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Essays on Frictional Labor and Housing Markets

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Essays on Frictional Labor and Housing Markets

DISSERTATION

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DOCTOR OF PHILOSOPHY

in Economics

by

Bessy Liao

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Professor Jan K. Brueckner,Co-Chair
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2024

DEDICATION

To my grandpa Liji,
who taught me perseverance during hardship and the joy of pursuit of knowledge.

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ABSTRACT OF THE DISSERTATION

Essays on Frictional Labor and Housing Markets

By

Bessy Liao

Doctor of Philosophy in Economics

University of California, Irvine, 2024

Jan K. Brueckner, Co-Chair
Guillaume Rocheteau, Co-Chair

This dissertation will study how frictions in labor and housing markets affect outcomes in each market since labor markets and housing markets are highly frictional. The first study empirically analyzes the role that information friction plays in the housing tenure choice when households move to a new location. The results show that information friction reduces the likelihood of homeownership. The second study theoretically and quantitatively analyzes the role that search friction plays in the labor market, in particular, in the Great Divergence phenomenon. I find that the search friction in the labor market moderates the Great Divergence and provides more incentive for low-skill workers to move to productive locations. The third study examines the consequences of rent control policies on the frictional rental housing market in a search and matching framework. The model suggests that an increase in productivity growth rate would reduce the optimal occupancy duration and raise the vacancy rate. In contrast, an increase in the eviction cost would have the opposite effect.

Chapter 1

Introduction

This collection of papers studies the role that frictions in the labor and housing market play in shaping the outcomes in each market.

The first and third studies in the dissertation concern the housing market. Chapter 2 tries to understand how information friction affects migrants' housing tenure choices. The empirical results show that geographic proximity and social connectedness mitigate information friction and improve the likelihood of homeownership of migrants who have moved for more than 50 miles. Occupational ties do not mitigate information friction for migrants who have moved for more than 50 miles but do so for shorter-distance migrants.

Chapter 4 focuses on a specific housing market policy, namely, rent control, and investigates its effects on the rental housing market for landlords and tenants. This chapter develops a search and matching model with a frictional rental housing market with search friction, productivity growth, and rent control shocks. It also studies the effects on the supply of rental housing. The model shows that an increase in productivity growth rate would reduce optimal occupancy duration and raise the unhoused rates and vacancy rates. In contrast, an increase in the eviction cost would have the opposite effect. More stringent land use regulations would raise the market tightness as well as the unhoused rate. Comparing the model with rent control policies to the one without, I find that rent control raises the price of non-rent-controlled units, reduces market tightness, and reduces the rate of unhoused renters.

Chapter 3 studies search frictions in the labor market and how the search frictions changed our understanding of the divergence of US cities since the 1980s. I document novel empirical facts about this "Great Divergence", showing that high-skill, high-rent cities also experience a reduction in long-run unemployment rates. Since wage and unemployment rates are jointly determined, incorporating geographic variation in unemployment rates is quintessential in understanding the welfare implication of this divergence. This chapter develops a spatial equilibrium model with frictional labor markets that give rise to unemployment, featuring workers of different skill levels that share a housing market. I calibrate the model to the US economy between 2005 and 2019 and find that the worker population is inefficiently small in high-wage, high-rent locations. The share of high-skill workers in these locations is inefficiently high. This misallocation is caused by the distortion resulting from the inseparability between the labor market and housing market location. Comparing the model to its competitive counterpart without unemployment shows that search frictions moderate the divergence, allowing an additional channel to balance the spatial equilibrium, leading to smaller utility differences between high- and low-skill workers. Policies that encourage low-skill workers to relocate to high-wage locations improve aggregate welfare.

The ambition of this dissertation is to extend our understanding of how information friction and search friction affect highly frictional and spatial markets. I find that information and search friction play a huge role in shaping the markets.

Chapter 2

Information barriers and housing tenure choice: Do local ties matter?

2.1 Introduction

Decades of research have been devoted to uncovering the complexity of housing tenure choice, in which a large amount of transaction costs are involved. In particular, a great deal of scholarly attention has been paid to the forward-looking nature of the investment decisions and the importance of uncertainties that can make households hesitant to purchase their own home. Previous studies, however, have tended to focus on future employment or income uncertainties and their impacts on housing tenure choice. Relatively little is known about other sources of uncertainties involved in the process of home purchase decision-making, including the role of ties to the destination. The lack of ties or connections to the destination could act as an important source of uncertainties, leading to households having limited information about the destination, resulting in LD movers in having less confidence in becoming owners, and therefore causing a significant difference in ownership rate among LD movers.

This study attempts to examine whether, and to what extent, local ties matter in shaping housing tenure choice among recent movers[1] in the United States. The goal is two-fold: 1) to draw attention to various kinds uncertainties and barriers to home purchase, especially to the ones that stem from the lack of local

[1]Household who have moved to their new home within a year.

ties, that have been largely neglected in the literature; and 2) to present a way to investigate the channels through which the housing tenure decision is affected by local ties. If we can understand the roles that local ties play in housing tenure choice, urban planners and other policy makers as well as the real estate industry could potentially make qualitative information about the destination more accessible for movers with weak local ties and help them avoid unnecessary transaction or moving costs.

More specifically, we look into the following channels through which the movers can establish local ties: geographic proximity, social connectedness, and occupational ties. We divide movers into long distance (LD) movers, defined as those who have moved more than 50 miles[2], and short distance (SD) mover because we think LD movers' and SD movers' tenure choices are differentially affected by channels of local ties. By definition, LD movers have weak ties through the geographic proximity channel since they moved from further away. However, such deterrent effects could be mitigated when households have other forms of local ties. For movers who are already embedded in a location and have spent time there, their ties to the destination is no longer hindered by the geographic proximity channel or social connectedness channel. In their case, occupational ties to the local housing market could play a bigger role in their housing tenure choices.

With the prevalence of online services that real estate corporations provide, it has become easier than ever for households who are looking for a new home to obtain information about future homes in even distant parts of the country. Nevertheless, a crude look at homeowner rates since the beginning of online real estate services [3] suggests that the share of owners among recent movers did not increase with the rise of online real estate services.

If the geographic proximity plays a significant role in reducing the probability of home purchase, one would expect to see a negative relationship between migration distance and home purchase rates. This can be seen in Figure 2.1, which illustrates the relationship between migration distance and homeownership rates among recent movers, without controlling for local ties, households or destination characteristics.[4] This pattern suggests that a lack of local ties, in particular through the channel of geographic proximity, may play a significant role in explaining the ownership gap among movers. Table 2.1 shows the share of owners by migration distance groups.[5] We can see that SD movers (< 50 mi) have the highest share of owners among

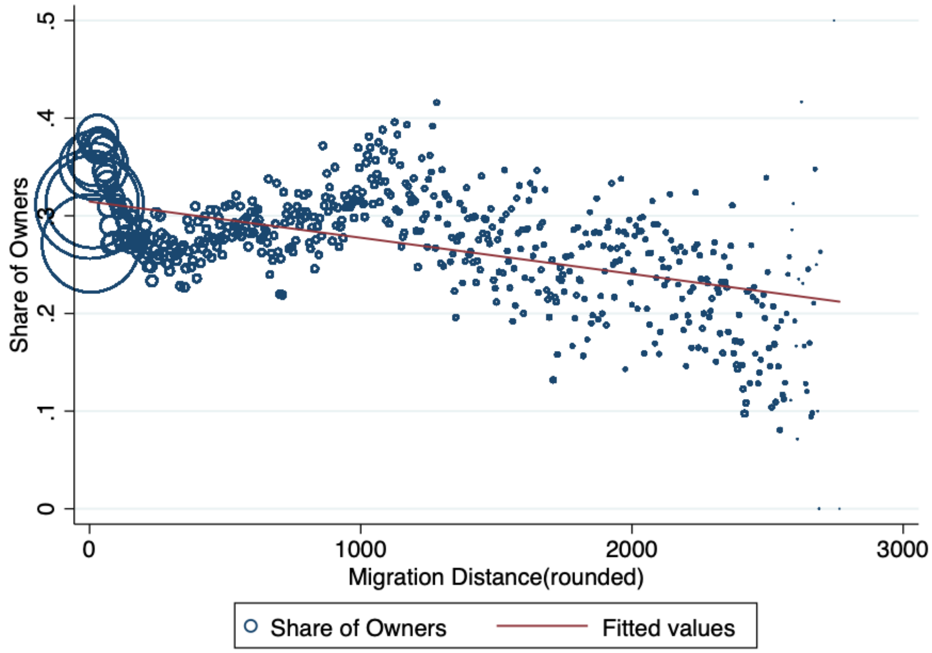
[2]similar definition to Ha et al. [2021], these movers are unlikely to be households who have made their initial long-distance move and later re-adjusted their residence in the new destination.

[3]As measurement for the time at which these services first became available, we use the date at which online real estate corporations applied for relevant patent application for the first time. For examples Redfin Corporation's (sought or received) their patent for "Online marketplace for real estate transactions" in 2005 (Eraker and Eraker [U.S. Patent 9,105,061.]), NetLeaseX IP Holdings LLC's with "Online real estate transaction system" in 2005 (Zimmerman and Donenfeld [U.S. Patent Application 11/061,921.])

[4]The size of the circle represents the size of the migrating population. This pattern does not appear to be monotonic, but it follows a general downward trend.

[5]Numbers in parentheses are the standard deviation, and N indicates sample size for each distance group.

Figure 2.1: Migration distance and Ownership



all distance groups whereas those who have moved over 2000 miles have the lowest share of owners.

Table 2.1: Share of owners across distance groups

Migration Dist.	< 50 mi	50-500 mi	500-1k mi	1k-1.5k mi	1.5k-2k mi	> 2k mi
Share of	0.300	0.244	0.267	0.299	0.221	0.187
Owners	(0.458)	(0.429)	(0.442)	(0.458)	(0.415)	(0.390)
	N = 171,771	N =16,108	N = 5,955	N = 3,432	N =2,342	N =3,336

To explore the role of local ties among movers more systematically and thus provide deeper insights into the complexity of housing tenure decision-making, we conduct empirical analysis, using data from the one percent Public Use Microdata Sample of the 2012-2019 American Community Survey, with several proxies for capturing the aforementioned channels of local ties. It is our hypothesis that channels such as geographic proximity and social connectedness reflect the strength of one's local ties, and hence how much information one has about the destination, [6] consequently making a significant difference in tenure choice among recent

[6] For example, distance and share of migration flow are used to measure information flow in Alsan and Wanamaker(2017)

movers. Our empirical analysis provides support for this hypothesis by showing that LD households with stronger local ties are considerably more likely to own their home compared to those who have weaker ties. The findings also suggest that these channels of local ties affect LD and SD movers differently.

2.2 Related Literature

According to the 2019 Survey of Consumer Finance, 64.9 percent of households in the U.S. owned their home, whereas only 59 percent of households had savings and only 53 percent had any stock holdings (Bricker et al. [2017]). In other words, for most households in the U.S., home is their largest investment (Davidoff [2006]), and hence the importance of housing tenure choice can be hardly understated.

There is a large body of literature on housing tenure choice. Existing studies tend to focus on characteristics of the households[7] as the main determinant for a household's tenure choices. Many of these studies, however, do not pay explicit attention to the uncertainties that the households might consider.

Henderson and Ioannides [1983] provides one of the early theoretical economic models for the demand of housing and tenure choice with uncertainty, suggesting the over-investment of households in housing. Building on this framework, Fu [1991] introduces uncertainty in the form of housing price variation into the housing tenure choice model. Brueckner [1997] further analyzes how multiple risky assets, including housing, affect housing consumption.

As noted by Henderson and Ioannides [1983], one important source of uncertainty avoidance behavior is our limited ability to predict future trajectories of the economy, particularly inflation and housing market cycles. Turner and Seo [2007] examines the extent to which house-price uncertainty affects households' transition from renting to owning, and their findings suggest that this form of uncertainty can act as a significant barrier to the transition. Fu [1995] analyzes how the decision-making of illiquid households is affected by uncertain future prices. Rosen et al. [1983] and Turner [2003] also report empirical evidence that the volatility and uncertainty in house prices negatively affect homeownership in the US.

Another important source of risk aversion is the uncertain future income or employment status of individuals. Diaz-Serrano [2005] shows that risk aversion triggered by labor income uncertainty can lower homeownership

[7]Such characteristics are mostly demographic characteristics, see e.g., Kain and Quigley [1972], Eilbott and Binkowski [1985]; Gyourko and Linneman [1996]; Painter et al. [2001]; Hilber and Liu [2008]; Coulson and Dalton [2010]. Additionally, downpayment constraints and borrowing constraints are discussed in Brueckner [1986] and Linneman and Wachter [1989] respectively.

rates. Haurin and Gill [1987], Haurin [1991] and Robst et al. [1999] also investigate the extent to which income uncertainty affects housing tenure choice. These studies suggest that income uncertainty, in general, reduces the probability that a household purchases residential housing. Sinai and Souleles [2005] and Sinai and Souleles [2013] consider risks in both renting and home purchase. They suggest that ownership may function as a hedge against uncertainty in rental costs in the future (i.e., the probability of home purchase increases with net rent risk). Not surprisingly, the literature on the uncertainty-homeownership relationship suggests that an increase in risk and uncertainty reduces housing consumption in two ways. First, higher levels of uncertainty lead to a lower probability of being an owner in the housing tenure choice Hilber [2005]. Second, higher levels of uncertainty lead to a reduction in the quantity of housing consumption for owners.

Although these studies clearly highlight the importance of various sources of uncertainty ranging from rental price volatility to future labor income, the role that local ties play in housing tenure choice has not been examined.[8] The uncertainty this paper focuses on is caused by the information gap between LD movers with strong local ties and weak local ties, since the strength of ties can affect the information about the destination available to the movers, and hence their housing tenure decisions.

The most relevant research to this paper would be that of Ha et al. [2021], titled "Do long distance moves discourage homeownership? Evidence from England". Using data from the Survey of English Housing, they show that LD movers' probability of homeownership is 5.5 percentage points lower than that of SD movers. What remains unknown is whether distance matters among LD movers (i.e. those who moved more than 50 miles) grouped all together and then compared with SD movers in Ha et al. [2021]. It is also important to understand how social or professional ties would allow them to overcome the lack of geographic proximity that can make their home purchase more challenging. According to **?**, the workings of the housing market can be shaped by the information asymmetry between sellers and buyers on knowledge of neighborhood characteristics does matter, since the incumbent sellers have gained stronger local ties through embeddedness than buyers who could be new to the area. Other studies also indicate the importance of information gaps between local and out-of-town buyers. Chinco and Mayer [2016], for instance, show that out-of-town home buyers' behavior patterns are similar to those of misinformed speculators. Agarwal et al. [2018] report that foreign investors pay a premium in making real estate transactions, which may reflect the information disadvantage. More generally, Portes and Rey [2005], Coval and Moskowitz [2001] and Baik et al. [2010] treat distance as a valid measure of information asymmetry between informed and uninformed investors.

[8] Although there is a body of work that highlights the role that general knowledge about the housing market and housing transaction play for housing tenure choice. This thread of literature includes work by Henretta [1984], Dietz and Haurin [2003], and Haurin and Morrow-Jones [2006], just to name a few.

2.3 Empirical Analysis

2.3.1 Data

The main dataset we used is from the American Community Survey (ACS) Public Use Microdata Sample (PUMS) obtained from IPUMS USA Steven Ruggles and Schouweiler [2021]. We combined eight years (2012-2019) of microdata from this comprehensive, cross-sectional data source that provides rich information for approximately 1 percent of the households in the United States each year, including their migration, housing tenure, housing payment, income, and other sociodemographic characteristics. The ACS PUMS allowed us to identify who moved (as opposed to living in the same housing unit) with detailed household characteristics needed for a more complete set of control variables.

Although the location information available in the data source is only as precise as Public Use Microdata Areas (PUMA), it is possible to use GIS to compute the migration distance of each household. Given our main focus is on households who moved more than 50 miles, the geographic unit precision is of lesser concern. The ACS PUMS is advantageous in that its sample size (i.e., 1 percent sample of the U.S. population each year) offers data for a larger number of households who moved over a range of distances than any other (publicly available) alternative data sources can. Additionally, the data source provides information about each household's origin and destination, enabling us to control for possible effects of origins and destinations.

Table 2.2. summarizes all the variables used in this study, including our measurements of local ties explained in detail below.

2.3.2 Measurement of Local Ties

As mentioned in the introduction, not all movers have the same degree of ties to their destinations, and this variation might significantly influence housing tenure choice. It is important to stress that the local ties can be made in various ways. Our measurements of local ties attempt to capture these multiple possibilities. Specifically, in this study, we employ the following three categories of measurements: geographic proximity, social connectedness, and occupational ties, as detailed below. Summary statistics of these measurements are shown in Table 2.3.

Table 2.2: Definition of variables

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	Definition
Ownership	Dummy variable indicating ownership of home
Log Migration Distance	Log of distance between the origin PUMA and the destination PUMA
Birth-State Indicator	Dummy variable indicating whether the household head lives in the state he/she was born in
County Flow Share	The 7-year average share of households who migrated from the same origin county to the same destination county as a percentage of the population of the destination county
Real Estate Occupation	Dummy variable indicating whether the household's head works in real estate industry
Household Income (k$)	Household income in thousand dollars
Age	Age of household head
Gender	Dummy variable indicating household head gender
Education	Number of years of education the household head received
Employment Status	Dummy variable indicating household head employment status
Race	Categorical variable of the race of the household head
Ethnicity	Dummy variable indicating if household head is Hispanic
Marital Status	Dummy variable indicating household head marital status
Metro Status	Dummy variable indicating if household lives in a metro area
Metro Status Change	Dummy variable indicating whether the metro status changed
Moved Within State	Dummy variable indicating whether migrated within the same state.
PUMA-level Ownership	PUMA average ownership rate
PUMA-level HH Income (k$)	PUMA average household income in thousand dollars
PUMA-level Home Value (k$)	PUMA average home value in thousand dollars
PUMA-level Housing Cost Ratio	PUMA average ratio between home value and household income
PUMA-level Units in Structure	PUMA average number of dwelling units in the home
PUMA-level Number of Rooms	PUMA average number of rooms in the home

Geographic Proximity

First and foremost, we use the log transformation of migration distance to capture each mover's geographic proximity to her/his destination. The distance used is the geographical distance between the center of the migrant's origin PUMA and the center of the destination PUMA, computed using ArcGIS. Although not perfect (since instead of the coordinate of the household, we only have information about the PUMA(s) that the household moved from and to), this measure provides a useful way to discern movers with varying degrees of geographic proximity to their destination places. As noted above, the imprecision in distance is relatively negligible, especially when it comes to long-distance migration. Given that a long distance can act as an impediment to the development of local ties, we hypothesize that the longer the migration distance, the weaker the local ties.

Social Connectedness

Some people may overcome difficulties in developing local ties over a long distance through other mechanisms, while others can't. Social connectedness is an important enabler of such possibilities. We use the following two metrics to capture the social ties the movers have with their destinations.

Birth-State Indicator

This metric is a dummy variable that indicates whether the household head moved to the state he/she was born in. Among long-distance movers, 31.7 percent moved to/within the state they were born in. If the individual was born in the state, he/she may have some family or social connections with the locality, and these families and social connections can grant the individual stronger social ties than someone who was not born in the state that they moved into. It would have been desirable if we could have had a smaller geographic unit of birthplace than the state, but it is the lowest geographic division of birthplace available in the data.

County Flow Share

Another channel through which we measure social connectedness is through earlier migrants be- tween the same origin and destination. We capture this channel of social connectedness by measuring the 7-year average share of households who migrated from the same origin county to the same destination county, relative to the population of the destination county. For example, for a household who lived in county A and moved to county B in 2017, we calculate the average number of people moving from A to B between 2010 and 2016 as a percentage of the population of county A. This metric enables us to discern origin-destination pairs with varying migration flows, and we assume the higher the county flow share, the tighter the two communities are connected, and hence the stronger the social ties.

$$\text{County Flow Share}_{AB,2017} = \frac{\frac{1}{7}\sum_{i=2010}^{2016} \text{Size of Movers}_{AB,i}}{\text{Pop}_A}$$

Occupational Ties

Local ties can also be developed through one's occupation. We use whether a mover has a real estate job as a measurement of occupational ties. In other words, we create a dummy variable indicating "Real Estate Job" holders – i.e., 1 if the occupation code in the ACS PUMS indicates the individual's occupation as either "Property, Real Estate, and Community Associated Managers", "Real Estate Broker and Sales Agents" or "Appraisers and Assessors of Real Estate" or if the individual works in the industry of "Real Estate", 0 otherwise. Among long-distance movers, only 0.85 percent of the individuals are identified as a real estate job holder. Given the occupational ties, these movers might have better access to information about their destinations (or the local housing market) and show distinct tenure choice patterns.

2.3.3 Households and PUMA characteristics

Apart from the traditional control variables for demographics and household characteristics,[9] we also include destination control variables in our empirical analysis, such as destination state fixed effects, PUMA-level household income, PUMA-level homeownership rates, and PUMA-level housing characteristics.[10] We also control for survey year fixed effects.

[9] summarized as "Household Controls" in regression tables

[10] summarized as "Destination Controls" in regression tables.

Table 2.3: Descriptive Statistics

	SD Movers	LD Movers	50-500 mi	500-1k mi	1k-1.5k mi	1.5k-2k mi	> 2k mi
Ownership	0.300	0.249	0.244	0.267	0.299	0.221	0.187
	(0.458)	(0.432)	(0.429)	(0.442)	(0.458)	(0.415)	(0.390)
	N = 171,771	N = 31,175	N =16108	N = 5955	N = 3432	N =2342	N =3336
Migration Distance	10.69	710.8	198.0	749.5	1,206	1,748	2,316
	(9.560)	(690.0)	(124.7)	(151.4)	(151.5)	(134.1)	(163.8)
	N = 171,771	N = 31,175	N =16108	N = 5955	N = 3432	N =2342	N =3336
Birth-State Indicator	0.472	0.262	0.373	0.150	0.132	0.126	0.123
	(0.499)	(0.440)	(0.483)	(0.357)	(0.339)	(0.332)	(0.328)
	N = 171,771	N = 31,175	N =16108	N = 5955	N = 3432	N =2342	N =3336
County Flow Share	0.0346	0.0208	0.0343	0.00562	0.00613	0.00430	0.00576
	(0.121)	(0.0550)	(0.0723)	(0.0104)	(0.0106)	(0.00793)	(0.0117)
	N = 171,771	N = 31,175	N =16108	N = 5955	N = 3432	N =2342	N =3336
Real Estate Occupation	0.0107	0.00953	0.00986	0.00896	0.0111	0.00761	0.00819
	(0.103)	(0.0972)	(0.0988)	(0.0942)	(0.105)	(0.0869)	(0.0902)
	N = 171,771	N = 31,175	N =16108	N = 5955	N = 3432	N =2342	N =3336

2.3.4 Empirical Strategy

Our empirical analysis attempts to answer the following: (1) what are the channels through which long-distance movers could strengthen local ties? and (2) do the same channels of local ties affect short-distance movers' housing tenure choice?

The effect of local ties on homeownership

Since our outcome variable of interest, housing tenure choice, is binary, we use a logit model to estimate the following equation:

$$\Pr(\text{own} = 1) = \Lambda(\beta_0 + X'_{LT}\beta_1 + Z'_{HH}\beta_2 + Z'_{dest}\beta_3 + \beta_4 Z_t)$$

where $\Lambda(.)$ is the logistic cumulative distribution function, X_{LT} is a vector of local ties variables as defined in section 2.3.2. Z_{HH} is a vector of household characteristics control variables including age, gender, race, ethnicity, education level, marital status, household income, employment status, metro status, and their origin state. It is referred to as "Household Controls" in the outcome tables. We are only considering individual characteristics of the head of household since we assume that moving and housing tenure choice are household-level decisions, and that the characteristics of the head are more relevant in making these decisions. Z_{dest} is a vector of destination characteristics control variables, including destination state, percentage of homeowners in destination PUMA, average household income in destination PUMA, destination PUMA average housing-cost-to-income ratio, average number of rooms per home in destination PUMA, and average home(s) per building in destination PUMA. It is referred to as "Destination Controls" in the result tables. Lastly, Z_t controls for survey year.

In order to understand the channels through which local ties affect long distance migrants' housing tenure decision, we estimated the logistic regression model with various specifications and presented the results in Tables 4 and 5. This analysis uses all long distance migrant households as sample. In terms of the local ties specification, Table 4 uses level variables, whereas Table 5 includes interaction terms between log migration distance and the other local ties variables.[11]

[11]For sensitivity analysis, we classify movers into several groups based on their moving distance to look more closely at the effect of distance at different distance levels. The sensitivity analysis results are shown in Table A.3 in appendix A.1.2. Additionally, we look at households who moved from eastern (western) coastal counties to western (eastern) coastal counties, since even though coast-to-coast migration is always long in distance (at least 2000 miles), distance alone may not accurately reflect the information flow between the origin and destination, given that much of this migration is between large coastal cities which sometimes have strong ties with one another (Badger and Bui). The results are shown in Table A.4 in appendix A.1.2.

For each table, we test five model specifications by including our measurements of local ties incrementally. They are the following: model (1) uses the baseline specification with log migration distance as a measurement of local ties; models (2-4) add the birth state indicator, county flow share, and the indicator variable for real estate job holders, respectively; model (5) includes all the four variables of local ties. All models control for household characteristics Z_{dest}, destination characteristics Z_{dest}, and year fixed effects Z_t, as indicated at the bottom of the tables.

Do the same channels of local ties affect short distance movers' housing tenure choice?

While our focus is on the contributions of local ties to long-distance movers' housing tenure choice, the measurements of local ties can also affect short distance movers. In order to understand whether short distance movers' housing tenure decisions are affected similarly or differently from the aforementioned channels of local ties, we have estimated our model with both long-distance migrants and short-distance migrants. In this analysis, we have created a dummy variable "LD indicator" that equals to 1 if the household moved more than 50 miles, and we have included the interaction term between the dummy variable and each measurement of local ties. In terms of model specification, this analysis is similar to our earlier analysis (Table 4) except for the inclusion of all of the interaction terms, enabling us to assess whether the same channels of local ties that affect long distance movers' housing tenure choices, affect short distance movers' tenure choices in the same manner.

2.4 Results

We report the empirical results in two parts, corresponding to the two sections in empirical strategy.[12]

2.4.1 Empirical evidence on the effect of local ties on LD movers' housing tenure choice

The impact of local ties on homeownership.

Table 4 shows the logistic regression results with the metrics of local ties and control variables. Among others, our results strongly suggest that a longer migration distance (logged) reduces the chances of home

[12]The result tables in this section focuses on the coefficients of local ties variables. For coefficients on the control variable, please see appendix A.1

purchase by about 1.41%-1.76%. As shown in Table 4, the strong negative effect of migration distance is found fairly consistent, while its magnitude turns out to be relatively smaller when the county flow share is included in the model.

Two of the three other metrics also show statistically significant effects. More specifically, the birth-state indicator exhibits a significant positive coefficient, suggesting that migrating into or within one's birth-state increases the chances of home purchase by 5.61%-5.67%. The marginal effect of county flow share is also positive, indicating it increases the chances of purchasing homes by 23.7%-24.3%. However, the metric of occupational ties (i.e., Real estate job holders) is not found to have a statistically significant impact among LD movers.

Interaction effects (How other measurements of local ties moderate or amplify the effect of migration distance)

In addition to the marginal effects of the variables presented in Table 4, Table 5 explores whether other local ties variables mitigate the effect of migration distance on LD movers' housing tenure choice. The coefficient of the interaction term between log migration distance and the birth-state indicator suggests that migrants who were born in their destination states tend to have a significantly reduced impact of migration distance (i.e., -0.0184+0.0163 and -0.0146+0.0142 in the second and fifth columns, respectively) compared to those who were not born in destination states. This result may imply strong birth state effects which allow LD movers to overcome the lack of geographic proximity through an alternative channel of local ties. As discussed earlier, the birth-state indicator may indicate the presence of family or social connections with the destination place, enabling the LD movers to access more information or have more confidence in making home purchase than someone who has to make a long distance move without such ties.

The county flow share variable, however, does not show the same moderating effect. Rather, the interaction term between log migration distance and this metric shows a negative sign, whereas the variable itself has a significant, positive impact on home purchase. This finding suggests that the positive impact of more migrant flows between the origin and destination tend to decrease as migration distance increases. In other words, the longer the migration distance, the smaller the marginal effect of county flow share is on home purchase. Again, the estimation results for occupational ties do not show any evidence of statistical significance, indicating that real estate job holders do not have a significantly higher or lower rate of home purchase when moving over a long distance.

Table 2.4: LD Movers With Level Variables

	(1)	(2)	(3)	(4)	(5)
	dy/dx	dy/dx	dy/dx	dy/dx	dy/dx
Local Ties	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)
Log Migration Distance	-0.0175***	-0.0176***	-0.0140***	-0.0175***	-0.0141***
	(0.00180)	(0.00197)	(0.00190)	(0.00180)	(0.00205)
Birth State Effect		0.0561***			0.0567***
		(0.00633)			(0.00630)
County Flow Share			0.237***		0.243***
			(0.0424)		(0.0422)
Real Estate Occupation				-0.00468	-0.00494
				(0.0192)	(0.0193)
Observations	31,175	31,175	31,175	31,175	31,175
Household Controls	Yes	Yes	Yes	Yes	Yes
Destination Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Overall, we find that migration distance has a negative impact on the probability of homeownership, whereas moving to the birth state or a high level of county flow share appears to have a positive impact. The effects of both of these are clear and consistent across different specifications, suggesting that social connectedness as well as geographic proximity does matter. The impact of holding a real estate job, however, is not statistically significant for any specifications. Additionally, the interaction terms in Table 5 shows that the effect of birth state supplements the effect of geographic proximity on ownership, whereas the effect of county flow share complements the effect of geographic proximity on ownership.

2.4.2 Empirical evidence on the effects of local ties on short-distance vs. long-distance movers

Table 2.6 presents how channels of local ties affect housing tenure choice for short-distance movers in comparison with their effects on long-distance movers. As explained earlier, this is accomplished by creating a dummy variable "LD Indicator" to reflect whether a mover is a long-distance mover and using the interaction term between the indicator and the metrics of local ties to assess if these ties have differential effects for long-distance movers and short-distance movers. The estimated coefficients for the level variables "Log Migration Distance","Birth-State Indicator", "County Flow Share", "Real Estate Occupation" capture the effects of local ties on SD mover's housing tenure choice, and the effects for LD movers are the sum of the coefficient for the level variable and that for the interaction term.

As shown in the table, the sign of the coefficient "Log Migration Distance" is not consistent across model specifications and is often not statistically significant. The results indicate that, unlike for LD movers, the effects of migration distance on the housing tenure choice of short-distance movers who moved less than 50 miles. One way to think about this finding is that, within a short distance, quality and quantity of local ties is no longer reflected via geographic proximity, since the households are already embedded in the destination location.

