

Lawrence Berkeley National Laboratory

LBL Publications

Title

THE ESTIMATION OF SCANNING EFFICIENCIES FOR EXPERIMENTS USING VISUAL DETECTORS

Permalink

<https://escholarship.org/uc/item/62h8d5tg>

Authors

Derenzo, Stephen E.
Hildebrand, Roger H.

Publication Date

1968-12-01

cy. 2

THE ESTIMATION OF SCANNING EFFICIENCIES
FOR EXPERIMENTS USING VISUAL DETECTORS

LIBRARY
DOCUMENT

Stephen E. Derenzo and Roger H. Hildebrand

December 1968

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

cy. 2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Nuclear Instruments
and Methods

UCRL-18638
Preprint

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

THE ESTIMATION OF SCANNING EFFICIENCIES
FOR EXPERIMENTS USING VISUAL DETECTORS

Stephen E. Derenzo and Roger H. Hildebrand

December 1968

-1-

THE ESTIMATION OF SCANNING EFFICIENCIES
FOR EXPERIMENTS USING VISUAL DETECTORS*

Stephen E. Derenzo

Lawrence Radiation Laboratory, University of California
Berkeley, California 94720, U.S.A.

and

Roger H. Hildebrand†

Lawrence Radiation Laboratory

and

The University of Chicago, Chicago, Illinois 60637, U.S.A.

ABSTRACT

A new technique is presented for estimating scanning efficiencies. The special feature of this technique is that it allows for the tendency of different scanners to miss the same events. Each event is assumed to have a visibility, v , depending both on its own characteristics and on the perceptiveness of the scanners. The distribution of events in visibility is described by a function $F(v)$ whose parameters are determined by comparing the numbers of new events found in each of three or more successive scans. The technique is applied to a study of μ^+ decay events, which have been scanned four times. The "true" number of events, $N_T = \int_0^1 F(v)dv$, found by this technique and a lower limit to N_T found by an alternate technique indicate that the inefficiency estimated by the usual double-scanning method can be low by a large factor.

1. Introduction

It is well recognized that scanning efficiencies for visual detector experiments may vary with significant experimental parameters such as scattering angle. To allow for this, efficiencies are usually calculated after the events have been measured and grouped into suitable intervals¹). It is also recognized that efficiencies often depend on incidental parameters such as vertex location and this is usually handled by establishing appropriate geometrical cut-offs. But what is often overlooked or ignored is that after these precautions have been taken the efficiencies obtained by comparing the records of two independent scans may still lead to systematic errors in the calculated event rates. Whenever the recognition of interesting events is influenced by factors such as the distribution and configuration of nearby track patterns, the events missed by one scanner tend to be the same ones which are missed by another so that scanning efficiencies are systematically overestimated^{2, 3}).

We shall briefly review the assumptions and consequences of the usual double scanning technique and shall then present a new technique which allows for differences in visibility.

2. The Geiger-Werner Coincidence Technique

2.1 TWO SCANS

Scanning efficiencies are usually estimated by the coincidence technique developed by Geiger and Werner⁴) for their experiments in α -particle scattering.

The Geiger-Werner method as applied say, to a bubble chamber or spark chamber experiment, may be described as follows: Film containing a true (unknown) number of events N_T is scanned separately by two scanners using

the same instructions. If N_1 and N_2 are the numbers of events found respectively by scanners 1 and 2, if λ_1 and λ_2 are the individual scanning efficiencies as defined by the expressions

$$\lambda_1 = N_1/N_T \quad \text{and} \quad \lambda_2 = N_2/N_T, \quad (1)$$

and if we assume that

$$\lambda_1 \cdot \lambda_2 = \text{probability that an event will be seen by both scanners}, \quad (2)$$

then we expect the number of events, N_{12} , seen by the two scanners in common (i.e., the number of "coincidences") to be

$$N_{12} = \lambda_1 \cdot \lambda_2 \cdot N_T. \quad (3)$$

Solving (1) and (3) for the three unknowns, we have

$$\lambda_1 = N_{12}/N_2, \quad \lambda_2 = N_{12}/N_1, \quad (4)$$

and

$$N_T = \frac{N_1 \cdot N_2}{N_{12}}. \quad (5)$$

On the same basis the "combined inefficiency", I_{GW} , (i.e., the probability that 1 and 2 will both miss an event) should be given by

