Parameter Estimation of the MCI and Related Models: Revisited

Masao Nakanishi
Kwansei Gakuin University
and
Lee G. Cooper
University of California, Los Angeles

University of California, Los Angeles
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ABSTRACT

Recent developments on least-squares estimation of the parameters of the Multiplicative Competitive Interaction Model and the Multinomial Logit Model are discussed. Log-linear regression models with dummy variables are shown to be useful tools. Also the extensions of the MCI model permit differential parameter values across choice objects are proposed.

INTRODUCTION

Six years have passed since we proposed a least-squares approach for estimating parameters of models of the following general type (which we called th Multiplicative Competitive Interaction (MCI) model) (Nakanishi and Cooper (1974):

$$(1) \qquad \pi_{ij} = \prod_{h=1}^{H} X_{hij}^{\beta_h} \Big/ \sum_{j=1}^{J} \prod_{h=1}^{H} X_{hij}^{\beta_h}$$

where: π_{ij} = the probability that a consumer in the i th choice situation (period and/or area) selects the j th object (i = 1, 2, ..., I; j = 1, 2, ... J),

X_{hij} = the value of the h th variable for object j in choice situation i ($X_{hij} \geq 0$, h = 1, 2, ..., H),

β_h = the parameter for the sensitivity of π_{ij} with respect to variable h.

Our proposal was based on the fact that model (1) may be transformed into a linear form in parameters by applying the following transformation to π_{ij}.

$$(2) \qquad \log(\pi_{ij}/\tilde{\pi}_{i.}) = \sum_{h=1}^{H} \beta_h \log(X_{hij}/\tilde{X}_{hi.})$$

where $\tilde{\pi}_{i.}$ and $\tilde{X}_{hi.}$ are the geometric means of π_{ij} and X_{hij} over j in choice situation i, respectively. The above transformation will be referred to as "log-centering" hereafter.

Since then a number of studies which appeared in the marketing literature (e.g., Bultez and Naert 1975, Stanley and Sewall 1976, and Mahajan, Jain, and Ratchford 1978) made use of the MCI model or the least-squares technique we proposed. Yet we feel that there still exist some misunderstandings concerning the model and the estimation technique which are preventing their more widespread use. The purpose of this note is to summerize several recent developments, some unpublished, associated with parameter estimation for the MCI and related models. Some empirical results are given for illustrative purposes.

ESTIMATION BY DUMMY VARIABLE REGRESSION

In the previous article, we derived a regression model of the following form from (2).

$$\log(p_{ij}/\tilde{p}_{i.}) = \sum_{h=1}^{H} \beta_h \log(X_{hij}/\tilde{X}_{hi.}) + \varepsilon_{ij} \tag{3}$$

where: p_{ij} = an estimate of π_{ij} ($p_{ij} > 0$),

$\tilde{p}_{i.}$ = the geometric mean of p_{ij} over j in situation i,

ε_{ij} = the stochastic disturbance term,

and considered the properties of ε_{ij} under several assumptions. But it can be shown that the estimates of β_h ($h = 1, 2, \ldots, H$) from (3) are numerically equivalent to those obtained from the following dummy variable regression model.

$$\log p_{ij} = \sum_{i'=1}^{I} \alpha_{i'} D_{i'} + \sum_{h=1}^{H} \beta_h \log X_{hij} + \varepsilon_{ij} \tag{4}$$

where: $D_{i'}$ = a dummy variable which is equal to 1 if $i' = i$ and 0 otherwise,

$\alpha_{i'}$ = the regression coefficient for $D_{i'}$.

If we take logarithms of both sides of (1), it may be written as

$$\log\pi_{ij} = \sum_{h=1}^{H} \beta_h \log X_{hij} + C_i$$

where $C_i = \log(\Sigma\Pi X_{hij}^{\beta_h})$. Since C_i does not change over j in a situation, dummy variable $D_{i'}$ (i' = i) absorbs its effect on π_{ij}. A formal proof is given in Appendix A. Note that ε_{ij} in (4) has a simpler specification than that in (3) because (4) does not involve the centering operation.

The equivalence between regression models (3) and (4) may be further demonstrated by the existence of a common transformation which converts them back to specification (1). Let the dependent variable in (3) be y^*_{ij}, that is $y^*_{ij} = \log(p_{ij}/\tilde{p}_{i.})$, and $\hat{y}^*_{ij}$ be its estimate. It is easy to show that, ignoring the ε_{ij} term,

$$(5) \qquad \hat{\pi}_{ij} = \exp(\hat{y}^*_{ij}) \Big/ \sum_{j=1}^{J} \exp(\hat{y}^*_{ij})$$

is equal to the estimate of π_{ij} calculated by substituting the estimated values of the β_h's into (1). The operation above may be termed the "inverse log-centering" transformation. If we let $\hat{y}_{ij}$ be the estimate of $\log p_{ij}$ from (4) and take its inverse log-centering transform,

$$\hat{\pi}_{ij} = \exp(\hat{y}_{ij}) \Big/ \sum_{j=1}^{J} \exp(\hat{y}_{ij})$$

gives the same estimate of π_{ij} as (5). Thus it has become clear that the inclusion of dummy variables in model (4) serves the same function as centering $\log p_{ij}$ for each i.

