	(86)	0.03	9.9
25	252	100	0	0	100	0	358						
26	158	14	68	17	0	15	109						
27	56	29	61	4	29	7	112						

(TABLE 2 Cont.)

## Notes to Table 2

Column 1. Firm identification

Column 2. Number of trading days on which both contributing and fully paid shares traded.

Column 3. Proportion of observations for which price of fully paid shares is less than par value.

Columns 4-7. The proportions of price observations for which the contributing and fully paid shares fall above and below the no-call and par values respectively.

Column 8. The average ratio of contributing share price to the no-call value.

Columns 9-10. The proportions of par value remaining unpaid before and after the call.

Column 11. The contributing share price at the first price observation after the call.

Column 12. The realized loss relative to the pre-call no-call value. The loss is measured as the difference between the value of the appropriate fraction of a fully paid share and the amount contributed.

Columns 12, 13. Cash raised by the call per share and as a proportion of the aggregate value of contributing and fully paid shares.



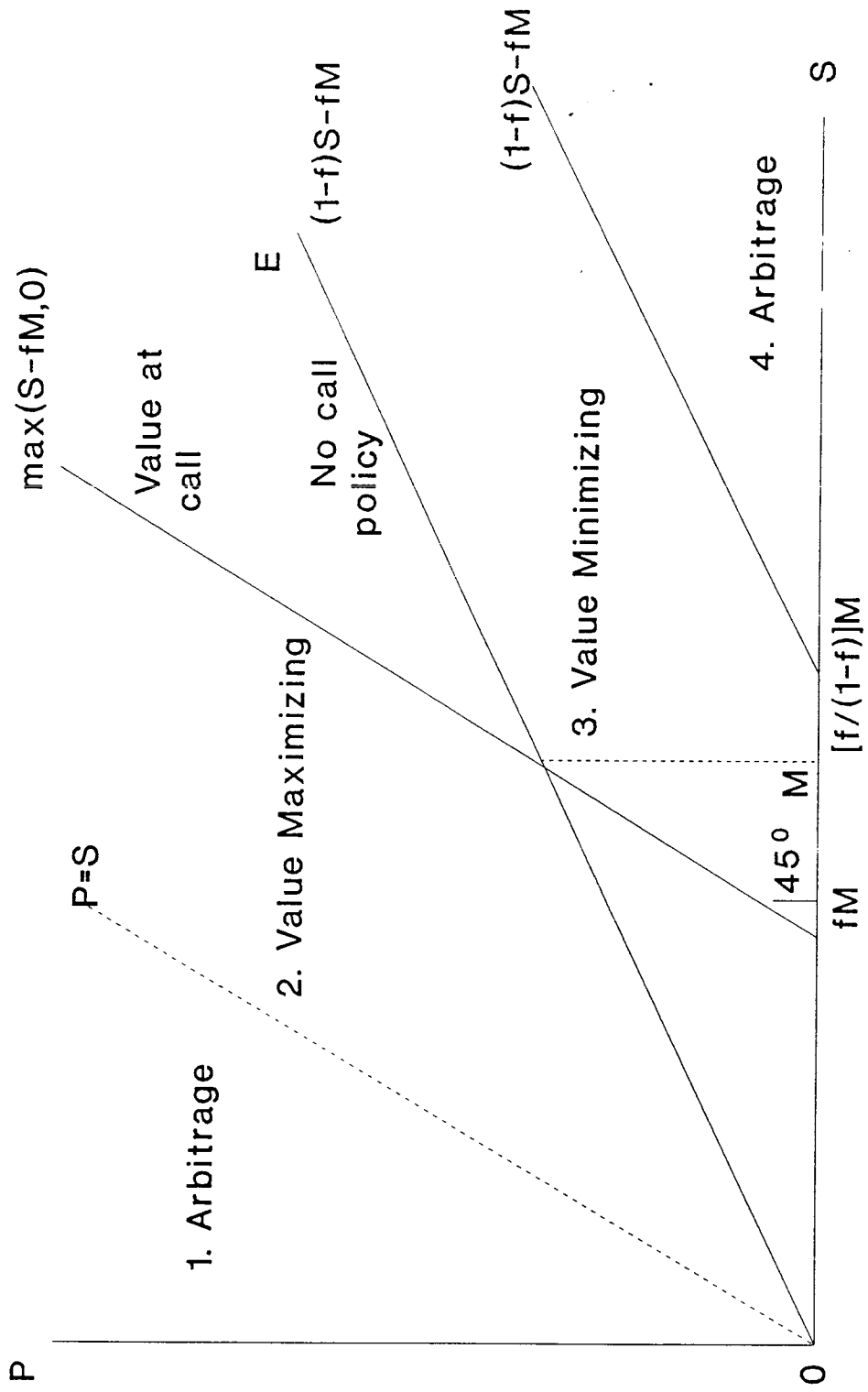
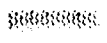