$$I_{GW} = (1 - \lambda_1)(1 - \lambda_2) \quad (6a)$$

and the "combined efficiency" should be

$$\lambda_c = 1 - (1 - \lambda_1)(1 - \lambda_2). \quad (6b)$$

2.2 THREE OR MORE SCANS

The analysis of section 2.1 may be extended to the case of n independent scans. If

$$\prod_{i=1}^n (1 - \lambda_i) = \text{probability that all } n \text{ scanners will miss a particular event} \quad (7)$$

then

$$\lambda_c(n) = 1 - \prod_{i=1}^n (1 - \lambda_i) \quad (8)$$

is the combined efficiency after n scans.

Note that if (7) is valid then there should be no difficulty in defining individual efficiencies. We expect that aside from statistical fluctuations

$$\lambda_i = \frac{N_{ij}}{N_j} = \frac{N_{ijk}}{N_{jk}} = \dots = \frac{N_{ik}}{N_k} = \frac{N_{ikl}}{N_{kl}} = \dots \quad (9)$$

2.3 BASIC ASSUMPTION

The basic assumption which underlies the Geiger-Werner technique and is implied in (2) and (8) is that all events have equal a priori probability of being observed. That is, the chance that a particular event will be observed is assumed to depend only on the perceptiveness of the scanner (assumed constant); not on the perceptibility of the event.

3. Test of Geiger-Werner technique

3.1 NUMBER OF NEW EVENTS FOUND IN n^{th} SCAN, M_n

In order to test the validity of the Geiger-Werner technique for a particular experiment (after taking the precautions mentioned in section 1) one must make three or more scans on at least a portion of the film. The adequacy of the efficiency estimates may then be tested by comparing the various values for λ_i given by (9).

A more meaningful test, however, and one which will be useful in overcoming the restriction of the Geiger-Werner assumption (section 2.3) is to compare the numbers of new events M_1, M_2, \dots, M_n found in each of several scans. Since each scan in turn may be considered the "first," the "second," etc., without regard to chronological order we may average the differences between scanners by immediately replacing M_1, M_2, \dots, M_n , with average values $\bar{M}_1, \bar{M}_2, \dots, \bar{M}_n$.

With four scans, for example, we would have

$$\bar{M}_1 = (1/4)[A + B + C + D] , \quad (10a)$$

$$\begin{aligned} \bar{M}_2 = (1/12)[(A\bar{B} + A\bar{C} + A\bar{D}) + (B\bar{A} + B\bar{C} + B\bar{D}) \\ + (C\bar{A} + C\bar{B} + C\bar{D}) + (D\bar{A} + D\bar{B} + D\bar{C})] , \end{aligned} \quad (10b)$$

$$\bar{M}_3 = (1/12)[(A\bar{B}\bar{C} + A\bar{B}\bar{D} + A\bar{C}\bar{D}) + (B\bar{A}\bar{C} + B\bar{A}\bar{D} + B\bar{C}\bar{D}) \quad (10c)$$

$$+ (C\bar{A}\bar{B} + C\bar{A}\bar{D} + C\bar{B}\bar{D}) + (D\bar{A}\bar{B} + D\bar{A}\bar{C} + D\bar{B}\bar{C})] , \quad (10c)$$

and

$$\bar{M}_4 = (1/4)[A\bar{B}\bar{C}\bar{D} + B\bar{A}\bar{C}\bar{D} + C\bar{A}\bar{B}\bar{D} + D\bar{A}\bar{B}\bar{C}] , \quad (10d)$$

where A, B, C, and D are the numbers of events⁵⁾ found by scanners

A, B, C and D; $A\bar{B}$ is the number of events seen by A but missed by B;

$A\bar{B}\bar{C}$ is the number of events seen by A but missed by both B and C, etc.

These expressions may readily be extended to accommodate any number of scans.

3.2 COMPARISON OF THE RATIOS $(\bar{M}_i/\bar{M}_{i-1})$ FOR $i = 1$ TO n

If equation (8) is valid then the series $\bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \dots$ should converge rapidly. For all values of i we expect the ratio $(\bar{M}_i/\bar{M}_{i-1})$ to have the constant value $(1-\lambda)$ where λ is the average scanning efficiency.