For the purpose of the regression analysis then one may use either (3) or (4) depending on the number of choice situations (periods and/or areas). When the total number of choice situations, I, is small, one should be indifferent between the two regression models; when I is large, the pre-reduction of data by log-centering reduces the number of parameters to be estimated (with a corresponding reduction in the number of degrees of freedom due to the centering operation).[1]

COMPARISON WITH RELATED MODELS

Log-Linear Models

An advantage of the dummy variables regression model (4) is that it is directly comparable with other log-linear models. Model (4) clearly is a special case of the log-linear regression model of the form

$$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log X_{hij} + \varepsilon_{ij} \quad . \tag{6}$$

Model (4) differs from (6) in that a separate intercept is assumed for each choice situation. There are occasions, especially in cross-sectional and or time-series analyses, in which the inclusion in (6) of separate intercepts for different situations is desirable, for one may wish to account for the changes in economic conditions, competitive environment, the size of choice sets, the characteristics of buyers, etc. Usually, if there were not a significant improvement in fit associated with dummy variables, D_{i}, one would choose to represent all the data with a single overall intercept as in (6). But we suggest that model (4) should always be preferred to (6) if one's purpose is to obtain "logically consistent" estimates of the π_{ij}'s from (4).

The logical consistency requirement for market share models has been extensively discussed by others (e.g., Naert and Bultez 1973 and McGuire, Weiss and Houston 1977). Restated in the present context, the estimates of π_{ij} are said to be logically consistent if they satisfy the conditions that

$$\sum_{j=1}^{J} \hat{\pi}_{ij} = 1 \text{ and } 0 \leq \hat{\pi}_{ij} \leq 1 \text{ for all } j \quad .$$

It is clear that the estimated values of π_{ij} from (6) (i.e., $\exp(\widehat{\log p}_{ij})$) do not satisfy this requirement. Model (4) in itself does not generate logically consistent $\hat{\pi}_{ij}$'s, but, since the estimates of the β_h's from (4) are those of the MCI model (1), one can obtain logically consistent estimates through the inverse log-centering transformation.

To illustrate the advantage of logically consistent models, regression models (4) and (6) are fitted to the same set of data shown in Appendix B[2], along wth two variations of (6), namely,

(7.a) $$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log(X_{hij}/\sum_{j=1}^{J} X_{hij}) + \varepsilon_{ij} \text{ and}$$

(7.b) $$\log p_{ij} = \alpha_0 + \sum_{h=1}^{H} \beta_h \log(X_{hij}/\bar{X}_{hi.}) + \varepsilon_{ij},$$

where $\bar{X}_{hi.}$ is the arithmetic mean of X_{hij} over j in choice situation i. The explanatory variables in (7.a) are in a share form and those in (7.b) are in a normalized form. Both have been used as market share models before (e.g., Lambin 1972, Weiss 1968, and Wildt 1974). We note that regression models (6), (7.a) and (7.b) are parameter estimating equations for the following multiplicative models respectively.

(8.a) $$\pi_{ij} = \exp(\alpha_0) \sum_{h=1}^{H} X_{hij}^{\beta_h},$$

(8.b) $$\pi_{ij} = \exp(\alpha_0) \sum_{h=1}^{H} (X_{hij}/\sum_{j=1}^{J} X_{hij})^{\beta_h} \quad ,$$

(8.c) $$\pi_{ij} = \exp(\alpha_0) \sum_{h=1}^{H} (X_{hij}/\bar{X}_{hi.})^{\beta_h}$$

None of those models satisfy the logical consistency requirement.

(Table 1 About Here)

Table 1 shows the OLS estimates of the parameters of the four models. Model (4) gives a marginally better fit as indicated in the R^2-values. Since (4) includes dummy variables, it has fewer degrees of freedom than other models, and its better fit is no indication of its superiority. Yet when the estimates of π_{ij}'s are computed from respective models (by inverse log-centering in the case of (4)), marked differences emerge. Table 2 shows the mean squared deviation between $\hat{\pi}_{ij}$ and p_{ij} (analogous to the

(Table 2 About Here)

variance of estimation errors) for each model. The mean squared deviation for the MCI model is by far the smallest. Note also that this is not due to a larger number of parameters in (4) because specification (1) requires only the estimates of β_h's. It is not difficult to see why fit improves with logical consistency. The maximum absolute deviation for the MCI model is bounded at 1.0 and the sum of squared deviations per choice situation is also bounded at 2.0. The bounding of the sum of squared deviations tends to enhance fit, as measured by the mean squared deviation.

A natural question arises at this point as to if one should compute $\hat{\pi}_{ij}$, from models (6), (7.a) and (7.b), not only by simply taking $\exp(\widehat{\log p}_{ij})$, but by taking the inverse log-centering transform of $\log p_{ij}$. This practice will improve fit as measured by the mean squared deviation, but we posit that it is theoretically unjustifiable. The MCI model specified by (1) and regression model (4) are related with each other through the log-centering and inverse log-centering transformations, but models (6), (7.a) and (7.b) and their original specifications, (8.a) through (8.c), are not. In fact the application of the log-centering transform to the π_{ij}'s in (8.a) through (8.c) results in an expression identical to (2).[3] Conversely, the application of the inverse log-centering transformation to $\log p_{ij}$ in (6), (7.a) and (7.b) does not recover the original specifications (8.a) through (8.c). Thus taking the inverse log-centering transform of the dependent variable is a hardly defensible practice for models (6), (7.a) and (7.b). If one wishes to have logically consistent estimates of the π_{ij}'s, one should select the MCI model over specifications (8.a) through (8.c).

The relationships among log-linear models are further clarified by noting that including dummy variables for choice situations in models (6), (7.a) and (7.b) leads to an identical set of the β_h estimates from them. Table 3 shows the estimated parameters when dummy variables for i = 2 and

(Table 3 About Here)

3 are added to each equation. It also shows the estimates of the β_h's for (4). Clearly all models give an identical fit and $\hat{\beta}_h$-values. The values of estimated α_0 and α_i's differ from one model to the next, reflecting trivial differences in the specification for each model. Those results are not surprising in view of the fact that (8.a) through (8.c) are transformed into (2) by log-centering. If for some reason one finds it necessary to add dummy variables for choice situations to regression models (6), (7.a) and (7.b) in cross-sectional or time-series analyses, then the MCI model (1) is de facto specified and estimation and prediction should proceed accordingly.

