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The Optimality of Interest Rate Ceilings and Floors in Lending Contracts: A Note

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In recent years, lenders have developed indexed loans, whose payments reflect changing market conditions. Because indexed loans that follow the market create payment risks for borrowers, as characterized by the phrase "payment shock," it has become commonplace to see adjustable rate loan contracts with payment ceilings (maximums) and floors (minimums).[1]

The current paper addresses the optimality of loans with *fully unrestricted* adjustable payments versus *constrained* adjustable payments.[2] Our analysis, using the principles of stochastic dominance developed by Hadar and Russell (1969) and Hanoch and Levy (1969), evaluates the optimality of payment ceilings and floors in adjustable rate loans. We show that the optimal adjustable rate lending instrument is likely to contain a ceiling and a floor.

Analysis

A fully unrestricted adjustable rate loan refers to a loan where any changes in market interest rates are translated into loan payments. In Figure 1, the curve AB represents the cumulative probability distribution of loan payments for a fully adjustable rate loan without a floor or a ceiling.

In Figure 1, we consider a restricted adjustable rate loan with floor, $-l$, and a ceiling, $-c$ (negative signs denote payment flow from borrowers to lenders). Given the ceiling $-c$, the borrower's expected dollar benefit from the ceiling during high interest rate periods is represented by the area

[1] These payment ceilings and floors come in several forms, the most frequently used relate to annual possible changes and/or lifetime changes for the loan instrument.

[2] Earlier studies that have addressed consumer optimal debt choices between adjustable rate and fixed rate debt instruments include Arvan and Brueckner (1986), Smith (1987), and Dokko and Edelstein (1991).

C in Figure 1. Given the floor $-l$, the borrower's expected dollar cost of increased payments during low interest rate periods is represented by the area L. Let $p(x)$ be the cumulative probability function for loan payments that would occur for a fully adjustable rate loan. We express C and L as[3]

$$C = \int_{-\infty}^{-c} p(x)dx, \tag{1a}$$

$$L = -l - \int_{-l}^{0} p(x)dx. \tag{1b}$$

Note that

$$\frac{\partial C}{\partial c} = p(-c) > 0,$$
$$\frac{\partial L}{\partial l} = -(1 - p(-l)) < 0.$$

If the loan program is self-funding, then

$$(1+\alpha)C = -L \tag{2}$$

where $\alpha(\geq 0)$ is the lender's charge for risk and administrative cost, i.e., α is the charge above actuarial expectations.

The borrower's expected utility under a restricted adjustable rate loan, U^*, can be expressed as

$$U^* = U(Y-c)p(-c) + \int_{-c}^{-l} U(Y+x)f(x)dx + U(Y-l)(1-p(-l))$$

where U is the borrower's utility function with $U' > 0$ and $U'' < 0$, and Y is the borrower's income (treated as exogenous). The first term in the right

[3]For $x < -c$, the household's dollar gain is $-x-c$. Hence, $C = \int_{-\infty}^{-c}(-x-c)f(x)dx$ where $f(x)$ is the probability density function (i.e., $\frac{dp(x)}{dx} = f(x)$). So, $C = -\int_{-\infty}^{-c} x dp(x) - cp(-c) = -\left[\int_{-\infty}^{-c} d(xp(x)) - \int_{-\infty}^{-c} p(x)dx\right] - cp(-c) = -[-cp(-c) - 0] + \int_{-\infty}^{-c} p(x)dx - cp(-c)$, which leads to equation (1a). For $x > -l$, the household's dollar loss is $-x-l$. Hence, $L = \int_{-l}^{0}(-x-l)f(x)dx$, which leads to equation (1b).

hand side of the expected utility function is the expected utility related to the payment ceiling $-c$; the second term is the expected utility generated by the variable payments schedule between the floor and the ceiling; and the third term is the expected utility caused by the floor $-l$. In equilibrium, expected utility maximization requires that

$$U'(Y-c)p(-c) \quad = \quad U'(Y-l)(1-p(-l)). \tag{3}$$

The left hand side of equation (3) is the expected marginal utility gain from "small" increases in the ceiling which equals the expected marginal utility loss from small increases in the floor (the right hand side). Figure 2 is a graphical representation of the borrower's expected utility maximization, assuming a non-corner solution. Borrower equilibrium occurs on the indifference curve II at point $T(-c^*, -l^*)$, tangent to the lender iso-profit function, the curve XX.[4]

Suppose that the loan contract has no minimum payment clause, and, in return for no floor, the lender charges a higher interest rate to increase loan payments by the constant amount δ in all outcomes.[5,6] Figure 3 shows the addition of δ by shifting the AB curve horizontally to the left. The maximum loan payment becomes $-(c+\delta)$. Since the AB curve shifts horizontally, the area C^* in Figure 3 is equal to the area C in Figure 1 such that the area L^* in Figure 3 is equal to the area L in Figure 1. While the lender is indifferent between the two plans, the borrower is not.