Thus in an experiment where the first scan yields 900 events and the second

adds 90, one would expect a third scan to give 9 more (ignoring statistical fluctuations) leaving only one to be discovered. But if the third number were 30 instead of 9 one would know immediately that the technique of section 2 was inadequate.

Whenever some events are harder to find than others, as is usually the case, one will have instead of a constant ratio the set of inequalities

$$(\bar{M}_n/\bar{M}_{n-1}) > (\bar{M}_{n-1}/\bar{M}_{n-2}) > \dots > (\bar{M}_2/\bar{M}_1) .$$

The problem then is to find the sum $N_T = \bar{M}_1 + \bar{M}_2 + \dots + \bar{M}_\infty$ when one knows only a few terms whose rate of convergence indicates that the remaining terms are non-negligible.

4. The New Technique

4.1 THE VISIBILITY, v

We define the visibility, v , of a particular event to be the probability that it will be found by an average scanner.

Although an event may have characteristics which make it highly conspicuous ($v \rightarrow 1$), invisible ($v=0$), or anything in between, we do not attempt to distinguish between the "inherent visibility" and the perceptiveness of the scanner since neither of these factors can be precisely defined without reference to the other. The visibility we have defined may be regarded as the product of these related factors.

We may define the average scanner (of, say, a group of four) by the following hypothetical scanning procedure. Suppose each scanner is identified by an index (1, 2, 3, 4) and each is working at a different machine. Whenever one of them finds an event a cyclic permutation is performed on the indices. The process is continued until each roll of

film has been scanned once on each of the four machines. The record of any one of the four hypothetical scanners labeled 1, 2, 3, and 4 will be that of an "average" scanner for that group. It will be seen that this procedure is essentially equivalent to the averaging process defined by equations 10a-10d.

4.2 THE VISIBILITY FUNCTION, $F(v)$: RELATIONSHIP TO \bar{M}_i

Let $F(v)dv$ be the number of events having visibilities in the range v to $v+dv$. Then the total number of events should be

$$N_T = \int_0^1 F(v)dv \quad (11)$$

According to the assumption of section 2.3, $F(v)$ would be a δ -function at the average scanning efficiency. In general, however, $F(v)$ is an unknown function which we shall seek to estimate.

Our method is to infer $F(v)$ from the known quantities $\bar{M}_1, \bar{M}_2, \dots, \bar{M}_n$.

The distribution of undetected events remaining after one scan, two scans, and $(i-1)$ scans will be $F(v)(1-v)$, $F(v)(1-v)^2$ and $F(v)(1-v)^{i-1}$ (see figure 1). Hence the number of new events found in the i^{th} scan will be

$$\bar{M}_i = \int_0^1 vF(v)(1-v)^{i-1} dv \quad (12)$$

The n equations (12) ($i=1$ to n) can be used to evaluate $F(v)$ if we assume for $F(v)$ an expression with not more than n parameters.

4.3 CHOICE OF THE FUNCTION $F(v)$

For pictures of good quality in which minimum ionization tracks are visible anywhere within the fiducial volume an event will have low visibility only if it is hidden by its own unfavorable configuration or

by superimposed tracks. Since it is unlikely that one track will exactly cover or smoothly join another (especially when seen in several views), we assume that $F(v)$ should approach zero as v approaches zero.

A function which can satisfy this condition, which accommodates a wide range of possibilities (see figure 2), and which is found to give a good fit to our own data, is the expression⁶⁾

$$F(v) = Kv^{\alpha}(1-v)^{\beta} \quad (13)$$

Since it should usually be feasible to make four scans on a portion of the film one may overdetermine the three parameters and thus test the adequacy of the function.

Other functions should, of course, be tried if (13) fails to give a satisfactory fit.

4.4 TWO-PARAMETER FIT

If $F(v)$ has a first moment at a high value of v , say $v > 0.8$, then one may achieve a fair representation of the distribution with a two-parameter fit. We may, for example, use (13) with $\beta=0$. Equation (11) then gives $N_T = K/(\alpha+1)$ and with (12) this becomes

$$N_T = \bar{M}_1(\bar{M}_1 - \bar{M}_2) / (\bar{M}_1 - 2\bar{M}_2) \quad (14)$$

or, in the notation of section 2,

$$N_T = N_{12}(N_1 + N_2) / (4N_{12} - N_1 - N_2) \quad (15)$$

We emphasize, however, that a third scan is necessary in order to test the validity of this or any other two-parameter method.