Multinomial Logit Model

Recently the multinomial logit (MNL) model:

(9) $$\pi_{ij} = \exp\left(\sum_{h=1}^{H} \beta_h X_{hij}\right) / \sum_{j=1}^{J} \exp\left(\sum_{h=1}^{H} \beta_h X_{hij}\right)$$

has been given some attention in the marketing literature (e.g., Punj and Staelin 1976 and Gensch and Becker 1979). It is known that the parameters,

β_h, in (9) may be estimated by a log-linear regression procedure under some conditions, especially when an appropriate estimate of π_{ij} such that $p_{ij}>0$ is available (McGuire, Weiss, and Houston 1977). Note first that the application of the log-centering transformation to π_{ij} in (9) results in

$$\text{(10)} \qquad \log(\pi_{ij}/\hat{\pi}_{i.}) = \sum_{h=1}^{H} \beta_h(X_{hij} - \bar{X}_{hi.})$$

where $\bar{X}_{hi.}$ is the arithmetic mean of X_{hij} over j in situation i. Since (10) is linear in the β_h's, one may estimate them by regression models.[4]

$$\text{(11.a)} \qquad \log(p_{ij}/\tilde{p}_{i.}) = \sum_{h=1}^{H} \beta_h(X_{hij} - \bar{X}_{hi.}) + \varepsilon_{ij}$$

or

$$\text{(11.b)} \qquad \log p_{ij} = \sum_{i'=1}^{I} \alpha_{i'}D_{i'} + \sum_{h=1}^{H} \beta_h X_{hij} + \varepsilon_{ij} \quad .$$

Logically consistent estimates of the π_{ij}'s then may be obtained from (11.a) or (11.b) through the inverse log-centering transformation.

When one compares (11.a) and (11.b) with (3) and (4), it becomes clear the basic difference between the MCI and MNL models is in the scaling of explanatory variables; the MCI model accepts only ratio-scaled variables, but the MNL model accepts both interval-and ratio-scaled variables. Thus the choice between the two models is dictated partly by the nature of explanatory variables. Another important consideration is the elasticity of π_{ij} with respect to each explantory variable implied by each model. If we let E^h_{ij} be the elasticity of π_{ij} with respect to variable X_h, from (1) and (9) we have

$$E^h_{ij} = \begin{cases} \beta_h\,(1 - \pi_{ij}) & \text{for the MCI model} \\ \beta_h X_{hij}\,(1 - \pi_{ij}) & \text{for the MNL mdoel.} \end{cases}$$

Similarly, the cross elasticity of π_{ij} with respect to X_{hik} (the value of variable X_h for object k ($k \neq j$)) may be defined as

$$E_{ik}^{h} = \begin{cases} -\beta_h \pi_{ik} & \text{for the MCI model} \\ -\beta_h X_{hik} \pi_{ik} & \text{for the MNL model.} \end{cases}$$

If there are a priori reasons for selecting one elasticity expression or another, then the choice between the MCI and MNL models can be made more logically. We suggest that one should freely select a mixture of the two models when the occasion requires one to do so.

In some applications of the MNL model, an estimate of π_{ij} such that $p_{ij}>0$ may not be available. In those applications (e.g., the conditional logit model of Punj and Staelin (1976)) where the dependent variable is valued either zero or one, regression models (11.a) and (11.b) cannot be used and a maximum likelihood approach will have to be employed. But this problem is not unique to the MNL model. When the data contain only zeroes or ones, the parameters of the MCI model will also have to be estimated by a maximum likelihood or minimum X^2-technique. The least-squares technique is limited to those cases where ratio-scaled estimates of choice probabilities such that $p_{ij}>0$ exist.

EXTENSIONS OF LOG-LINEAR REGRESSION MODEL

A common assumption about the MCI model is that parameter β_h is equal across choice objects (e.g., brands), but this assumption is not necessary. One may for example specify that

$$(12) \qquad \pi_{ij} = \prod_{h=1}^{H} X_{hij}^{\beta_{hjj}} \Big/ \sum_{j=1}^{J} \prod^{H} X_{hij}^{\beta_{hjj}}$$

where β_{hjj} = the parameter for the sensitivity of π_{ij} with respect to X_{hij}. Since (12) may be written as

(12) $$\pi_{ij} = \alpha_i \prod_{h=1}^{H} X_{hjj}^{\beta_{hjj}} ,$$

a dummy variable regression model of the following form will estimate the β_{hjj}'s for all objects.

(13) $$\log p_{ij} = \sum_{i'=1}^{I} \alpha_{i'} D_{i'} + \sum_{h=1}^{H} \left(\sum_{j'=1}^{J} \beta_{hj'j'} d_{j'} \right) \log X_{hij} + \varepsilon_{ij}$$

where $d_{j'}$ = a dummy variable which is equal to 1 if $j' = j$ and 0 otherwise. In the context of market share models, (12) assumes that β_h is different for each brand. Such differences may be created by more effective promotional and/or distributional methods for some brands. Regression model (13) is useful in testing hypotheses on differential effectiveness of marketing efforts.

In practice, model (13) may pose some difficulties when the number of choice situations, I, is large, since it requires the estimation of the α_i's ($i = 1, 2, \ldots ,I$). Fortunately, the proof in Appendix A shows that the inclusion of the D_i's in (13) serves the same function as centering both the dependent and explanatory variables for each i. Hence if we define a set of new variables as

$$z_{hj'ij} = d_{j'} \log X_{hij} ,$$

the following equivalent regression model may be used in place of (13).