Figure 2: Restrictions on Rational Pricing under Different Call Policies  
 Under the MM call policy the price of contributing shares must fall below the line OE



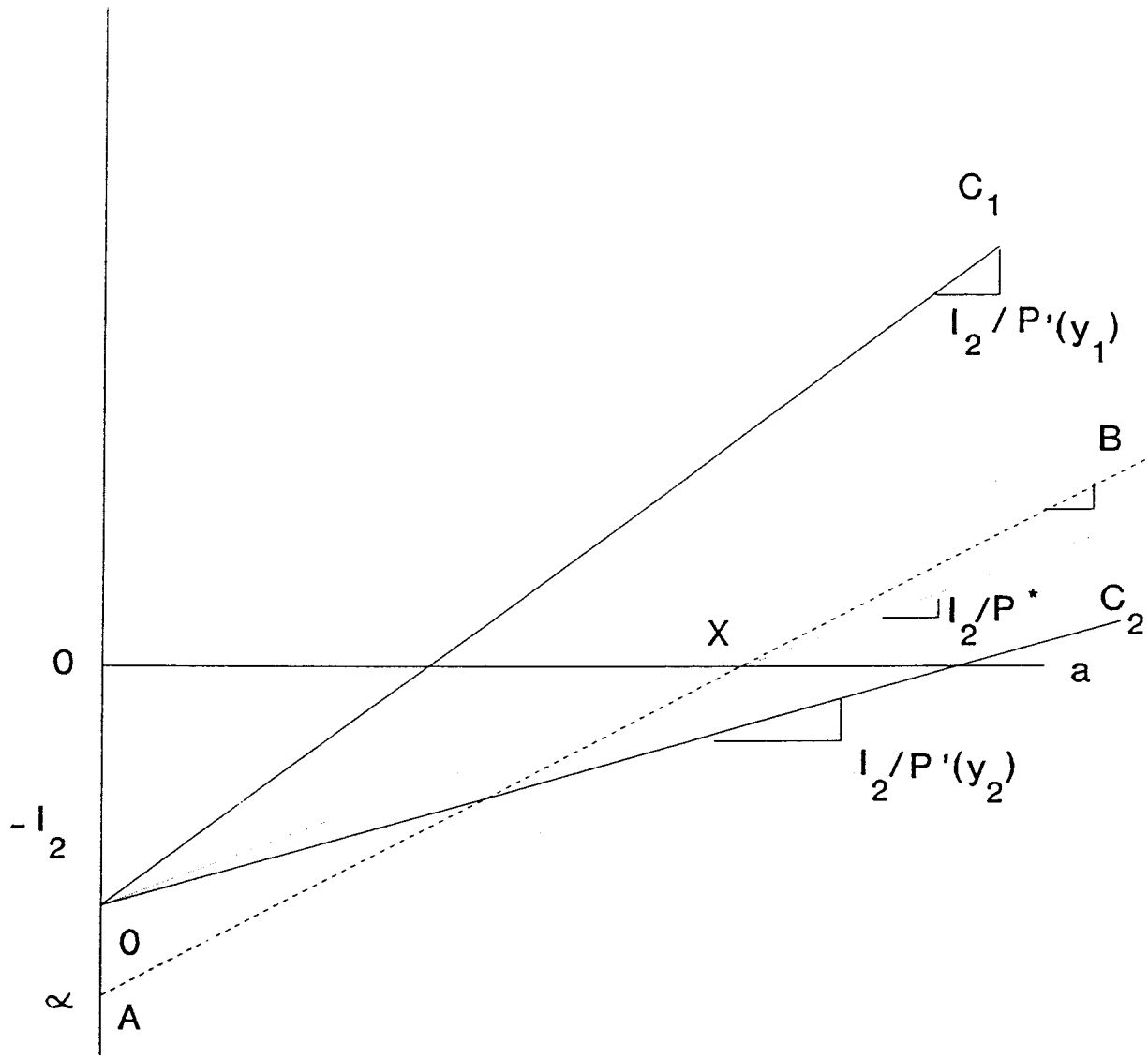


Figure 1: The Myers-Majluf Problem with Perfectly Correlated Asset Values and a Pre-Arranged Stock Sale Price.