5. Lower Limit for N_T

Since the function $F(v)$ inferred from $\bar{M}_1, \dots, \bar{M}_n$ is only an approximation it is useful to find a limit to N_T by an independent method.

It is easily shown (see Appendix) that for any function $F(v)$ which is integrable in the region $0 \leq v \leq 1$, and for all values of \bar{M}_i ($i = 1$ to ∞) which satisfy eq. (12) we have the inequality

$$(\bar{M}_{i+2}/\bar{M}_{i+1}) \geq (\bar{M}_{i+1}/\bar{M}_i) .$$

Hence after n scans we may assume

$$\begin{aligned} & \bar{M}_{n+1} + \bar{M}_{n+2} + \bar{M}_{n+3} + \dots \\ &= \bar{M}_n \frac{\bar{M}_{n+1}}{\bar{M}_n} + \bar{M}_n \frac{\bar{M}_{n+1}}{\bar{M}_n} \frac{\bar{M}_{n+2}}{\bar{M}_{n+1}} + \bar{M}_n \frac{\bar{M}_{n+1}}{\bar{M}_n} \frac{\bar{M}_{n+2}}{\bar{M}_{n+1}} \frac{\bar{M}_{n+3}}{\bar{M}_{n+2}} + \dots \\ &\geq \bar{M}_n \frac{\bar{M}_n}{\bar{M}_{n-1}} \left[1 + \frac{\bar{M}_n}{\bar{M}_{n-1}} + \left(\frac{\bar{M}_n}{\bar{M}_{n-1}} \right)^2 + \dots \right] , \end{aligned}$$

and we obtain for N_T the limit

$$N_T \geq \left[\sum_{i=1}^n \bar{M}_i \right] + (\bar{M}_n^2 / \bar{M}_{n-1}) [1 - (\bar{M}_n / \bar{M}_{n-1})]^{-1} \quad (16)$$

subject only to statistical fluctuations in the measured quantities $\bar{M}_1, \dots, \bar{M}_n$.

6. Application to an Experiment on μ^+ Decay

To illustrate the technique we shall give the scanning data for an experiment on μ^+ decay⁷). In this experiment the scanners were instructed to record all e^+ tracks whose radii of curvature corresponded to projected momenta below 7 MeV/c. All of the film was scanned at least twice and about one fifth was scanned four times. The data presented in Tables 1

and 2 are for the 1147 events from this 1/5 sample whose measured momenta were below 6.8 MeV/c.

The data for the 1084 events found in the first two scans of the 1/5 sample indicated a combined scanning efficiency $\lambda_c = 97.8\%$ [eq. (6b)]. From this it appeared that the true number of events was 1109 and that only 25 had been missed.

The number of new events actually observed in the third scan, however, was 43. Hence it was clear without explicitly carrying out the test of section 3 that the efficiency of the first two scans was much lower than had been estimated. Applying the method of section 5 to the three-scan data, the true number, N_T , could be set above 1141 and applying the method of section 4 to the same data, N_T was estimated to be 1160 indicating an efficiency of only 93.4% for the first two scans.

The best estimate, using the data of all four scans and again the method of section 4, was 1168 indicating a 92.8% efficiency for the first two scans (or an inefficiency 3.3 times the original estimate).

Since the estimates 1160 and 1168 from the three- and four-scan data were in fair agreement, and since the three parameter function (13) gave a good fit to the four measured quantities \bar{M}_1 , \bar{M}_2 , \bar{M}_3 , and \bar{M}_4 (see Table 2), we concluded that we had an adequate representation of the true visibility distribution.

To obtain the true number of events in the remaining 4/5 of the film the number found in the two scans of that portion was corrected by the factor $1/0.928$ indicated by our best estimate. In anticipation of this procedure the rolls of film chosen for quadruple scanning had been taken at even intervals from the entire sample.