(14) $$\log(p_{ij}/\tilde{p}_{i.}) = \sum_{h=1}^{H} \sum_{j'=1}^{J} \beta_{hj'j'} (z_{hj'ij} - \bar{z}_{hj'i.}) + \varepsilon_{ij} ,$$

where $\bar{z}_{hj'i.}$ is the arithmetic mean of $z_{hj'ij}$ over j in choice situation i[5].

We may further extend (12) to allow attributes of objects other than j to have a direct influence on π_{ij}. Again in the context of market share models expressions (1) and (12) show that the market share of one brand is affected by marketing efforts of another brand only directly through the denominator on the right-hand side of the respective expressions. Let

(15) $$\pi_{ij} = \prod_{h=1}^{H} \prod_{k=1}^{J} X_{hik}^{\beta_{hjk}} \Big/ \sum_{j=1}^{J} \prod_{h=1}^{H} \prod_{k=1}^{J} X_{hik}^{\beta_{hjk}}$$

where X_{hik} = the value of variable h for object k in choice situation i

β_{hjk} = the parameter for the sensitivity of π_{ij} with respect to X_{hik} (k = 1, 3, ..., J).

This model is important theoretically, if not practically, because it permits marketing efforts of brand k to have direct influence on the market share of brand j, and specifies that the extent of influence may be different for each (j, k) combination.

The parameters of (15), however, are not estimable by a dummy variable regression model such as (4) and (13). Since the same set of explanatory variables, X_{hik}(k = 1, 2, ..., J), is repeated for all J objects in a situation, dummy variables, $D_{i'}$ (i' = 1, 2, ..., I), become jointly collinear with every explanatory variable. But McGuire, Weiss and Houston (1977) have shown that, by taking the log-centering transform of π_{ij} in (15), we have

(16) $$\log(\pi_{ij}/\tilde{\pi}_{i.}) = \sum_{h=1}^{H} \sum_{j=1}^{J} (\beta_{hjk} - \bar{\beta}_{h.k}) \log X_{hik},$$

where $\bar{\beta}_{h.k}$ is the arithmetic mean of β_{hjk} over j. Thus, with ratio-scaled estimates of the π_{ij}'s and adequate degrees of freedom to estimate JxJxH parameters, a regression model of the following form is suggested.

(17) $$\log(p_{ij}/\tilde{p}_{i.}) = \sum_{h=1}^{H} \sum_{k=1}^{J} \Big(\sum_{j'=1}^{J} \beta^{*}_{hj'k} d_{j'}\Big) \log X_{hik} + \varepsilon_{ij}$$

where $\beta^{*}_{hj'k} = \beta_{hj'k} - \bar{\beta}_{h.k}$, and $d_{j'}$ is the dummy variable defined for (13)[6].

With model (17) one cannot estimate β_{hjk} per se, but one needs only the estimates of the β^{*}_{hjk}'s for the many practical purposes. For example, the estimates of π_{ij} may be obtained directly from (17) by taking the inverse log-centering transform of the dependent variable $(\log(p_{ij}/\tilde{p}_{i.}))$, without the knowledge of the β_{hjk}'s. It is instructive at this point to derive the elasticities of π_{ij} in (15) with respect to X_{hik} $(k = 1, 2, \ldots, J)$. First take the partial derivative of π_{ij} with respect to X_{hik},

$$\frac{\partial \pi_{ij}}{\partial X_{hik}} = \frac{\pi_{ij}}{X_{hik}} (\beta_{hjk} - \sum_{j'=1}^{J} \beta_{hj'k} \pi_{ij'}).$$

Hence the elasticity of π_{ij} with respect to X_{hik} is given by

(18)

$$E^{h}_{ijk} = \beta_{hjk} - \sum_{j'=1}^{J} \beta_{hj'k} \pi_{ij'}.$$

Expression (18) shows that the influence of X_{hik} on π_{ij} is modified by the weighted average of influence on all objects.[7] Now substitute β^{*}_{hjk} for β_{hjk} in (18). We have

$$\beta^{*}_{hjk} - \sum_{j'=1}^{J} \beta^{*}_{hj'k} \pi_{ij'} = (\beta_{hjk} - \bar{\beta}_{h.k}) - \sum_{j'=1}^{J} (\beta_{hj'k} - \bar{\beta}_{h.k}) \pi_{ij'}$$

$$= \beta_{hjk} - \sum_{j'=1}^{J} \beta_{hj'k} \pi_{ij'} = E^{h}_{ijk}$$

since $\sum_{j'=1}^{J} \pi_{ij'} = 1$. Thus we have shown that the knowledge of the β^{*}_{hjk}'s is sufficient for computing E^{h}_{hijk}.

Similar extensions of the MNL model paralleling (12) and (15) are clearly possible, but we only note that the parameters of the extended MNL models may be estimated by (13) and (17), if we replace $\log X_{hij}$ and $\log X_{hik}$ in those equations by X_{hij} and X_{hik}, and that the expression for the elasticity of π_{ij} with respect to X_{hik} $(k = 1, 2, \ldots, J)$ for the fully extended MNL model (analogous to (15)) is given by

$$E^{h}_{ijk} = X_{hik} (\beta_{hjk} - \sum_{j'=1}^{J} \beta_{hj'k} \pi_{ij'}).$$ [8]

MICELLANEOUS TOPICS

Problem of $p_{ij} = 0$

We have already stated that the least-squares technique for estimating parameters of the MCI and MNL models is limited to the cases where ratio-scaled estimates of choices probabilities such that $p_{ij}>0$ are available. But even in those cases, situations arise where an estimated probability, p_{ij}, is zero for some (i,j) combination. The recommended procedure for such situations is to discard those (i,j) combinations from the data set and use the remaining p_{ij}'s which are greater than zero for estimating the β_h's (Young and Young 1975). Though discarding zero p_{ij}'s reduces the total degrees of freedom available to the analysis, we note that the maximum likelihood approach is no different in the disuse of the observed zeroes (i.e., no choices). After all, if $p_{ij} = 0$ for an (i,j) combination, one should perhaps assume that object j is not in the choice set for consumers in choice situation i. The structure of regression model (4) shows that it is application when there are two or more alternative objects per choice situation for which $p_{ij}>0$. Thus the loss in degrees of freedom due to the discarding of observations may be compensated by a careful research design.