Table 2.5: LD Movers With Interaction Variables

	(1)	(2)	(3)	(4)	(5)
	dy/dx	dy/dx	dy/dx	dy/dx	dy/dx
Local Ties	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)
Log Migration Distance	-0.0175***	-0.0184***	-0.0131***	-0.0177***	-0.0146***
	(0.00180)	(0.00214)	(0.00193)	(0.00181)	(0.00224)
Birth-State Indicator		-0.0472			-0.0334
		(0.0401)			(0.0400)
Log Migration Distance * Birth-State Indicator		0.0163***			0.0142**
		(0.00625)			(0.00625)
County Flow Share			1.053***		0.974***
			(0.307)		(0.311)
Log Migration Distance * County Flow Share			-0.188***		-0.171**
			(0.0690)		(0.0698)
Real Estate Occupation				-0.106	-0.115
				(0.0970)	(0.0949)
Log Migration Distance * Real Estate Occupation				0.0170	0.0187
				(0.0161)	(0.0158)
Observations	31,175	31,175	31,175	31,175	31,175
Household Controls	Yes	Yes	Yes	Yes	Yes
Destination Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 2.6: Movers With Level Variables

	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Local Knowledge	Ownership	Ownership	Ownership	Ownership	Ownership	Ownership
Log Migration Distance	0.0311***	0.0358***	-0.0112	-0.000273	0.0358***	-0.0111
	(0.00410)	(0.00411)	(0.00716)	(0.00627)	(0.00411)	(0.00718)
Long Distance Indicator * Log Migration Distance	-0.160***	-0.169***	-0.154***	-0.163***	-0.170***	-0.157***
	(0.00334)	(0.00365)	(0.00621)	(0.00517)	(0.00366)	(0.00630)
Birth State Effect		0.208***	0.173***	0.157***	0.208***	0.173***
		(0.00997)	(0.0129)	(0.0124)	(0.00997)	(0.0129)
Long Distance Indicator * Birth State Effect		0.227***	0.190***	0.261***	0.226***	0.189***
		(0.0210)	(0.0386)	(0.0293)	(0.0210)	(0.0386)
County Flow Share			0.397***			0.395***
			(0.0451)			(0.0451)
Long Distance Indicator * County Flow Share			2.171***			2.116***
			(0.330)			(0.330)
Past Year House Value Variance				3.12e-08		3.13e-08
				(3.40e-08)		(3.42e-08)
Long Distance Indicator * Past Year House Value Variance				2.10e-07***		1.88e-07***
				(6.03e-08)		(6.48e-08)
Real Estate Occupation					0.131***	0.151***
					(0.0475)	(0.0561)
Long Distance Indicator * Real Estate Occupation					0.133	-0.00236
					(0.111)	(0.174)
Observations	349,030	349,030	202,946	227,371	349,030	202,784
Household Controls	YES	YES	YES	YES	YES	YES
PUMA level Controls	YES	YES	YES	YES	YES	YES
Origin and Destination States	YES	YES	YES	YES	YES	YES

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The two metrics of social connections show dissimilar impacts. For SD movers the coefficients for the Birth-State indicator are positive, but no longer statistically significant. The county flow share variable, however, has positive and statistically significant effects. Compared to LD movers, the effects of these two variables are weaker. That is to say, social connectedness increases the likelihood of home purchase for LD households more than for SD households, all other things being equal.

Last but not least, the coefficient for real estate occupation holders is positive and statistically significant for SD movers, whereas it is insignificant when only LD movers are in the sample (as presented in Table 4). In other words, the occupational ties captured via this variable make a difference among locally embedded short-distance movers. As noted above, however, this channel of local ties does not appear to act a substitute for geographic proximity for LD movers.

In summary, the channels through which local ties affect housing tenure choice are very different for LD movers than those for SD movers. Migration distance plays a smaller role for SD movers, while social connectedness still has positive effect on SD movers, although smaller in magnitude compared to the effect for LD movers. Having a real estate job seems to increase the probability of home purchase for SD movers, but this does not significantly affect housing tenure choice of LD movers.

Regarding household characteristics, we find that higher values of household income, age, and education levels have positive effects on the likelihood of home purchase.[13] Households with married heads are more likely to own.[14] All of these results are consistent with the literature. As for destination PUMA-level characteristics, PUMA-level ownership has a very strong positive effect on home purchase, which may be due to the fact that PUMAs vary in their offerings of housing mix (e.g., single-family housing vs. multi-family units). PUMA-level average household income does not have a statistically significant impact, while a higher average home-value-to-income-ratio lowers the probability that the household owns a home as anticipated.

2.5 Conclusion and Discussion

While homeownership has long been promoted in the United States and many other countries, buying a house has remained as one of the most complicated decisions faced by households for generations. This is particularly true nowadays. The experience of the 2008 subprime mortgage crisis and resultant foreclosures

[13]Regression coefficients using level local ties variable for household characteristics and PUMA-level characteristics are reported on Table A.1 and Table A.2 in appendix A.1.

[14]In the long distance sample, all household heads are employed and are living in metropolitan areas.

has made it reasonable, if not necessary, to be cautious about purchasing a home. On the other hand, the rapid housing price recovery (and further escalation) in recent years has made it difficult for prospective home buyers to make the investment decision in a prudent manner.

In the literature, much attention has been paid to the nature of this important, complicated decision making, but our understanding is still incomplete. This study attempts to expand the literature by showing the additional challenges long-distance movers have and examining the role of local ties in shaping their decision-making process. More specifically, we explore the mechanisms by which the presence or absence of local ties can affect housing tenure choice using indicators of geographic proximity, social connectedness and occupational ties.

Our empirical analysis results show the following. First, for LD movers, having stronger levels of local ties raises the propensity to purchase their home than those with weaker local ties. Second, channels of local ties are of different impact on LD movers versus SD movers on their housing tenure choices. Migration distance and house value growth are more important factors for LD movers whereas having a real estate occupation is a more important factor for SD movers.

These finding are consistent across different specifications of the logit regression model using various measurements for local ties. Given the long-lived nature of home purchase (and large transaction costs involved), the lack of local ties can pose a significant challenge to migrants and result in a lower level of confidence (or a higher level of uncertainty), which makes renting a more desirable (or safer) option for them when they move into a new region.

Admittedly, the present study is not without limitations, with the most obvious one being that cross-sectional data limits our ability to establish causality. Additionally, this study does not address the motivation of migration, giving no consideration to what motivated a migrant to move (over a short or long distance) and paying no attention to what motivated them to move over a short vs. long distance. In other words, long-distance movers may be different from short-distance movers in aspects we could not observe. A more complete understanding could be obtained when the housing tenure and 'move or not' decisions are jointly modeled and analyzed in future research. Longitudinal data sets would also enable researchers to unravel the complexity of migrants' housing tenure choice. Nevertheless, this work offers a meaningful step towards gaining deeper insights into the complexity of housing tenure choice among migrants.

Chapter 3

The Great Divergence with Frictional Labor Markets

3.1 Introduction

Divergence in wages between high-skill and low-skill workers has been well documented since the 1980s, most notably in works by Katz and Murphy [1992] and Goldin and Katz [2008]. Recent research has drawn attention to the spatial dimension of wage divergence, whereby Ganong and Shoag [2017], Hsieh and Moretti [2019] and Austin et al. [2018] document the wage divergence in terms of geography. Together, these trends in labor income reinforce each other, leading to a polarization of cities described by Moretti [2012] as “the Great Divergence”, where an abundance of high-skill workers are clustered in high-wage and high-rent cities and the low-wage and low-rent cities bear a larger share of low-skill workers. Since wages are only observed conditional on employment, what matters for workers is the product of wages and employment probabilities. Therefore, understanding the welfare implications of the Great Divergence necessitates incorporating the spatial variation of unemployment rates caused by search frictions in the labor markets.

How do search frictions in the labor markets contribute to the Great Divergence? In this paper, I document the dispersion of unemployment rates in the US in terms of geography by skill level. Between 2005 and 2019, there was considerable geographic variation in unemployment rates, particularly for low-skill workers. Furthermore, the cities that have grown in high-skill concentration and real wages also experienced decreased unemployment rates for both skill types. I then develop and calibrate a spatial equilibrium model with heterogeneous workers in frictional labor markets and local housing markets to understand the implications of frictional labor markets on the location decisions of high- and low-skill workers. I ask how high-skill workers' location choices affect low-skill workers' location choices and vice versa. Using the calibrated model, I find the optimal skill composition of workers across space with search frictions in labor markets.

Locations fundamentally differ in their production function and housing supply in my model, which generates an equilibrium with two types of locations - locations with large shares of high-skill workers (H) feature high-wage, high-rent with low unemployment rates, and locations with small shares of high-skill workers (L) feature low-wage, low-rent with high unemployment rates. The negative association between wages and unemployment rates across locations results from the model's job creation condition since firms have incentives to create more jobs where the per-worker output is higher.

High-skill and low-skill workers affect one another through the following channels: First, due to the complementarity between high- and low-skill workers in the production process, locations with more high-skill workers pay higher wages for low-skill workers. This is the agglomeration force that creates incentives for high- and low-skill workers to co-locate. Second, due to the limited housing supply, high- and low-skill workers compete on the common housing market, raising the cost of living in high-wage locations, which is a dispersing force. The relative strength of these opposing forces determines the equilibrium size of labor markets as well as the skill composition in each of them.

I find that search frictions in the labor market moderate the divergence, resulting in high-wage, high-rent cities having a smaller share of high-skill workers compared to its competitive counterfactual. With search friction, the expected income gaps between high-wage, high-rent cities and low-wage, low-rent cities are narrowed, especially for low-skill workers. This is because high-wage, high-rent locations also feature much lower unemployment rates for low-skill workers, raising their expected wages and creating incentives for the low-skill workers to move there.

The decentralized equilibrium is never efficient, even when the Hosios [1990] efficiency condition is satisfied. The local housing markets distort workers' location choices, complicating the congestion externality and leading to workers' misallocation across locations. In random search models, inefficiency arises due to the missing price of market tightness. One way to implement the Hosios [1990] condition and to restore efficiency is charging an entry fee to workers such that the cost of participating in the labor market equals the cost of the congestion they create for other workers. In my model, however, the "entry fee" workers pay to participate is housing rent due to the inseparability of work and home location. Therefore, the entry fee price is not based on market tightness but is determined solely by a land clearing condition. Thus, even when the Hosios [1990] condition is imposed, the size of the de facto entry fee is distorted by the housing price. Therefore, market tightness is still mispriced by the housing rent, leading to the misallocation of workers across locations.

A calibrated version of the model with two representative locations, one high-skill intensive (location H) and one low-skill-intensive (location L), shows that search friction in the labor market lowers the share of high-skill workers in H by 1.6%, reduces the real wage gap between locations by around 30% for both skill groups, and shrinks the location housing rent gap by 14%. Hence, search friction in the labor market moderates the Great Divergence. Comparing the outcomes of the planner's problem with the decentralized allocation, we can see that the labor force is inefficiently small in the more productive location, and the unemployment rate is inefficiently high for high-skill workers in both locations, whereas inefficiently low for low-skill workers in both locations. The constrained efficient allocation thus produces 5% more aggregate output than the decentralized allocation. Given the inefficiencies, I conduct a counterfactual experiment by giving low-skill workers relocation subsidies to encourage them to live in location H. The subsidy improves aggregate welfare and moves the equilibrium towards the constrained-efficient outcome.

The rest of the paper unfolds as follows. The remaining parts of the introduction present some motivating facts on the divergence of local labor markets and related literature. Section 3.2 presents a baseline model to illustrate intuition. Section 3.3 characterizes the equilibrium. Section 3.4 discusses how labor market search friction affects the Great Divergence, and Section 3.5 solves the planner's problem. Section 3.6 presents quantitative analyses. Lastly, Section 3.7 concludes.

3.1.1 Descriptive facts

This section presents some descriptive facts. I illustrate that between 2005-2019 (i) There are notable unemployment rates dispersions across metropolitan areas. The range of variation is much wider for low-skill workers than for high-skill workers; (ii) For both skill groups, growth in the share of the high-skill labor force is associated with the decrease in local unemployment rates; (iii) For both skill groups, the growth in wages is associated with the decrease in local unemployment rates.

Two definitions of high-skill workers are applied here. The first is defined through the worker's occupation;

more detailed methodology can be found in Section 3.6.1. The worker's educational attainment defines the second one. A worker with a college degree or above is considered a high-skill worker and otherwise is a low-skill worker.

Figure 3.1 shows the variation of unemployment rates across locations. Panel (a) maps the high-skill workers' unemployment rates by MSAs and panel (b) maps low-skill workers' unemployment rates by MSAs. We can see that the 2005-2019 average MSA unemployment rate for high-skill workers ranges from 0.89% - 6.72%, whereas for low-skill workers, the range is much wider, ranging from 2%-13.98%.

Figure 3.2 shows that the growth of high-skill worker share is negatively associated with changes in unemployment rates. From panels (a) and (b), we can see that the relationship between the growth of the share of high-skill workers and the unemployment rate is stronger for low-skill workers than for high-skill workers.

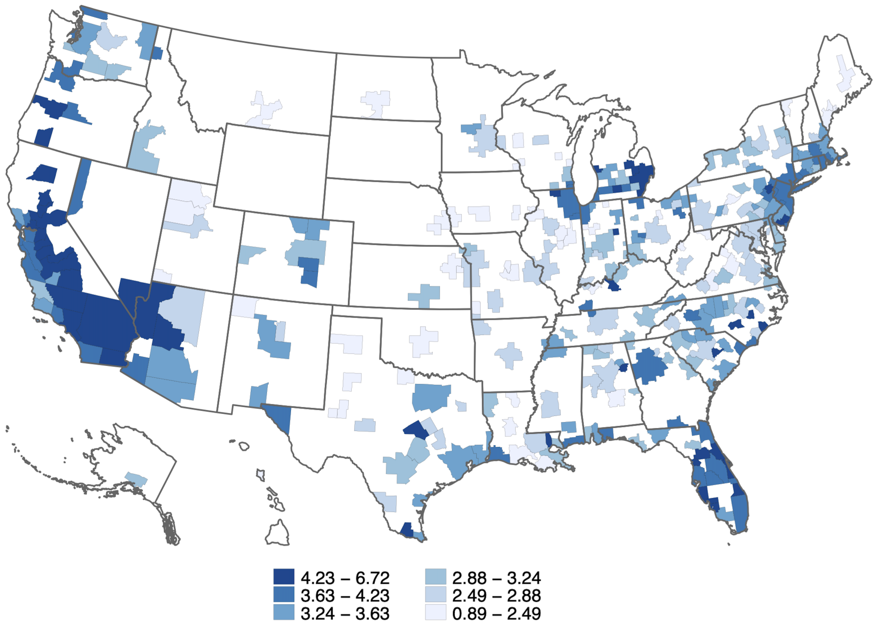
Figure 3.3 and Figure 3.4 show that the growth of nominal and real wages[1] are negatively associated with changes in unemployment rates. From panels (a) and (b) in both figures, we can see that similar to the spatial pattern presented in Figure 3.2, the relationship between the growth of real wages and changes in unemployment is stronger for low-skill workers than for high-skill workers. Together, Figure 3.2 - 3.4 show that unemployment rates have decreased in locations that have become more concentrated with high-skill workers and experienced growth in wages.

I ran one set of regressions to tease out the effect of high-skill share from the MSA fixed effects. Table 3.1 presents the effects of high-skill share on unemployment rates. Columns (1) and (2) show results for high-skill workers, whereas columns (3) and (4) show results for low-skill workers. Columns (1) and (3) use OLS regression to estimate the effect of the high-skill worker share on the unemployment rate, whereas column (2) and (4) uses local per capita patent counts as instruments for the share of high-skill workers. We can see from the negative coefficients that unemployment rates for both skill types are negatively correlated with the share of high-skill workers. The sizes of the coefficients are smaller for high-skill workers than for low-skill workers. Compared to IV regression outcomes in columns (2) and (4), the OLS regressions have a downward bias for both skill types. [2]Note that the regression outcomes are consistent with the graphical representations illustrated earlier in this section. Increases in the share of high-skill workers correlate with reduced unemployment rates for both skill types.

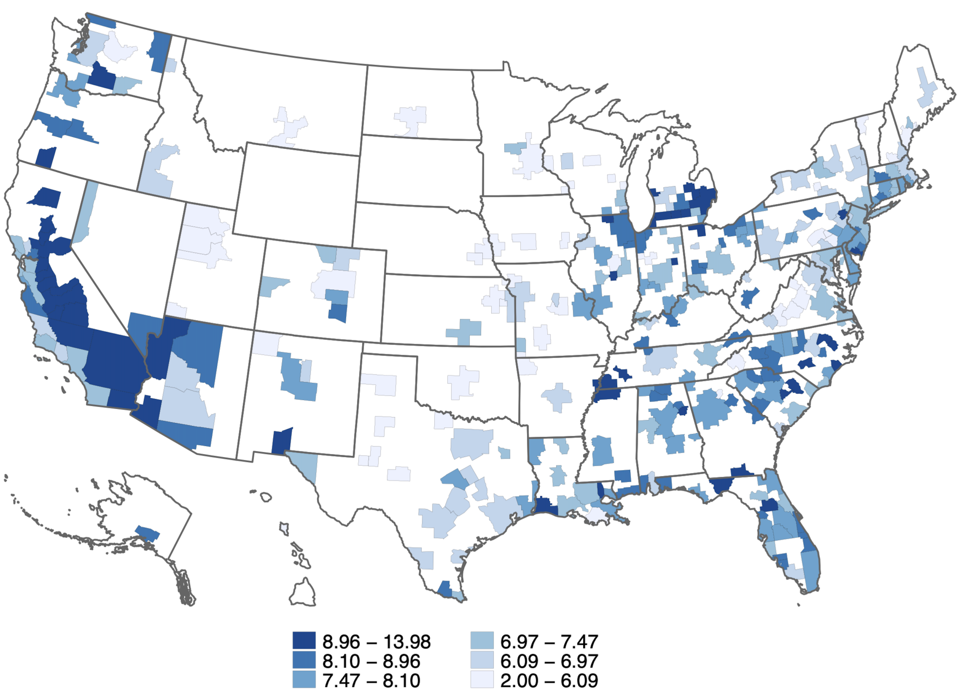
[1]Real wages calculated by discounting nominal wages by local housing prices.

[2]These two patterns persist for the education-based skill definition, referring to Table B.1 in appendix B.1.1

Figure 3.1: Local unemployment rate by skills, 2005-2019 average



(a) High-skill workers



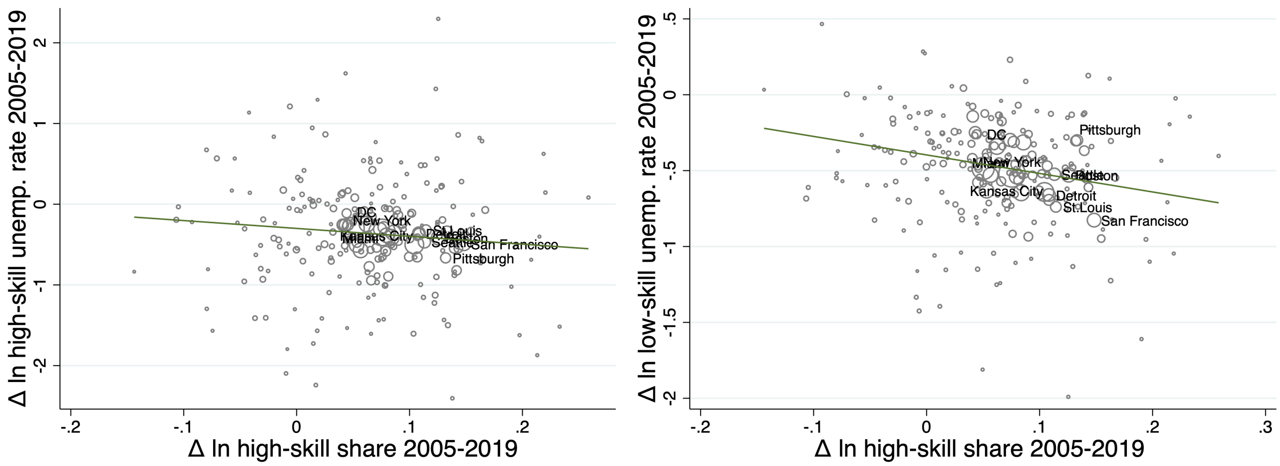
(b) Low-skill workers

Notes: This map uses American Community Survey data from 2005-2019. Each block represents a metropolitan area (MSA). The skill definition used in the graph is occupation-based.

Figure 3.2: Changes in Share of High-Skill Workers and Unemployment Rates by Skill Types, 2005-2019

(a) High-skill workers

(b) Low-skill workers

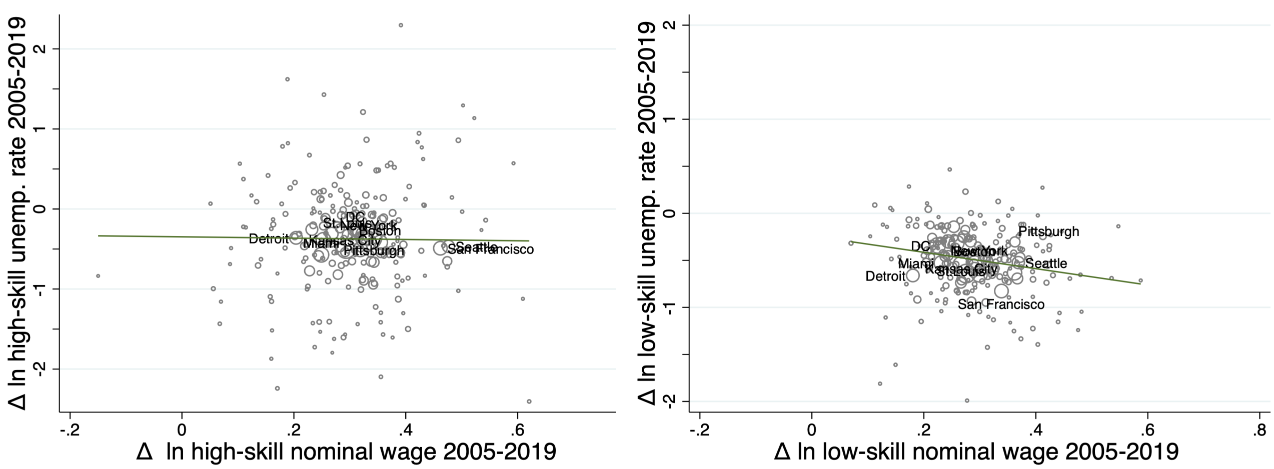


Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs using the education-based skill definition can be found in appendix B.1.

Figure 3.3: Changes in Nominal Wages and Unemployment Rate by Skill Types, 2005-2019

(a) High-skill workers

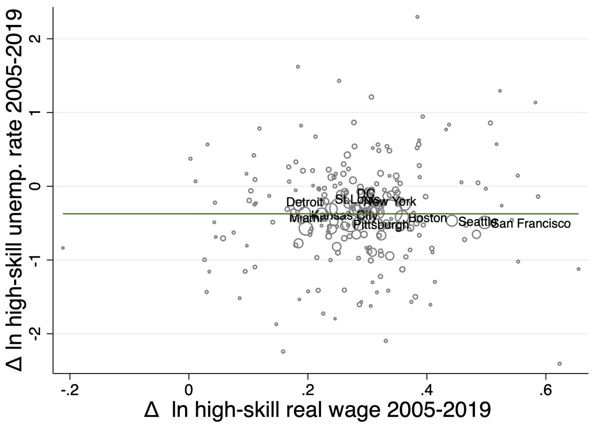
(b) Low-skill workers



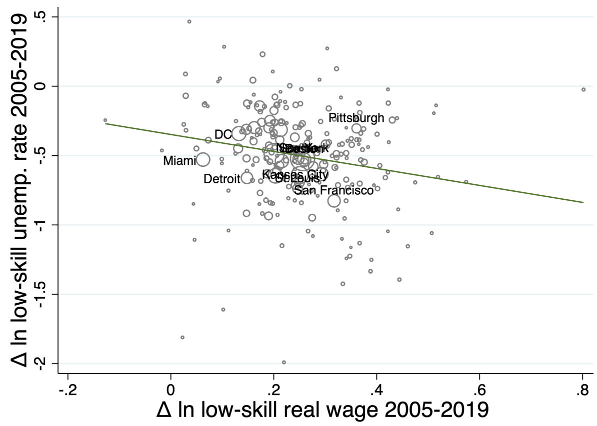
Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs of education-based skill definition can be found in appendix B.1.

Figure 3.4: Changes in Real Wages and Unemployment Rate by Skill Types, 2005-2019

(a) High-skill workers



(b) Low-skill workers



Notes: This figure uses American Community Survey data from 2005-2019. Each circle represents a metropolitan area (MSA). The data points are weighted by the 2005 labor force size. The red line is the linear fit. The skill definition used in the graph is occupation-based. Graphs of education-based skill definition can be found in appendix B.1.

Table 3.1: Share of High-Skill Worker and Unemployment Rates

	(1)	(2)	(3)	(4)
Unemployment Rate	High-Skill	High-Skill	Low -Skill	Low -Skill
	OLS	IV	OLS	IV
Log Share of High-Skill Worker (Occ)	-0.339***	-0.221	-0.434***	-0.227***
	(0.0625)	(0.100)	(0.0369)	(0.0593)
Observations	2,635	2,575	2,643	2,583
R-squared	0.321	0.316	0.407	0.406
MSA FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

3.1.2 Related Literature

This paper speaks to three threads of literature. First, this paper contributes to the body of work that studies the divergence between high-skill and low-skill workers regarding location choices. Shapiro [2006], Berry and Glaeser [2005], Moretti [2012], Eeckhout et al. [2021], Eckert et al. [2020] Giannone [2017] find that the critical driver of the spatial dispersion is due to productivity channels. They closely examine the sources of productivity changes in different cities, such as the concentration of college graduates, type of jobs, skill agglomeration, IT investment, skill- and information-intensive service industries, and skill-based technological changes. They find that productivity growth significantly affects wage premiums for high-skill workers but produces a much smaller effect for low-skill workers.

In addition to the productivity channels, Ganong and Shoag [2017], Gyourko et al. [2013] and Glaeser and Gyourko [2018] show that housing also plays a crucial role in the divergence of skill composition. They find that housing costs, housing price appreciation, and housing supply elasticity significantly contribute to the divergence of skill composition since, in highly productive locations, the prohibitively high housing prices crowd out lower-income households. In particular, the divergence is mainly explained by the highly inelastic land supply in the more attractive locations. These papers show that heterogeneity in productivity and housing supply matters for the divergence across locations. Thus, I incorporate such heterogeneity in a theoretical framework with frictional labor markets.

Second, this paper speaks to the literature on spatial differences in unemployment, where frictional local labor markets are studied in a geographic framework. This literature includes Kline and Moretti [2013], Kuhn et al. [2021], Bilal [2023] and Deschamps and Wilemme [2021]. These papers all study variations of the Diamond [1982] - Mortensen [1979] - Pissarides [1985] embedded in a Rosen [1979] - Roback [1982] spatial equilibrium. In particular, Kuhn et al. [2021] and Bilal [2023] emphasize the spatial differentials of job creation and job destruction. This paper focuses on the different spatial patterns of unemployment for high-skill and low-skill workers, a novel feature of this thread of literature. It examines their effects on the Great Divergence.

Lastly, this paper speaks to the body of work on spatial mismatch and optimal allocation of workers. Desmet and Rossi-Hansberg [2013] study the optimal city size, Fajgelbaum and Gaubert [2020] study the optimal allocation of workers across space. Acemoglu [2001] shows the skill composition of jobs can be inefficient when two types of jobs are created for one type of workers. Recent work by Hsieh and Moretti [2019] and Fournier [2020] shows that both interurban and intra-urban spatial misallocation leads to inefficiency. In contrast, Marinescu and Rathelot [2018] and Şahin et al. [2014] present evidence that geographical mismatch is present but is a minor driver in terms of the aggregate unemployment rate. This paper aims to enrich our understanding of the impact of spatial misallocation by bringing the heterogeneous skill levels and frictional labor market to the discussion. This has non-trivial welfare implications as the scale of misallocation can be masked by the heterogeneity of skill levels and employment status.

3.2 Environment

Time is continuous and indexed by $t \in R_+$. There are J locations. Each location $j \in \{1, ..., J\}$ is characterized by production technology and a housing supply Q_j.

Production

Three types of goods are produced in each location: one final good Z freely traded across locations and two intermediate goods Y^s,Y^n produced by high-skill and low-skill workers, respectively. The final good Z is treated as the numeraire and has a price of one. The production function of the final good is CES:

$$Z_j = \left[\sigma_j (Y_j^s)^\rho + (1 - \sigma_j)(Y_j^n)^\rho\right]^{1/\rho} \ ,$$

where σ_j is an exogenous parameter that indicates the relative importance of high-skill intermediate goods, and $1/(1-\rho)$ is the elasticity of substitution between the high-skill and low-skill input. The bigger the σ_j, the more important is the high-skill input. Therefore, if $\sigma_j > \sigma_k$, we say that location j is high-skill-intensive and location k is low-skill-intensive. The intermediate goods are non-storable and sold in competitive markets. There is a continuum of intermediate goods firms, and each chooses to produce either the s or n type of goods and hire one worker of that type. Intermediate goods productivity is denoted by y^χ, where high-skill workers have higher intermediate goods productivity than low-skill workers, i.e., $y^s > y^n$. The total output of intermediate goods in location j is equal to the sum of individual firms' production. This is equal to

$$Y_j^\chi = y^\chi (1 - u_j^\chi) L_j^\chi,$$

where u_j^χ denotes the unemployment rate for workers of skill type χ at location j, and L_j^χ is the labor force size of skill type χ at location j. Both u_j^χ and L_j^χ are determined by the equilibrium. Since the two intermediate goods are sold in competitive markets, their prices are equal to their marginal products in the production of the final good:

$$p_j^s = \sigma_j \left(Y_j^s\right)^{\rho-1} Z_j^{1-\rho}, \tag{3.1}$$

$$p_j^n = (1-\sigma_j) \left(Y_j^n\right)^{\rho-1} Z_j^{1-\rho}. \tag{3.2}$$

Workers

Workers are risk-neutral and discount the future at a rate $r > 0$. Their preferences over non-housing consumption c_t and housing h_t are

$$E \int_0^\infty e^{-rt} \left(\frac{c_t}{1-\eta}\right)^{1-\eta} \left(\frac{h_t}{\eta}\right)^{\eta} dt. \tag{3.3}$$

There is a unit measure of workers of both skills. The total measure of high-skill workers is denoted by ξ, which is less than one, and the local labor force share of high-skill workers is ζ_j, hence

$$\zeta_j = \frac{L_j^s}{L_j}, \tag{3.4}$$

$$1 = \sum_j L_j, \tag{3.5}$$

$$\xi = \sum_j \zeta_j L_j. \tag{3.6}$$

Matching function

The labor markets are segregated so that workers of skill type χ can only work for an intermediate firm of skill type χ. For convenience, I use ϕ to denote the aggregate state of skill level and location $\phi = \{j \times \chi\}$. The matching function $m(\theta_\phi)$ between workers and intermediate goods firms depends on market tightness θ_ϕ of each ϕ, where $\theta_\phi \equiv \frac{v_\phi}{u_\phi}$. The matching function is Cobb-Douglas

$$m(u_\phi, v_\phi) = A u_\phi^\alpha v_\phi^{1-\alpha},$$

where A is the matching parameter, u_ϕ is the unemployment rate for type ϕ, and v_ϕ is the job vacancy rate. Since the matching function is homogeneous of degree of 1, the job finding rate $f(\theta_\phi)$ and vacancy filling rate $q(\theta_\phi)$ are

$$f(\theta_\phi) = \frac{m(u_\phi, v_\phi)}{u_\phi} = m(1, \theta_\phi); \quad q(\theta_\phi) = \frac{m(u_\phi, v_\phi)}{v_\phi} = m(1/\theta_\phi, 1). \tag{3.7}$$

Unemployed workers are free to move when unemployed and can only look for jobs where they live. Employed workers cannot move between locations but could quit their jobs and move.

Housing clearing condition

Each location has a housing supply Q_j, and absentee landowners own the land. They collect rents from workers in location j and use them to enjoy non-housing consumption c_j^O. In each location, j, total land supply equals total land demand. Therefore, the land-clearing condition for each location j is

$$Q_j = \sum_{\phi} [h_{\phi}^b u_{\phi} + h_{\phi}(1 - u_{\phi})] L_{\phi} \ , \tag{3.8}$$

where h_{ϕ}^b denotes housing consumption of unemployed worker of type ϕ, and h_{ϕ} denotes housing consumption of employed worker of type ϕ.

3.3 Equilibrium

The description of the equilibrium is presented as follows. I start by discussing the consumption decisions in Section 3.3.1, then I define the flow Bellman equations in Section 3.3.2 and Section 3.3.3 describes the spatial equilibrium. Section 3.3.4 discusses wage bargaining. Section 3.3.5 discusses the equilibrium conditions. Section 3.3.6 then defines a steady-state equilibrium. Section 3.3.7 discusses the implications of the equilibrium. And lastly, Section 3.3.8 and Section 3.3.9 discuss cross-skill interaction and present comparative statics.