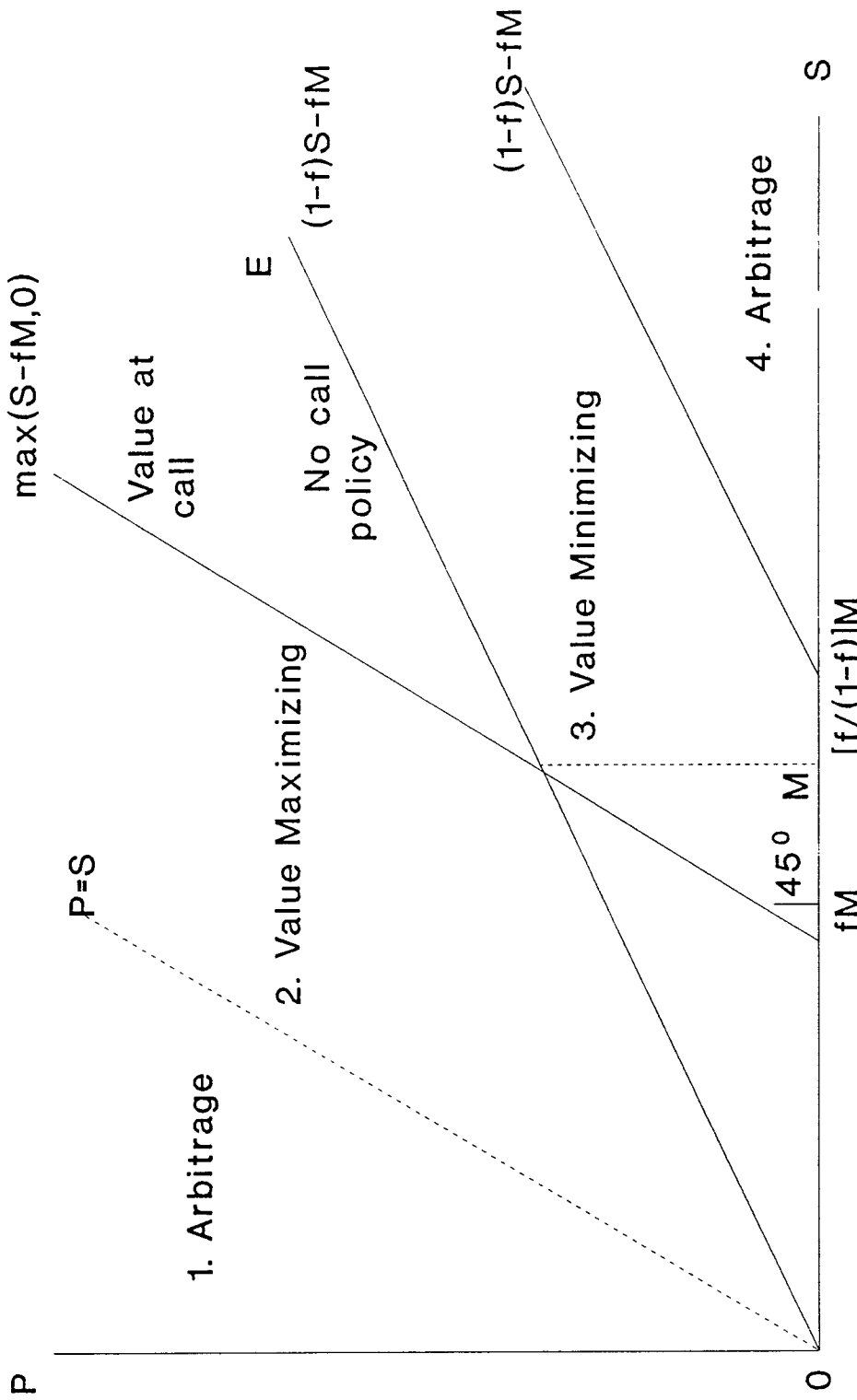


Figure 2: Restrictions on Rational Pricing under Different Call Policies

Under the MM call policy the price of contributing shares must fall below the line OE