The easiest way to increase the total degrees of freedom is to increase the number of choice situations, I. If the p_{ij}'s are generated by the usual multinomial sampling process, increasing the sample size per choice situation reduces the probability that $p_{ij} = 0$. Also there are types of probability estimates (e.g., "odds-ratio" estimates) which will guarantee p_{ij} to be greater than zero. By combining those techniques, the problem of zero p_{ij}'s should pose little difficulty to the users of the least-squares estimation technique.

Choice of Explanatory Variables

The original specification of the MCI model, (1), has an interesting property: If each explanatory variable is multipled (or divided) by an arbitrary constant, possibly unique for each choice situation, the estimated values of the β_h's are unchanged. This property, not shared by the extended models (12) and (15), gives the research a flexibility in selecting explanatory variables.[9] For example, in studying shopper spatial behavior, Huff (1963) used travel time and shopping center size as explanatory variables. But the above mentioned property of the MCI model suggests that, to the extent that travel time is proportional to distance, they may be used interchangeably since the parameter estimates from them will be equal. This also makes the practice of estimating travel time by dividing distance by an average speed superfluous.

A similar comment applied to shopping center size. This variable is usually measured in terms of selling floor space, but one should realize that Huff used floor space as a surrogate for the width of merchandise assortment, presumably for the lack of a better measure. Thus any measure which is proportional to the width of assortment is a theoretically justifiable alternative for selling floor space. The estimate of β_h will be approximately the same for any such measure.

Handling of Binary Variables

Mahajan, Jain and Ratchford (1978) gave a comprehensive treatment on the use of binary-coded variables in the MCI model. They showed that the easiest method of handling binary variables in the MCI model is to use their exponential transforms, that is if X_{hij} is a binary-coded variables, to use exp (X_{hij}). This in fact changes the MCI model into the MNL model with respect to those variables, but, considering the close relationships between the two models, there should be no hesitation in mixing them. The exponential transform of a binary-coded variable will appear as a usual dummy variable in regression models (4), (13), and (16), thereby simplifying calculations.

Another method for handling binary-coded variable is the "index of distinctness" devised by Nakanishi, Cooper, and Kassarjian (1974). Their index (NCK index) is defined as

$$(19) \qquad z_{hij} = \begin{cases} 1/r_{hi} & \text{if object } j \text{ posesses attribute } h, \\ (1 - r_{hi}) & \text{otherwise} \end{cases}$$

where r_{hi} = the proportion of objects in situation i which possesses attribute h. Mahajan, and Jain (1977) showed that this index, after the log-centering transformation, becomes

$$(20) \quad \log(Z_{hij}/\tilde{Z}_{hi.}) = \begin{cases} [-\log r_{hi}(1 - r_{hi})]\ (1 - r_{hi}) & \text{if object } j \text{ possesses attribute } h, \\ [-\log r_{hi}(1 - r_{hi})]\ (-r_{hi}) & \text{otherwise} \end{cases}$$

where $\tilde{Z}_{hi.}$ is the geometric mean of Z_{hi} over j in situation i. Compare this with the log-centered form of an exponentially transformed binary variable.

$$(21) \quad \log(X^e_{hij}/\tilde{X}^e_{hi.}) = \begin{cases} (1 - r_{hi}) & \text{if object } j \text{ possesses attribute } h, \\ -r_{hi} & \text{otherwise,} \end{cases}$$

where $\tilde{X}^e_{hi.} = \exp(\bar{X}_{hi.}) = \exp(r_{hi})$. The difference between (20) and (21) is just the factor $[-\log r_{hi}(1 - r_{hi})]$. Since this factor changes over h and i, Mahajan, Jain and Ratchford question the wisdom of its inclusion in the model in cross-sectional analyses, for fear of misinterpreting the results (1978, p. 214).

However, it can be shown that (19) is a special case of a new standardizing transformation. We usually standardize a variable (X_{hij}, say) by computing

$$z_{hij} = (X_{hij} - \bar{X}_{hi.})/S_{hi.}$$

where $\bar{X}_{hi.}$ = the arithmetic mean of X_{hij} over j in situation i,

$S_{hi.}$ = the standard deviation of X_{hij} in situation i.

But z_{hij} cannot be used in the MCI model because it is not log-transformable. We propose the following transformation as an alternative method of standardizing variables.

$$(22) \qquad z_{hij} = \begin{cases} 1 + z_{hij}^2 & \text{if } z_{hij} \geq 0, \\ (1 + z_{hij}^2)^{-1} & \text{if } z_{hij} < 0. \end{cases}$$

This transformation is log-transformable and may be used in the MCI model. That transformation (19) is a special case of (22) can be seen if one realizes that $\bar{X}_{hi.} = r_{hi}$ and $S_{hi.} = r_{hi}(1 - r_{hi})$ for a binary-coded variable X_{hij}, and

$$(23) \qquad z_{hij} = \begin{cases} (1 - r_{hi}) / \sqrt{r_{hi}(1 - r_{hi})} & \text{if } X_{hij} = 1, \\ -r_{hi} / \sqrt{r_{hi}(1 - r_{hi})} & \text{if } X_{hij} = 0. \end{cases}$$

Substituting the above z_{hij}'s into (22) yields (19). Thus we have shown that the NCK index of distinctness standardizes, though in an unconventional way, a binary-coded variable for each choice situation separately. We posit that cross-sectional comparisons are facilitated, rather than hindred, by the use of standardized variables. Whether or not our proposition is correct is partly a matter to be resolved through further empircal testing.