The result of this analysis was to increase by 5% our estimate of the fraction of the positron spectrum below 6.8 MeV, a correction which thoroughly justified the 20% extra scanning effort required. In experiments with lower scanning efficiency there may be even more to gain by applying the technique we have described.

It appears from our experience that the analysis of scanning efficiency deserves more care than it usually receives and that the extra effort required for a careful analysis should usually be a small fraction of the total investment in the experiment. A third scan of a small portion of the film should be sufficient to indicate whether more is necessary and the amount of additional scanning can be adjusted accordingly. If the result of the experiment can be strongly influenced by a variation in efficiency with some quantity such as momentum or scattering angle so that it is necessary to subdivide the efficiency analysis in to many bins, then one is likely to need a larger portion of three- or four-scan data in order to obtain statistically significant estimates. Whatever the circumstances the effort devoted to multiple scanning should be such that the remaining uncertainty in scanning efficiency is suitably balanced with the other systematic and statistical errors.

Appendix

PROOF OF THE RELATIONSHIP $(\bar{M}_{i+2}/\bar{M}_{i+1}) \geq (\bar{M}_{i+1}/\bar{M}_i)$

If the distribution $F(v)$ is integrable over the interval $(0,1)$,

then

$$\frac{\bar{M}_{i+2}}{\bar{M}_{i+1}} \geq \frac{\bar{M}_{i+1}}{\bar{M}_i}, \quad i = 1, \dots, \infty$$

where

$$\bar{M}_i = \int_0^1 vF(v)(1-v)^{i-1} dv. \quad (A1)$$

Proof:

Define the function $G_i(v) = F(v)v(1-v)^{i-1}$.

The relation

$$\frac{\int_0^1 G_i(v)(1-v)^2 dv}{\int_0^1 G_i(v) dv} \geq \left[\frac{\int_0^1 G_i(v)(1-v) dv}{\int_0^1 G_i(v) dv} \right]^2$$

which may be written $\langle (1-v)^2 \rangle \geq \langle 1-v \rangle^2$ follows from the fact that the weighting function $G_i(v)$ is non-negative since in that case

$$\langle (1-v)^2 \rangle - \langle 1-v \rangle^2 = \langle [1(1-v) - \langle 1-v \rangle]^2 \rangle \geq 0.$$

Hence

$$\frac{\int_0^1 F(v)v(1-v)^{i+1} dv}{\int_0^1 F(v)v(1-v)^i dv} \geq \frac{\int_0^1 F(v)v(1-v)^i dv}{\int_0^1 F(v)v(1-v)^{i-1} dv}$$

and with (A1) we obtain

$$\bar{M}_{i+2}/\bar{M}_{i+1} \geq \bar{M}_{i+1}/\bar{M}_i.$$

The equality holds only when $F(v)$ is a δ -function.

We are grateful to S. M. Flatté, J. H. Klems, L. Sanathanan, and R. Tsutakawa for stimulating discussions. It is a pleasure to acknowledge support from a Shell Foundation Fellowship (S.E.D.) and from a John Simon Guggenheim Fellowship (R.H.H.) during portions of this work.

One of us (R.H.H.) wishes to express his gratitude to the Lawrence Radiation Laboratory and especially to Professors E. Segrè and O. Chamberlain for their hospitality during his sojourn in Berkeley.

References

- *) Work supported by the National Science Foundation (grants GP-6135, GP-9093, and GP-6464) and by the Office of Naval Research, contract Nonr-2121(25).
- †) John Simon Guggenheim Fellow, 1968-69. Permanent address: Univ. of Chicago.
- 1) The measuring efficiency itself must be estimated as a function of experimental parameters. See S. E. Derenzo, and R. H. Hildebrand, Nuclear Instruments and Methods 58, 13 (1968).
- 2) C. J. Waddington, Suppl. Nuovo Cimento 19, 37 (1961).
- 3) E. Marquit, Nuovo Cimento 23, 781 (1962).
- 4) H. Geiger and A. Werner, Zeit. f. Phys. 21, 187 (1924). Note that these authors understood clearly the limitations of their technique.
- 5) We avoid the notation N_1, N_2, \dots, N_i , used in section 2, in order not to suggest that scanner i has any special relationship to \bar{M}_i .
- 6) $\int_0^1 K v^\alpha (1-v)^\beta dv = K \Gamma(\alpha+1) \Gamma(\beta+1) / \Gamma(\alpha+\beta+2)$ for $\alpha, \beta > -1$.
- 7) S. E. Derenzo and R. H. Hildebrand, Phys. Rev. Letters 20, 614 (1968) and Physics Letters (Dec. 31, 1968), to be published.