# PRICE RATIO OF FULLY PAID SHARES AROUND DATE OF CONTRIBUTION

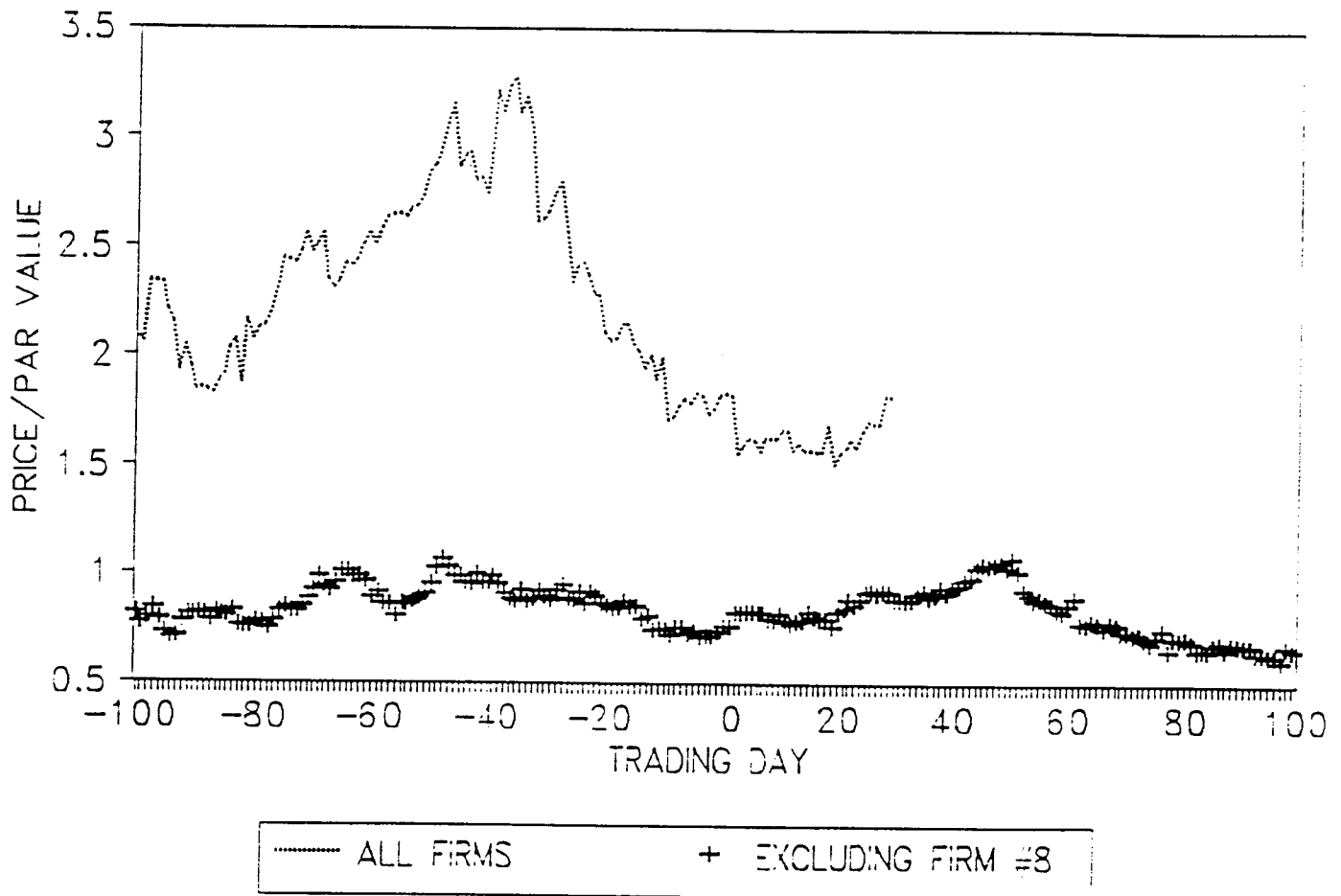


FIGURE 3