CONCLUSION

We have reviewed in this note a number of recent developments related to the parameter estimation for the MCI and MNL models. We hope that the material contained here will provide those who are interested in utilizing those potentially powerful models with a useful reference on the types of problems they might encounter in their applications.

APPENDIX A

We are to prove that the regression estimates of the β_h's obtained from (3) and (4) are numerically equivalent. Let

$$
\begin{aligned}
y_{ij} &= \log p_{ij} \\
x_{hij} &= \log X_{hij} \\
\underline{Y}_i &= (y_{i1}\ y_{i2}\ \cdots\ y_{iJ})' \\
\underline{Y} &= (\underline{Y}_1'\ \ \underline{Y}_2'\ \ \cdots\ \ \underline{Y}_I')' \\
\underline{X}_i &= \begin{pmatrix} x_{1i1} & x_{2i1} & \cdots & x_{Hi1} \\ x_{1i2} & x_{2i2} & \cdots & x_{Hi2} \\ \cdots & \cdots & \cdots & \cdots \\ x_{1iJ} & x_{2iJ} & \cdots & x_{HiJ} \end{pmatrix} \\
\underline{X} &= (\underline{X}_1'\ \ \underline{X}_2'\ \cdots\ \ \underline{X}_I')'
\end{aligned}
$$

First, it can be shown (Nakanishi and Cooper 1974) that the OLS estimate of $\underline{\beta} = (\beta_1, \beta_2, \ldots \beta_H)'$ obtainable from (3) may be written as:

$$\underline{\beta}_1 = [\sum_{i=1}^{I} \underline{X}_i'(\underline{I} - \frac{1}{J}\underline{J})\,\underline{X}_i]^{-1}\,[\sum_{i=1}^{I} \underline{X}_i'(I - \frac{1}{J}\underline{J})\underline{Y}_i]\ ,$$

where: $\underline{I}$ = J x J indentity matrix

$\underline{J}$ = J x J matrix of 1's.

But $\underline{\beta}_1$ may be written, using a matrix of dummy variables containing only 1's and 0's as follows.

$$\underline{\beta}_1 = [\underline{X}'\ (\underline{I} - \underline{D}(\underline{D}'\underline{D})^{-1}\underline{D}')\underline{X}]^{-1}[\underline{X}'(\underline{I} - \underline{D}(\underline{D}'\underline{D})^{-1}\underline{D}')\underline{Y}]$$

where

$$\underline{D} = \begin{pmatrix} \underline{1} & \underline{0} & \cdots & \underline{0} \\ \underline{0} & \underline{1} & \cdots & \underline{0} \\ \underline{0} & \underline{0} & \cdots & \underline{1} \end{pmatrix}$$

= (I x J)x I matrix of dummy variables. $\underline{1}$ and $\underline{0}$ are 1's and 0's with dimension J.

Next, the estimate of $\underline{\beta}$ and $\underline{\alpha} = (\alpha_1, \alpha_2, \ldots \alpha_i)'$ from (4) may be written as

$$\begin{pmatrix} \hat{\underline{\alpha}} \\ \hat{\underline{\beta}}_2 \end{pmatrix} = [(\underline{D} \mid \underline{X})' (\underline{D} \mid \underline{X})]^{-1} (\underline{D} \mid \underline{X})' \underline{Y}$$

But

$$[(\underline{D} \mid \underline{X})' (\underline{D} \mid \underline{X})]^{-1} = \left(\begin{array}{c|c} \underline{D}'\underline{D} & \underline{D}'\underline{X} \\ \hline \underline{X}'\underline{D} & \underline{X}'\underline{X} \end{array}\right)^{-1}$$

$$= \left(\begin{array}{c|c} (\underline{D}'\underline{D})^{-1}[\underline{I} + \underline{D}'\underline{X}\underline{H}^{-1}\underline{X}'\underline{D}](\underline{D}'\underline{D})^{-1} & -(\underline{D}'\underline{D})^{-1}\underline{D}'\underline{X}\underline{H}^{-1} \\ \hline -\underline{H}^{-1}\underline{X}'\underline{D}(\underline{D}'\underline{D})^{-1} & \underline{H}^{-1} \end{array}\right)$$

where $\underline{H} = \underline{X}'\underline{X} - \underline{X}'\underline{D}(\underline{D}'\underline{D})^{-1}\underline{D}'\underline{X}$.

Hence

$$\begin{aligned} \underline{\beta}_2 &= -\underline{H}^{-1}\underline{X}'\underline{D}(\underline{D}'\underline{D})^{-1}\underline{D}'\underline{Y} \quad \underline{H}^{-1}\underline{X}'\underline{Y} \\ &= \underline{H}^{-1}[\underline{X}'\underline{Y} - \underline{X}'\underline{D}(\underline{D}'\underline{D})^{-1}\underline{D}'\underline{Y}] = \underline{\beta}_1 \qquad \text{(Q.E.D)} \end{aligned}$$

APPENDIX B

SAMPLE DATA FOR PARAMETER ESTIMATION[a]