TABLE 1

Scanning data for μ -decay experiment. A, B, C, and D are the numbers of events found by scanners A, B, C and D; $\bar{A}\bar{B}$ is the number of events seen by A but missed by B; $\bar{A}\bar{B}\bar{C}$ is the number of events seen by A but missed by both B and C, etc.^a

A=969	$\bar{A}\bar{B}$ =172	$\bar{A}\bar{B}\bar{C}$ =38	$\bar{A}\bar{B}\bar{C}\bar{D}$ = 8
B=912	$\bar{A}\bar{C}$ = 85	$\bar{A}\bar{C}\bar{D}$ =20	$\bar{B}\bar{A}\bar{C}\bar{D}$ =13
C=1014	$\bar{A}\bar{D}$ = 64	$\bar{A}\bar{B}\bar{D}$ =26	$\bar{C}\bar{A}\bar{B}\bar{D}$ =18
D=1037	$\bar{B}\bar{A}$ =115	$\bar{B}\bar{A}\bar{C}$ =28	$\bar{D}\bar{A}\bar{B}\bar{C}$ =20
	$\bar{B}\bar{C}$ = 75	$\bar{B}\bar{C}\bar{D}$ =25	
	$\bar{B}\bar{D}$ = 66	$\bar{B}\bar{A}\bar{D}$ =28	
	$\bar{C}\bar{A}$ =130	$\bar{C}\bar{A}\bar{B}$ =43	
	$\bar{C}\bar{B}$ =177	$\bar{C}\bar{B}\bar{D}$ =36	
	$\bar{C}\bar{D}$ = 77	$\bar{C}\bar{A}\bar{D}$ =33	
	$\bar{D}\bar{A}$ =132	$\bar{D}\bar{A}\bar{B}$ =45	
	$\bar{D}\bar{B}$ =191	$\bar{D}\bar{B}\bar{C}$ =50	
	$\bar{D}\bar{C}$ =100	$\bar{D}\bar{A}\bar{C}$ =35	

^a In the notation of section 2, $N_1=A=969$, $N_2=B=912$, $N_{12}=A-\bar{A}\bar{B}=B-\bar{B}\bar{A}=797$.

TABLE 2

Fits of the visibility function $F(v) = Kv^\alpha(1-v)^\beta$ to the data of Table 1 using the technique of section 4.2: Estimates of N_T .

	first 2 scans		first 3 scans		all 4 scans	
	observed ^a	fit ^b	observed ^a	fit ^b	observed ^a	fit ^b
\bar{M}_1	940.5	940.5	965.0	965.0	983.0	983.0
\bar{M}_2	143.5	143.5	125.7	125.7	115.3	115.0
\bar{M}_3	-----	19.0	36.3	36.3	33.9	34.5
\bar{M}_4	-----	2.2	-----	14.5	14.8	14.5
Total Seen	1084		1127		1147	
K		5219 ^c		1349		829
α		3.55 ^c		1.89		1.38
β		$\equiv 0^c$		-0.42		-0.55
N_T^d		1147 ^c		1160		1168 ^g
$N_T(GW)^e$		1109				
$N_T(Min)^f$		1109		1141		1158

^a Equation (10)

^b Equation (12)

^c See section 4.4

^d Equation (11)

^e Equation (5)