3.3.1 Housing and non-housing consumption

All variables in the model are functions of t. To simplify notation, the t argument is suppressed from now on. Since the utility function is Cobb-Douglas, the worker's consumption maximization problem would result in the share of spending on housing and non-housing consumption being fixed and governed by a parameter η. [3]Therefore, non-housing consumption c_{ϕ}^b for an unemployed worker and c_{ϕ} for an employed worker are

$$c_{\phi}^b = (1 - \eta) b^{\chi}, \quad c_{\phi} = (1 - \eta) w_{\phi} \ , \tag{3.9}$$

[3]The derivation can be found in appendix B.2.1

where w_ϕ is the wage, and b^χ is the unemployment benefit for workers of skill type χ. Housing consumption h_ϕ^b for an unemployed worker and h_ϕ for an employed worker are

$$h_\phi^b = \frac{\eta b^\chi}{R_j}, \quad h_\phi = \frac{\eta w_\phi}{R_j}, \tag{3.10}$$

where R_j is rent in location j, determined by plugging the housing consumption equation (3.10) into the housing clearing condition equation (3.8):

$$R_j = \frac{\eta L_j}{Q_j}\left[[b^n u_j^n(1-\zeta_j) + b^s u_j^s \zeta_j] + w_j^s(1-u_j^s)\zeta_j + w_j^n(1-u_j^n)(1-\zeta_j)\right]. \tag{3.11}$$

Plugging the expression of optimal housing consumption, equation (3.9) and optimal non-housing consumption (3.10) into the utility function, equation (3.3), the indirect utility of an employed worker becomes $w_\phi R_j^{-\eta}$ and the indirect utility of an unemployed worker becomes $b^\chi R_j^{-\eta}$.

3.3.2 Bellman Equations

Let U_ϕ, W_ϕ, V_ϕ, J_ϕ denote the value of the unemployed, the employed, an intermediate goods firm vacancy and a filled intermediate firm job for each ϕ.[4] The Bellman equations involving these variables are:

$$rW_\phi = w_\phi R_j^{-\eta} + s^\chi(U_\phi - W_\phi), \tag{3.12}$$

$$rU_\phi = \max_j\{b^\chi R_j^{-\eta} + f(\theta_\phi)(W_\phi - U_\phi)\}, \tag{3.13}$$

$$rV_\phi = \max_j\{-k^\chi + q(\theta_\phi)(J_\phi - V_\phi)\}, \tag{3.14}$$

$$rJ_\phi = p_\phi y^\chi - w_\phi + s^\chi(V_\phi - J_\phi). \tag{3.15}$$

The first Bellman equation is an employed worker's flow value. The first term on the right-hand side is an employed worker's indirect utility, as discussed in Section 3.3.1. With probability s^χ, a worker becomes unemployed and separated from her job. The second Bellman equation is an unemployed worker's flow value. Since an unemployed worker can move between locations, the worker chooses a location j that maximizes utility. Like an employed worker, the first term on the right-hand side is the indirect utility of an unemployed worker. An unemployed worker meets a firm at rate $f(\theta_\phi)$.

[4]Note that only the intermediate goods firms need to match with workers in the frictional labor markets and the intermediate goods are sold in a competitive market in the production of final goods.

The third Bellman equation is a vacant firm's flow value. Vacant firms are also free to choose where to locate, so they will choose location j to maximize their profit. Once they settle in a location, they need to pay a vacancy cost k^{χ} that depends on the skill type. It is more costly to open a high-skill vacancy than a low-skill vacancy, i.e., $k^s > k^n$. A vacant firm meets an unemployed worker at rate $q(\theta_\phi)$. The last Bellman equation is the flow value of a filled firm. The firm's profit is the value of the output $p_\phi y^{\chi}$ less the wage paid to the worker. A match is exogenously destroyed at the rate s. The free entry condition of the firms implies $V_\phi = 0$; hence the max operator drops out of the equation (3.14). Using the last two Bellman equations, J_ϕ must satisfy both of the following:

$$J_\phi = \frac{k^{\chi}}{q(\theta_\phi)}; J_\phi = \frac{p_\phi y^{\chi} - w_\phi}{r + s^{\chi}}. \tag{3.16}$$

Rearranging equation (3.12) and equation (3.13) yields

$$(r + s^{\chi})(W_\phi - U_\phi) = (w_\phi - b^{\chi}) R_j^{-\eta} - f(\theta_\phi)(W_\phi - U_\phi). \tag{3.17}$$

3.3.3 Spatial Equilibrium

Since unemployed workers are free to move between locations, if there are unemployed workers in different locations in equilibrium, they should be indifferent between the locations. Hence, their value will be the same for all locations, i.e., $U_j^{\chi} = U_{j'}^{\chi} = \overline{U}^{\chi}, \forall j, j' \in J$ where $\overline{U}^{\chi}$ denotes the common value for the unemployed worker of skill type χ. Therefore, the max operator regarding location drops out of equation (3.13).

3.3.4 Wage Bargaining

Following Bilal [2023], I use an adjusted surplus, where the surplus for the worker is adjusted by the level of local rents so that the marginal utility of a dollar is equalized between the worker and firm since the worker's wage is discounted by it. The adjusted surplus is formulated as follows:

$$S_\phi = J_\phi + R_j^{\eta}[W_\phi - U_\phi].$$

Nash Bargaining determines the wage with an adjusted surplus, which yields

$$(1 - \beta) R_j^{\eta}(W_\phi - U_\phi) = \beta(J_\phi - V_\phi), \tag{3.18}$$

where β is worker bargaining power, and $V_\phi = 0$. Rearranging and plugging equation (3.16) into equation (3.18) yields

$$W_\phi - U_\phi = \frac{\beta}{(1-\beta)R_j^\eta}\frac{k^\chi}{q(\theta_\phi)}. \tag{3.19}$$

Plugging equation (3.17) into equation (3.19) yields

$$(1-\beta)R_j^\eta[(w_\phi - b^\chi)R_j^{-\eta} - f(\theta_\phi)\frac{\beta}{(1-\beta)R_j^\eta}\frac{k^\chi}{q(\theta_\phi)}] = \beta(p_\phi y^\chi - w_\phi). \tag{3.20}$$

Therefore, using equation (3.7) to eliminate $f(\theta_\phi)$ and $q(\theta_\phi)$, the expression for the wage is

$$w_\phi = \beta p_\phi y^\chi + [(1-\beta)b^\chi + \beta\theta_\phi k^\chi]. \tag{3.21}$$

3.3.5 Equilibrium Conditions

Plugging equation (3.16), (3.19) and (3.21) into equation (3.18), we have the job creation condition for each skill location group ϕ:

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\beta)p_\phi y^\chi - [(1-\beta)b^\chi + \beta\theta_\phi k^\chi]}{r + s^\chi}. \tag{3.22}$$

The left-hand side is the firm's expected cost of hiring a worker, where the location-specific vacancy cost is adjusted by the expected time to find a worker. The right-hand side is the firm's expected gain from opening the vacancy. The job creation condition thus shows that firms keep entering the market until the expected profit of a vacancy equals the expected cost.

Spatial Equilibrium Condition

The spatial equilibrium condition equates the value of an unemployed worker across locations. Unemployed workers of type χ enter location j until their indirect utility is equalized across locations. Plugging expression of $(W_\phi - U_\phi)$, as in equation (3.19), into equation (3.13) yields the spatial equilibrium condition:

$$\overline{U}^\chi = \left(b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_j^\chi\right)R_j^{-\eta}; \quad \forall j \in J. \tag{3.23}$$

The unemployed worker will choose their location based on market tightness and housing prices. Since equation (3.22) relates productivity ($p_\phi y^\chi$) and market tightness θ_ϕ, even though ($p_\phi y^\chi$) does not show up in the spatial equilibrium condition, it can be inferred the value of market tightness. The bigger the market tightness, the more likely they will be employed; the higher the rent, the more expensive it is to live there, and hence lower indirect utility. The unemployed workers will allocate themselves until this expression is equalized across locations.

Beveridge Curve

The Beveridge curve is given by the following:

$$u_\phi = \frac{s^\chi}{s^\chi + f(\theta_\phi)}. \tag{3.24}$$

3.3.6 Definition of equilibrium

A steady-state equilibrium is $\{w_\phi, u_\phi, \theta_\phi, p_\phi, \zeta_j, L_j, R_j\}$ for $\phi \in J \times \{s, n\}$ and $j \in J$ such that: equations (3.1),(3.2), (3.5),(3.6),(3.11),(3.22),(3.23),(3.24) are satisfied for each ϕ and j.

3.3.7 Equilibrium Properties

This section further explores the equilibrium conditions and the equilibrium properties. Plotting the spatial equilibrium condition in the top panel of Figure 3.5, we can see that within each skill type, workers are indifferent among points on an upward-sloping curve relating rent to market tightness. If workers choose location j with a higher market tightness, then they are facing a higher R_j^η, so that if $\theta_j^\chi > \theta_{j'}^\chi$, then $R_j > R_{j'}$. Plotting the Beveridge Curve in the bottom panel of Figure 3.5, we can see that the unemployment rate is lower within the same skill group in locations with bigger market tightness. Therefore if $R_j > R_{j'}$, then $u_j^\chi < u_{j'}^\chi$.

Within each skill type, rent and market tightness are positively related across locations, while unemployment decreases with market tightness. Therefore, rent and unemployment rate are negatively related across locations, i.e., if $R_{j'} > R_j$, then $\theta_{j'}^\chi > \theta_j^\chi$, and $u_{j'}^\chi < u_j^\chi$.

Proof. See Appendix B.6.1 □

Next, I examine the implication of the equilibrium on the relationship between the ranking of wages and unemployment rates across locations. For the job creation condition (3.22) to hold, an increase in the market tightness θ_ϕ would raise the price of the intermediate goods p_ϕ. Therefore, within each skill type χ, if $\theta_j^\chi > \theta_k^\chi$, then $p_j^\chi > p_k^\chi$. By wage equation (3.21), we can see that w_ϕ increases with p_ϕ and θ_ϕ. Since we already know that p_ϕ also increases with θ_ϕ, we can say that w_ϕ increases with θ_ϕ, hence if $\theta_j^\chi > \theta_k^\chi$, then $w_j^\chi > w_k^\chi$. By the Beveridge Curve (3.24), we know that the unemployment rate is decreasing in market tightness. Therefore, we can see that workers receive higher wages for the same skill level in locations with lower unemployment rates: $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$ $\forall j, j' \in J$ and $\chi \in \{s, n\}$.

Within each skill type, a location with a lower unemployment rate has a higher nominal wage. i.e. if $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$. $\forall j, j' \in J$ and $\chi \in \{s, n\}$.

Proof. See Appendix B.6.2 □

Within each skill type, a location with a lower unemployment rate has a higher real wage if its $\frac{p_j^\chi y^\chi}{R_j^\eta}$ is bigger. i.e. If $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$, then $u_j^\chi < u_{j'}^\chi$, then $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$.

Proof. See Appendix B.6.3 □

Corollary 3.3.7 establishes the relationship between the ranking of wages and the ranking of unemployment within each skill type across locations. It is critical to understand how the dispersion of unemployment rates shapes the great divergence. It shows that unemployment rates are lower within each skill group in locations where wages are higher, which maps the descriptive facts shown in Figure 3.3. Corollary 3.3.7 shows that the theoretical relationship between real wages ($\tilde{w}_j^\chi$) and unemployment rates is less conclusive, and the relationship depends on the size of the parameters. The intuition for Corollary 3.3.7 is that if the output difference dominates the rent difference between locations, then, for both types of workers, the location with higher real wages features lower unemployment rates. On the other hand, if the rent difference dominates the output difference between locations, then, for both types of workers, the location with higher real wages features lower unemployment rates. Section 3.6 presents the relationship between real wages and unemployment rates the calibrated model generated. It matches with Figure 3.3 presented in Section 3.1.1.

3.3.8 Comparative Statics

Considering the case of two locations $j \in \{H, L\}$, where H is a high-skill-intensive location and L is a low-skill-intensive location, with $\sigma_H > \sigma_L$. I am interested in understanding how location- and skill-based parameters affect the equilibrium outcomes, particularly their effects on market tightness, labor force sizes, high-skill share, and unemployment rates. Proposition 3.3.8 summarizes the comparative statics for location parameters, and Proposition 3.3.8 summarizes comparative statics for skill parameters.

Assuming skill dependent parameters are symmetrical, i.e. $\xi = 0.5; y^s = y^n; b^s = b^n; k^s = k^n$, comparative statics regarding the location parameters are summarized in Table 3.2.

Proof. See Appendix B.6.4 □

As the housing supply in location j, Q_j, increases relative to $Q_{j'}$, the only thing in the equilibrium affected is housing rent R_j, which decreases with Q_j. Therefore, more workers of both types will move to location j; hence, L_j increases relative to $L_{j'}$. Market tightness and unemployment rates are not affected, as indicated by the horizontal arrows.

As σ_j increases relative to $\sigma_{j'}$, there is more demand for high-skill workers. Hence, firms will create more high-skill openings in location j and location j'. More high-skill workers move to location j; hence, the unemployment rate is lower for high-skill workers, particularly those in location j. However, since there are fewer job openings for low-skill workers and the population share of low-skill workers is fixed, the unemployment rates are higher for low-skill workers in both locations. The relative labor force size is pinned down using spatial equilibrium conditions, which say that workers of both types are indifferent between locations when the disadvantages of costly rent balance the advantages of market tightness. Since location j has higher market tightness for both types, more workers will enter location j despite its higher rent. Therefore, location j's worker size expands as σ_j increases.

Assuming location-based parameters are equal, i.e. $\sigma_j = \sigma_{j'} = 0.5$, $Q_j = Q_{j'}$, comparative statics regarding the skill parameters are summarized in Table 3.3.

Proof. See Appendix B.6.5 □

As y^s increases relative to y^n, surplus for both types of matches increases at the same rate since $\sigma_H = \sigma_L =$

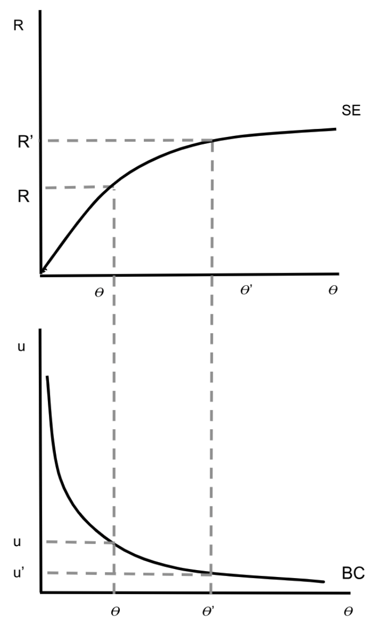


Figure 3.5: Spatial Equilibrium and Beveridge Curve

Table 3.2: Comparative Statics of Location Parameters

	θ_j^s/θ_j^n	u_j^s/u_j^n	$L_j/L_{j'}$	$\zeta_j/\zeta_{j'}$
$Q_j/Q_{j'} \uparrow$	$\rightarrow$	$\rightarrow$	$\uparrow$	$\rightarrow$
$\sigma_j/\sigma_{j'} \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$

Table 3.3: Comparative Statics of Skill Parameters

	θ_j^s/θ_j^n	u_j^s/u_j^n	$L_j/L_{j'}$	$\zeta_j/\zeta_{j'}$
$y^s/y^n \uparrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$b^s/b^n \uparrow$	$\downarrow$	$\uparrow$	$\rightarrow$	$\rightarrow$
$k^s/k^n \uparrow$	$\downarrow$	$\uparrow$	$\rightarrow$	$\rightarrow$

0.5. Therefore, market tightness increases for all skill-location groups, and unemployment decreases. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

As b^s increases relative to b^n, unemployment becomes more attractive for high-skill workers. Hence, the unemployment rate becomes higher for them. Therefore, market tightness decreases for high-skill workers. Due to the complementarity of high-skill output and low-skill output, being employed becomes less attractive for low-skill workers since fewer high-skill workers are employed. Therefore, the unemployment rate for low-skill workers also decreases, but since it is the second-order effect of the increase in b^s, the magnitude of the decreases is much smaller than for high-skill workers. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

As k^s increases relative to k^n, opening a high-skill vacancy becomes more expensive. Therefore, market tightness decreases, and unemployment increases for high-skill workers. Due to the complementarity of high-skill output and low-skill output, higher unemployment of high-skill workers means that employment becomes less attractive for low-skill workers. Therefore, the unemployment rate for low-skill workers also decreases, but since it is a second-order effect of the increase in k^s, the magnitude of the decrease is much smaller than for high-skill workers. Since the locations are symmetrical, there will be an equal number of workers in each location, and the share of high-skill workers is equalized between locations.

The comparative statics show that only the location-related parameters affect the skill composition and worker allocation across locations. In the absence of asymmetry of location-related parameters, skill-related parameters only affect differences in unemployment rates and market tightness between skill levels but do not affect the allocation of workers across locations.

3.3.9 Cross-skill Interaction

The labor markets are segregated by skill level. Therefore, the high-skill and low-skill workers will not be creating labor market congestions for workers of the other skill type. That is to say, a high-skill worker's decision to look for jobs in location j does not make it less likely for a low-skill worker to find a job in location j, and vice versa. Nevertheless, high(low)-skill workers' decision to look for jobs in location j can still affect the labor market outcome for low(high)-skill workers. The cross-skill interaction still occurs through two channels. First, it occurs through the production channel.

The final good is produced using both high-skill intermediate goods and low-skill intermediate goods, and they are complementary in final goods production. More high-skill workers employed in a location increases the marginal productivity of low-skill intermediate goods, which augments demand for low-skill workers. This is an attraction force that encourages high- and low-skill workers to co-locate.

On the other hand, high-skill workers and low-skill workers share a common housing market. Without an unlimited housing supply, they raise the housing cost for each other, limiting the size of the labor market in a location. This is the dispersing force. The relative strength of these two forces pins down the labor market sizes as well as the skill composition in each one of them.

3.4 Search Frictions and the Great Divergence

To understand whether frictions in the labor market exacerbate or alleviate the concentration of high-skill workers in highly productive locations, I compare the model with search friction with the models where the labor market is competitive[5]. In the competitive labor market, wages are

$$\breve{w}_j^s = \sigma_j(\breve{L}_j^s)^{\rho-1}(y^s)^{\rho}(y^n\breve{L}_j^n)^{1-\rho} = \breve{p}_j^s y^s, \tag{3.25}$$

$$\breve{w}_j^n = (1-\sigma_j)(\breve{L}_j^n)^{-\rho}(y^n)^{1-\rho}(y^s\breve{L}_j^s)^{\rho} = \breve{p}_j^n y^n. \tag{3.26}$$

Rent is

$$\breve{R}_j = \frac{\eta\breve{L}_j}{Q_j}[\breve{w}_j^s\zeta_j + \breve{w}_j^n(1-\zeta_j)]. \tag{3.27}$$

The labor clearing condition is the same as in the model with frictional labor markets

$$\sum_j \breve{L}_j = 1; \quad \sum_j \breve{L}_j\zeta_j = \xi. \tag{3.28}$$

The spatial equilibrium condition states that, within the same skill type, the worker's utility is the same in all locations

$$\breve{U}^{\chi} = \breve{w}_j^{\chi}\breve{R}_j^{-\eta}. \tag{3.29}$$

[5]Note that the competitive labor market models where wages are the marginal product of labor are different from a model where matching efficiency reaches infinity, which still preserves the wage bargaining structure.

A steady-state equilibrium with competitive labor markets is $\{\breve{w}_\phi, \breve{\zeta}_j, \breve{L}_j, \breve{R}_j\}$ for $\phi \in J \times \{s, n\}$ such that equations (3.25),(3.26),(3.27),(3.28),(3.33) are satisfied.

In an economy without labor market search frictions, wages depend on the marginal output of the worker. Therefore, the wage gap between the locations will be

$$\Delta \breve{w}^\chi = (\breve{p}_j^\chi - \breve{p}_{j'}^\chi) y^\chi = \Delta \breve{p}^\chi y^\chi. \tag{3.30}$$

However, in an economy with labor market search frictions, wages depend on both the marginal output of the worker and the market tightness. Therefore, the wage gap between the locations depends on both the gap of marginal productivity and the gap between market tightness.

$$\Delta w^\chi = \beta y^\chi (p_j^\chi - p_{j'}^\chi) + \beta k^\chi (\theta_j^\chi - \theta_{j'}^\chi) = \beta y^\chi \Delta p^\chi + \beta k^\chi \Delta \theta^\chi. \tag{3.31}$$

Since $\beta < 1$, the contribution of marginal output in the wage gap is smaller than that in the competitive version. The differences in market tightness between locations also contribute to the wage gap.

Therefore, the wage gap between locations is bigger in the competitive labor market if $\Delta \breve{p}^\chi y^\chi > \beta y^\chi \Delta p^\chi + \beta k^\chi \Delta \theta^\chi$. And the wage gap between locations is smaller in the competitive labor market if $\Delta \breve{p}^\chi y^\chi < \beta y^\chi \Delta p^\chi + \beta k^\chi \Delta \theta^\chi$. It is summarized in Proposition 3.4.

For both skill types, the wage gap between locations is bigger in the frictional labor market model if

$$\Delta \breve{p}^\chi y^\chi - [\beta \Delta p^\chi y^\chi - \beta k^\chi \Delta \theta^\chi] > 0.$$

Otherwise, the wage gap between locations is smaller in the frictional labor market model.

Proof. See Appendix B.6.6. □

In the competitive labor market, the spatial equilibrium condition indicates that workers' location choices are based on the relative sizes of wages and housing prices. The relative size of the nominal wage and housing price pins down the spatial allocation of high- and low-skill workers. Nevertheless, in the benchmark economy with search frictions, the spatial equilibrium condition indicates that workers' location choices depend on the relative size of housing prices and market tightness, which affects both wages and unemployment rates. Re-arrange the job creation condition, and we can see that the spatial equilibrium condition for the economy with search friction is

$$\overline{U}^{\chi} = \left[p_{\phi} y^{\chi} - \frac{r + s^{\chi}}{1-\beta} \frac{k^{\chi}}{q(\theta_{\phi})} \right] R_j^{-\eta} \tag{3.32}$$

The spatial equilibrium for the economy without search friction is

$$\breve{\overline{U}}^{\chi} = \left[\breve{p}_{\phi} y^{\chi} \right] \breve{R}_j^{-\eta}. \tag{3.33}$$

since $\breve{w}_{\phi} = \breve{p}_{\phi} y^{\chi}$. We can see that in the frictional model, both productivity and market tightness play roles in determining the spatial equilibrium. However, the differences in market tightness are dampening the productivity differences between locations since the second term that contains that market tightness in equation (3.32) is subtracted from the first term in the equation. Section 3.6.4 quantitatively studies how the allocation of workers across space in labor markets with search friction differs from that in competitive labor markets.

3.5 Planner's problem

The social planner aims to maximize a social welfare function subject to a resource constraint and the law of motion of unemployment. The social welfare function assigns equal welfare weights for the three types of agents: high-skill workers, low-skill workers, and absentee landlords. Let N_{ϕ} denote the number of unemployed workers of each skill-location group ϕ and E_{ϕ} denote the number of employed workers.

The planner's objective function is

$$\omega = \int_0^{\infty} e^{-rt} \left(\sum_{\phi} \left[\left(\frac{c_{\phi}^E}{1-\eta} \right)^{1-\eta} \left(\frac{h_{\phi}^E}{\eta} \right)^{\eta} \times E_{\phi} + \left(\frac{c_{\phi}^U}{1-\eta} \right)^{1-\eta} \left(\frac{h_{\phi}^U}{\eta} \right)^{\eta} \times N_{\phi} \right] + \sum_j c_j^O \right) dt,$$

where the first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of absentee landlords. Since the housing supply is fixed in each location, no additional social cost is incurred to the planner, no matter how the housing is allocated among the workers.

The planner chooses vacancy V_{ϕ} and size of unemployed worker N_{ϕ} for each ϕ, along with housing and non-housing consumption for the workers and landlord $(c_{\phi}^E, c_{\phi}^U, h_{\phi}^E, h_{\phi}^U, c_j^O)$. The constraints the planner faces are:

1) Law of motion for employment for each ϕ,

$$\dot{E}_\phi = m(N_\phi, V_\phi) - sE_\phi \tag{3.34}$$

where $\dot{E}_j^\chi$ is the evolution of employed worker

2) Land clearing for each location,

$$Q_j = \sum_\chi \left[N_j h_j^{\chi,U} + E_j h_j^{\chi,E} \right] \tag{3.35}$$

3) Resource constraint of the planner,

$$0 = \sum_j Z_j + \sum_\phi (N_\phi b^\chi - k^\chi V_\phi) - \left[\sum_\phi c_\phi^E \times E_\phi + c_\phi^U \times N_\phi \right] - \sum_j c_j^O \tag{3.36}$$

4) High-skill worker size and population constraints,

$$\xi = \sum_j E_j^s + N_j^s;\ 1 - \xi = \sum_j E_j^n + N_j^n \tag{3.37}$$

For each ϕ, the size of the labor force L_ϕ equals the sum of the employed and the unemployed workers, i.e., $L_\phi = E_\phi + N_\phi$. The economy-wide resource constraint (equation 3.35) pins down the total level of consumption by absentee landlords. The derivation of the planner's solution is explained in more detail in Appendix B.2.2.

3.5.1 Comparison between Planner's and Decentralized Equilibrium

Job Creation Condition

Using the first order condition for θ_ϕ and the equation for the co-state variable u_ϕ, the planner's version of the job creation condition for each market ϕ is

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\alpha)p_\phi y^\chi - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r + s^\chi}, \tag{3.38}$$

whereas the decentralized job creation condition is

$$\frac{k^{\chi}}{q(\theta_{\phi})} = \frac{(1-\beta)p_{\phi}y^{\chi} - [(1-\beta)b^{\chi} + \beta\theta_{\phi}k^{\chi}]}{r + s^{\chi}}. \tag{3.39}$$

Comparing the planner's job creation condition and the decentralized job creation condition within each market, one can easily see that the equivalence between them requires the following conditions,

$$\alpha = \beta,$$

where α is the matching function elasticity and β is the bargaining power of workers. This is the within market Hosios [1990] condition, common in the random search literature. As in Şahin et al. [2014], imposing the standard Hosios [1990] condition eliminates within-market congestion externality for each market ϕ.

Spatial Optimality Condition

With multiple locations, the planner needs to choose how to allocate workers across locations. Using the first order condition for N_{ϕ}, the planner's spatial optimality condition is

$$\overline{U}^{*\chi} = b^{\chi} + \frac{\alpha}{1-\alpha} k^{\chi}\theta_j^{\chi} \tag{3.40}$$

This condition states that the planner would allocate unemployed workers to a labor market until their contribution to locations is equalized. On the other hand, recall the decentralized spatial equilibrium condition equalizes the indirect utility of an unemployed worker across locations,

$$\overline{U}^{\chi} = \left(b^{\chi} + \frac{\beta}{1-\beta} k^{\chi}\theta_j^{\chi}\right) R_j^{-\eta}; \quad \forall j \in J, \quad \chi \in \{s, n\}. \tag{3.41}$$

The two expressions generally do not coincide. The addition of the housing market distorts the allocation since the planner and the unemployed worker have different valuations for residing in a location. The planner's unemployed worker allocation decision only concerns the effect an additional unemployed worker has on the market tightness, but the unemployed workers themselves care about not only the differences in tightness but also how the cost of living differs by location. The workers' indirect utility takes into account the housing cost, whereas the planner's optimal spatial condition does not. Therefore, even when the within-market standard Hosios [1990] condition ($\alpha = \beta$) is satisfied, the two spatial conditions coincide only when $\eta = 0$ or $R_j = R_{j'} = 0$.

Proposition 3.5.1 states the conditions when the decentralized equilibrium coincides with the planner's solution

The bargaining power parameters of workers β_j^χ need to satisfy the following conditions for the decentralized allocation to coincide with the constrained efficient allocation.

1. For the job creation conditions within each labor market to coincide $\quad \alpha_\phi = \beta_\phi$

2. For the spatial equilibrium conditions to coincide

$$\beta_j^\chi = 1 - \left[1 + \frac{R_j^\eta \left(b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi\right) - b^\chi}{k^\chi \theta_j^\chi}\right]^{-1}$$

These conditions are simultaneously satisfied when $\eta = 0$ or $R_j = R_{j'}$ and $\alpha_\phi = \beta_\phi$.

Proof. See appendix B.6.7 □

Note that the inefficiency is still caused by congestion externality. However, the addition of housing markets distorts the allocation even when within market Hosios [1990] condition is satisfied. Within market Hosios [1990] condition guarantees efficient job creation in the absence of housing market consideration since the housing supply is fixed, and housing is not directly related to the final good production. Nevertheless, the current utility function ties housing consumption to market tightness, which affects total output. Hence, even if the housing markets are frictionless, it complicates the existing congestion externality in the frictional labor market via the inseparability of job finding and housing consumption location, making the competitive housing market relevant.

In random search models, inefficiency arises due to the missing price of market tightness. One way to implement the Hosios [1990] condition and to restore efficiency is charging an entry fee to workers such that the cost of participating in the labor market is equal to the cost of the congestion they create for other workers. In my model, however, the "entry fee" workers pay to participate is the housing rent due to the inseparability of work and home location. Yet, the size of the housing rent does not equal the price of market tightness. Therefore, the market tightness is mispriced, leading to discrepancies between the planner's equilibrium conditions and the decentralized equilibrium conditions.

Additionally, the two skill levels further complicate the problem since two market tightnesses collectively affect the common housing rent, and despite differences in contribution to output, high-skill, and low-skill workers face the same price to enter the location. The expected cost and benefit of being in a local labor market are further distorted between the decentralized equilibrium and the planner's solution. Therefore, the common housing market forces two market tightnesses to affect each other, even if the within market standard Hosios [1990] condition is satisfied within each skill-location labor market ϕ, the between-skill interactions of the market tightness still leads to misallocation since the within market Hosios [1990] condition only eliminate within-market congestion by equating the costs of congestion and benefits of participation within a local labor market; therefore, even when satisfied, the additional congestion cost from the housing market distorts the de facto cost of congestion. The cost of congestion no longer equals the benefit the high-skill worker's participation generates, but it equals the cost of congestion their participation generates plus the change in housing prices due to their participation. Therefore, the market tightness is still mispriced by the housing rent.

Workers of both skill types have incentives to locate in the more productive location; they will do so in the absence of the housing market. However, the common local housing market disproportionally discourages low-skill workers from living in more productive locations. It allocates more workers of both skill types to the less-productive location, leading to inefficiency.

As shown in Proposition 3.5.1, with the current common housing market, only when $\eta = 0$ or $R_j = R_{j'}$ could the within market Hosios [1990] condition restore efficiency.

3.6 Quantitative Analysis

This section presents the calibrated version of the model to compare the decentralized and constrained efficient allocations, compare the frictional labor market with the competitive labor market, and perform counterfactual policy experiments. First, I introduce the data used for the quantitative exercises in Section 3.6.1. In Section 3.6.2, I introduce a few modifications to the model that are unique to its quantitative version. Section 3.6.3 details the calibration strategy. Section **??** studies the effects search frictions have on the Great Divergence. Section 3.6.5 compares the decentralized and constrained efficient allocations. Lastly, Section 3.6.6 performs policy experiments.

3.6.1 Data

The model is calibrated to a representative high-skill-intensive location H, using data from San Francisco-Oakland-Hayward MSA, and a representative low-skill-intensive location L, using data from Detroit-Warren-Dearborn MSA. The period is from 2005 to 2019. The primary data set used for the quantitative exercises is the American Community Survey (ACS), obtained from IPUMS Steven Ruggles and Sobek [Accessed Aug 1st, 2022].