Area (i)	Other (j)	Probability Estimates (p_{ij})	Independent Variables: Size (X_1)	Time (X_2)	Dummy Variables (D_1)	(D_2)	(D_3)
1	1	.89873	239	2.8	1	0	0
	5	.06329	1250	15.4	1	0	0
	6	.01266	281	14.0	1	0	0
	11	.01266	502	15.7	1	0	0
	13	.01266	134	10.8	1	0	0
2	1	.67890	239	3.6	0	1	0
	2	.08716	236	6.8	0	1	0
	3	.01835	326	17.0	0	1	0
	5	.17431	1250	16.1	0	1	0
	7	.00917	338	17.4	0	1	0
	10	.01835	222	17.8	0	1	0
	11	.00917	502	19.2	0	1	0
	14	.00459	121	9.4	0	1	0
3	1	.70443	239	4.2	0	0	1
	2	.02956	236	8.7	0	0	1
	3	.00985	326	14.3	0	0	1
	4	.00985	97	8.6	0	0	1
	5	.10345	1250	20.5	0	0	1
	6	.03448	281	15.2	0	0	1
	7	.02956	228	11.5	0	0	1
	8	.00985	326	15.5	0	0	1
	10	.01478	222	27.1	0	0	1
	11	.01478	502	17.4	0	0	1
	12	.00985	425	25.8	0	0	1
	13	.02956	134	5.2	0	0	1

[a]Adopted from Huff (1963). Objects of choice are shopping centers, and independent variables are shopping center size (X_1) in thousands of square feet and travel time (X_2) in minutes.

FOOTNOTES

1. Because one degree of freedom is "used up" in estimating the mean for each choice situation (period/area).

2. The data are taken from Huff (1963, pp. 453-4). The estimated values of choice probabilities are $p_{ij} = n_{ij}/n_i$, where n_i is the same size in situation i and n_{ij} is the number of respondents who chose object j (a shopping center in this case).

3. In fact any multiplicative model which may be written as

$$\pi_{ij} = \alpha_i \prod_{h=1}^{H} X_{hij}^{\beta_h},$$

where α_i is any constant for choice situation i, has a log-centered from identical to (2). (1) and (8.a) through (8.c) are clearly in this form.

4. The error terms in (11.a) and (11.b) have slightly different structures due to the log-centering of the p_{ij}'s in (11.a). If we let ε_{ij}^1 and ε_{ij}^2 be the error terms in (11.a) and (11.b), respectively, the relationship between them is expressed as $\varepsilon_{ij}^1 = \varepsilon_{ij}^2 - \bar{\varepsilon}_{i.}^2$ where $\bar{\varepsilon}_{i.}^2$ is the arithmetic mean of ε_{ij}^2 over j in situation i.

5. The error term in (14) is the centered form (i.e., the mean per i is subtracted) of that in (13).

6. Model (17) may be reformulated as a multivariate regression model. Let

$$x_{hij} = \log X_{hij}$$

$$y_{ij}^* = \log(p_{ij}/\tilde{p}_{i.})$$

$$\underline{Y}^* = \begin{pmatrix} y_{11}^* & y_{21}^* & \cdots & y_{I1}^* \\ y_{12}^* & y_{22}^* & \cdots & y_{I2}^* \\ \cdots & \cdots & \cdots & \cdots \\ y_{1J}^* & y_{2J}^* & \cdots & y_{IJ}^* \end{pmatrix} \quad \underline{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{21} & \cdots & \varepsilon_{I1} \\ \varepsilon_{12} & \varepsilon_{22} & \cdots & \varepsilon_{I2} \\ \cdots & \cdots & \cdots & \cdots \\ \varepsilon_{1J} & \varepsilon_{2J} & \cdots & \varepsilon_{IJ} \end{pmatrix}$$

$$\underline{X}^T = \begin{pmatrix} x_{111} & x_{112} & \cdots & x_{11J} & x_{211} & \cdots & x_{21J} & x_{H11} & \cdots & x_{H1J} \\ x_{121} & x_{122} & \cdots & x_{12J} & x_{221} & \cdots & x_{22J} & x_{H21} & \cdots & x_{H2J} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{1I1} & x_{1I2} & \cdots & x_{1IJ} & x_{2IJ} & \cdots & x_{2IJ} & x_{HI1} & \cdots & x_{HIJ} \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} \beta^*_{111} & \beta^*_{112} & \cdots & \beta^*_{11J} & \beta^*_{211} & \cdots & \beta^*_{21J} & \beta^*_{H11} & \cdots & \beta^*_{H1J} \\ \beta^*_{121} & \beta^*_{122} & \cdots & \beta^*_{12J} & \beta^*_{221} & \cdots & \beta^*_{22J} & \beta^*_{H21} & \cdots & \beta^*_{H2J} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta^*_{1J1} & \beta^*_{1J2} & \cdots & \beta^*_{1JJ} & \beta^*_{2J1} & \cdots & \beta^*_{2JJ} & \beta^*_{HJ1} & \cdots & \beta^*_{HJJ} \end{pmatrix}$$

The multivariate regression model equivalent to (17) is expressed as

$$\underline{Y}^* = \underline{B}\underline{X} + \underline{\varepsilon}.$$

If one adopts the classical regression model in which only contemporaneous correlations of ε_{ij} exist (i.e., $E\varepsilon_{ij}\varepsilon_{ij'} = \sigma_{jj'}$), then the OLS estimate of $\underline{B}$ is its BLUE (Goldberger 1964, pp. 246-8). When "sampling errors" (Nakanishi and Cooper 1974) are present, the assumptions of the classical regression model do not hold and some form of generalized least squares estimates will be called for.

7. We may add that the elasticity expression for specifications (1) and (12) are special cases of (18). If we let $\beta_{hjk} = 0$ for all $k \neq j$, we have E^h_{ijk} for (12), that is

$$E^h_{ijk} = \begin{cases} \beta_{hjj}(1 - \pi_{ij}) & \text{if } k = j, \\ -\beta_{hkk}\pi_{ik} & \text{if } k \neq j. \end{cases}$$

If we further assume that $\beta_{hjj} = \beta_h$ for all j, we obtain the elasticity expression for (1) derived in the preceeding section.