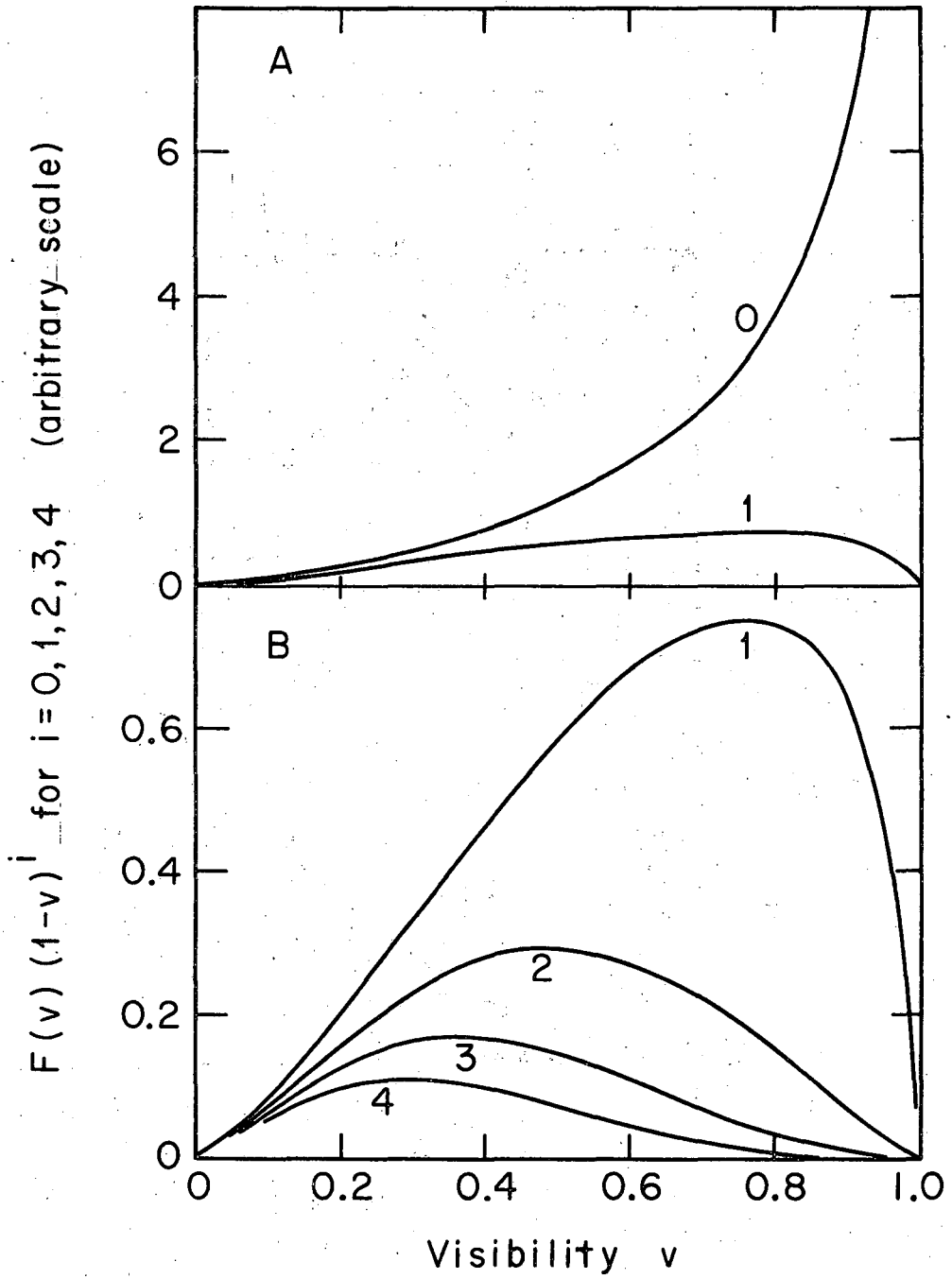
^f Equation (16)