In the quantitative version of the model, high-skill versus low-skill workers are defined based on the worker's occupation, using the task index created by Autor and Dorn (2013). For each occupation, I construct a skill index AM for each occupation k, which is defined as the following:

$$AM_k = \frac{\left(T^A_{k,1980} - T^M_{k,1980}\right) - \underline{AM}}{\overline{AM} - \underline{AM}},$$

where $T^A_{k,1980}$ is abstract task input, defined as the average of the Dictionary of Occupational Titles (DOT) variable for "direction control and planning" which measures managerial and interactive tasks and "GED Math", measuring mathematical and formal reasoning requirements. $T^M_{k,1980}$ is the manual task input, defined as the DOT variable for an occupation's demand for "eye-hand-foot coordination". The AM index's goal is to capture each occupation's skill level. $\underline{AM}$ and $\overline{AM}$ are defined as follows for normalization purposes

$$\underline{AM} \equiv \min\left\{T^A_{1,1980} - T^M_{1,1980}, \ldots, T^A_{K,1980} - T^M_{K,1980}\right\}$$

$$\overline{AM} \equiv \max\left\{T^A_{1,1980} - T^M_{1,1980}, \ldots, T^A_{K,1980} - T^M_{K,1980}\right\}$$

If $AM_k > 0.618$, occupation k is considered a high-skill occupation; otherwise, k is considered a low-skill occupation. Using this categorization, I find the share of high-skill workers in the sample is $\xi = 0.4513$. More information about AM can be found in appendix B.4.

3.6.2 Quantitative version of the model

I introduce two differences in the quantitative version of the model compared to the environment in Section 3.2. First, I generalize the model by endogenizing the job destruction decision. Worker productivity becomes idiosyncratic and is drawn from a distribution F_ϕ that depends on the location and skill pair. Firms optimally choose reservation productivity y^*_ϕ and destroy jobs with productivity less than it, where the value of a filled job with reservation productivity equals zero. At rate λ, employed workers re-draw their productivity. If the newly drawn productivity is less than the reservation productivity, the match is destroyed, the worker becomes unemployed, and the firm becomes vacant.

This extension preserves the basic structure of the equilibrium presented in Section 3.3. The main differences are the following. First, the equilibrium has an additional element, reservation productivity y^*_ϕ. Second, an additional equilibrium condition, the Job Destruction condition, is introduced. The Job Creation condition and the Job Destruction condition jointly pin down the reservation productivity and the market tightness for each skill location pair. Third, the variation in unemployment comes from differences in the job finding rate and the endogenous separation rate. Since the critical condition for worker allocation, the spatial equilibrium condition, does not explicitly involve reservation productivity in the extended model, it is identical to the spatial equilibrium condition presented in Section 3.3. The analytical results from Section 3.3 hold.

Additionally, I allow matching efficiency parameter A_j to differ by location and the unemployment benefit b_ϕ to be different for each skill-location group. Flexibility in these parameters allows the calibration to be more precise. Details and derivation of the quantitative version of the model can be found in appendix B.3.

3.6.3 Calibration

The calibration uses the following parameters from the literature. The rate of productivity shock is set to be $\lambda = 0.085$, following Fujita and Ramey [2012]. Following Petrongolo and Pissarides [2001], the elasticity of the matching function is set to $\alpha = 0.5$, which is in line with empirical evidence. The worker's bargaining power is then set to $\beta = 0.5$ to implement the Hosios [1990] condition. Following Krusell et al. [2000], the elasticity of substitution between high-skill and low-skill workers is set to $\rho = 0.4$.

From the data sample, the total share of high-skill workers in the two locations is $\xi = 0.4513$. The discount rate is r= 0.0143, the average annual interest rate during the period. I can find the average market tightness for each location using the US Job Openings and Labor Turnover Survey (JOLTS) MSA level data from January 2005 to December 2019. Since the expression of job finding rate is $f(\theta_\phi) = A_j \theta_\phi^\alpha$, A_j can be backed out where $A_H = 0.74$ and $A_L = 0.67$. Following the affordable housing guideline (Health and Code [1977]), I use 30 percent as the share of income spent on housing, $\eta = 0.3$.

I use the land area as a proxy for the housing supply in each location. I normalized the land area of location H to be 1. Census Bureau's data of land areas indicates that the land area in L is 57% bigger

Table 3.4: Parameter Value

Parameter		Value	Source
I. From Literature			
Matching function elasticity	α	0.5	Petrongolo and Pissarides [2001]
Worker bargaining power	β	0.5	Hosios [1990] Efficiency Condition
Productivity shock	λ	0.085	Fujita and Ramey [2012]
Elasticity of substitution	$\frac{1}{1-\rho}(\rho)$	1.67(0.4)	Krusell et al. [2000]
Share of spending on housing	η	0.3	Health and Code [1977]
II. From Data			
Discounting rate	r	0.0143	Annual federal funds rate
Total share of skilled labor	ξ	0.4513	Share of high-skill occupation, ACS
High-skill worker weight in H	σ_H	0.648	High-skill labor income share
High-skill worker weight in L	σ_L	0.476	High-skill labor income share
Matching Efficiency in location H	A_H	0.74	Job Finding Prob. in H, JOLTS
Matching Efficiency in location L	A_L	0.67	Job Finding Prob. in L, JOLTS
Pareto dist. scale parameter (H, high-skill)	$y^s_{H,m}$	1.2	Minimum schooling level, ACS
Pareto dist. scale parameter (H, low-skill)	$y^n_{H,m}$	0.5	Minimum schooling level, ACS
Pareto dist. scale parameter (L, high-skill)	$y^s_{L,m}$	1.1	Minimum schooling level, ACS
Pareto dist. scale parameter (L, low-skill)	$y^n_{L,m}$	0.5	Minimum schooling level, ACS
Land area in location H	T_H	1	Normalization
Land area in location L	T_L	1.57	Census Bureau

than the land area in H; therefore, $Q_L = 1.57$. Following Krusell et al. [2000] and using the high-skill labor income share for each location, the weight of high-skill workers in the final goods production function is $\sigma_H = 0.648$ and $\sigma_L = 0.476$. Productivities of both skill types are assumed to follow Pareto Distributions $F \sim Pareto(y_{m,\phi}, \alpha_\phi)$ where $y_{m,\phi}$ is the scale parameter for the skill location group and α_ϕ is its shape parameter. Since the scale parameter in the Pareto distribution reflects the lower bound of the distribution, it is obtained from the minimum level of schooling of each skill location group, where $y^s_{m,H} = 1.2$, $y^n_{m,H} = 0.5$, $y^s_{m,L} = 1.1$, $y^n_{m,L} = 0.5$. [6] The shape parameter is calibrated by using the mean wage generated by the model to back out the mean productivity for each skill location group, where the expression of the mean productivity involves only the shape and scale parameter of the Pareto distribution. The results are $\alpha^s_H = 1.28, \alpha^n_H = 1.45, \alpha^s_L = 1.2$ and $\alpha^n_L = 1.43$. Table 3.4 summarizes the parameter values.

The remaining parameters are calibrated as follows. Unemployment insurance b_ϕ for each skill location group is calibrated using the replacement rate, where $\frac{b_\phi}{\bar{w}_\phi} = 0.71$, following Hall and Milgrom [2008], where $\bar{w}_\phi$ is

[6] The minimum schooling level for high-skill workers in location H, low-skill workers in location H, high-skill workers in location L and low-skill worker in location L are twelve years, five years, eleven years and five years respectively.

Table 3.5: Calibrated Parameters

Parameter		Calibrated Value
Unemployment utility (high-skill, location H)	b_H^s	0.832
Unemployment utility (low-skill, location H)	b_H^n	0.383
Unemployment utility (high-skill, location L)	b_L^s	0.799
Unemployment utility (low-skill, location L)	b_L^n	0.369
Flow vacancy cost (high-skill)	k^s	1.95
Flow vacancy cost (low-skill)	k^n	0.98
Shape parameter of Pareto dist. (high-skill, location H)	a_H^s	1.38
Shape parameter of Pareto dist. (low-skill, location H)	a_H^n	1.5
Shape parameter of Pareto dist. (high-skill, location L)	a_L^s	1.3
Shape parameter of Pareto dist. (low-skill, location L)	a_L^n	1.5
Upper bound for productivity (high-skill, location H)	$\overline{y}_H^s$	91.49
Upper bound for productivity (low-skill, location H)	$\overline{y}_H^n$	54.68
Upper bound for productivity (high-skill, location L)	$\overline{y}_L^s$	67.10
Upper bound for productivity (low-skill, location L)	$\overline{y}_L^n$	46.76

Table 3.6: Targeted Moments

	Data	Model		Data	Model
Replacement Rate (Hs)	0.71	0.710	Replacement Rate (Ls)	0.71	0.710
Replacement Rate (Hn)	0.71	0.709	Replacement Rate (Ln)	0.71	0.709
Mm wage ratio(Hs)	2.498	2.467	Mm wage ratio(Ls)	2.479	2.461
Mm wage ratio(Hn)	2.502	2.491	Mm wage ratio(Ln)	2.532	2.524
90-10 percentile ratio (Hs)	7.946	7.951	90-10 percentile ratio (Ls)	6.903	6.906
90-10 percentile ratio (Hn)	12.79	12.79	90-10 percentile ratio (Ln)	12.64	12.64

the average wage for each skill location group generated by the model. Flow vacancy cost is calibrated to match its share of average labor productivity for each skill level, following Hagedorn and Manovskii [2008]. I use the mean-min (Mm) wage ratio for each skill location group to calibrate the shape parameter of the Pareto distribution where $a_H^s = 1.38$, $a_H^n = 1.5$, $a_L^s = 1.3$, $a_L^n = 1.5$. Lastly, I used the $90-10$ percentile wage ratio for each skill location group to calibrate the upper bound of match-specific productivity by skill location group, where $\overline{y}_H^s = 91.49, \overline{y}_H^n = 54.68, \overline{y}_L^s = 67.1$ and $\overline{y}_L^n = 46.76$. The results of the calibrated parameters are summarized in Table 3.5. Table 3.6 illustrates that the model closely matches the empirical targets.

To assess the model's performance, I look at several non-targeted empirical moments that are believed to be

Table 3.7: Non-Targeted Moments

		Data	Model
(a) Labor Market Composition			
Population share of high-skill location	L_H	0.6043	0.5384
Share of high-skill worker in high-skill place	ζ_H	0.518	0.5828
Share of high-skill worker in low-skill place	ζ_L	0.3481	0.2979
(b) Unemployment Ratio			
Unemployment rate Δ% for high-skill worker	$(u^s_H - u^s_L)/u^s_L$	-14.35%	-21.7%
Unemployment rate Δ% for low-skill worker	$(u^n_H - u^n_L)/u^n_L$	-30.12%	-21.5%

particularly important for the model. First, I look at the composition of labor markets. Panel (a) of Table 3.7 compares labor market compositions between the data and the model. The model predicts 53.84% of the workers are in location H; among them, 58.28% are high-skill workers. 29.79% of the labor force in location L are high-skill workers. The model performs well in matching the labor market compositions in the data as we can see that the difference between the data and the model are narrow for L_H, ζ_H, and ζ_L. Panel (b) of Table 3.7 compares the unemployment ratio between locations. For both skill groups, the model predicts that the unemployment rate is lower in location H than in location L. Overall, the calibration matches the labor force composition and the relationship between locations for wages and unemployment rates, as we have seen in descriptive facts presented in Section 3.1.1.

3.6.4 Search Frictions and the Great Divergence

This section quantitatively assesses the effect of labor market search friction on the great divergence. As discussed in Section 3.4, in the model with competitive labor markets, the spatial equilibrium conditions, equation **??** indicate that workers' location choices are based on the relative sizes of wages and housing prices. However, in the case of labor markets with search frictions, what determines the spatial equilibrium are the marginal product of labor, market tightness, and housing prices, as shown in **??**. The presence of market tightness in the spatial equilibrium condition indicates that the allocation of workers will be different for the frictional labor market and the competitive labor market. Table 3.8 summarizes the allocation for the two different labor markets.

Compared to the frictional model, the competitive labor market equilibrium places a higher share of high-skill workers but fewer workers in location H. The location wage gap is also higher in the competitive labor market model. It is 27.3% higher for high-skill workers and 28.9% higher for low-skill workers. Again, in the competitive model, wages for both skill types in H must be much higher to attract workers there. Lastly, the location rent gap in the competitive model is 13.6% higher than in the frictional model, resulting from the bigger location wage gaps. Therefore, we can say that the model with labor market search friction moderates the great divergence. Lastly, a back-of-the-envelope calculation for the utility of high- and low-skill workers suggests that the utility gap for high-skill workers and low-skill workers in the competitive labor market is about 4% bigger than in the model with frictional labor markets.

3.6.5 Planner's vs. decentralized allocation

In this section, I compare the constrained efficient and decentralized allocations under the parameters and calibrated parameters presented in Table 3.4 and Table 3.5. For the constrained efficient allocation, I compute the planner's choice of reservation productivity y^*_ϕ, market tightness θ_ϕ, number of workers L_ϕ to maximize the sum of steady-state net output of location H and L. Table 3.9 summarizes the results.

Column (1) of Table 3.9 shows the decentralized allocation. For both high-skill and low-skill workers, the unemployment rate is higher in location L. Within each location, the unemployment rate of high-skill workers is lower than the unemployment rate of low-skill workers. The pattern of unemployment rates matches descriptive facts presented in Figure 3.2. Regarding the distribution of workers, the calibrated model suggests that location H has more high-skill workers than low-skill workers. In contrast, location L has more low-skill workers than high-skill workers.

Column (3) of Table 3.9 shows the percentage differences between decentralized and constrained efficient allocations. The constrained efficient allocation exhibits a higher level of reservation productivity for both groups in both locations. The planner allocates more workers of both skill types to location H. For high-skill workers in both locations, the constrained efficient allocation shows higher market tightness relative to the decentralized version, whereas the reverse is true for low-skill workers. Finally, the aggregate output of the constrained efficient allocation is 4.794% higher than the decentralized allocation.

The discrepancies between the decentralized and constrained efficient outcomes arise from the inefficiencies discussed in Section 3.5. [7] Without considering housing costs, more low-skill workers moved into location H. Hence L_H increased, and ζ_H decreased. Leading to a higher low-skill unemployment rate in location H. Therefore, compared to the constrained efficient equilibrium, the decentralized equilibrium allocates

[7] Note that even though the baseline model which Section 3.5 is based on is different from the endogenous separation version of the model that the quantitative exercises are based on, the intuition of inefficiencies are similar and is a result of the differences in the spatial equilibrium condition.

Table 3.8: Comparison of allocations

	Competitive Labor Market	Frictional Labor Market	%Diff
Share of high-skill worker in H	0.5917	0.5839	1.3349%
Labor force in H	0.5070	0.5376	-5.6911%
Share of high-skill worker in L	0.3069	0.2971	3.2933%
Labor force in L	0.4930	0.4624	6.6167%
Location wage ratio (high-skill workers)	1.3423	1.0542	27.3292%
Location wage ratio (low-skill worker)	1.3423	1.0417	28.8634%
Location housing price ratio	2.6681	2.3487	13.5994%

Table 3.9: Allocation Comparison

		(1) Decentralized	(2) Centralized	(3) % Difference
Reservation Productivity				
	y_H^{s*}	4.7017	5.9492	26.5326%
	y_H^{n*}	1.6961	2.0840	22.8709%
	y_L^{s*}	3.9492	5.3072	34.3857%
	y_L^{n*}	1.4454	1.5488	7.1504%
Market Tightness				
	θ_H^s	2.3340	3.2395	38.7986%
	θ_H^n	2.0679	1.9374	-6.3074%
	θ_L^s	1.7284	3.2112	85.7887%
	θ_L^n	1.5276	1.5768	3.2247%
Distribution of workers				
	L_H	0.5376	0.8659	61.0711%
	ζ_H	0.5839	0.4986	-14.6111%
	L_L	0.4624	0.1341	-71.0027%
	ζ_L	0.2971	0.1458	-50.9410%
Unemployment Rate				
	u_H^s	0.0599	0.0538	-10.3179%
	u_H^n	0.0629	0.0679	7.9636%
	u_L^s	0.0725	0.0581	-19.9231%
	u_L^n	0.0756	0.0762	0.8310%
Output				
	$Z_H + Z_L$	0.3775	0.3956	4.794%

inefficiently small amounts of workers of both skill types in location H.

3.6.6 Policy Experiments

In this section, I study the effects of policies that aim at correcting the inefficiencies caused by the externalities. Table 3.10 contains the results of the experiment, where Column (1) is the allocation of the decentralized equilibrium, Column (2) shows results from the counterfactual experiment, and Column (3) compare the difference between the decentralized equilibrium and the allocation with the policy.

Low-skill worker relocation subsidy

Since Table 3.9 shows that inefficiently low numbers of workers, in particular, low-skill workers, choose location H, the first policy experiment studies the effect of lump-sum subsidy for low-skill workers in location H. A fixed subsidy τ^m is given to all low-skill workers in location H regardless of employment status. The subsidies are financed by a lump-sum tax τ^c on all workers, regardless of employment status, skill type, or location. The subsidy's size equals 10 percent of housing spending an unemployed low-skill worker in location H would pay [8].

As seen in column (2), when low-skills workers in location H are subsidized by all firms, the labor force increases in location H, and it becomes slightly less concentrated in high-skill workers. The allocation of worker distribution is moving toward the constrained efficient allocation. Compared to the benchmark decentralized allocations, this policy experiment creates more jobs in location H, and unemployment rates are lower for all skill-location groups. Putting equal weights on all skill-location groups, the aggregate welfare is 0.05613% higher under this policy experiment.

3.7 Conclusion

This paper documents the geographic dispersion of unemployment rates in the US for workers of different skill levels. I then develop a model featuring frictional labor markets in a spatial equilibrium to study how the frictional labor market shapes the great divergence across US cities and its effect on the optimal allocation of

[8] Details of the policy experiment equilibrium can be found in appendix B.5.

Table 3.10: Policy Experiments

		(1) Benchmark	(2) Worker	subsidy
			Level	% Difference
Reservation Productivity				
	y_H^{s*}	4.7017	4.6959	-0.1242%
	y_H^{n*}	1.6961	1.6902	-0.3486%
	y_L^{s*}	3.9492	3.9419	-0.1859%
	y_L^{n*}	1.4454	1.4413	-0.2848%
Unemployment rates				
	u_H^s	0.0599	0.0599	-0.0291%
	u_H^n	0.0629	0.0629	0.0809%
	u_L^s	0.0725	0.0725	0.0442%
	u_L^n	0.0756	0.0755	-0.1175%
Worker Distribution				
	L_H	0.5376	0.5394	0.3366%
	ζ_H	0.5839	0.5826	-0.2321%
	L_L	0.4624	0.4606	-0.3942%
	ζ_L	0.2971	0.2976	0.1523%
Aggregate Welfare		0.3563	0.3565	0.05613%

heterogeneous workers. The model generates theoretical results that explain the empirical pattern of wages and unemployment rates for high- and low-skill workers and the skill composition across labor markets.

Comparing the model with labor market frictions with the model with competitive labor markets shows that frictional labor markets moderate the divergence in high-skill worker concentration and the wage gap between locations compared to its full employment counterpart. The high-wage location also features low unemployment rates, particularly for low-skill workers. A bigger wage gap is required to obtain the spatial equilibrium without friction in the labor market. A normative analysis shows that the decentralized equilibrium is never efficient even if the standard within market Hosios [1990] condition holds, but can be efficient if a generalized version of the Hosios [1990] condition holds. The additional inefficiency is caused by distortions resulting from the housing market since the housing rent takes on the additional role of an entry fee to labor markets but is not priced accordingly. A calibrated version of the model using representative high-skill-intensive and low-skill-intensive locations shows that inefficient amounts of workers of both skill types choose to stay in the low-skill-intensive location due to the high housing cost of the high-skill-intensive location. Additionally, the amount of jobs created in high-skill-intensive locations is inefficiently low for both skill types. Subsidies incentivizing workers to locate to high-skill-intensive locations raise aggregate welfare.

Chapter 4

Creative Destruction in Rental Housing Market

4.1 Introduction

How does the introduction of rent control policies shape rental housing market outcomes? Recent economic research has provided substantial and valuable empirical evidence on the effects of rent control expansion (Diamond et al. [2019]), and the effects of ending rent control (Sims [2007] and Autor et al. [2014]), in particular on the units under rent control. Nevertheless, little is known about the effects of rent control policies beyond the units that are under rent control. What are the general equilibrium consequences of rent control policy on the rental housing market on the rest of the rental housing market? This is an important question since units under rent control only make up a small share of the total rental housing supply.

This paper studies the consequences of rent control policies on the frictional rental housing market using a Diamond [1982] - Mortensen [1979]-Pissarides [1985] style random matching model. This model provides a microstructure of the rental housing market, where we can explicitly model the behaviors of renters and landlords with different rent control statuses. The general equilibrium modeling approach also allows the number of rent-control units, non-rent-control units, and vacant units to be determined by the interaction of housing entry, match destruction, and free entry. It can also provide mappings between theoretical predictions and empirical facts for key elements of the rental housing market, including rents, occupancy duration, tenant turnover, and tenant eviction. Additionally, this model allows for connecting rent control policies to land regulation policies. The former policies affect the incentives for entry into the rental housing market, and the latter determine the elasticity of supply, painting a more comprehensive picture of the supply side of rental housing.

The rental housing market is highly frictional. Figure 4.1 from the Current Population Survey shows that between 1968-2022, the average rental housing vacancy rate is about 8%, which is about five times the vacancy rate of owner-occupied housing. The persistent rental vacancy rate shows that the rental housing market is indeed frictional. Not to mention, anyone who has rented an apartment knows looking for rental housing takes time, and the units are hardly homogenous. Even in the case of large apartment buildings where the floor plans are identical, the uniqueness of each unit comes from the fact that they are located on different floors or have different lighting.

The rest of the paper unfolds as follows. The remaining parts of the introduction present the history of rent control and related literature. Section 4.2 presents a baseline model without rent control shocks. Section 4.3 presents a model with rent control shocks, characterizes the equilibrium, and shows comparative statics. Lastly, Section 4.4 concludes.

4.1.1 The History and Present of Rent Control

Regulations are widespread in housing markets, and rent controls are arguably among the most important historically (Friedman and Stigler [1946] and Gyourko and Glaeser [2008]). The modern era of US rent controls began as a part of World War II era price controls and as a reaction to housing shortages following demographic changes immediately after the war (Fetter [2016]). These "hard price controls" that directly regulate the nominal price of housing have been replaced by newer policies that limit rent increases Arnott [1995]. However, the details of rent increase limits vary by jurisdiction.

In terms of geographic coverage, as of February 2022, California, Oregon, and the District of Columbia have statewide rent control policies. Maine, Maryland, New Jersey, and New York have county and city-level laws in effect. Twenty-five states have preemptive rent control rules. To qualify as a rent control unit, the unit needs to reach a certain age, and the number of units in the structure needs to exceed five. The model in Section 4.3 follows most closely to the New York City style rent control policies.

4.1.2 Related Literature

This project speaks to three threads of literature.

Housing Search

First and foremost, it speaks to the housing search literature. The model of this paper follows the use of search and matching models to study frictions in the owner-occupied housing market, as established by Wheaton [1990]. Han and Strange [2015] provides a survey of the literature. More recently, Gabrovski and Ortego-Marti [2019] introduced the double entry of buyers and sellers into the housing market to capture the cyclical behavior of the housing market. Nevertheless, the majority of models with frictional housing markets focus on the owner market, and the rental housing market is often left out of the models. Recent papers by Halket and di Custoza [2015], Ioannides and Zabel [2019], Bo [2022] and Han et al. [2023], explicitly consider search and matching in both ownership and rental markets.

Their objectives differ from those of this paper. They focus instead on issues such as the Beveridge curve in the housing market and the relationship between price-to-rent ratios and homeownership rates across sub-markets. This paper focuses solely on the rental market and how rent control shocks change the bargaining process in the rental housing market.

Rent Control

This project is related to the rent control literature. There is a lot of empirical literature on the topic of rent control. Sims [2007] and Autor et al. [2014] study the effects of ending rent control in the Boston metropolitan area. Sims [2007] finds that rent control had little effect on the construction of new housing but did encourage owners to shift units away from rental status and reduced rents substantially. Diamond et al. [2019] exploits a quasi-experimental variation in rent control assignment in San Francisco to study its impact on tenants and landlords of rent control units. Asquith [2019b] examines how controlled landlords change their housing supply in response to demand increases. Asquith [2019a] find that when the overall price increases, landlords withdraw rent-controlled units as a consequence of a local demand shock. A list of studies finding longer tenant durations in rent control units includes Linneman [1987]; Gyourko and Linneman [1989]; Munch and Svarer [2002]; Nagy [1995], Nagy [1997]; Ault et al. [1994]; Krol and Svorny [2005]; Svarer et al. [2005].

In terms of theory, there is an older literature on rent control combining applied theory with cross-sectional empirical methods: Early [2000], Glaeser and Luttmer [2003], Gyourko and Linneman [1989] just to name a few. These papers' goals are to test whether the data are consistent with the theory being studied but usually do not quantify the causal effects of rent control. Additionally, McFarlane [2003] uses an urban growth model to directly model a controlled market with vacancy decontrol, showing that under stable price growth, the long-run supply would not be much affected by rent control since the landlord can freely set the base rent. However, rent control hastens construction and re-development. Nagy [1997] employs price, vacancy decontrol, and rent control in a partial equilibrium model. Basu and Emerson [2000] studies the adverse selection of tenants under control and shows that landlords prefer short-stay tenants.

This paper provides a general equilibrium model in a frictional rental market where rent is bargained between renters and landlords. Additionally, a rent control shock, the transitions between rent-control and non-rent-control units, and evictions are explicitly modeled.

Creative Destruction

The last thread of literature this project speaks to is the Creative Destruction literature. Most of the creative destruction literature is in the labor market context. Aghion and Howitt [1994] and Mortensen and Pissarides [1998], ask how economic growth affects unemployment in the long run. This paper's model is closest to the latter but applied in a rental housing market context. In this model, price increases are embodied in new apartment contract rents but disembodied in rent-controlled apartments since rent increases are limited and can no longer be bargained.

4.2 Model without rent control

4.2.1 General Environment

Time is continuous. The economy grows at the rate g, and $G(t)$ is a general productivity measure,

$$G(t) = e^{gt}.$$

Following Mortensen and Pissarides [1998], utility $\varepsilon(t)$ and all the costs grow at the same rate as productivity to ensure the existence of a steady state with balanced growth:

$$\varepsilon(t) = G(t)\varepsilon_0; \quad c^v(t) = G(t)c_0^v; \quad c^s(t) = G(t)c_0^s; \quad c^k(t) = G(t)c_0^k; \quad c^e(t) = G(t)c_0^e;$$

where c_0^v, c_0^s, c_0^k, and c_0^e are the initial vacancy cost, search cost, construction cost, and eviction cost, respectively. There are two types of agents: renters and landlords. The total size of renters is 1. Landlords own rental units, and they rent them out to renters.

Landlords pay builders to build housing. Builders provide new housing vacancies at a cost of $c^k(t)$ to landlords. Construction cost depends on the common cost of construction ($\bar{k}$) and local zoning cost z:

$$c^k(t) = c_0^k G(t) \quad c_0^k = z\ln(\bar{k}). \tag{4.1}$$

c^k is increasing in the stringency of local land use regulation, which is captured by z. Market tightness is defined as $\theta = \frac{u}{v}$ where u denotes the rate of unhoused renters, and v denotes the vacancy rate. The matching function is defined as follows,

$$M(u,v) = Au^{1-\alpha}v^{\alpha}, \quad m(\theta) = A\theta^{-\alpha},$$

where A is the matching efficiency parameter, α is the matching elasticity, and $m(\theta)$ is the rate at which an unhoused renter meets a vacancy.

4.2.2 Environment - Renters and Landlords

Unhoused Renter:

$U(t)$ is the value function for an unhoused renter. She pays search cost $c^s(t)$ to look for rental properties. With probability $m(\theta)$, she is matched with a vacancy and becomes a housed renter with value function $Q(t)$.

Housed Renter:

$Q(t)$ is the value function of a housed renter. She receives utility $\varepsilon(t)$ from enjoying the rental property and pays a price $\rho(t)$. At rate $s + \delta$, the match is destroyed, and the housed renter becomes an unhoused renter again. Note that s is the exogenous destruction rate for the match, and δ is the depreciation rate for the housing unit. The depreciation rate is added to the separation rate to account for the probability of match destruction.

Landlord:
$L(t)$ is the value function of the landlord of an occupied unit. She receives rent $\rho(t)$ from the renter while the unit is occupied. With probability s, the renter is separated from the unit, and the landlord has a vacancy again. Rent is bargained every period. The unit depreciates at rate δ.

Vacancy:
$V(t)$ is the value function of a vacancy. A vacancy landlord pays a vacancy cost of $c^v(t)$. She meets unhoused renters at rate $\theta m(\theta)$ to let out their housing unit at price $\rho(t)$. The unit depreciates at rate δ.
Figure 4.2 illustrates the transitions of tenants and landlords in the baseline model.

4.2.3 Bellman Equations

Unhoused Renter $[U(t)]$

$$rU(t) = -c^s(t) + m(\theta)[Q(t) - U(t)] + \dot{U}(t) \tag{4.2}$$

Housed Renter $[Q(t)]$

$$rQ(t) = \varepsilon(t) - \rho(t) + (s + \delta)[U(t) - Q(t)] + \dot{Q}(t) \tag{4.3}$$

Vacancy $[V(t)]$

$$rV(t) = -c^v(t) + \theta m(\theta)[L(t) - V(t)] - \delta V(t) + \dot{V}(t) \tag{4.4}$$

Landlord $[L(t)]$

$$rL(t) = \rho(t) + s[V(t) - L(t)] - \delta L(t) + \dot{L}(t) \tag{4.5}$$

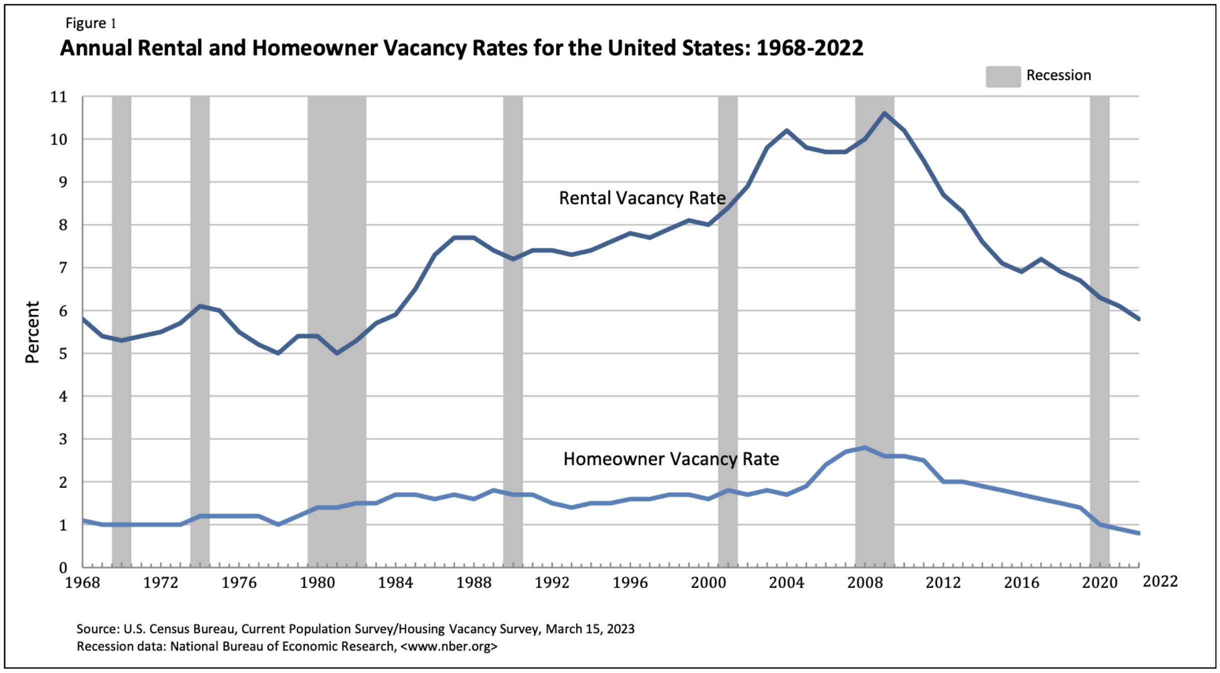


Figure 4.1: Annual Renter and Vacancy Rates for the US: 1968 - 2022

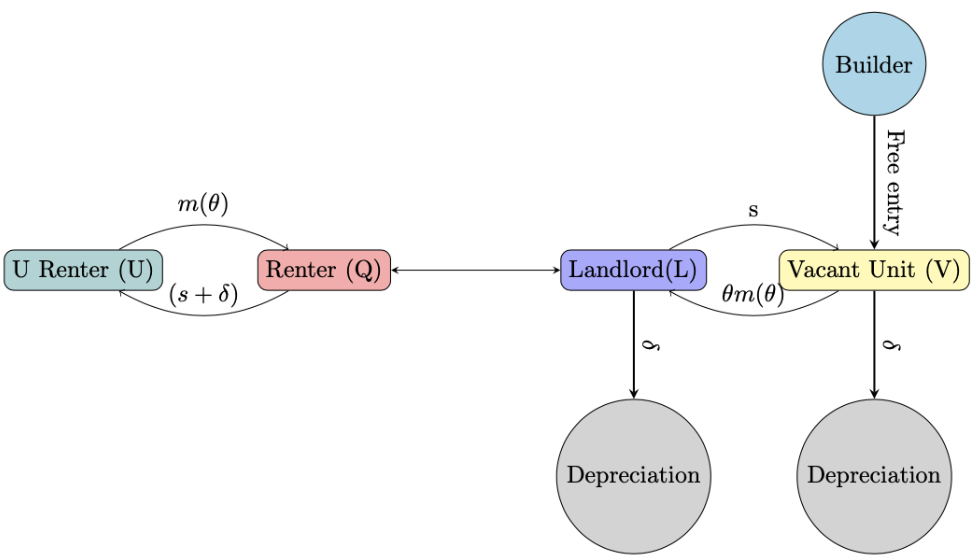


Figure 4.2: Transitions for Tenants and Landlords, without Rent Control

4.2.4 Equilibrium Conditions

Nash Bargaining

Bargaining between a housed renter and a landlord is ongoing and determines the price $\rho(t)$,

$$\rho(t) = \arg max[Q(t) - U(t)]^{\beta}[L(t) - V(t)]^{1-\beta} \tag{4.6}$$

Re-arrange the Bellman equation for the landlords and the renter, and using the bargaining rule,

$$[Q(t) - U(t)] = \frac{\beta}{1-\beta}[L(t) - V(t)]. \tag{4.7}$$

In equilibrium, vacancies enter the market until the value of vacancy equals the construction cost, i.e. $V(t) = c^k(t)$. Therefore, the price is

$$\rho(t) = (1-\beta)[\varepsilon(t) + c^s(t)] + \beta(1 - \frac{1}{\theta})(r+\delta)c^k(t) - \beta\frac{1}{\theta}c^k(t).$$

The price at time $t = 0$ is

$$\rho_0 = (1-\beta)[\varepsilon_0 + c^s_0] + \beta(1 - \frac{1}{\theta})(r+\delta)c^k_0 - \beta\frac{1}{\theta}c^k_0. \tag{4.8}$$

We can see from here that the price is positively related to market tightness, and the price is also positively related to construction cost $c^k(t)$. We can see that $\rho(t)$ grows at the same rate as $\varepsilon(t), c^v(t), c^s(t)$.