# PRICE RATIO OF CONTRIBUTING SHARES AROUND DATE OF CONTRIBUTION

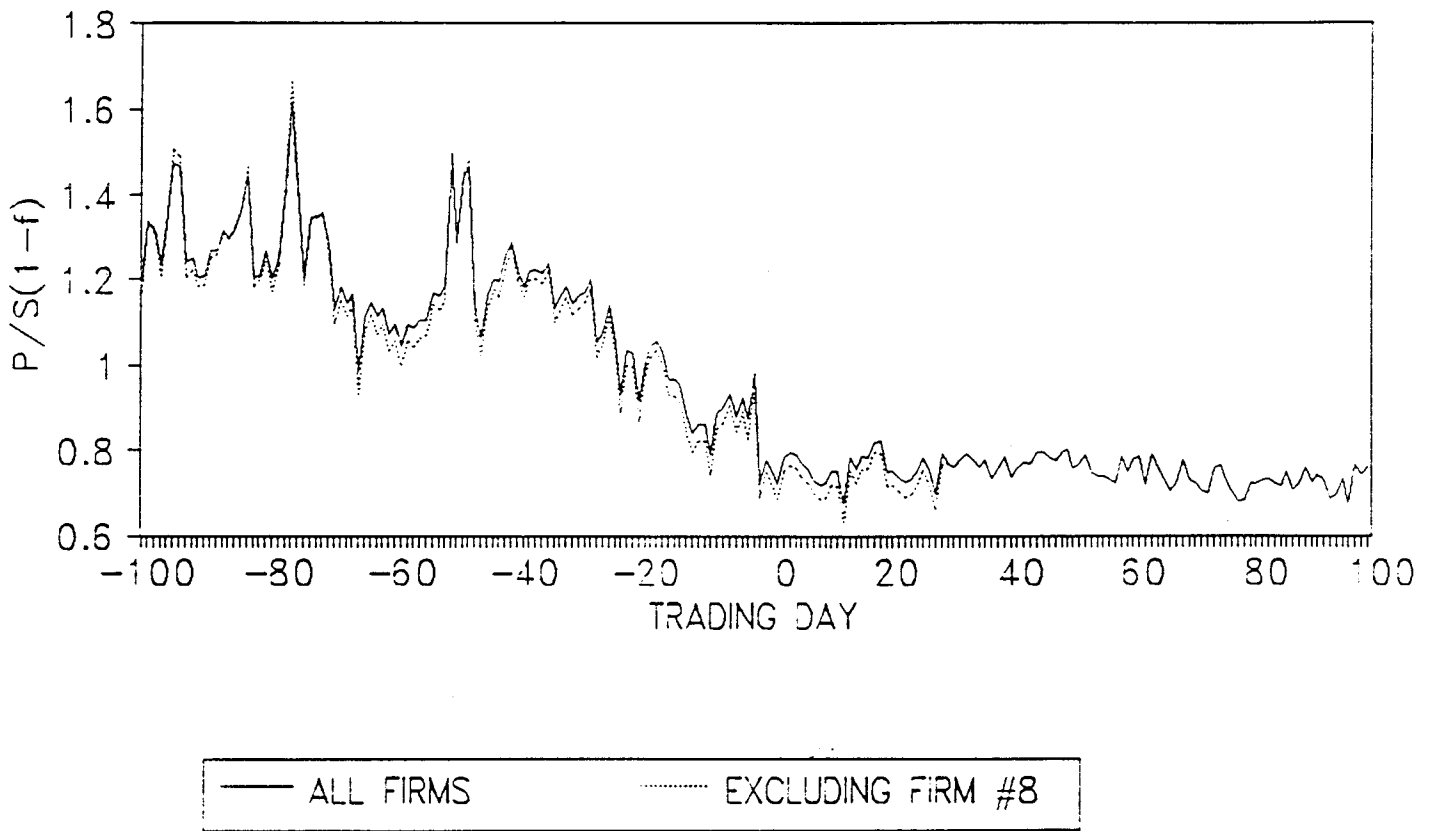


FIGURE 4