^g Best estimate

Figure Captions

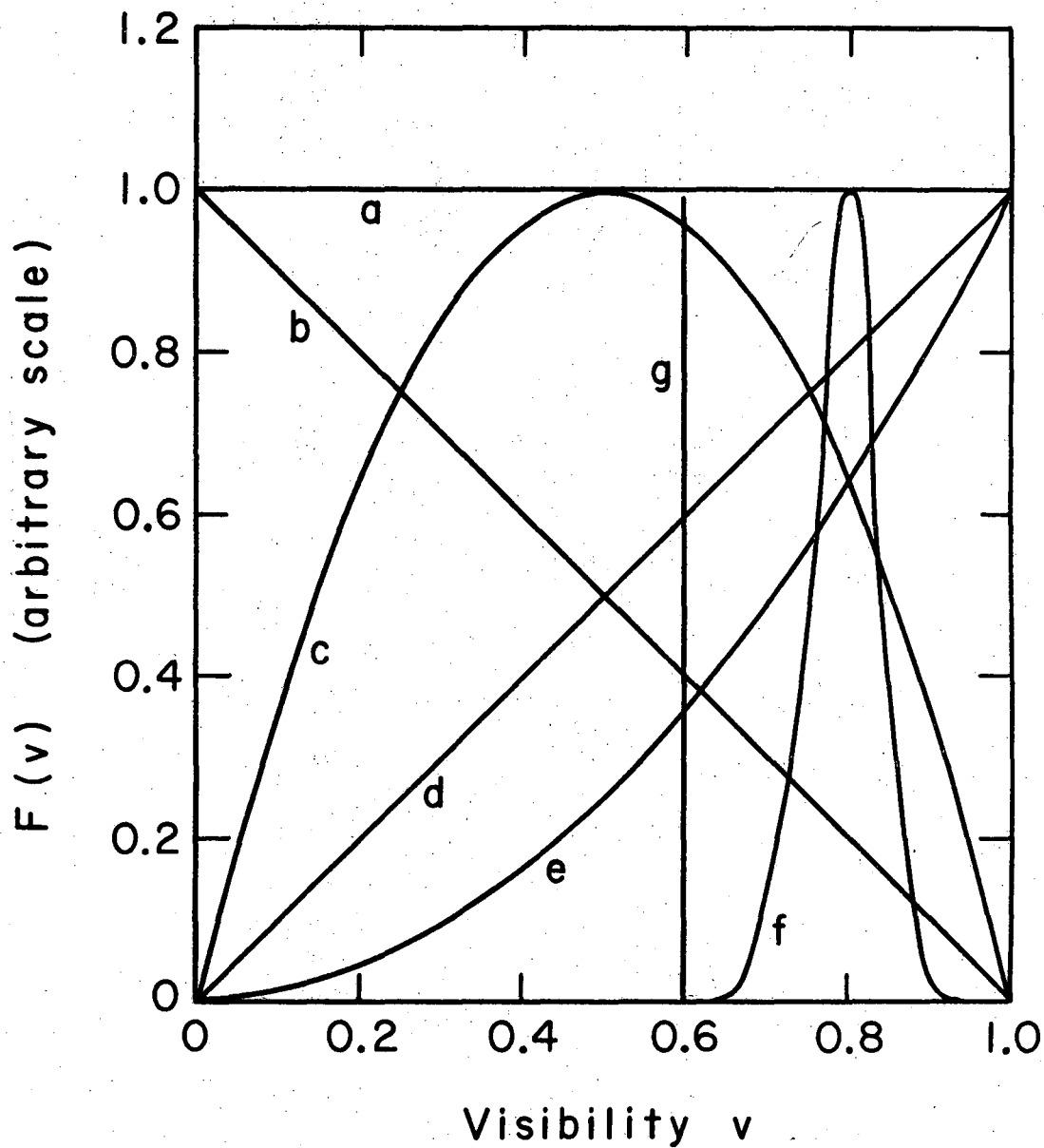
Figure 1. Distribution of μ -Decay Events after 0, 1, 2, 3, and 4 Successive Scans. (Figures A and B drawn to different scales.)
Curve 0: $F(v) = 829v^{1.38}(1-v)^{-0.55}$ = initial distribution estimated as in section 4 by fit of expression (13) to scan data for the 1147 measured events. Curve 1: $F(v)(1-v)$ = distribution remaining after first scan.
Curve 2: $F(v)(1-v)^2$ = distribution remaining after second scan.
etc.

Figure 2. The Function $F(v) = Kv^\alpha(1-v)^\beta$. The versatility of the function is illustrated by curves corresponding to various choices of the parameters α and β . a: $\alpha=\beta=0$. b: $\alpha=0, \beta=1$. c: $\alpha=\beta=1$. d: $\alpha=1, \beta=0$. e: $\alpha=2, \beta=0$. f: $\alpha=80, \beta=20$. g: $(\alpha/\beta)=(3/2), \alpha \rightarrow \infty$.



XBL6812-7354

Fig. 1



XBL6812-7353

Fig. 2

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or*
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.*

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720