Housing Entry Condition

Free entry condition of vacancy $V(t) = c^k(t)$ helps us get the housing entry condition (HE) condition by re-arranging the Bellman equation for Vacancy and Landlord:

$$\frac{(r+\delta)c^k(t) + c^v(t)}{\theta m(\theta)} = \frac{\rho(t)}{r+s+\delta} - \frac{(r+\delta)c^k(t)}{r+s+\delta}. \tag{4.9}$$

The left-hand side is the expected cost of finding a renter, and the right-hand side is the landlord's surplus. The housing entry condition shows that the landlord keeps entering the market until the profit from being a landlord is just enough to cover the cost of finding a renter.

Housing Beveridge Curve

The Housing Beveridge curve provides the relationship between the unhoused rate u and the market tightness θ,

$$u = \frac{s + \delta}{s + \delta + m(\theta)}. \tag{4.10}$$

Note that, in theory, the landlord could choose to evict the tenant and become vacant again. However, the landlords's surplus,

$$L(t) - V(t) = \frac{(r + \delta)c^k(t) + c^v(t)}{\theta m(\theta)},$$

is always positive. Therefore, in the baseline model without rent control, a landlord would never choose to evict a renter.

4.2.5 Equilibrium

An equilibrium without rent control shock is $\{p_0, \theta, u\}$ such that equation (4.8), (4.9) and (4.10) are satisfied. Since the Price Equation is increasing in θ and Housing Entry Condition is decreasing on the (θ, ρ_0) coordinate plane, the Housing Entry Condition (Equation 4.9) and Price Equation (Equation 4.8) jointly pin down market tightness θ and initial price ρ_0, as illustrated in Figure 4.3. Figure 4.4 illustrates the determination of unhoused rate u and vacancy rate v by the Housing Entry Condition (Equation 4.9) and Housing Beveridge Curve (Equation 4.10).

4.3 Model with Rent Control shocks

4.3.1 Environment

The general environment described in Section 4.2.1 also applies to this version of the model. In addition, a rent control shock occurs with probability σ. A rent control shock only occurs to occupied units that are not rent-controlled. Once a housing unit becomes rent-controlled, it stays rent-controlled until the match is dissolved. $\omega \in \{0, 1\}$ indicates the rent control status of the value function. $\omega = 1$ indicates that Rent Control applies to the housing unit, whereas $\omega = 0$ indicates that Rent Control does not apply to the housing unit.

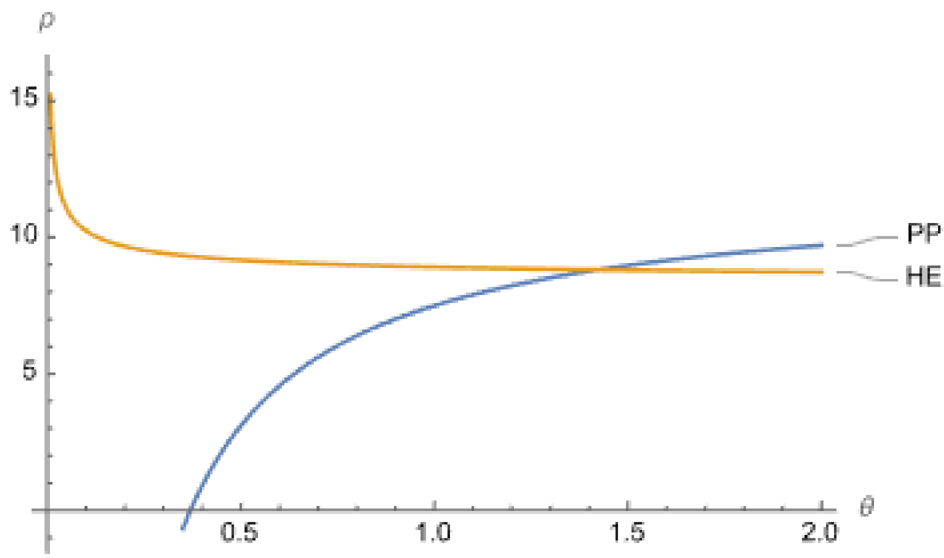


Figure 4.3: HE and PP curves

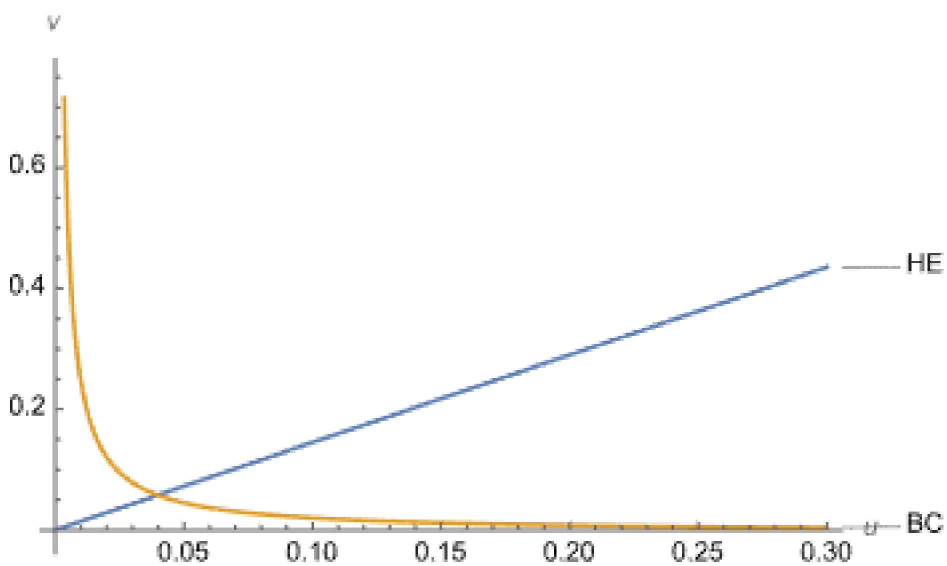


Figure 4.4: HE and BC curves

4.3.2 Environment - Renters and Landlords

Unhoused Renter

$U(t)$ is the value function of the unhoused renter. She pays search cost $c^s(t)$ to look for rental properties. With probability $m(\theta)$, she is matched with a vacancy and becomes a non-rent controlled renter with value function (Q_0).

Non-Rent Control Renter

$Q_0(t)$ is the value function of a renter in non-rent-controlled units. She receives utility $\varepsilon(t)$ from enjoying the rental property and pays a price $p(t)$. At rate $(s + \delta)$, she is exogenously separated (or rental unit is destroyed) from her rental unit and becomes an unhoused renter again. The non-rent-control housing unit is subject to rent-control (RC) shocks, with probability σ, the unit becomes rent-controlled, and hence she becomes a rent-controlled tenant, with value function $Q_1(p(t), t)$.

Rent-Controlled Renter

$Q_1(p(\tau), t)$ is the value function of a renter in a rent-controlled unit. τ is the time at which the RC shock occurred. $p(\tau)$ is the rent at the time τ, and is fixed for $t > \tau$. Note that Q_1 is a function of $p(\tau)$ since the RC rent $p(\tau)$ affects the value of the rent-controlled tenant. She receives utility ε from enjoying the rental property and pays a price $p(\tau)$. At rate $(s + \delta)$, she is exogenously separated from her unit and becomes an unhoused renter looking for housing again. At time t = T, which is endogenously determined from the equilibrium, she is evicted from her unit by the landlord and becomes an unhoused renter again.

Vacancy

$V(t)$ is the value function of a vacancy. A landlord with a vacancy needs to pay a vacancy cost of $c^v(t)$. With probability $\theta m(\theta)$, the vacancy is matched with an unhoused renter, and the landlord with vacancy becomes a landlord with a non-rent-controlled unit, with value function $Q_0(t)$. The vacancy depreciates at rate δ.

Non-Rent-Controlled Landlord

$L_0(t)$ is the value function of a non-rent-controlled landlord. She receives rent $p(t)$ from a renter. With probability s, the renter is exogenously separated from the unit. The non-rent-control landlord is subject to

RC shocks. With probability σ, the unit becomes a rent-controlled unit, and hence, the landlord becomes a rent-controlled landlord. The unit depreciates at rate δ.

Rent-Controlled Landlord

$L_1(p(\tau), t)$ is the value function of a rent-controlled landlord. τ is the time at which the RC shock occurred, and $p(\tau)$ is the rent at the time τ. Similar to the rent-controlled renter, L_1 is a function of $p(\tau)$, since the RC rent $p(\tau)$ affects the value of the rent-controlled landlord.

The landlord receives rent $p(\tau)$ from the renter. With probability s, the renter is separated from the unit, and the landlord becomes a vacancy again. The landlord chooses whether to keep her renter or evict the renter. Note that a non-rent-controlled landlord would never choose to evict the renter since the landlord's surplus is always positive when rent is bargained. However, the rent-controlled landlord's surplus is no longer always positive since the rent is fixed at $p(\tau)$ and the landlord will be receiving low rents from a long-standing renter. In this case, she needs to evaluate if she should stay with the current renter or pay an eviction cost of $c^e(t)$ and let the unit be decontrolled and become a vacancy again.

Eviction can be thought of as capturing all of the supply of units re-entering the rental housing market since the unit is no longer under rent control. The unit depreciates at rate δ.

Figure 4.5 illustrates the transitions of tenants and landlords.

4.3.3 Bellman Equations

Unhoused Renters $[U]$

$$rU(t) = -c^s(t) + m(\theta)[Q_0(t) - U(t)] + \dot{U}(t) \tag{4.11}$$

Non-rent Control Renters $[Q_0]$

$$rQ_0(t) = \varepsilon(t) - p(t) + (s + \delta)[U(t) - Q_0(t)] + \sigma[Q_1(p(t), t) - Q_0(t)] + \dot{Q}_0(t) \tag{4.12}$$

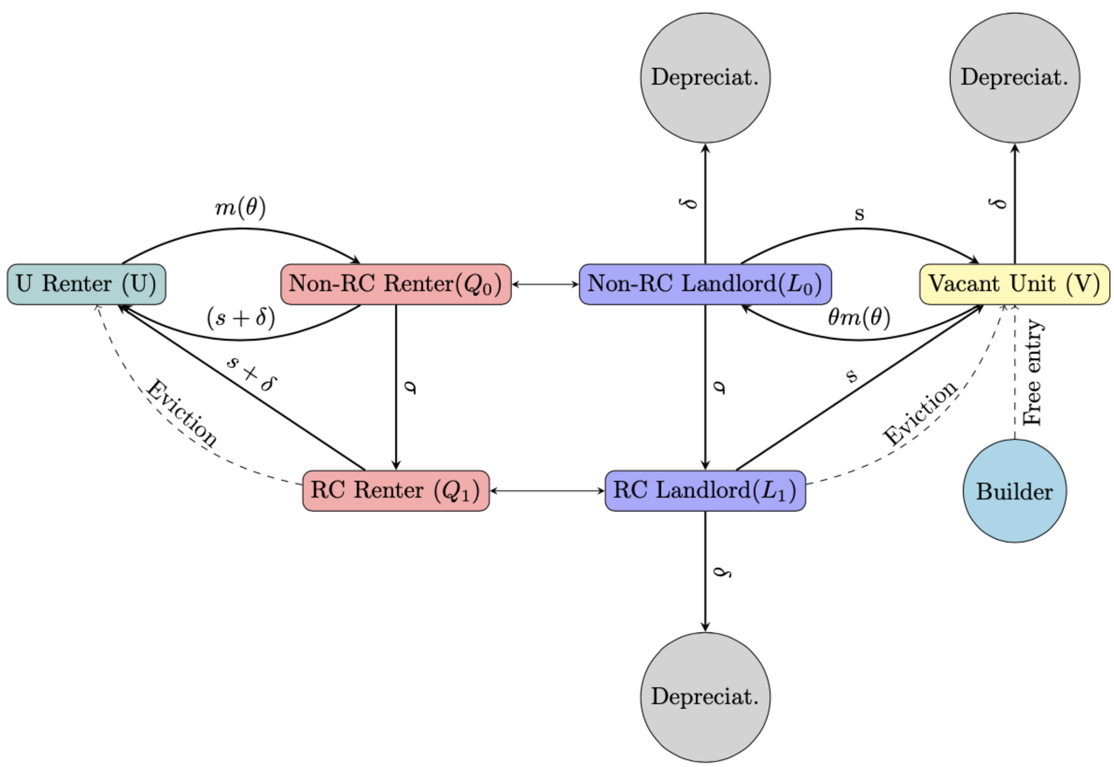


Figure 4.5: Transitions for Tenants and Landlords

Rent Control Renters $[Q_1]$

$$rQ_1(p(\tau),t) = \varepsilon(t) - p(\tau) + (s+\delta)[U(t) - Q_1(p(\tau),t)] + \dot{Q}_1(p(\tau),t) \tag{4.13}$$

Vacancy $[V]$

$$rV(t) = -c^v(t) + \theta m(\theta)[L_0(t) - V(t)] - \delta V(t) + \dot{V}(t) \tag{4.14}$$

Non-rent control landlord $[L_0]$

$$rL_0(t) = p(t) + s[V(t) - L_0(t)] + \sigma[L_1(p(t),t) - L_0(t)] - \delta L_0(t) + \dot{L}_0(t) \tag{4.15}$$

Rent control landlord $[L_1]$

$$rL_1(p(\tau),t) = \max\{p(\tau) + s[V(t) - L_1(p(\tau),t)], V(t) - c^e(t)\} - \delta L_1(p(\tau),t) + \dot{L}_1(p(\tau),t) \tag{4.16}$$

4.3.4 Equilibrium Conditions

Nash Bargaining

Bargaining between a non-rent control tenant and a non-rent-controlled landlord determines rent price $p(t)$,

$$p(t) = arg\max[Q_0(t) - U(t)]^{\beta}[L_0(t) - V(t)]^{1-\beta}.$$

Since the values for rent control tenant $Q_1(p(\tau),t)$ and rent control landlord $L_1(p(\tau),t)$ are functions of the price $p(\tau)$ when RC shock occurred at time $t=\tau$, the partial derivatives of Q_1 and L_1 with respect to the rent control price are not zero. Therefore, the bargaining rule becomes the following,

$$\beta[L_0(t) - V(t)]\left[-1 + \sigma\frac{\partial Q_1(p(t),t))}{\partial p(t)}\right] + (1-\beta)[Q_0(t) - U(t)]\left[1 + \sigma\frac{\partial L_1(p(t),t))}{\partial p(t)}\right] = 0, \tag{4.17}$$

which is different from the Equation(4.7), the bargaining rule introduced in Section 4.2. Therefore, the price is,

$$p(t) = \frac{1}{\beta Z + (1-\beta)} \left\{ (1-\beta)(\varepsilon + c^s) + \beta Z \left[(1-\frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta}c^v \right] + [(1-\beta) - \beta Z] \frac{\sigma}{r+s+\delta+\sigma} \right\}, \tag{4.18}$$

where

$$Z = \frac{1 + \sigma \left[\frac{(s+\delta)m(\theta)}{r+m(\theta)} - (r+s+\delta) \right]}{\left[1 + \sigma \frac{1}{(r+\delta+s)} \right]}.$$

Details of the derivation can be found in C.1.1.

Match Destruction

Bellman equation for $L_1(p(\tau), t)$ for $t < t + T$,

$$rL_1(p(\tau), t) = p(\tau) + s[V(t) - L_1(p(\tau), t)] - \delta L_1(p(\tau), t) + \dot{L}_1(p(\tau), t).$$

The landlord chooses the life of the match to maximize its value. Let T be the optimal match duration. Hence, the optimal destruction age for vintage τ is $(\tau + T)$. The maximum value of a match formed at time t should satisfies

$$L_1(p(t), t) = \max_T \left\{ \int_t^{t+T} [p(t) + sc^k(x)] e^{-(r+s+\delta)(x-t)} dx - [V(t+T) - c^e(t+T)] e^{-(r+s+\delta)T} \right\}.$$

Therefore,

$$T = \frac{1}{g} \ln \left[\frac{\left(\frac{1}{\beta Z + (1-\beta)} \left\{ (1-\beta)(\varepsilon + c_0^s) + \beta Z \left[(1-\frac{1}{\theta})[(r+\delta)c_0^k - \frac{1}{\theta}c_0^v \right] + [(1-\beta) - \beta Z] \frac{\sigma}{r+s+\delta+\sigma} \right\} \right)}{(c_0^k - c_0^e)[g - (r+s+\delta)] - sc_0^k} \right], \tag{4.19}$$

where

$$Z = \frac{1 + \sigma \left[\frac{(s+\delta)m(\theta)}{r+m(\theta)} - (r+s+\delta) \right]}{\left[1 + \sigma \frac{1}{(r+\delta+s)} \right]}.$$

We can call this Match Destruction condition (MD). Details of the derivation can be found in C.1.2. From here, we can see that the lifetime of the match and growth rate are inversely related, and the lifetime of the match and market tightness are positively correlated.

Housing Entry Condition

Re-arrange the Bellman equation of vacancy,

$$L_0(t,t) = \theta \frac{(r+\delta)c^k(t) + c^v(t)}{m(\theta)} + V(t). \tag{4.20}$$

In order to get an expression for $L_0(t,t)$, we need to first find an expression for $L_1(p(t),t)$ since $L_1(p(t),t)$ can be expressed by $L_0(t,t)$ by re-arraning the value function of $L_0(t,t)$:

$$L_0(t,t) = \frac{1}{r+s+\delta+\sigma}[p(t) + sV(t) + \sigma L_1(p(t),t)]. \tag{4.21}$$

Re-arranging the Bellman equation for $L_1(p(t),t)$ for $t < \tau + T$:

$$L_1(p(t),t) = \frac{1}{r+s+\delta}[p(t) + sV(t)].$$

Plugging $L_1(p(t),t)$ into the Equation (4.21),

$$L_0(t,t) = \frac{1}{r+s+\delta+\sigma}\left(p(t) + sV(t) + \sigma\frac{1}{r+s+\delta}[p(t) + sV(t)]\right). \tag{4.22}$$

Equate Equation (4.20) and Equation (4.22):

$$\frac{(r+\delta)c^k(t) + c^v(t)}{\theta m(\theta)} + V(t) = \frac{1}{r+s+\delta+\sigma}[p(t) + sV(t) + \sigma\frac{1}{r+s+\delta}[p(t) + sV(t)]].$$

Plug in the free entry condition $V(t) = c^k(t)$:

$$\frac{(r+\delta)c^k(t) + c^v(t)}{\theta m(\theta)} + \frac{(r+\delta)c^k(t)}{r+s+\delta} = \frac{1}{r+s+\delta+\sigma}[p(t) + \frac{\sigma}{r+s+\delta}p(t)].$$

Re-arrange, and the housing entry condition is

$$\frac{[(r+\delta)c^k(t) + c^v(t)]}{\theta m(\theta)} = \frac{p(t)}{r+s+\delta} - \frac{(r+\delta)c^k(t)}{r+s+\delta}. \tag{4.23}$$

Housing Beveridge Curve

The fraction of rent control match surviving to age T is $e^{-(s+\delta)T}$. Number of matches created at time t is

$C(t) = m(\theta)u(t)$, and number of matches destructed at time t is $D(t) = \sigma e^{-(s+\delta)T}C(t-T)+(s+\delta)[1-u(t)]$, which must equal to C(t), therefore, the housing Beveridge curve is

$$u = \frac{s+\delta}{s+\delta+[1-\sigma e^{-(s+\delta)T}]m(\theta)}. \tag{4.24}$$

A decrease in T (shorter lifetime of match) would shift the BC to the right, leading to higher vacancy and more renters without housing. Compare this expression to the Housing Beveridge Curve equation in the case without rent control; we can see that the unhoused rate will be higher in this case since $[1-\sigma e^{-(s+\delta)T}] < 1, \forall T \geq 0$, which is a direct "creative destruction effect" on u.

Note that, similar to Section 4.2, in theory, the non-rent-controlled landlord could choose to evict the tenant and become vacant again. However, non-rent-controlled landlords's surplus,

$$L_0(t) - V(t) = \frac{(r+\delta)c^k(t) + c^v(t)}{\theta m(\theta)} + \frac{\sigma}{r+s+\delta+\sigma},$$

is always positive. Therefore, the non-rent-controlled landlord would never choose to evict the renter.

4.3.5 Equilibrium

An equilibrium with rent control shock is $\{p_0, \theta, u, T\}$ such that equation (4.18), (4.19), (4.23) and (4.24) are satisfied. As illustrated in Figure 4.6, Housing Entry Condition (Equation 4.23) and Price Equation (Equation 4.18) jointly pin down market tightness θ and initial price p_0, since the Price Equation is increasing in θ and the Housing Entry Condition is decreasing in θ in the (θ, p_0) coordinate plane.

Figure 4.7 illustrates the determination of the optimal destruction time T by the Housing Entry Condition (equation 4.23) and Match Destruction Conditions (equation 4.19). The Match Destruction Condition provides a positive relationship between market tightness θ and optimal destruction time T. The Housing Entry Condition is invariant to the optimal destruction time T. Figure 4.8 illustrates the determination of unhoused rate u and vacancy rate v by the Housing Entry Condition (equation 4.23) and Housing Beveridge Curve (equation 4.24). The Housing Beveridge Curve pins down the unhoused rate u and the vacancy rate v using the optimal destruction time and market tightness determined from the above figures.

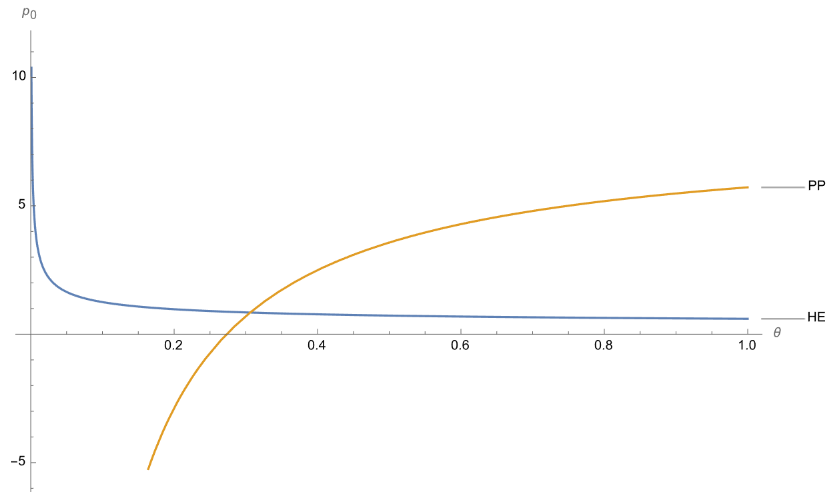


Figure 4.6: HE and PP curves

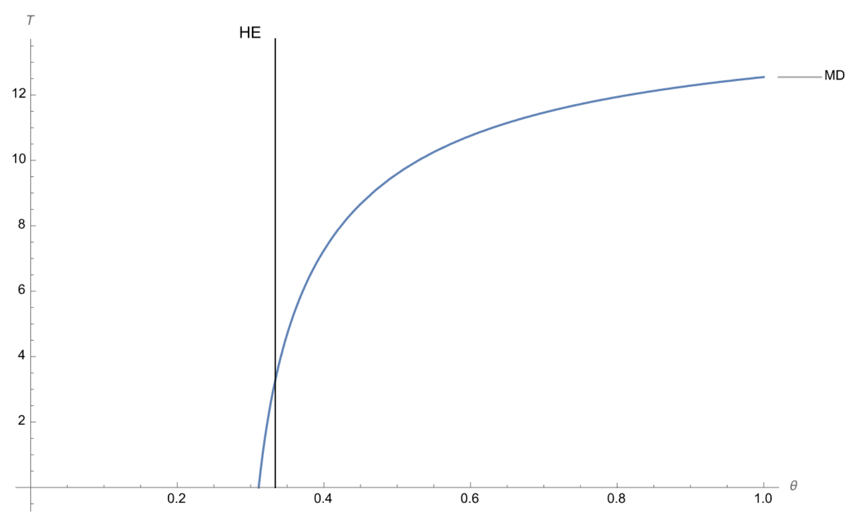


Figure 4.7: HE and MD curves

4.3.6 Comparative Statics

We are interested in how changes in (1) growth rate g, (2) eviction cost c_0^e, (3) construction cost c_0^k, and (4) RC shock frequency σ affect the equilibrium outcomes. Table 4.1 summarizes the comparative statics.

The effect of growth rate increase

Change of growth rate g has no effects on the Housing Entry Condition or the Price Question. Therefore, θ and p_0 do not change. An increase in the growth rate means that the economy is growing at a faster rate; hence, it takes less time for the match to be not ideal for the rent-controlled landlords. Therefore, the rent-controlled landlord would have an incentive to evict the renters sooner. Figure 4.9a shows that an increase in g shifts the Match Destruction Curve downwards; therefore, the optimal destruction time decreases as the growth rate increases. Since T decreases as a consequence of an increase in g, the Housing Beveridge Curve is shifted upwards since a short match duration means a higher unhoused rate and a higher vacancy rate. Therefore, both the unhoused rate u and the vacancy rate v increase as a consequence of the increase of growth rate g, as shown in Figure 4.9b.

The effects of eviction cost increases

Similar to changes in the growth rate g, the change of eviction cost c_0^e has no effects on the Housing Entry Condition or the Price Equation. Therefore, θ and p_0 do not change. An increase in the eviction rate means that it is costlier for the rent-controlled landlord to evict the tenants; hence the landlord would want the renter to stay in the unit for a longer period if the eviction cost rises. Figure 4.10a shows that an increase in eviction cost c_0^e shifts the Match Destruction Curve upwards. Therefore, the optimal destruction time increases as eviction costs increase. Since T increases as a consequence of an increase in c_0^e, the Housing Beveridge Curve is shifted downwards since a longer match duration means a lower unhoused rate and a lower vacancy rate. Therefore, both the unhoused rate u and the vacancy rate v are reduced as a consequence of the increase of eviction cost c_0^e.