8. The elasticity expression derived by Gensch and Recker (1979, p. 129, Eq. 11) is different from ours, but their expression should probably read

$$E_{ij}^{k\ell} = \left| X_{ij}^{\ell} \left[\sum_{q \epsilon A} p_i (q:A) \frac{\partial V_j^q}{\partial X_{ij}^{\ell}} - \frac{\partial V_i^k}{\partial X_{ij}^{\ell}} \right] \right| ,$$

which is equivalent to ours except that the above is in the absolute value.

9. This property is mathematically called the homogeneity of degree 0 (McGuire, Weiss and Houston 1977, p. 129). The fully extended model (15) becomes homogeneous of degree 0 in variable X_h by imposing an additional condition that

$$\sum_{k=1}^{J} \beta_{hjk} = \text{constant for all } j.$$

But this condition is equivalent to the condition that

$$\sum_{k=1}^{J} \beta_{hjk}^{*} = 0 \text{ for all } j.$$

It is possible to impose this last condition on the parameters estimated by regression model (17). See Goldberger (1964), pp. 255-7) for the discussion of restricted least-squares estimation.

Table 1

PARAMETER ESTIMATES FOR LOG-LINEAR MODELS (WITHOUT DUMMY VARIABLES)

	Parameters						
	α_0	α_1	α_2	α_3	β_1	β_2	R^2
Model (4)[b]							
Estimate	-	-6.2199[a]	-5.8228[a]	-5.7882[a]	1.4616[a]	-2.4089[a]	.784
(Std. Error)		(1.3849)	(1.3332)	(1.3118)	(.2596)	(.2936)	
Model (6)[c]							
Estimate	-5.7262[a]	-	-	-	1.4041[a]	-2.3405[a]	.771
(Std. Error)	(1.2819)				(.2488)	(.2807)	
Model (7.a)[d]							
Estimate	-4.4819[a]	-	-	-	1.6027[a]	-2.1121[a]	.689
(Std. Error)	(.6816)				(.2916)	(.2154)	
Model (7.b)[e]							
Estimate	-3.4515[a]	-	-	-	1.4430[a]	-2.4004[a]	.764
(Std. Error)	(.1608)				(.2583)	(.2924)	

[a] Significant at the $\alpha = .05$ level

[b] $$\log p_{ij} = \sum_{i'=1}^{3} \alpha_{i'} D_{i'} + \sum_{h=1}^{2} \beta_h \log X_{hij} + \varepsilon_{ij}$$

[c] $$\log p_{ij} = \alpha_0 + \sum_{h=1}^{2} \beta_h \log X_{hij} + \varepsilon_{ij}.$$

[d] $$\log p_{ij} = \alpha_0 + \sum_{h=1}^{2} \beta_h \log(X_{hij} / \sum_{j=1}^{J} X_{hij}) + \varepsilon_{ij}.$$

[e] $$\log p_{ij} = \alpha_0 + \sum_{h=1}^{2} \beta_h \log(X_{hij}/\bar{X}_{hi.}) + \varepsilon_{ij}.$$

Table 2

MEAN SQUARED ERROR BETWEEN
ACTUAL AND ESTIMATED MARKET SHARES

Model	Sum of Squared Residuals	Mean Squared Error[a]
6	.3928	.0171
7.a	.9015	.0392
7.b	.5243	.0228
MCI[b]	.1006	.0050

[a] Degrees of freedom are 20 for the MCI model and 23 for other models.

[b] Estimated market shares are computed through the inverse log-centering transformation.

Table 3

PARAMETER ESTIMATES FOR LOG-LINEAR MODELS (WITH DUMMY VARIABLES)

	Parameters						
	α_0	α_1	α_2	α_3	β_1	β_2	R^2
Model (4)[b]							
Estimate	-	-6.2199[a]	-5.8228[a]	-5.7882[a]	1.4616[a]	-2.4089[a]	.784
(Std. Error)		(1.3849)	(1.3332)	(1.3118)	(.2596)	(.2936)	
Model (6)[c]							
Estimate	-6.2199[a]	-	.3971	.4317	1.4616[a]	-2.4089[a]	.784
(Std. Error)	(1.3849)		(.4366)	(.4150)	(.2596)	(.2936)	
Model (7.a)[d]							
Estimate	-4.6509[a]	-	-.6743	-1.3475	1.4616[a]	-2.4089[a]	.784
(Std. Error)	(.6206)		(.4475)	(.4570)	(.2596)	(.2936)	
Model (7.b)[e]							
Estimate	-3.1261[a]	-	-.2290	-.5181	1.4616[a]	-2.4089[a]	.784
(Std. Error)	(.3445)		(.4301)	(.4020)	(.2596)	(.2936)	

a Significant at the $\alpha = .05$ level

b $\log p_{ij} = \sum_{i'=1}^{3} \alpha_{i'} D_{i'} + \sum_{h=1}^{2} \beta_h \log X_{hij} + \varepsilon_{ij}$

c $\log p_{ij} = \alpha_0 + \sum_{i'=2}^{3} \alpha_{i'} D_{i'} + \sum_{h=1}^{2} \beta_h \log X_{hij} + \varepsilon_{ij}$

d $\log p_{ij} = \alpha_0 + \sum_{i'=2}^{3} \alpha_{i'} D_{i'} + \sum_{h=1}^{2} \beta_h \log(X_{hij} / \sum_{j=1}^{J} X_{hij}) + \varepsilon_{ij}$

e $\log p_{ij} = \alpha_0 + \sum_{i'=2}^{3} \alpha_{i'} D_{i'} + \sum_{h=1}^{2} \beta_h \log(X_{hij}/\bar{X}_{hi.}) + \varepsilon_{ij}.$

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