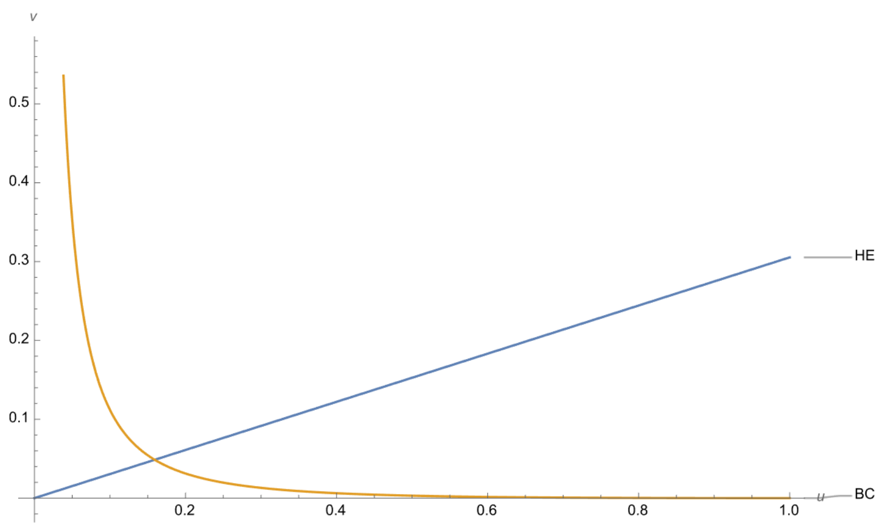
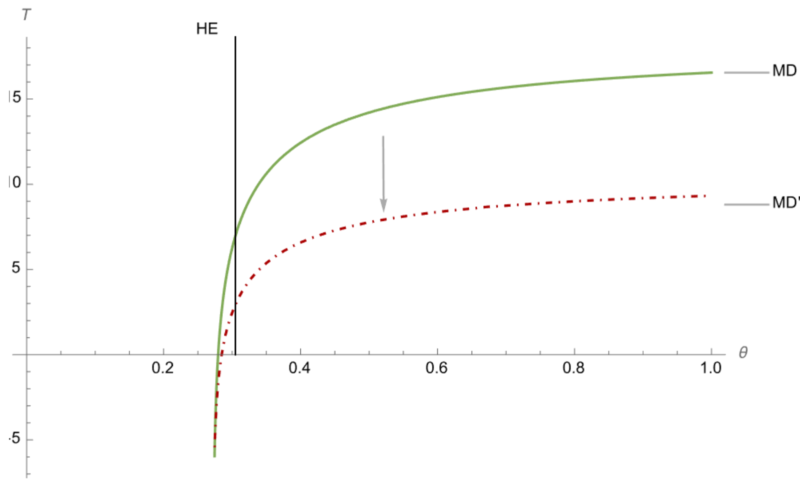


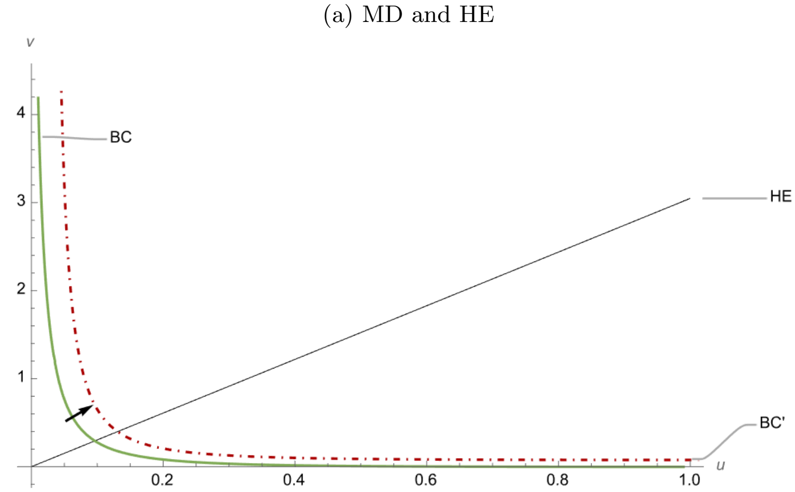
Figure 4.8: HE and BC curves

	T	θ	u	v	p_0
g	↓	-	↑	↑	-
c_0^e	↑	-	↓	↓	-
c_0^k	depends	↑	↑	depends	depends
σ	depends	↓	↓	depends	↑

Table 4.1: Comparative Statics

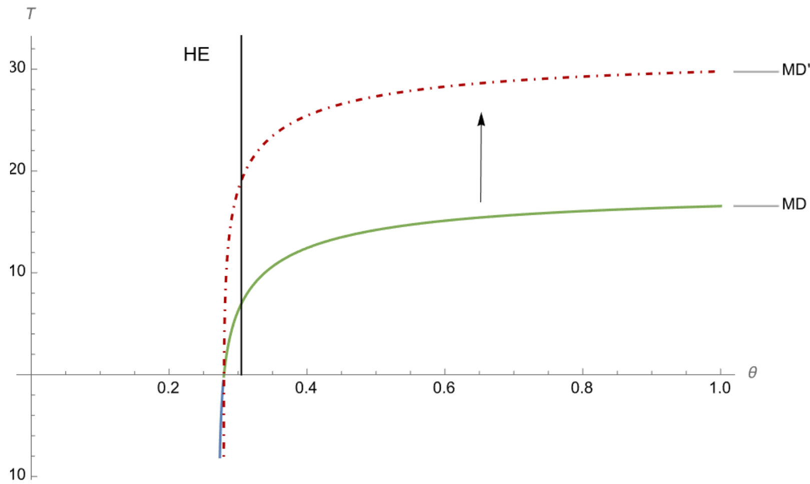


(a) MD and HE

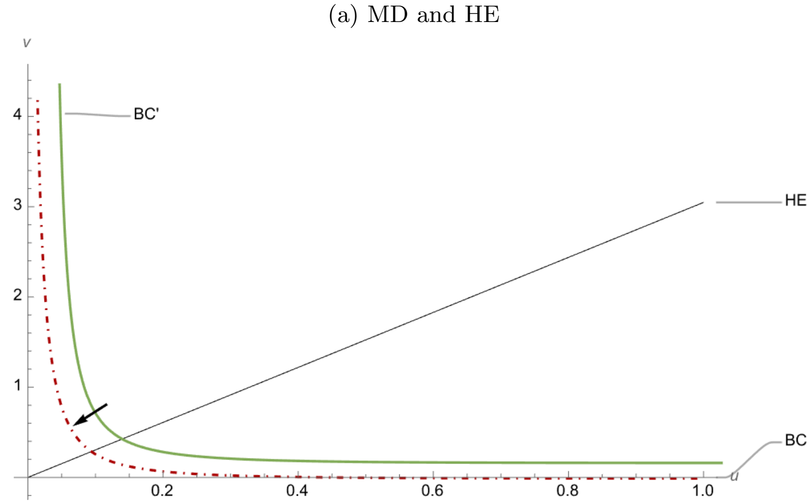


(b) BC and HE

Figure 4.9: The effect of growth rate increase on the equilibrium

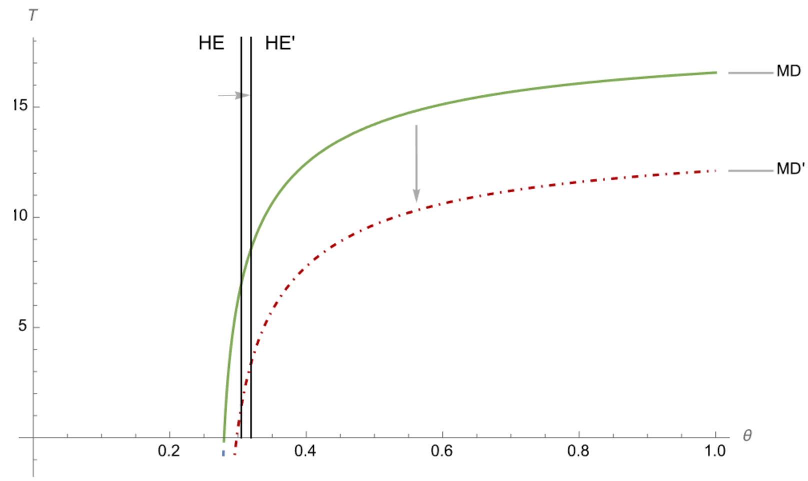


(a) MD and HE

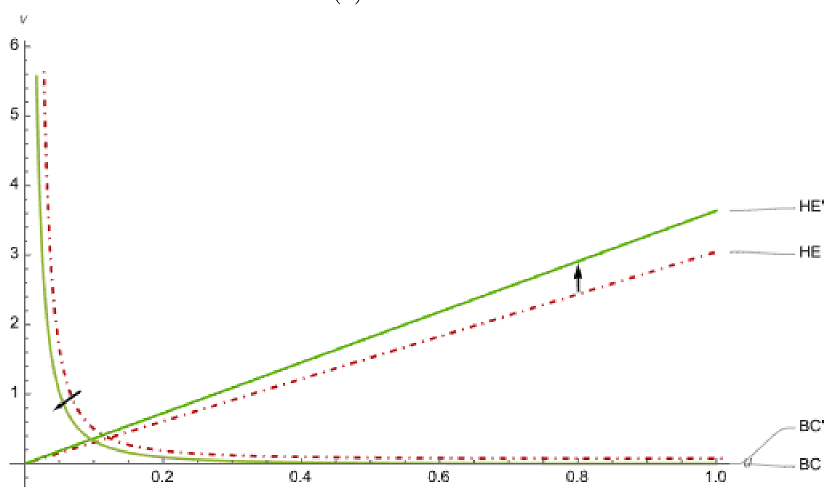


(b) BC and HE

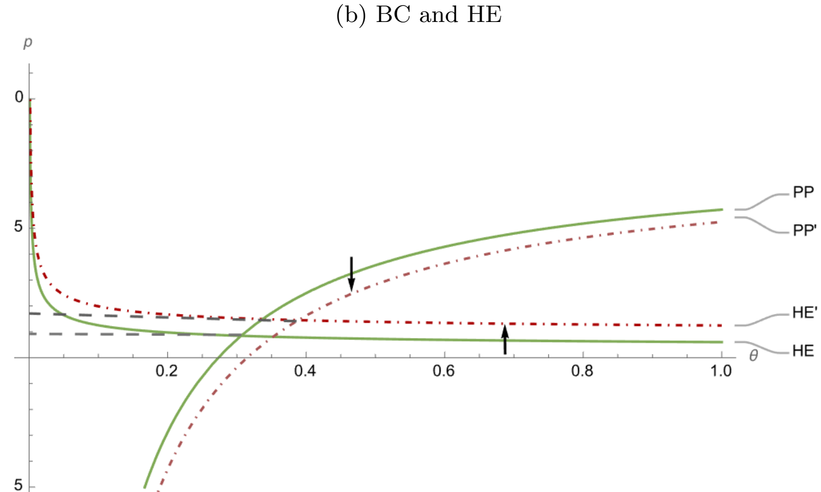
Figure 4.10: The effect of eviction costs increase on the equilibrium



(a) MD and HE



(b) BC and HE



(c) PP and HE

Figure 4.11: The effects of construction costs increase on the equilibrium

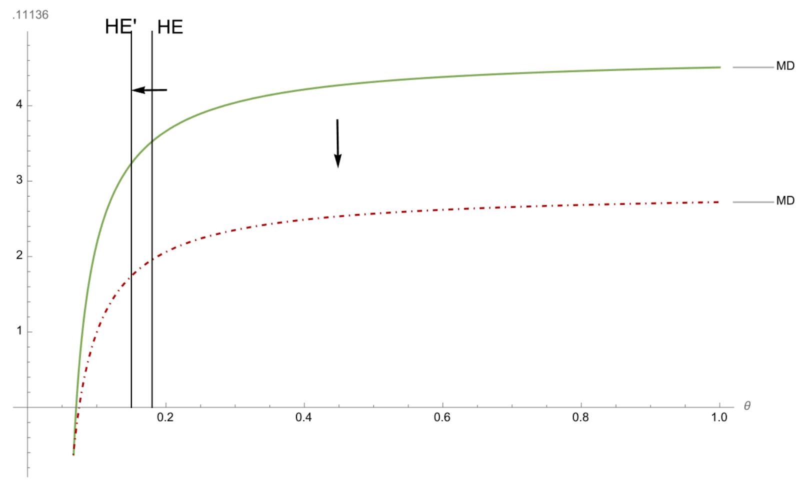
The effects of construction cost increases

An increase in the construction cost c_0^k means that market tightness increases since it becomes costlier to provide new housing. The increase in market tightness shifts the Match Destruction Curve downwards and the Housing Entry Curve to the right. Therefore, the market tightness increases, but the effect on optimal destruction time depends on the parameters. Figure 4.11a illustrates the scenario where the optimal destruction time decreases as the construction cost increases. The increase in market tightness due to an increase in the construction cost also means that for each vacancy, there are more unhoused renters, shifting the Housing Beveridge Curve downwards. Therefore, the unhoused rate u increases, but the effect on vacancy rate v depends on the parameters. Figure 4.11b illustrates the scenario where the vacancy rate increases as the construction cost increases. Lastly, the increase in market tightness due to an increase in the construction cost also means higher prices, shifting the Price curve downwards. Together with an upward shift of the Housing Entry Curve, the market tightness increases, but the effect on price depends on the parameters. Figure 4.11c illustrates the scenario where the price increases as the construction cost increases. As defined in Equation (4.1), the effects of higher construction costs can also be interpreted as the effects of more stringent land use regulations.

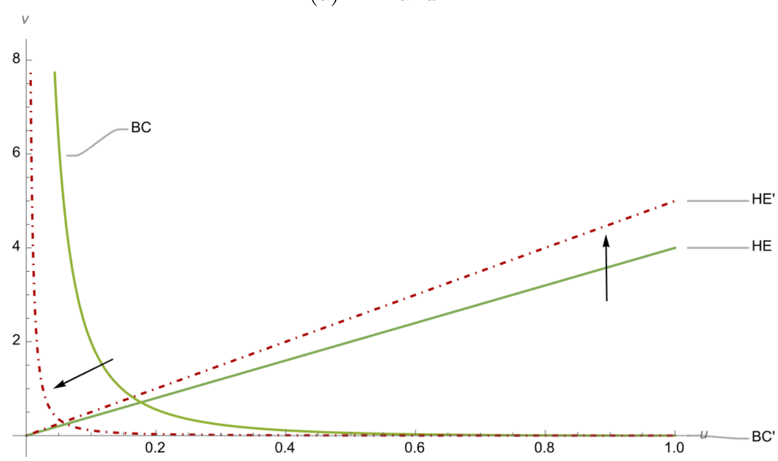
The effect of RC shock frequency increase

An increase in RC shock frequency means that the housing units are more likely to become rent-controlled; therefore, prices need to be set higher to offset the effect. Hence, an increase in RC shock frequency σ shifts the Price curve upwards, and the Housing Entry Condition on the (θ, p_0) coordinate plane does not move. Therefore, the market tightness decreases, and the price p_0 increases, as shown in Figure 4.12c. Since market tightness decreases as a consequence of an increase in σ, the Match Destruction Curve is shifted downwards, and the Housing Entry Curve on the (θ, T) coordinate plane to the left. However, the effects on the optimal destruction time depend on the parameters. Figure 4.12a illustrates the scenario where the optimal destruction time decreases as the RC shock frequency increases. Since market tightness decreases as a consequence of an increase in σ, the Housing Beveridge Curve is shifted downwards, and the Housing Entry is shifted upwards on the (u, v) coordinate plane. Therefore, the unhoused rate u increases, but the effect on vacancy rate v depends on the parameters. Figure 4.12b illustrates the scenario where the vacancy rate decreases as the RC shock frequency increases.

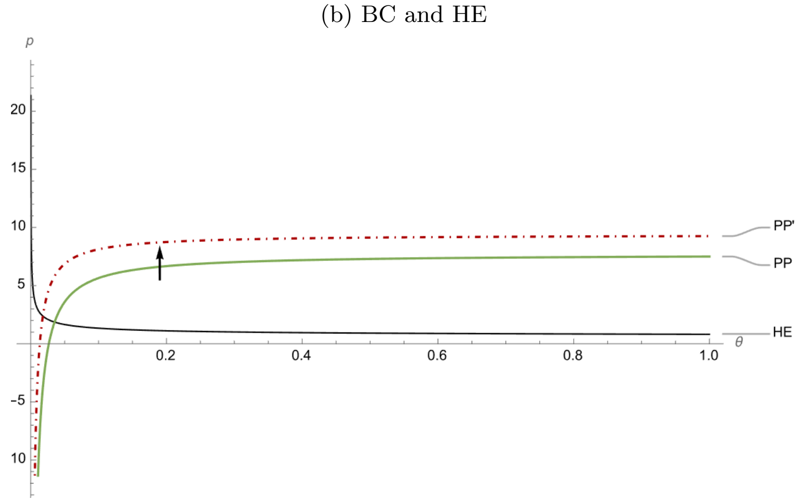
In the extreme case of $\sigma = 0$, the model becomes equivalent to the model without rent control shock, as



(a) MD and HE



(b) BC and HE



(c) PP and HE

Figure 4.12: The effect of RC shock frequency increase on the equilibrium

presented in Section 4.2.

4.4 Conclusion

This project answers the question of how rent control policies shape the rental housing market outcomes for renters and landlords of rent-controlled and non-rent-controlled rental units and their effects on the supply of rental housing. The paper develops a search and matching model of the rental housing market with search friction featuring productivity growth and rent control shocks.

The model shows that an increase in the growth rate would decrease optimal occupancy duration and raise the unhoused and vacancy rates. In contrast, an increase in the eviction cost would have the opposite effect. An increase in the construction cost would raise the market tightness as well as the unhoused rate. However, the effect of changes in construction cost on optimal destruction time, vacancy rate, and price depends on the parameters.

Comparing the model with rent control shock to the one without, I find that rent control raises the initial price, reduces market tightness, and reduces the rate of unhoused renters. However, the effect on vacancy rate depends on the parameters.

Chapter 5

Conclusion

In conclusion, this dissertation examines the impact of friction in the labor and housing markets. It provides novel empirical facts and theoretical frameworks for studying these frictional markets. The first study empirically investigates the role of information friction in housing tenure choices when households relocate, revealing that information friction reduces the likelihood of homeownership. The second study theoretically and quantitatively explores the role of search friction in the labor market, particularly in relation to the Great Divergence phenomenon. The findings indicate that search friction in the labor market mitigates the Great Divergence and incentivizes low-skill workers to move to more productive locations. The third study assesses the effects of rent control policies on the frictional rental housing market within a search and matching framework, demonstrating that increased productivity growth rates reduce optimal occupancy duration and raise vacancy rates. In contrast, higher eviction costs have the opposite effect. These insights collectively highlight the complex interactions between labor and housing market frictions and their broader economic implications.

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Appendix A

A.1 Additional Regression Tables

A.1.1 The effect of household and PUMA characteristics

Table A.1 and Table A.2 show the regression coefficients on different household characteristics and PUMA characteristics from the regression presented in Table 2.4. Each column corresponds to the model specification of the column with the same number in Table 2.4.

A.1.2 Senstivitiy Analysis

Distance Bins

Table A.3 breaks migration distance down into bins to look more closely at the effects of distance. We examine whether the effect of distance varies as distance increases. Here, we use migration distance without taking a log to compare the marginal impact of distance on home purchase rates at different distance levels. We divide movers into four different intervals, as shown in Table A.3. The estimation results confirm the strong negative association between migration distance and home purchase rates across specifications. The results also show a declining magnitude of the coefficient, suggesting the marginal impact of distance is

Table A.1: HH characteristics

HH characteristics	(1) dy/dx Pr(ownership)	(2) dy/dx Pr(ownership)	(3) dy/dx Pr(ownership)	(4) dy/dx Pr(ownership)	(5) dy/dx Pr(ownership)
Age	0.00466***	0.00477***	0.00465***	0.00466***	0.00476***
	(0.000153)	(0.000153)	(0.000153)	(0.000153)	(0.000153)
Male	-0.00400	-0.00251	-0.00424	-0.00402	-0.00278
	(0.00402)	(0.00402)	(0.00402)	(0.00402)	(0.00402)
Race: Black	-0.0817***	-0.0806***	-0.0826***	-0.0818***	-0.0815***
	(0.00810)	(0.00811)	(0.00809)	(0.00810)	(0.00809)
Race: Native American	-0.0550	-0.0565*	-0.0547	-0.0550	-0.0563*
	(0.0344)	(0.0341)	(0.0339)	(0.0344)	(0.0336)
Race: AAPI	-0.0341***	-0.0261***	-0.0340***	-0.0341***	-0.0261***
	(0.00691)	(0.00697)	(0.00690)	(0.00691)	(0.00696)
Race: Others	-0.00987	-0.00791	-0.00988	-0.00988	-0.00789
	(0.00828)	(0.00828)	(0.00829)	(0.00828)	(0.00829)
Education Attainment (Years)	0.00495***	0.00520***	0.00496***	0.00495***	0.00520***
	(0.000842)	(0.000845)	(0.000841)	(0.000842)	(0.000845)
Binary Marital Status	0.0798***	0.0820***	0.0801***	0.0798***	0.0823***
	(0.00413)	(0.00414)	(0.00413)	(0.00413)	(0.00414)
HH income (thousand $)	0.000677***	0.000675***	0.000672***	0.000677***	0.000670***
	(2.54e-05)	(2.54e-05)	(2.54e-05)	(2.54e-05)	(2.54e-05)
Observations	31,175	31,175	31,175	31,175	31,175
Local Ties	Yes	Yes	Yes	Yes	Yes
Destination Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table A.2: PUMA-level characteristics

	(1)	(2)	(3)	(4)	(5)
	dy/dx	dy/dx	dy/dx	dy/dx	dy/dx
PUMA-level charactersitics	Pr(ownership)	Pr(ownership)	Pr(ownership)	Pr(ownership)	Pr(ownership)
Puma-level avg.	0.568***	0.563***	0.544***	0.568***	0.539***
share of owners	(0.0438)	(0.0438)	(0.0440)	(0.0438)	(0.0440)
Puma-level avg.	-0.749	-0.778	-0.875	-0.762	-0.908
share of the employed	(4.261)	(4.267)	(4.214)	(4.261)	(4.221)
Puma-level avg	-0.00130***	-0.00125***	-0.00126***	-0.00129***	-0.00122***
HH income in (thousand $)	(0.000236)	(0.000235)	(0.000235)	(0.000236)	(0.000235)
Puma-level avg	-4.70e-05	-5.25e-05	-4.81e-05	-4.70e-05	-5.38e-05
house value (thousand $)	(3.78e-05)	(3.77e-05)	(3.76e-05)	(3.78e-05)	(3.76e-05)
Puma-level avg	0.0666***	0.0650***	0.0674***	0.0666***	0.0658***
own rent ratio	(0.0101)	(0.0101)	(0.0101)	(0.0101)	(0.0101)
Puma-level avg	0.00221	0.00295	0.00256	0.00222	0.00334
housing value to income ratio	(0.00439)	(0.00438)	(0.00437)	(0.00439)	(0.00437)
Puma-level share	0.0375	0.0360	0.0492	0.0375	0.0478
of single family homes	(0.0674)	(0.0674)	(0.0670)	(0.0674)	(0.0670)
Puma-level avg	0.0202*	0.0207*	0.0207*	0.0202*	0.0211*
unit in structure	(0.0118)	(0.0118)	(0.0117)	(0.0118)	(0.0117)
Puma-level avg	0.00842	0.00843	0.00982	0.00841	0.00985
room in housing	(0.00702)	(0.00702)	(0.00701)	(0.00702)	(0.00701)
Observations	31,175	31,175	31,175	31,175	31,175
Local Ties	Yes	Yes	Yes	Yes	Yes
Household Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

smaller for longer-distance LD movers than for shorter-distance migrants. Other measurements of local ties (except for the indicator for real estate occupation holders) are still salient in these specifications.

Coast-to-coast migrants

Additionally, as noted in section 2.3.4, we analyze households who moved from western (eastern) coastal counties to eastern (western) coastal counties due to strong ties between some large coastal cities (Badger and Bui). In Table A.4 we estimate the same empirical model as in Table 2.4, but using only coast-to-coast migrants. The results are largely consistent with our results from the full LD sample, but the coefficients on migration distance are no longer statistically significant, whereas the coefficients on the birth-state indicator are statistically significant across specifications. This finding seems to suggest that when it comes to migration over a very long distance (i.e., coast-to-coast migration), social ties play a more important role as a source of local ties compared to geographic proximity.

Table A.3: Long Distance Movers With Distance Intervals

	(1)	(2)	(3)	(4)	(5)
Access to Local Information	Ownership	Ownership	Ownership	Ownership	Ownership
Migration Distance (50-200 miles)	-0.00110***	-0.00110***	-9.66e-05	-0.00111***	-7.28e-05
	(0.000261)	(0.000263)	(0.000471)	(0.000261)	(0.000473)
Migration Distance (200-1000 miles)	-0.000739***	-0.000626***	-0.000341***	-0.000738***	-0.000263***
	(5.31e-05)	(5.35e-05)	(9.34e-05)	(5.31e-05)	(9.40e-05)
Migration Distance (1000-2000 miles)	-0.000383***	-0.000323***	-0.000212***	-0.000383***	-0.000165***
	(2.68e-05)	(2.72e-05)	(4.49e-05)	(2.68e-05)	(4.54e-05)
Migration Distance (above 2000 miles)	-0.000267***	-0.000233***	-0.000135***	-0.000267***	-0.000107***
	(2.42e-05)	(2.44e-05)	(3.48e-05)	(2.42e-05)	(3.51e-05)
Birth-State Indicator		0.352***			0.335***
		(0.0223)			(0.0411)
County Flow Share			2.206***		2.206***
			(0.360)		(0.359)
Real Estate Occupation				0.109	-0.0476
				(0.0987)	(0.166)
Observations	31,175	31,175	31,175	31,175	31,175
Household Controls	Yes	Yes	Yes	Yes	Yes
Destination Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table A.4: Coast-to-Coast Movers With Level Variables

	(1)	(2)	(3)	(4)	(5)
	dy/dx	dy/dx	dy/dx	dy/dx	dy/dx
Local Ties	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)	Pr(Ownership)
Log Migration Distance	0.0327	0.0144	0.0338	0.0307	0.0152
	(0.208)	(0.204)	(0.208)	(0.208)	(0.204)
Birth-State Indicator		0.0761***			0.0761***
		(0.0190)			(0.0191)
County Flow Share			-0.330		-0.367
			(0.643)		(0.640)
Real Estate Occupation				-0.0275	-0.0236
				(0.0694)	(0.0666)
Observations	1,323	1,323	1,323	1,323	1,323
Household Controls	Yes	Yes	Yes	Yes	Yes
Destination Controls	Yes	Yes	Yes	Yes	Yes
Survey Year	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Appendix B

B.1 Descriptive facts with alternative definition of skill

B.1.1 Tables

B.2 Derivation

B.2.1 Equilibrium Derivation from section 3.2

Consumption and housing decision

Worker's maximization problem is

$$\max_{c_\phi, h_\phi} \mathcal{U} = \left(\frac{c_\phi}{1-\eta}\right)^{1-\eta} \left(\frac{h_\phi}{\eta}\right)^{\eta},$$

$$s.t. \;\; w_\phi = c_\phi + R_j h_\phi.$$

First order conditions wrt (h_ϕ, c_ϕ) are

$$\frac{\partial H}{\partial c^E_\phi} = 0, \; \frac{\partial H}{\partial c^U_\phi} = 0 \Rightarrow E_\phi(\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta})^{-\eta} - E_\phi = 0; \;\; N_\phi(\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})^{-\eta} - N_\phi = 0; \tag{B.1}$$

$$\frac{\partial H}{\partial h^U_\phi} = 0, \frac{\partial H}{\partial h^U_\phi} = 0 \Rightarrow E_\phi(\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta})^{1-\eta} - \kappa_j E_\phi = 0; \;\; N_\phi(\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})^{1-\eta} - \kappa_j N_\phi = 0; \tag{B.2}$$

Figure B.1: Changes in Share of High-Skill Workers and Unemployment Rates by Skill Types, 2005-2019

(a) (b)

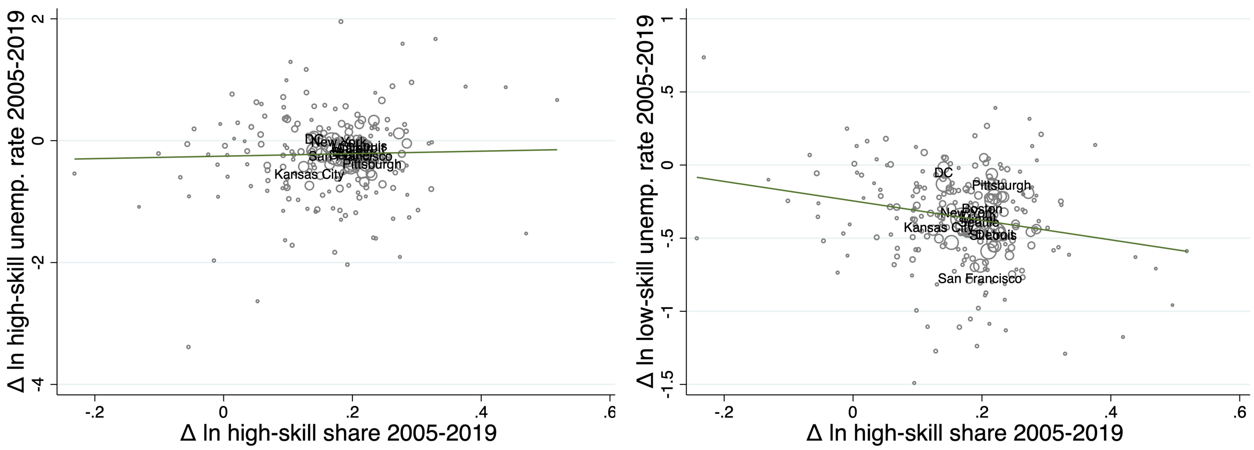

Figure B.2: Changes in Unemployment and Nominal Wages by Skill Types, 2005-2019

(a) (b)

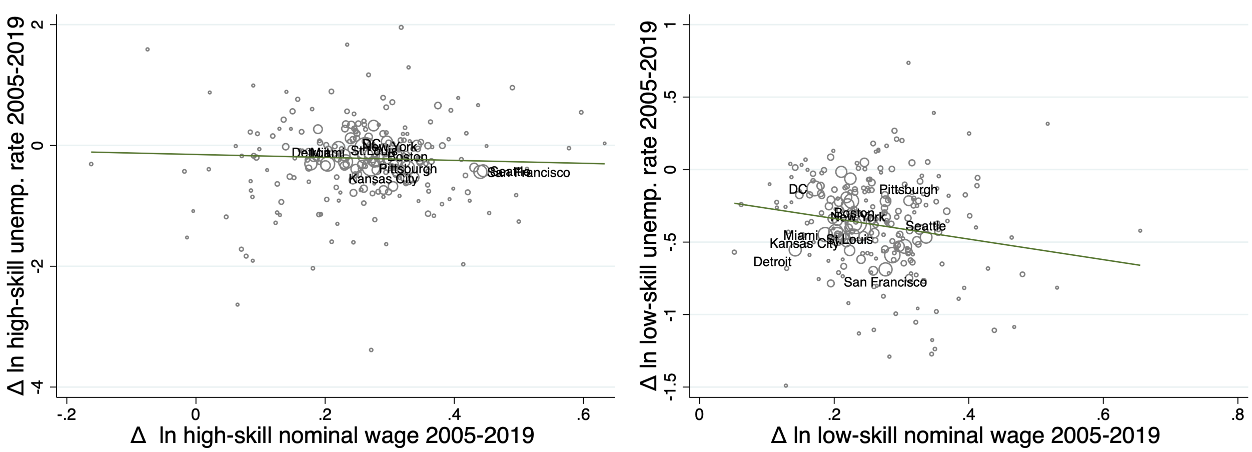

Figure B.3: Changes in Unemployment and Real Wages by Skill Types, 2005-2019

(a) (b)

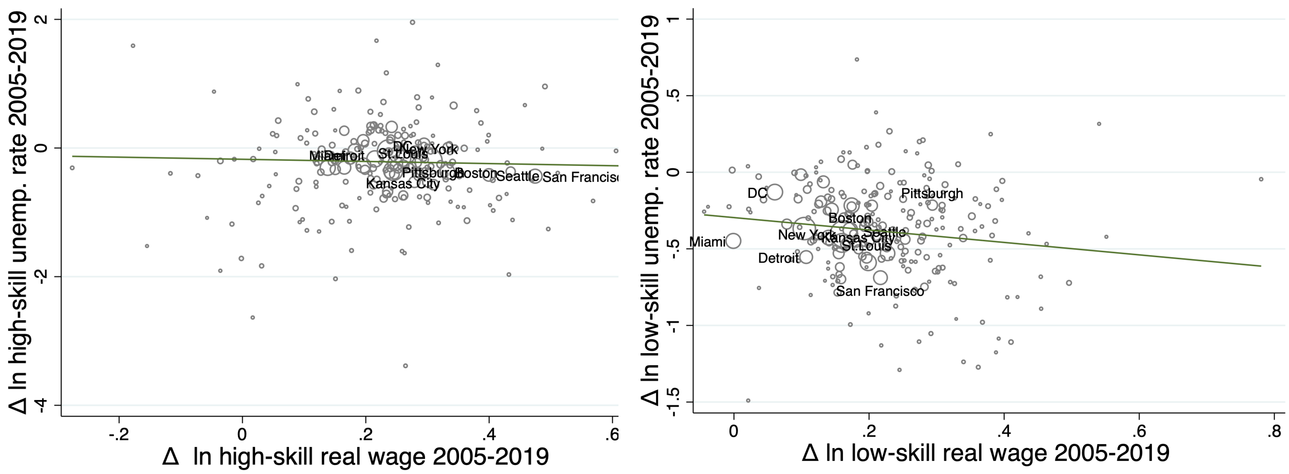

Table B.1: Share of High-Skill Worker and Unemployment Rates

	(1)	(2)	(3)	(4)
Log Unemployment Rate	High-Skill	Low-Skill	High-Skill	Low-Skill
	OLS	IV	OLS	IV
Log Share of High-Skill Worker (Educ)	0.0330	0.124*	-0.152***	-0.0200
	(0.0377)	(0.0646)	(0.0210)	(0.0360)
Observations	2,622	2,563	2,643	2,583
R-squared	0.295	0.295	0.459	0.458
MSA FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

$$\frac{\partial H}{\partial c_\phi^E} = 0 \Rightarrow \frac{1-\eta}{c_\phi^E}\mathcal{U}_\phi^E E_\phi - E_\phi = 0 \qquad \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow \frac{1-\eta}{c_\phi^U}\mathcal{U}_\phi^U N_\phi - N_\phi = 0; \tag{B.3}$$

$$\frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^E}\mathcal{U}_\phi^E E_\phi - \kappa_j E_\phi = 0; \qquad \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^U}\mathcal{U}_\phi^U N_\phi - \kappa_j N_\phi = 0. \tag{B.4}$$

Hence $\kappa_j = (\frac{c_\phi^E}{h_\phi^E}\frac{\eta}{1-\eta}) = (\frac{c_\phi^U}{h_\phi^U}\frac{\eta}{1-\eta})$ The first two F.O.C. leads to the following equation

$$h_\phi^E = \frac{c_\phi^E}{\kappa_j}\frac{\eta}{1-\eta}; \;\; h_\phi^U = \frac{c_\phi^U}{\kappa_j}\frac{\eta}{1-\eta}; \;\; \mathcal{U}_\phi^E = \frac{c_\phi^E}{1-\eta}; \;\; \mathcal{U}_\phi^U = \frac{c_\phi^U}{1-\eta}.$$

B.2.2 Planner's problem from section 3.5

The social planner aims to maximize a social welfare function subject to resource constraints and the law of motion of employment. The social welfare function puts equal welfare weights for the three groups of agents: two types of workers and absentee landlords. Let N_ϕ denote the number of unemployed workers of type ϕ, and let E_ϕ denote the number of employed workers of type ϕ.

The planner's objective function is

$$\omega = \int_0^\infty e^{-rt}\left(\sum_\phi\left[(\frac{c_\phi^E}{1-\eta})^{1-\eta}(\frac{h_\phi^E}{\eta})^\eta \times E_\phi + (\frac{c_\phi^U}{1-\eta})^{1-\eta}(\frac{h_\phi^U}{\eta})^\eta \times N_\phi\right] + \sum_j c_j^O\right) dt,$$

The first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of absentee landlords.

Planner chooses vacancy number V_ϕ and number of unemployed workers N_ϕ, for each ϕ, along with housing and non-housing consumption for the workers and landlord $(c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U, c_j^O)$. The constraints the planner faces are [1] the law of motion for employment for each ϕ, [2] land clearing for each location, [3] the resource constraint of the planner, and [4] high-skill worker size and population constraints,

$$\begin{aligned}
\text{LOM of employed worker} \quad & \dot{E}_j^{\chi} = m(N_\phi, V_\phi) - sE_\phi \\
\text{Local housing constraint} \quad & Q_j = \left[\sum_{\chi} N_j h_j^{\chi,U} + E_j h_j^{\chi,E}\right] \\
\text{Resource Contraint} \quad & \sum_j Z_j + \sum_\phi (N_\phi b^{\chi} - k^{\chi} V_\phi) - \left[\sum_\phi c_\phi^E \times E_\phi + c_\phi^U \times N_\phi\right] - \sum_j c_j^O = 0 \\
\text{Total workers constraint} \quad & \xi = \sum_j E_j^s + N_j^s;\ 1 - \xi = \sum_j E_j^n + N_j^n
\end{aligned}$$

The current-value Hamiltonian for the planner is

$$\begin{aligned}
&\mathcal{H}(E_\phi, N_\phi, V_\phi, c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U, \gamma_\phi, \mu_j, \phi^{\chi}) = \sum_\phi \left[(\frac{c_\phi^E}{1-\eta})^{1-\eta} (\frac{h_\phi^E}{\eta})^{\eta} \times E_\phi + (\frac{c_\phi^U}{1-\eta})^{1-\eta} (\frac{h_\phi^U}{\eta})^{\eta} \times N_\phi - \left(c_\phi^E \times E_\phi + c_\phi^U \times N_\phi\right)\right] \\
&+ \sum_j Z_j + \sum_\phi (N_\phi b^{\chi} - k^{\chi} V_\phi) + \sum_\phi \gamma_\phi \left[m(N_\phi, V_\phi) - sE_\phi\right] + \mu_j \left[Q_j - \left(\sum_\chi N_j h_j^{\chi,U} + E_j h_j^{\chi,E}\right)\right] \\
&+ \psi^s \left[\xi - \left(\sum_j E_j^s + N_j^s\right)\right] + \psi^n \left[1 - \xi - \left(\sum_j E_j^s + N_j^s\right)\right]
\end{aligned}$$

where E_ϕ are the state variables, $(N_\phi, V_\phi, c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U)$ are control variables, and $(\gamma_\phi, \mu_j, \phi^{\chi})$ are the co-state variables.

Optimal consumption and housing

First order conditions wrt (h_ϕ, c_ϕ) are

$$\frac{\partial H}{\partial c_\phi^E} = 0,\ \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow E_\phi (\frac{c_\phi^E}{h_\phi^E} \frac{\eta}{1-\eta})^{-\eta} - E_\phi = 0;\quad N_\phi (\frac{c_\phi^U}{h_\phi^U} \frac{\eta}{1-\eta})^{-\eta} - N_\phi = 0 \tag{B.5}$$

$$\frac{\partial H}{\partial h_\phi^U} = 0, \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow E_\phi (\frac{c_\phi^E}{h_\phi^E} \frac{\eta}{1-\eta})^{1-\eta} - \mu_j E_\phi = 0;\quad N_\phi (\frac{c_\phi^U}{h_\phi^U} \frac{\eta}{1-\eta})^{1-\eta} - \mu_j N_\phi = 0 \tag{B.6}$$

$$\frac{\partial H}{\partial c_\phi^E} = 0 \Rightarrow \frac{1-\eta}{c_\phi^E} \mathcal{U}_\phi^E E_\phi - E_\phi = 0 \qquad \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow \frac{1-\eta}{c_\phi^U} \mathcal{U}_\phi^U N_\phi - N_\phi = 0 \tag{B.7}$$

$$\frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^E} \mathcal{U}_\phi^E E_\phi - \mu_j E_\phi = 0 \qquad \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \frac{\eta}{h_\phi^U} \mathcal{U}_\phi^U N_\phi - \mu_j N_\phi = 0 \tag{B.8}$$

Hence $\mu_j = (\frac{c^E_\phi}{h^E_\phi}\frac{\eta}{1-\eta}) = (\frac{c^U_\phi}{h^U_\phi}\frac{\eta}{1-\eta})$. The first two F.O.C.s lead to the following equation

$$h^E_\phi = c^E_\phi \frac{\eta}{1-\eta}; \quad h^U_\phi = c^U_\phi \frac{\eta}{1-\eta}; \quad \mathcal{U}^E_\phi = \frac{c^E_\phi}{1-\eta}; \quad \mathcal{U}^U_\phi = \frac{c^U_\phi}{1-\eta}$$

therefore $\left(\mathcal{U}^E_\phi - \mu_j h^E_\phi - c^E_\phi\right) = 0$, $\left(\mathcal{U}^U_\phi - \mu_j h^U_\phi - c^U_\phi\right) = 0$

FOC wrt V_ϕ

$$0 = -k^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial V_\phi}$$

Therefore, $\gamma_\phi = \frac{k^\chi}{(1-\alpha)AN^\alpha_\phi V^{-\alpha}_\phi} = \frac{k^\chi}{(1-\alpha)q(\theta_\phi)}$, where ψ^χ is the shadow value of an additional worker of skill level χ in the unemployment pool regardless of location.

Co-state equation for E_ϕ

$$\frac{\partial H}{\partial E_\phi} = r\gamma_\phi - \dot{\gamma}_\phi \Rightarrow r\gamma_\phi - \dot{\gamma}_\phi = -\gamma_\phi s + \frac{\partial Z_j}{\partial E_\phi} - \psi^\chi + \left(\mathcal{U}^E_\phi - \mu_j h^E_\phi - c^E_\phi\right)$$

impose steady state condition $\dot{\gamma_\phi} = 0$, and plug in optimal housing consumption the expression becomes

$$\gamma_\phi\,(r+s) = -\psi^\chi + \frac{\partial Z_j}{\partial E_\phi}$$

Therefore, $\psi^\chi = \frac{\partial Z_j}{\partial E_\phi} - \gamma_\phi\,(r+s)$

FOC wrt N_ϕ

$$0 = b^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi} - \psi^\chi + \left(\mathcal{U}^U_\phi - \mu_j h^U_\phi - c^U_\phi\right)$$

plug in optimal housing consumption. Therefore, $\psi^\chi = b^\chi + \gamma_\phi \frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi}$

Equating the two expressions of ψ^χ and plugging in the expression of $\frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi}$, we have

$$(r + s + \alpha A N^{\alpha-1}_\phi V^{1-\alpha}_\phi)\frac{k^\chi}{(1-\alpha)AN^\alpha_\phi V^{-\alpha}_\phi} = \frac{\partial Z_j}{\partial E_\phi} - b^\chi$$

Let $\theta_\phi = \frac{V_\phi}{N_\phi}$, the expression becomes

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\alpha)\frac{\partial Z_j}{\partial E_\phi} - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r+s}$$

For the same ψ^χ

$$b^\chi + \gamma_j^\chi \frac{\partial m(N_j^\chi, V_j^\chi)}{\partial N_j^\chi} = b^\chi + \gamma_{j'}^\chi \frac{\partial m(N_{j'}^\chi, V_{j'}^\chi)}{\partial N_{j'}^\chi}$$

Plug in the expression for $\frac{\partial m(N_\phi, V_\phi)}{\partial N_\phi} = \alpha f(\theta_\phi)$ and γ_ϕ, the expression becomes

$$b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi = b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_{j'}^\chi$$

which is equivalent to $\theta_j^\chi = \theta_{j'}^\chi$

Social Planner's Solution

Summarizing, (N_ϕ, V_ϕ, E_ϕ) would solve

$$\begin{aligned}
\frac{k^\chi}{q(\theta_\phi)} &= \frac{(1-\alpha)p_\phi y^\chi - [(1-\alpha)b^\chi + \alpha\theta_\phi k^\chi]}{r+s} \\
b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi &= b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_{j'}^\chi \\
u_\phi &= \frac{s}{s + f(\theta_\phi)} \\
\xi = \sum_j E_j^s + N_j^s;\ 1 - \xi &= \sum_j E_j^n + N_j^n
\end{aligned}$$

where $\theta_\phi = V_\phi / N_\phi$.

B.3 Quantitative model in 3.6.2

B.3.1 Equilbrium

Bellman Equations

Let U_ϕ, W_ϕ, V_ϕ, J_ϕ denote the value function of the unemployed, the employed, a vacant job and a filled job for each location and skill level.

$$rW_\phi(y^\chi) = w_\phi(y^\chi)R_j^{-\eta} + \lambda \int_{\underline{y}_\phi}^{\bar{y}_\phi} \max\{U_\phi - W_\phi(y^\chi), W_\phi(x^\chi) - W_\phi(y^\chi)\} dF_\chi(x^\chi) \tag{B.9}$$

$$rU_\phi = \max_j \{b^\chi R_j^{-\eta} + f(\theta_\phi) \int_{\underline{y}_\phi}^{\bar{y_\phi}} \max\{W_\phi(y^\chi) - U_\phi, 0\} dF_\chi(x^\chi)\} \tag{B.10}$$

$$rV_\phi = \max_j \{-k^\chi + q(\theta_\phi) \int_{\underline{y}_\phi}^{\bar{y_\phi}} \max\{J_\phi(x^\chi) - V_\phi, 0\} dF_\chi(x^\chi)\} \tag{B.11}$$

$$rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{\underline{y}_\phi}^{\bar{y^\chi}} \max\{V_\phi - J_\phi(y^\chi), J_\phi(x^\chi) - J_\phi(y^\chi)\} dF_\chi(x^\chi) \tag{B.12}$$

where $F_\chi(y^\chi)$ is skill distribution for skill level χ.

The first Bellman equation is an employed worker's flow value. Since the worker's utility function is Cobb-Douglas, she spends η share of her income on housing. Hence, the flow value of income is her wage adjusted by rent. The probability of matching with a firm is $f(\theta_\phi)$ for an unemployed worker. Upon meeting the firm, she draws type-specific productivity y^χ from distribution $F_\chi(.)$. At rate λ, the worker redraws productivity $x^\chi \sim F_\chi(.)$. If $x^\chi < y_\phi^*$, the match is destroyed. The worker becomes unemployed, and the firm becomes vacant. If $x^\chi \geq y_\phi^*$, the match is not destroyed and the productivity becomes x^χ. The second Bellman equation is an unemployed worker's flow value. Since an unemployed worker can move between locations, the worker will choose a location that maximizes her utility. Like an employed worker, the unemployment benefit is adjusted by local rent R_j.

The third Bellman equation is a vacant firm's flow value. Vacant firms are also free to choose where to locate, so they will choose location j to maximize their profit. Once they settle in a location, they must pay a vacancy cost k^χ. A vacant firm meets an unemployed worker at rate $q(\theta_\phi)$. The last Bellman equation is the flow value of a filled firm. The firm's profit is the value of the output less the wage paid to the worker.

Similar to the Bellman equation of the employed worker, at rate λ, match productivity receives a shock $x^{\chi} \sim F_{\chi}(.)$. If $x^{\chi} < y^*_{\phi}$, the match is destroyed. The worker becomes unemployed, and the firm becomes vacant. If $x^{\chi} \geq y^*_{\phi}$, the match is not destroyed and the productivity becomes x^{χ}.

Reservation productivity y^*_{ϕ} is chosen such that if $y^{\chi} < y^*_{\phi}$, then the job is destroyed and if $y^{\chi} \geq y^*_{\phi}$,then the match is formed keep. The Bellman equations become

$$rW_{\phi}(y^{\chi}) = w_{\phi}(y^{\chi})R_j^{-\eta^{\chi}} + \lambda \int_{y^*_{\phi}}^{\bar{y^{\chi}}} [W_{\phi}(x^{\chi}) - W_{\phi}(y^{\chi})]dF(x^{\chi}) - \lambda F(y^*_{\phi})[W_{\phi}(y^{\chi}) - U_{\phi}], \tag{B.13}$$

$$rU_{\phi} = \max_j\{b^{\chi}R_j^{-\eta^{\chi}} + f(\theta_{\phi}) \int_{y^*_{\phi}}^{\bar{y^{\chi}}} [W_{\phi}(y^{\chi}) - U_{\phi}]dF(x^{\chi})\}, \tag{B.14}$$

$$rV_{\phi} = -k^{\chi} + q(\theta_{\phi}) \int_{y^*_{\phi}}^{\bar{y^{\chi}}} [J_{\phi}(x^{\chi}) - V_{\phi}]dF(x^{\chi}), \tag{B.15}$$

$$rJ_{\phi}(y^{\chi}) = p_{\phi}y^{\chi} - w_{\phi}(y^{\chi}) + \lambda \int_{y^*_{\phi}}^{\bar{y^{\chi}}} [J_{\phi}(x^{\chi}) - J_{\phi}(y^{\chi})]dF(x^{\chi}) - \lambda F(y^*_{\phi})J_{\phi}(y^{\chi}). \tag{B.16}$$

Use $J(y^*_{\phi}) = 0$ and $W(y^*_{\phi}) = U_{\phi}$ to get rid of integral, yields

$$(r+\lambda)J_{\phi}(y^{\chi}) = y^{\chi}p_{\phi} - w_{\phi}(y^{\chi}) + \lambda \int_{y^*_{\phi}}^{\bar{y^{\chi}}} J_{\phi}(x^{\chi})dF(x^{\chi}),$$

$$(r+\lambda)W_{\phi}(y^{\chi}) = [w_{\phi}(y^{\chi})]R_j^{-\eta^{\chi}} + \lambda \int_{y^*_{\phi}}^{\bar{y^{\chi}}} W_{\phi}(x^{\chi})dF(x^{\chi}) + \lambda F(y^*_{\phi})U_{\phi}.$$

Evaluate at $y^{\chi} = y^*_{\phi}$,

$$0 = (r+\lambda)J_{\phi}(y^*_{\phi}) = p_{\phi}y^*_{\phi} - w_{\phi}(y^*_{\phi}) + \lambda \int_{y^*_{\phi}}^{\bar{y^{\chi}}} J_{\phi}(x^{\chi})dF(x^{\chi}) \tag{B.17}$$

$$\Rightarrow (r+\lambda)J_{\phi}(y^{\chi}) = [p_{\phi}y^{\chi} - w_{\phi}(y^{\chi}_{\phi})] - [p_{\phi}y^*_{\phi} - w_{\phi}(y^*_{\phi})] \tag{B.18}$$

$$\Rightarrow (r+\lambda)J_{\phi}(y^{\chi}) = w_{\phi}(y^*_{\phi}) - w_{\phi}(y^{\chi}_{\phi}) + p_{\phi}[y^{\chi} - y^*_{\phi}]. \tag{B.19}$$

Wages

Following Bilal [2023], define adjusted surplus S_{ϕ} for match productivity to be

$$S_{\phi}(y^{\chi}) = J_{\phi}(y^{\chi}) + R_j^{\eta^{\chi}}[W_{\phi} - U_{\phi}].$$

Nash Bargaining

$$
\begin{aligned}
\beta J_\phi(y^\chi) &= (1-\beta) R_j^{\eta^\chi}[W_\phi(y^\chi) - U_\phi], \quad \forall y^\chi \geq y_\phi^* \\
\Rightarrow \beta \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi) dF_\phi(x^\chi) &= (1-\beta) \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(x^\chi) - U_\phi] dF_\phi(x^\chi).
\end{aligned}
$$

With free entry condition, $V_\phi = 0$ and equation (B.15),

$$
\frac{k^\chi}{q(\theta_\phi)} = \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi) dF(x^\chi).
$$

Plug the expression of J_ϕ into Nash bargaining rule to get expression of $\int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi] dF(x^\chi)$,

$$
\int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi] dF(x^\chi) = \frac{\beta}{1-\beta} \frac{k^\chi}{q(\theta_\phi)} R_j^{-\eta^\chi}.
$$

Plug this expression into the Bellman equation for the unemployed U_ϕ,

$$
\begin{aligned}
rU_\phi &= \max_j \{ b^\chi R_j^{-\eta^\chi} + f(\theta_\phi) \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi] dF(x^\chi) \}, \\
\Rightarrow rU_\phi &= \max_j (b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_\phi) R_j^{-\eta}.
\end{aligned}
$$

With spatial equilibrium $U_j^\chi = U_{j'}^\chi = \bar{U}^\chi, \ \forall j$, the Bellman equation for U_ϕ becomes

$$
r\bar{U}^\chi = (b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_\phi) R_j^{-\eta}. \tag{B.20}
$$

Subtract Bellman equations and re-arrange

$$
\begin{aligned}
r[W_\phi(y^\chi) - \bar{U}^\chi] = R_j^{-\eta^\chi} [w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta} \theta_\phi k^\chi] + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(x^\chi) - U_\phi] dF(x^\chi) \\
- \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi] dF(y^\chi) - \lambda F(y_\phi^*) [W_\phi(y^\chi) - U_\phi], \\
rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi) dF(x^\chi) \\
- \lambda \int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(y^\chi) dF(x^\chi) - \lambda F(y_\phi^*) J_\phi(y^\chi).
\end{aligned}
$$

Use Nash Bargaining $(1-\beta)R_j^{\eta^\chi}(W_\phi - U_\phi) = \beta(J_\phi - V)$,

$$\beta[p_\phi y^\chi - w_\phi(y^\chi)] = R_j^{\eta^\chi}(1-\beta)\{R_j^{-\eta^\chi}[w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta}\theta_\phi k^\chi]\}$$
$$\Rightarrow \beta[p_\phi y^\chi - w_\phi(y^\chi)] = (1-\beta)[w_\phi(y^\chi) - b^\chi - \frac{\beta}{1-\beta}\theta_\phi k^\chi]$$
$$\Rightarrow w_\phi(y^\chi) = \beta p_\phi y^\chi + (1-\beta)b^\chi + \beta\theta_\phi k^\chi.$$

Job Creation Condition

Evaluate $w_\phi(y^\chi)$ at $y^\chi = y_\phi^*$ and subtract it from $w_\phi(y^\chi)$ yields,

$$w_\phi(y^\chi) - w_\phi(y_\phi^*) = \beta p_\phi(y^\chi - y_\phi^*).$$

Plug the expression into equation(B.19)

$$(r+\lambda)J_\phi(y^\chi) = w_\phi(y_\phi^*) - w_\phi(y_\phi^\chi) + p_\phi[y^\chi - y_\phi^*]$$

$$\Rightarrow (r+\lambda)J_\phi(y^\chi) = (y^\chi - y_\phi^*)p_\phi(1-\beta)$$

Re-arrange equation (B.15),

$$rV_\phi = -k^\chi + q(\theta_\phi)\int_{y_\phi^*}^{\bar{y^\chi}}[J_\phi(x^\chi) - V_\phi]dF(x^\chi)$$
$$\Rightarrow k^\chi = q(\theta_\phi)(1-F(y_\chi^*))\int_{y_\phi^*}^{\bar{y^\chi}}[J_\phi(x^\chi) - V_\phi]\frac{dF(x^\chi)}{1-F(y_\chi^*)}$$
$$\Rightarrow k^\chi = q(\theta_\phi)(1-F(y_\phi^*))[J_\phi^e - V_\phi].$$

where $J_\phi^e = E[J_\phi(y^\chi)|y^\chi \geq y_\phi^*]$. Therefore, job creation condition is

$$\frac{k^\chi}{q(\theta_\phi)[1-F(y_\phi^*)]} = \frac{p_\phi(1-\beta)(y_\phi^e - y_\phi^*)}{r+\lambda}, \tag{B.21}$$

where $y_\phi^e = E[y_\phi|y_\phi \geq y_\phi^*]$.

Job Destruction Condition

Plug w into the Bellman equation of J_ϕ

$$(r+\lambda)J_\phi(y^\chi) = p_\phi y^\chi - (\beta p_\phi y^\chi + [(1-\beta)b + \beta\theta_\phi k^\chi]) + \lambda\int_{y_\phi^*}^{\bar{y^\chi}} J_\phi(x^\chi)dF(x^\chi). \tag{B.22}$$

Evaluate at $y^{\chi} = y^*_{\phi}$ and subtracting the resulting equation from equation (B.22),

$$(r+\lambda)J_{\phi}(y^{\chi}) = (1-\beta)p_{\phi}(y^{\chi} - y^*_{\phi}).$$

Plug this expression into J of equation (B.22),

$$(r+\lambda)J_{\phi}(y^{\chi}) = (1-\beta)p_{\phi}y^{\chi} - [(1-\beta)b^{\chi} + \beta\theta_{\phi}k^{\chi}] + (1-\beta)\frac{p_{\phi}\lambda}{r+\lambda}\int_{y^*_{\phi}}^{\bar{y^{\chi}}}(y^{\chi} - y^*_{\phi})dF(x^{\chi}).$$

Evaluate this equation at $y^{\chi} = y^*_{\phi}$, and use $J_{\phi}(y^*_{\phi}) = 0$ to get the Job Destruction Condition,

$$p_{\phi}y^*_{\phi} - [b^{\chi} + \frac{\beta}{1-\beta}\theta_{\phi}k^{\chi}] + \frac{p_{\phi}\lambda}{r+\lambda}\int_{y^*_{\phi}}^{\bar{y^{\chi}}}(y^{\chi} - y^*_{\phi})dF(x^{\chi}) = 0. \tag{B.23}$$

Equilibrium Conditions

Job Creation condition (B.21) and Job Destruction Condition (B.23) determine equilibrium $(\theta^*_{\phi}, y^{**}_{\phi})$ for each ϕ. JC: As $\theta_{\phi} \uparrow \Rightarrow q(\theta_{\phi}) \downarrow \Rightarrow y^*_{\phi} \downarrow$. JD: As $\theta_{\phi} \uparrow \Rightarrow y^*_{\phi} \uparrow$

Beverage Curve

$$u_{\phi} = \frac{\lambda F(y^*_{\phi})}{\lambda F(y^*_{\phi}) + f(\theta_{\phi})}. \tag{B.24}$$

The shape of the distribution $F(y^*_{\phi})$ affects the unemployment rate and hence the job finding rate. For the same reservation productivity y^*_{ϕ}, the fatter the tail of $F(y^*_{\phi})$, the smaller the value of $F(y^*_{\phi})$.

Spatial Equilibrium Condition

$$(\frac{R_j}{R_{j'}})^{-\eta} = \frac{(b^{\chi} + \frac{\beta}{1-\beta}k^{\chi}\theta^{\chi}_{j'})}{(b^{\chi} + \frac{\beta}{1-\beta}k^{\chi}\theta^{\chi}_{j})}, \tag{B.25}$$

therefore, the difference in housing price between the two locations is explained by the difference $\theta_{\phi}\mu_j$.

Market clearing condition for housing

$$R_j = \frac{\eta\{L_j^n[\bar{w_j^n}(1-u_j^n)+b^n u_j^n] + L_j^s[\bar{w_j^s}(1-u_j^s)+b^s u_j^s]\}}{Q_j}.$$

Market clearing condition for workers

$$\sum_j L_j = 1; \quad \xi = \sum_j \zeta_j L_j.$$

Equilibrium Equations

$$0 = p_\phi y_\phi^* - [b^\chi + \frac{\beta}{1-\beta}\theta_\phi k^\chi] + \frac{p_\phi \lambda}{r+\lambda}\int_{y_\phi^*}^{\bar{y^\chi}} [y^\chi - y_\phi^*] dF(x^\chi) \tag{B.26}$$

$$\frac{k^\chi}{q(\theta_\phi)[1-F(y_\chi^*)]} = \frac{p_\phi(1-\beta)[y_\phi^e - y_\phi^*]}{r+\lambda} \tag{B.27}$$

$$u_\phi = \frac{\lambda F(y_\phi^*)}{\lambda F(y_\phi^*) + f(\theta_\phi)} \tag{B.28}$$

$$R_j = \frac{\eta\{L_j^n[\bar{w_j^n}(1-u_j^n)+b^n u_j^n] + L_j^s[\bar{w_j^s}(1-u_j^s)+b^s u_j^s]\}}{Q_j} \tag{B.29}$$

$$(\frac{R_j}{R_{j'}})^{-\eta} = \frac{(b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_{j'}^\chi)}{(b^\chi + \frac{\beta}{1-\beta}k^\chi\theta_j^\chi)} \tag{B.30}$$

$$w_\phi(y^\chi) = \beta p_\phi y^\chi + (1-\beta)b^\chi + \beta\theta_\phi k^\chi \tag{B.31}$$

$$\sum_j L_j = 1; \quad \xi = \sum_j L_j \zeta_j \tag{B.32}$$

Defitinition of equilibrium

A steady-state equilibrium is $\{w_\phi, y_\phi^*, u_\phi, \theta_\phi, p_\phi, \zeta_j, L_j, R_j\}$ for $\phi \in J \times \{s, n\}$ and $j \in J$ such that: equations (3.2)- (3.1), (3.5)-(3.6),(3.11),(B.21),(B.23),(B.24),(B.25) are satisfied

B.3.2 Planner's Problem

The Social Planner's problem is very similar to the baseline version presented in Section 3.5. The derivation for the social planner's solution is summarized here. The social planner aims to maximize a social welfare function subject to resource constraints and the law of motion of unemployment. The social welfare function assigns equal welfare weights for the three groups of agents: two types of workers and absentee landlords. Let N_ϕ denote the number of unemployed workers of type ϕ, and let E_ϕ denote the number of employed workers of type ϕ.

The planner's objective function is

$$\omega = \int_0^\infty e^{-rt} \left(\sum_\phi \left[\left(\frac{c_\phi^E}{1-\eta} \right)^{1-\eta} \left(\frac{h_\phi^E}{\eta} \right)^\eta \times E_\phi + \left(\frac{c_\phi^U}{1-\eta} \right)^{1-\eta} \left(\frac{h_\phi^U}{\eta} \right)^\eta \times N_\phi \right] + \sum_j c_j^O \right) dt,$$

where the first component is the aggregate utility of the employed workers, the second component is the aggregate utility of the unemployed workers, and the last component is the consumption of out-of-town landlords.

The planner picks market tightness (θ_ϕ) reservation productivity y_ϕ^* and labor force size (L_ϕ) for each ϕ, as well as housing and non-housing consumption for workers and landlord ($c_\phi^E, c_\phi^U, h_\phi^E, h_\phi^U, c_j^O$). The constraints the planner faces are (1) the law of motion for unemployment (for each ϕ), (2) land clearing for each location (for each j), (3) resource constraint of the planner, (4) high-skilled worker size and population constraints. The current-value Hamiltonian for the planner is

$$\begin{aligned} H = & \sum_j [(\frac{c_j^{sE}}{1-\eta})^{1-\eta} (\frac{h_j^{sE}}{\eta})^\eta \zeta_j (1-u_j^s) + (\frac{c_j^{sU}}{1-\eta})^{1-\eta} (\frac{h_j^{sU}}{\eta})^\eta \zeta_j (1-u_j^s) + (\frac{c_j^{nE}}{1-\eta})^{1-\eta} (\frac{h_j^{nE}}{\eta})^\eta (1-\zeta_j)(1-u_j^n) \\ & + (\frac{c_j^{nU}}{1-\eta})^{1-\eta} (\frac{h_j^{nU}}{\eta})^\eta u_j^n (1-\zeta_j)] L_j + \sum_\phi \gamma_\phi \left[A\theta_\phi^{1-\alpha} u_\phi - \lambda F(y_\phi^*)(1-u_\phi) \right] \\ & + \sum_j \{ [y_j^{es}(1-u_j^s)\zeta_j]^{\sigma_j} [y_j^{en}(1-u_j^n)(1-\zeta_j)]^{1-\sigma_j} - c_j^{sE}\zeta_j(1-u_j^s) - c_j^{nE}(1-\zeta_j)(1-u_j^n) \\ & \quad + (b^s - k^s\theta_j^s - c_j^{sU})\zeta_j u_j^s + (b^n - k^n\theta_j^n - c_j^{nU})(1-\zeta_j)u_j^n \} L_j + \psi^s \left[\xi - \sum_j L_j^s \right] + \psi^n \left[1 - \xi - \sum_j L_j^n \right] \\ & + \sum_j \kappa_j \left(Q_j - L_j \left[h_j^{sE}\zeta_j(1-u_j^s) + h_j^{sU}\zeta_j u_j^s + h_j^{nE}(1-\zeta_j)(1-u_j^n) + h_j^{nU}(1-\zeta_j)u_j^n \right] \right) \end{aligned}$$

Optimal consumption and housing

First order conditions wrt (h_ϕ, c_ϕ)

$$
\begin{aligned}
&\frac{\partial H}{\partial c_\phi^E} = 0,\ \frac{\partial H}{\partial c_\phi^U} = 0 \Rightarrow 1 = \frac{1-\eta}{c_\phi^E}\mathcal{U}_\phi^E = \frac{1-\eta}{c_\phi^U}\mathcal{U}_\phi^U \\
&\frac{\partial H}{\partial h_\phi^U} = 0, \frac{\partial H}{\partial h_\phi^U} = 0 \Rightarrow \kappa_j = \frac{\eta}{h_\phi^E}\mathcal{U}_\phi^E = \frac{\eta}{h_\phi^U}\mathcal{U}_\phi^U
\end{aligned}
$$

The first two FOCs lead to the following equation

$$
h_\phi^E = \frac{c_\phi^E}{\kappa_j}\frac{\eta}{1-\eta};\quad h_\phi^U = \frac{c_\phi^U}{\kappa_j}\frac{\eta}{1-\eta};\quad \mathcal{U}_\phi^E = \frac{c_\phi^E}{1-\eta};\quad \mathcal{U}_\phi^U = \frac{c_\phi^U}{1-\eta}
$$

Planner's FOC wrt (θ_ϕ)

$$
\begin{aligned}
&\frac{\partial H}{\partial \theta_j^s} = 0 \Rightarrow -u_j^s \zeta_j L_\phi \mu_j k^\chi + \gamma_\phi (1-\alpha) A(\theta_j^s)^{-\alpha} u_\phi = 0 \Rightarrow \gamma_j^s = \frac{k^s \zeta_j L_j}{(1-\alpha)A\theta_\phi^{-\alpha}} \\
&\frac{\partial H}{\partial \theta_j^n} = 0 \Rightarrow -u_j^s (1-\zeta_j) L_j \mu_j k^\chi + \gamma_\phi (1-\alpha) A(\theta_j^n)^{-\alpha} u_\phi = 0 \Rightarrow \gamma_j^n = \frac{k^n (1-\zeta_j) L_j}{(1-\alpha)A\theta_\phi^{-\alpha}}
\end{aligned}
$$

Planner's FOC wrt (y_ϕ^*)

$$
\frac{\partial H}{\partial y_\phi^*} = 0 \Rightarrow p_\phi (1-u_\phi) L_\phi \frac{\partial y^e}{\partial y^*} - \gamma_\phi \lambda (1-u_\phi) \frac{\partial F(y_\phi^*)}{\partial y_\phi^*} = 0
$$

Note that,

$$
\begin{aligned}
\frac{\partial y^e}{\partial y_\phi^*} &= \frac{\partial}{\partial y_\phi^*}\left([1-F(y_\phi^*)]^{-1}\int_{y_\phi^*} y_\phi dF(y_\phi)\right) \\
&= f(y_\phi^*)[1-F(y_\phi^*)]^{-2}\int_{y_\phi^*} y_\phi dF(y_\phi) + [1-F(y_\phi^*)]^{-1}(-y_\phi^* f(y_\phi^*)) \\
&= f(y_\phi^*)[1-F(y_\phi^*)]^{-2}[1-F(y_\phi^*)]y_\phi^e - [1-F(y_\phi^*)]^{-1} y_\phi^* f(y_\phi^*) \\
&= f(y_\phi^*)[1-F(y_\phi^*)]^{-1}(y_\phi^e - y_\phi^*)
\end{aligned}
$$

$$\Rightarrow p_\phi(1-u_\phi)L_\phi f(y^*_\phi)[1-F(y^*_\phi)]^{-1}(y^e_\phi - y^*_\phi) - \gamma_\phi\lambda(1-u_\phi)f(y^*_\phi) = 0$$

Plug in γ_ϕ

$$\frac{k^\chi}{[1-F(y^*)]q(\theta_\phi)} = \frac{(1-\alpha)(y^e_\phi - y^*_\phi)}{r+\lambda}$$

Planner's FOC wrt (L_ϕ)

$$\frac{\partial H}{\partial L_\phi} = 0 \Rightarrow \psi^\chi = \frac{\partial Z_j}{\partial L^\chi_j} + (b^\chi_j - k^\chi\theta^\chi_j)u^\chi_j.$$

Plug in the expression for p_ϕ

$$p^s_j = \sigma_j\left(Y^s_j\right)^{\rho-1} Z_j^{1-\rho}; \quad p^n_j = (1-\sigma_j)\left(Y^n_j\right)^{\rho-1} Z_j^{1-\rho}.$$

Therefore, the spatial optimality condition is

$$p^\chi_j y^{e,\chi}_j(1-u^\chi_j) + (b^\chi_j - k^\chi\theta^\chi_j)u^\chi_j = p^\chi_{j'} y^{e,\chi}_{j'}(1-u^\chi_{j'}) + (b^\chi_{j'} - k^\chi\theta^\chi_{j'})u^\chi_{j'}, \quad \forall\chi.$$

Equation for co-state variable u_ϕ

$$\frac{\partial H}{\partial u_\phi} = r\gamma_\phi - \dot\gamma_\phi \Rightarrow r\gamma_\phi - \dot\gamma_\phi = -\gamma_\phi[A(\theta_\phi)^{1-\alpha} + \lambda F(y^*_\phi) + s] + \frac{\partial Z_j}{\partial u_\phi} + L_\phi(b^\chi - k^\chi\theta_\phi)$$

Plug in γ_ϕ and impose steady state condition $\dot{\gamma_\phi} = 0$ and re-arrange,

$$0 = p_\phi y^*_\phi - [b_\phi + \frac{\alpha}{1-\alpha}\theta_\phi k^\chi] + \frac{p_\phi\lambda}{r+\lambda}\int_{y^*_\phi}^{\bar{y^\chi}}(y^\chi - y^*_\phi)dF(x^\chi).$$

Planner's optimal choice of $\{\theta_\phi, y^*_\phi, L_\phi\}, \forall\phi$ will satisfy the following conditions

$$0 = p_\phi y^*_\phi - [b_\phi + \frac{\alpha}{1-\alpha}\theta_\phi k^\chi] + \frac{p_\phi\lambda}{r+\lambda}\int_{y^*_\phi}^{\bar{y^\chi}}(y^\chi - y^*_\phi)dF(x^\chi),$$

$$\frac{k^\chi}{[1-F(y^*)]q(\theta_\phi)} = \frac{(1-\alpha)(y^e_\phi - y^*_\phi)}{[1-F(y^*)](r+\lambda)},$$

$$p_j^\chi y_j^{e,\chi}(1-u_j^\chi)+(b_j^\chi-k_j\theta_j^\chi)u_j^\chi=p_{j'}^\chi y_{j'}^{e,\chi}(1-u_{j'}^\chi)+(b_{j'}^\chi-k^\chi\theta_{j'}^\chi)u_{j'}^\chi,$$

$$u_\phi=\frac{\lambda F(y_\phi^*)}{\lambda F(y_\phi^*)+f(\theta_\phi)},$$

$$1=\sum_j L_j;\ \xi=\sum_j L_j\zeta_j$$

B.4 Data

B.4.1 Occupation-based skill definition

Using the AM measure, the occupation with the highest and lowest skills would be

Table 4: Occupation with the highest and lowest AM

Highest 20	AM	Lowest 20	AM
Physical Scientist	1	Dancers	0
Chemical Engineers	0.983	Parking Lot Attendant	0.222
Chemists	0.952	Paving, surfacing, and tamping equipment operators	0.253
Actuaries	0.944	Operating Engineers of construction equipment	0.273
Dietitians and Nutritionists	0.942	Fire Fighting	0.273
Metallurgical and Materials Engineers	0.926	Excavating and Loading Machine Operators	0.281
Mechanical Engineers	0.926	Bus Driver	0.283
Funeral Directors	0.924	Truck, Delivery, and Tractor Drivers	0.283
Accountants and Auditors	0.922	Taxi Cab Driver	0.285
Petroleum, Mining and Geological Engineers	0.921	Roofer and Slaters	0.291
Managers of Medicine	0.914	Crane, derrick, winch, and hoist operators	0.291
Financial Managers	0.911	Structural Metal Workers	0.302
Aerospace Engineer	0.897	Plasterers	0.306
Atmospheric and Space Scientists	0.895	Textile and Sewing Machine Operator	0.343
Other Financial Specialist	0.893	Garbage and Recyclable Material Collector	0.343
Subject Instructor (HS/College)	0.892	Driller of Earth	0.361
Managers and Specialists in Marketing, Advertising, and Public relations	0.883	Railroad brake, coupler, and switch operators	0.362
Biological Scientists	0.882	Millwrights	0.370
Computer Software Developer	0.879	Carpenter	0.371

B.5 Policy Experiment Equilibrium

B.5.1 Policy Experiments

Relocation subsidies

A relocation subsidy τ^m for low-skill workers in location H. The subsidies are financed by lump-sum tax τ^c on workers, regardless of employment status. The size of the subsidy equals 10 percent of housing spending an unemployed low-skill worker in location H would pay.

The Bellman equations become

$$rW_\phi(y^\chi) = [\mathbf{1}_{j=H,\chi=n}\tau^m - \tau^c + w_\phi(y^\chi)]R_j^{-\eta^\chi} + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(x^\chi) - W_\phi(y^\chi)]dF(x^\chi) - \lambda F(y_\phi^*)[W_\phi(y^\chi) - U_\phi], \tag{B.33}$$

$$rU_\phi = \max_j \left\{ [\mathbf{1}_{j=H,\chi=n}\tau^m - \tau^c + b_\phi]R_j^{-\eta^\chi} + f(\theta_\phi) \int_{y_\phi^*}^{\bar{y^\chi}} [W_\phi(y^\chi) - U_\phi]dF(x^\chi) \right\}. \tag{B.34}$$

$$rV_\phi = -k^\chi + q(\theta_\phi) \int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - V_\phi]dF(x^\chi), \tag{B.35}$$

$$rJ_\phi(y^\chi) = p_\phi y^\chi - w_\phi(y^\chi) + \lambda \int_{y_\phi^*}^{\bar{y^\chi}} [J_\phi(x^\chi) - J_\phi(y^\chi)]dF(x^\chi) - \lambda F(y_\phi^*)J_\phi(y^\chi), \tag{B.36}$$

where $\mathbf{1}_{j=H,\chi=n}$ is an indicator function that equals to 1 if location $j = H$ and skill type $\chi = n$, and equals to 0 otherwise. The wage equation becomes

$$w_\phi(y^\chi) = \beta p_\phi y^\chi + [(1-\beta)b_\phi + \beta\theta_\phi k^\chi] \tag{B.37}$$

The equilibrium conditions with policy instruments are the following

$$r\bar{U}^{\chi} = [\mathbf{1}_{j=H,\chi=n}\tau^{m} - \tau^{c} + b_{\phi} + \frac{\beta}{1-\beta}(k^{\chi} + \tau^{c})\theta_{\phi}]R_j^{-\eta}, \tag{B.38}$$

$$\frac{k^{\chi}}{q(\theta_{\phi})[1-F(y_{\chi}^{*})]} = \frac{p_{\phi}(1-\beta)(y_{\phi}^{e} - y_{\phi}^{*})}{r+\lambda}. \tag{B.39}$$

$$0 = p_{\phi}y_{\phi}^{*} - [b_{\phi} + \frac{\beta}{1-\beta}\theta_{\phi}k^{\chi}] + \frac{p_{\phi}\lambda}{r+\lambda}\int_{y_{\phi}^{*}}^{\bar{y^{\chi}}}[y^{\chi} - y_{\phi}^{*}]dF(x^{\chi}), \tag{B.40}$$

$$u_{\phi} = \frac{\lambda F\left(y_{\phi}^{*}\right)}{\lambda F\left(y_{\phi}^{*}\right) + f\left(\theta_{\phi}\right)}. \tag{B.41}$$

The subsidies for the workers are financed by a lump-sum tax τ^{c} on workers, regardless of employment status, skill, or location. The subsidy is given to the workers such that the size of housing consumption is

$$t^{m} = 0.1 \times b_{H}^{n}\eta$$

$$t^{c} = t^{m}[(L_{H}(1-\zeta_{H})]$$

B.6 Proofs and Discussions

B.6.1 Proof of Proposition 3.3.7

By the spatial equilibrium condition, reproduced here for convenience,

$$\overline{U}^{\chi} = \left(b^{\chi} + \frac{\beta}{1-\beta}k^{\chi}\theta_j^{\chi}\right)R_j^{-\eta}, \tag{B.42}$$

we can see that within each skill type, workers are indifferent between locations. If one location's market tightness is higher, i.e., $\theta_j^{\chi} > \theta_{j'}^{\chi}$, then $R_j > R_{j'}$ must be true to maintain the spatial equilibrium condition since the rest of the elements in the equations do not vary by location. Additionally, the Beverage Curve dictates a negative relationship between market tightness and unemployment rate, i.e., if $\theta_j^{\chi} > \theta_{j'}^{\chi}$, then $u_j^{\chi} < u_{j'}^{\chi}$. Combining these two inequalities, we can see that if the location with a higher rent also features a lower unemployment rate for each skill type, i.e., if $R_j > R_{j'}$, then $u_j^{\chi} < u_{j'}^{\chi}$.

B.6.2 Proof of Corollary 3.3.7

The job creation condition, reproduced here for convenience,

$$\frac{k^\chi}{q(\theta_\phi)} = \frac{(1-\beta)p_\phi y^\chi - [(1-\beta)b^\chi + \beta\theta_\phi k^\chi]}{r+s},$$

which shows that when market tightness θ_ϕ increases, the price of the intermediate goods p_ϕ must also increase. Therefore, within each skill type χ, if the market tightness is bigger in one location, then the intermediate goods' price must be higher in that location, i.e., if $\theta_j^\chi > \theta_k^\chi$, then $p_j^\chi > p_k^\chi$. The wage equation, reproduced here for convenience,

$$w_\phi = \beta p_\phi y^\chi + [(1-\beta)b^\chi + \beta\theta_\phi k^\chi],$$

shows that wage increases in both the market tightness and the intermediate goods' price. Since we already know that p_ϕ also increases with θ_ϕ, we can say that if $\theta_j^\chi > \theta_k^\chi$, then $w_j^\chi > w_k^\chi$. By the Beveridge Curve (3.24), we know that the unemployment rate is decreasing in market tightness, therefore if $u_j^\chi < u_{j'}^\chi$, then $w_j^\chi > w_{j'}^\chi$ $\forall j, j' \in J$ and $\chi \in \{s, n\}$

B.6.3 Proof of Corollary 3.3.7

The real wage's expression is $\tilde{w}_j^\chi = \frac{w_j^\chi}{R_j^\eta}$. Plug the spatial equilibrium into the wage equation, and then plug in the job creation condition

$$\begin{aligned} w_\phi &= \beta p_\phi y^\chi + [(1-\beta)b^\chi + \beta\theta_\phi k^\chi] && \text{(B.43)} \\ &= \beta p_\phi y^\chi + \frac{1}{1-\beta}\overline{U}^\chi R_j^\eta && \text{(B.44)} \\ \Rightarrow \overline{U}^\chi &= \tilde{w}_j^\chi - \frac{\beta}{1-\beta}R_j^{-\eta}\left[(r+s^\chi)\frac{k^\chi}{q(\theta_j^\chi)} + (1-\beta)b^\chi + \beta\theta_j^\chi k^\chi\right] && \text{(B.45)} \end{aligned}$$

Since $\overline{U}^\chi$ does not vary across space, for both $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$ and $\theta_j > \theta_{j'}$ to be satisfied, it must be true that $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$. By the Beveridge Curve (24), we know that the unemployment rate is decreasing in market tightness; therefore, $u_j^\chi < u_{j'}^\chi$, and $\tilde{w}_j^\chi > \tilde{w}_{j'}^\chi$ when $\frac{p_j^\chi y^\chi}{R_j^\eta} > \frac{p_{j'}^\chi y^\chi}{R_{j'}^\eta}$. The theoretical relationship between real wages and unemployment rates is less conclusive.

B.6.4 Proof of proposition 3.3.8

1. Case 1: $\sigma_j = \sigma_k$, $Q_j > T_k$. We can implement $\uparrow \frac{\sigma_j}{\sigma_k}$ by raising σ_j while holding σ_k constant. In this case, the production side is symmetrical i.e. $p_j^\chi = p_k^\chi$, $\zeta_j = \zeta_k$, but the housing market side is different. Since more land is available in location j, from the Spatial Equilibrium equation (3.23), it must be that $R_j = R_k$. By housing cost equation (3.11), since $R_j = R_k$, $\zeta_j = \zeta_k, w_j^\chi = w_k^\chi, u_j^\chi = u_k^\chi$, hence it must be that $L_j > L_k$ to balance the difference in $Q_j > T_k$. All the other variables have the same value for each ϕ.

2. Case 2: $\sigma_j > \sigma_k$, $Q_j = T_k$. We can implement $\uparrow \frac{Q_j}{T_k}$ by raising Q_j while holding T_k constant. From spatial equilibrium for high-skill worker

$$\left(b^s + \frac{\beta}{1-\beta} k^s \theta_j^s\right) R_j^{-\eta} = \left(b^s + \frac{\beta}{1-\beta} k^s \theta_k^s\right) R_k^{-\eta}.$$

From spatial equilibrium for low-skill worker

$$\left(b^n + \frac{\beta}{1-\beta} k^n \theta_j^n\right) R_j^{-\eta} = \left(b^n + \frac{\beta}{1-\beta} k^n \theta_k^n\right) R_k^{-\eta}$$

.

Hence

$$\frac{b^s + \frac{\beta}{1-\beta} k^s \theta_j^s}{b^s + \frac{\beta}{1-\beta} k^s \theta_k^s} = \frac{b^n + \frac{\beta}{1-\beta} k^n \theta_j^n}{b^n + \frac{\beta}{1-\beta} k^n \theta_k^n}.$$

Since $b^s = b^n, k^s = k^n$, then it must be that $\theta_j^s = \theta_k^s$ and $\theta_j^n = \theta_k^n$, therefore $u_j^s = u_k^s, u_j^n = u_k^n, p_j^s = p_k^s, p_j^n = p_k^n$. Going back to the spatial equilibrium condition, $R_j = R_k$.

Since $p_j^s = p_k^s$ and $\sigma_j > \sigma_k$, using the price equation 3.2 and 3.1, it must be that the $\zeta_j > \zeta_k$.

Since $p_j^s = p_k^s$, the ratio between p_j^s and p_k^s is

$$1 = \frac{\sigma_j \rho_j}{\sigma_k \rho_k} \left(\frac{1-u_j^s}{1-u_j^n}\right)^{\sigma_j \rho_j - \sigma_j \rho_k} \frac{\left(\frac{\zeta_j}{1-\zeta_j}\right)^{\sigma_j \rho_j - 1}}{\left(\frac{\zeta_k}{1-\zeta_k}\right)^{\sigma_k \rho_k - 1}}.$$

Since $\sigma_j > \sigma_k$ and $\zeta_j > \zeta_k$, it must be that $u_j^s < u_j^n$, and hence due to the Beveridge Curve, it must be true that $\theta_j^s > \theta_j^n$.

B.6.5 Proof of proposition 3.3.8

Since the location-dependent parameters are symmetrical, the production functions are the same across locations, and so are the housing supplies. Therefore, the skill composition and total worker size will also be symmetrical across locations. and $L_j/L_{j'}$ and $\zeta_j/\zeta_{j'}$ will not change even if skill dependent parameter changes.

1. Case 1: $y^s > y^n, b^s = b^n, k^s = k^n$ We can implement $\uparrow \frac{y^s}{y^n}$ by raising y^s while holding y^n constant. As y^s increases, surpluses for both types of matches increase. Therefore, market tightness increases for all ϕ, and unemployment rates decrease for all ϕ. Since $b^s = b^n, k^n = k^s$, the ratio for θ_j^s/θ_j^n and u_j^s/u_j^n stay the same.

2. Case 2: $y^s = y^n, b^s > b^n, k^s = k^n$. We can implement $\uparrow \frac{b^s}{b^n}$ by raising b^s while holding b^n constant. By the job creation condition, as b^s increases, θ_j^s decreases. Since θ_j^n stays the same, $\frac{\theta_j^s}{\theta_j^n}$ decreases. By the Beveridge Curve, we can see that when θ_ϕ increases, u_ϕ decreases. Therefore, $\frac{u^s}{u^n}$ increases as $\frac{b^s}{b^n}$ decreases.

3. Case 3: $y^s = y^n, b^s = b^n, k^s > k^n$. We can implement $\uparrow \frac{k^s}{k^n}$ by raising k^s while holding k^n constant. By the job creation condition, as k^s increases, θ_j^s decreases. Since θ_j^n stays the same, $\frac{\theta_j^s}{\theta_j^n}$ decreases. By the Beveridge Curve, we can see that when θ_ϕ increases, u_ϕ decreases. Therefore, $\frac{u^s}{u^n}$ increases as $\frac{k^s}{k^n}$ decreases.

By the spatial equilibrium condition, reproduced here for convenience,

$$\overline{U}^\chi = \left(b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_j^\chi\right) R_j^{-\eta}, \tag{B.46}$$

we can see that within each skill type, workers are indifferent between locations. If one location's market tightness is higher, i.e., $\theta_j^\chi > \theta_{j'}^\chi$, then $R_j > R_{j'}$ must be true to maintain the spatial equilibrium condition since the rest of the elements in the equations do not vary by location.

The Beverage Curve in the extended model has a different expression than in the baseline model, reproduced here for convenience,

$$u_\phi = \frac{\lambda F(y_\phi^*)}{\lambda F(y_\phi^*) + f(\theta_\phi)}.$$

Since we already know that if $R_j > R_{j'}$, then $\theta_j^\chi > \theta_{j'}^\chi$, in order for the result from Proposition 3.3.7 to hold, we need to show that

Suppose $\theta^{\chi}_{j} = \theta^{\chi}_{j'} + \Delta$

Additionally, the Beverage Curve dictates a negative relationship between market tightness and unemployment rate, i.e., if $\theta^{\chi}_{j} > \theta^{\chi}_{j'}$, then $u^{\chi}_{j} < u^{\chi}_{j'}$. Combining these two inequalities, we can see that if the location with a higher rent also features a lower unemployment rate for each skill type, i.e., if $R_j > R_{j'}$, then $u^{\chi}_{j} < u^{\chi}_{j'}$.

B.6.6 Proof of Proposition 3.4

Subtract the wage difference in the frictional labor market (Equation 3.31) from the wage difference in the competitive labor market (Equation 3.30).

$$\Delta\breve{w}^{\chi} - \Delta w^{\chi} = \Delta\breve{p}^{\chi} y^{\chi} - [\beta y^{\chi}\Delta p^{\chi} + \beta k^{\chi}\Delta\theta^{\chi}]$$

Therefore, if $\Delta\breve{p}^{\chi} y^{\chi} - [\beta y^{\chi}\Delta p^{\chi} + \beta k^{\chi}\Delta\theta^{\chi}] > 0$, then $\Delta\breve{w}^{\chi} > \Delta w^{\chi}$, the location wage gap is bigger in the competitive labor market than the frictional labor market; Otherwise, $\Delta\breve{w}^{\chi} < \Delta w^{\chi}$, the location wage gap is bigger in the frictional labor market than in the competitive labor market.

B.6.7 Proof of Proposition 3.5.1

The first condition,

$$\alpha_{\phi} = \beta_{\phi} \tag{B.47}$$

can be easily obtained by comparing the job creation condition in the decentralized equilibrium and the planner's equilibrium condition, allowing the bargaining power and the matching function elasticity to vary by location-skill groups. The second condition is obtained by equating the spatial equilibrium condition of the decentralized equilibrium and the spatial optimal condition of the planner

$$b^{\chi} + \frac{\alpha}{1-\alpha} k^{\chi}\theta^{\chi}_{j} = \left(b^{\chi} + \frac{\beta}{1-\beta} k^{\chi}\theta^{\chi}_{j}\right) R_j^{-\eta}$$

Re-arrange the equation and express it in terms of β_j^χ, the equation becomes

$$\beta_j^\chi = 1 - \left[1 + \frac{R_j^\eta \left(b^\chi + \frac{\alpha}{1-\alpha} k^\chi \theta_j^\chi\right) - b^\chi}{k^\chi \theta_j^\chi}\right]^{-1} \tag{B.48}$$

Nevertheless, equation (B.47) and equation (B.48) can only be simultaneously satisfied if the decentralized spatial equilibrium condition becomes

$$\overline{U}^\chi = \left(b^\chi + \frac{\beta}{1-\beta} k^\chi \theta_j^\chi\right)$$

which happens when $R_j^{-\eta} = R_{j'}^{-\eta}$ since $R_j^{-\eta}$ will be dropped out of the spatial equilibrium condition. There are two possibilities for $R_j^{-\eta} = R_{j'}^{-\eta}$ to hold, we need either $\eta = 0$ or $R_j = R_{j'}$. Therefore, equation (B.47) and equation (B.48) can only be simultaneously satisfied either $\eta = 0$ or $R_j = R_{j'}$ holds.

Appendix C

C.1 Equilibrium Derivation

C.1.1 Bargainig with Rent Control

Bargaining between a non-rent-controlled renter and a non-rent-controlled landlord determines price $p(t)$,

$$p(t) = arg\max[Q_0(t) - U(t)]^{\beta}[L_0(t) - V(t)]^{1-\beta}.$$

Re-arrange the Bellman equations for vacancy and landlord,

$$(r + s + \delta)[L_0(t) - V(t)] = p(t) + c^v(t) + \sigma[L_1(p(t), t) - L_0(t)] - \theta m(\theta)[L_0(t) - V(t)].$$

Re-arrange the Bellman equations for non-rent-controlled renter and an unhoused renter,

$$(r + s + \delta)[Q_0(t) - U(t)] = \varepsilon(t) - p(t) + c^s(t) + \sigma[Q_1(p(t), t) - Q_0(t)] - m(\theta)[Q_0(t) - U(t)].$$

p(t) solves

$$\beta[L_0(t) - V(t)]\left[-1 + \sigma\frac{\partial Q_1(p(t), t))}{\partial p(t)}\right] + (1 - \beta)[Q_0(t) - U(t)]\left[1 + \sigma\frac{\partial L_1(p(t), t))}{\partial p(t)}\right] = 0. \tag{C.1}$$

Denote

$$Z = -\frac{\left[-1 + \sigma\frac{\partial Q_1(p(t),t))}{\partial p(t)}\right]}{\left[1 + \sigma\frac{\partial L_1(p(t),t))}{\partial p(t)}\right]}.$$

Re-arrange equation (4.11), (4.12), and (4.13), and differentiate $Q_1(p(t),t)$ wrt $p(t)$. Re-arrange equation (4.14), (4.15), and (4.16), and differentiate $L_1(p(t),t)$ wrt $p(t)$,

$$\frac{\partial Q_1(p(t),t)}{\partial p(t)} = -\left[\frac{(s+\delta)m(\theta)}{r+m(\theta)} - (r+s+\delta)\right]; \quad \frac{\partial L_1(p(t),t)}{\partial p(t)} = \frac{1}{(r+\delta+s)}.$$

Plug these expressions into Z,

$$Z = -\frac{\left[-1+\sigma\frac{\partial Q_1(p(t),t))}{\partial p(t)}\right]}{\left[1+\sigma\frac{\partial L_1(p(t),t))}{\partial p(t)}\right]} = -\frac{\left[-1-\sigma\left[\frac{(s+\delta)m(\theta)}{r+m(\theta)} - (r+s+\delta)\right]\right]}{\left[1+\sigma\frac{1}{(r+\delta+s)}\right]} = \frac{1+\sigma\left[\frac{(s+\delta)m(\theta)}{r+m(\theta)} - (r+s+\delta)\right]}{\left[1+\sigma\frac{1}{(r+\delta+s)}\right]}.$$

Re-arrange equation (C.1),

$$[Q_0(t) - U(t)] = \frac{\beta}{1-\beta}[L_0(t) - V(t)]Z$$

Note that the last term is how the bargaining rule is different from the case without rent control. Plug in the expression for $[L_0(t) - V(t)]$ and $[Q_0(t) - U(t)]$,

$$\begin{aligned} &Z\beta\{p(t) + c^v(t) + \sigma[L_1(p(t),t) - L_0(t)] - \theta m(\theta)[L_0(t) - V(t)]\} \\ =&(1-\beta)\{\varepsilon(t) - p(t) + c^s(t) + \sigma[Q_1(p(t),t) - Q_0(t)] - m(\theta)[Q_0(t) - U(t)]\}. \end{aligned}$$

Plug in the expression of $[L_1(p(t),t) - L_0(t)]$ and $[Q_1(t,t) - Q_0(t)]$, the expression becomes

$$\begin{aligned} &Z\beta\left[p(t) + c^v(t) + \frac{\sigma}{r+s+\delta+\sigma} - \theta m(\theta)[L_0(t) - V(t)]\right] \\ =&(1-\beta)\left[\varepsilon(t) - p(t) + c^s(t) + \frac{\sigma}{r+s+\delta+\sigma} - m(\theta)\frac{\beta}{1-\beta}[L_0(t) - V(t)]Z\right]. \end{aligned}$$

Re-arrange,

$$\begin{aligned} &Z\beta\left[p(t) + c^v(t) + \frac{\sigma}{r+s+\delta+\sigma}\right] - \beta Z\theta m(\theta)[L_0(t) - V(t)] \\ =&(1-\beta)\left[\varepsilon(t) - p(t) + c^s(t) + \frac{\sigma}{r+s+\delta+\sigma}\right] - \beta Z m(\theta)[L_0(t) - V(t)]. \end{aligned}$$

Therefore, the price is

$$
\begin{aligned}
p(t) &= \frac{1}{\beta Z + (1-\beta)}[-\beta Z \left[c^v(t) + \frac{\sigma}{r+s+\delta+\sigma}\right] + \beta Z \theta m(\theta)[L_0(t) - V(t)] \\
&\quad + (1-\beta)\left[\varepsilon(t) + c^s(t) + \frac{\sigma}{r+s+\delta+\sigma}\right] - \beta Z m(\theta)[L_0(t) - V(t)]] \\
\Rightarrow p(t) &= \frac{-\beta Z c^v(t) + (1-\beta)\left[\varepsilon(t) + c^s(t)\right] + \beta Z m(\theta)(\theta - 1)[L_0(t) - V(t)]}{\beta Z + (1-\beta)} + \frac{-\beta Z + (1-\beta)}{\beta Z + (1-\beta)} \frac{\sigma}{r+s+\delta+\sigma} \\
\Rightarrow p(t) &= \frac{-\beta Z c^v(t) + (1-\beta)[\varepsilon(t) + c^s(t)]}{\beta Z + (1-\beta)} - \frac{\beta Z m(\theta)(1-\theta)[L_0(t) - V(t)])}{\beta Z + (1-\beta)} + \frac{-\beta Z + (1-\beta)}{\beta Z + (1-\beta)} \frac{\sigma}{r+s+\delta+\sigma} \\
\Rightarrow p(t) &= \frac{1}{\beta Z + (1-\beta)} \left\{ (1-\beta)(\varepsilon + c^s) + \beta Z \left[(1 - \frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta} c^v \right] + [(1-\beta) - \beta Z] \frac{\sigma}{r+s+\delta+\sigma} \right\}.
\end{aligned}
$$

C.1.2 Match Destruction Condition

Match Destruction: Look at the Bellman equation for $L_1(p(\tau), t)$ for $t < t + T$,

$$
rL_1(\tau, t) = p(\tau) + s[V(t) - L_1(\tau, t)] - \delta L_1(p(\tau), t) + \dot{L}_1(p(\tau), t),
$$

The landlord chooses the life of the match to maximize its value. Let T be the optimal match duration. Hence, the optimal destruction age for vintage τ is $(\tau + T)$. The maximum value of a match formed at time t should satisfies

$$
L_1(p(t), t) = \max_T \left\{ \int_t^{t+T} [p(t) + sc^k(x)] e^{-(r+s+\delta)(x-t)} dx - [V(t+T) - c^e(t+T)] e^{-(r+s+\delta)T} \right\}.
$$

The price $p(t)$ is given by:

$$
p(t) = \frac{1}{\beta Z + (1-\beta)} \left\{ (1-\beta)(\varepsilon + c^s) + \beta Z \left[(1 - \frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta} c^v \right] + [(1-\beta) - \beta Z] \frac{\sigma}{r+s+\delta+\sigma} \right\}.
$$

Plug the expression for $p(t)$ into $L_1(p(t), t)$,

$$
\begin{aligned}
L_1(p(t), t) = \max_T \int_t^{t+T} &\left[\frac{1}{\beta Z + (1-\beta)} \left\{ (1-\beta)(\varepsilon + c^s) + \beta Z \left[(1 - \frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta} c^v \right] + \frac{\sigma[(1-\beta) - \beta Z]}{r+s+\delta+\sigma} \right\} + sc^k(x) \right] e^{-(r+s+\delta)(x-t)} dx \\
&- [V(t+T) - c^e(t+T)] e^{-(r+s+\delta)T}.
\end{aligned}
$$

Take the first order condition with respect to T,

$$0 = \left(\frac{1}{\beta Z + (1-\beta)}\left\{(1-\beta)(\varepsilon + c^s) + \beta Z\left[(1-\frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta}c^v\right] + \frac{[(1-\beta)-\beta Z]\sigma}{r+s+\delta+\sigma}\right\}\right)e^{gt-(r+s+\delta)T}$$
$$+ sc_0^k e^{g(t+T)-(r+s+\delta)T} - (c_0^k - c^e)[g-(r+s+\delta)]e^{g(t+T)-(r+s+\delta)T}$$

$$\Rightarrow 0 = \left(\frac{1}{\beta Z + (1-\beta)}\left\{(1-\beta)(\varepsilon + c^s) + \beta Z\left[(1-\frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta}c^v\right] + [(1-\beta)-\beta Z]\frac{\sigma}{r+s+\delta+\sigma}\right\}\right)$$
$$+ [sc_0^k - (c_0^k - c^e)[g-(r+s+\delta)]e^{gT}.$$

Therefore,

$$T = \frac{1}{g}\ln\left[\frac{\left(\frac{1}{\beta Z+(1-\beta)}\left\{(1-\beta)(\varepsilon + c^s) + \beta Z\left[(1-\frac{1}{\theta})[(r+\delta)c^k - \frac{1}{\theta}c^v\right] + [(1-\beta)-\beta Z]\frac{\sigma}{r+s+\delta+\sigma}\right\}\right)}{(c_0^k - c^e)[g-(r+s+\delta)] - sc_0^k}\right],$$

where

$$Z = (1-\beta)\left[\varepsilon(t) - p(t) + c^s(t) + \frac{\sigma}{r+s+\delta+\sigma} - m(\theta)\frac{\beta}{1-\beta}[L_0(t) - V(t)]Z\right].$$