[4]For the given value of α, there will be one joint solution for l and c because c is a decreasing function of l in equation (2), and c is an increasing function of l in equation (3).

[5]In our single period analysis, we abstract from prepayment and default risks across different types of loans.

[6]We also assume that α, the lender's charge for risk and administrative cost, is the same for each loan.

L (under the floor-ceiling plan) is collected when the marginal utility is lower than when L^* (under the no floor plan) is collected. By the second degree of stochastic dominance principle of Hadar and Russell (1969) and Hanoch and Levy (1969), L is a utility-superior plan to L^*, implying that an adjustable rate loan with both a floor and a ceiling is preferred to an adjustable rate loan with a ceiling but no floor.

In conclusion, our analysis demonstrates that adjustable rate loans with ceilings and floors are likely to be optimal for many borrowers. In particular, adjustable rate loans with a ceiling and a floor are likely to dominate unrestricted adjustable rate loans as well as adjustable rate loans only with ceilings.

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Figure 1: Payments
Unrestricted versus Ceiling-Floor Loans

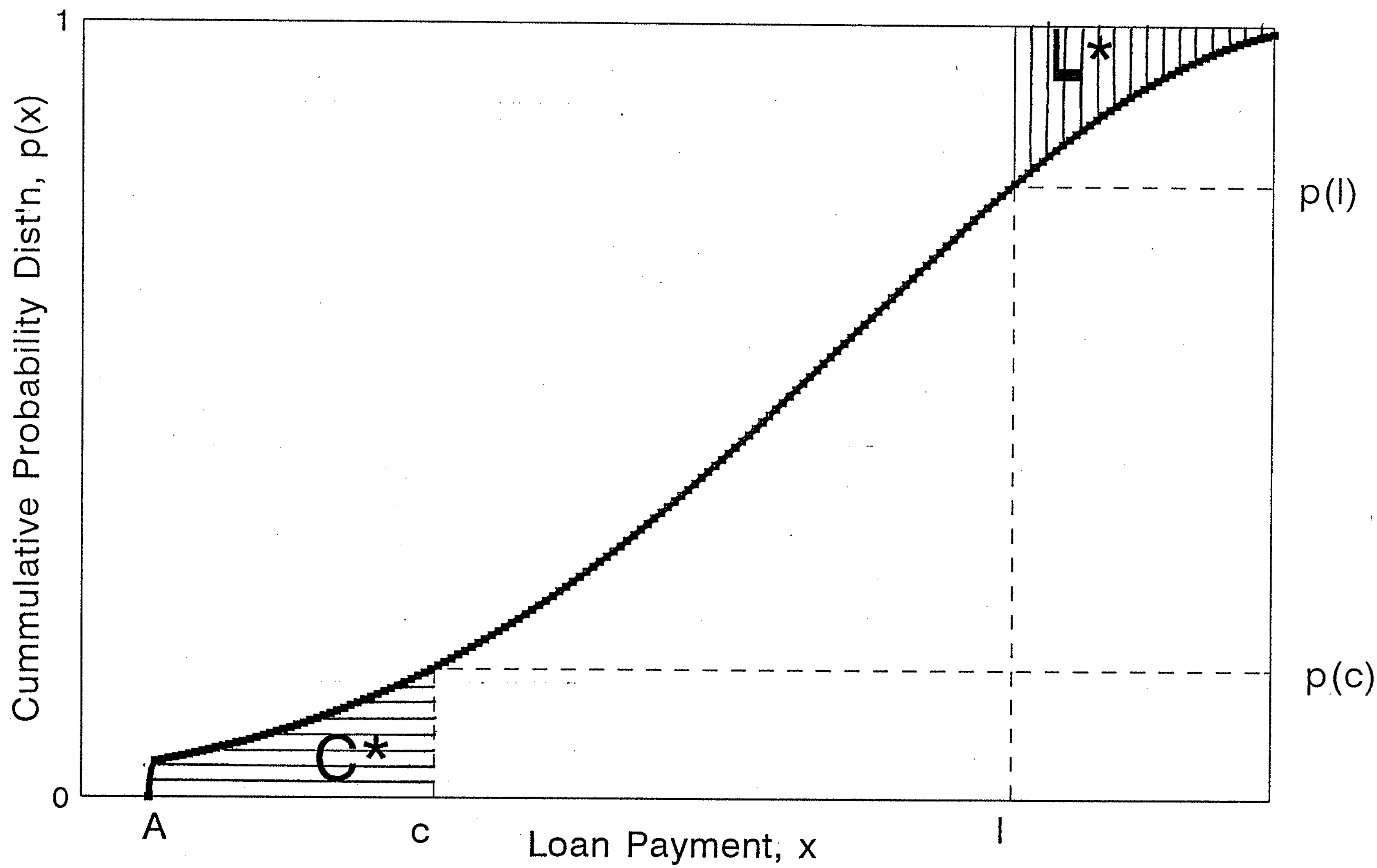
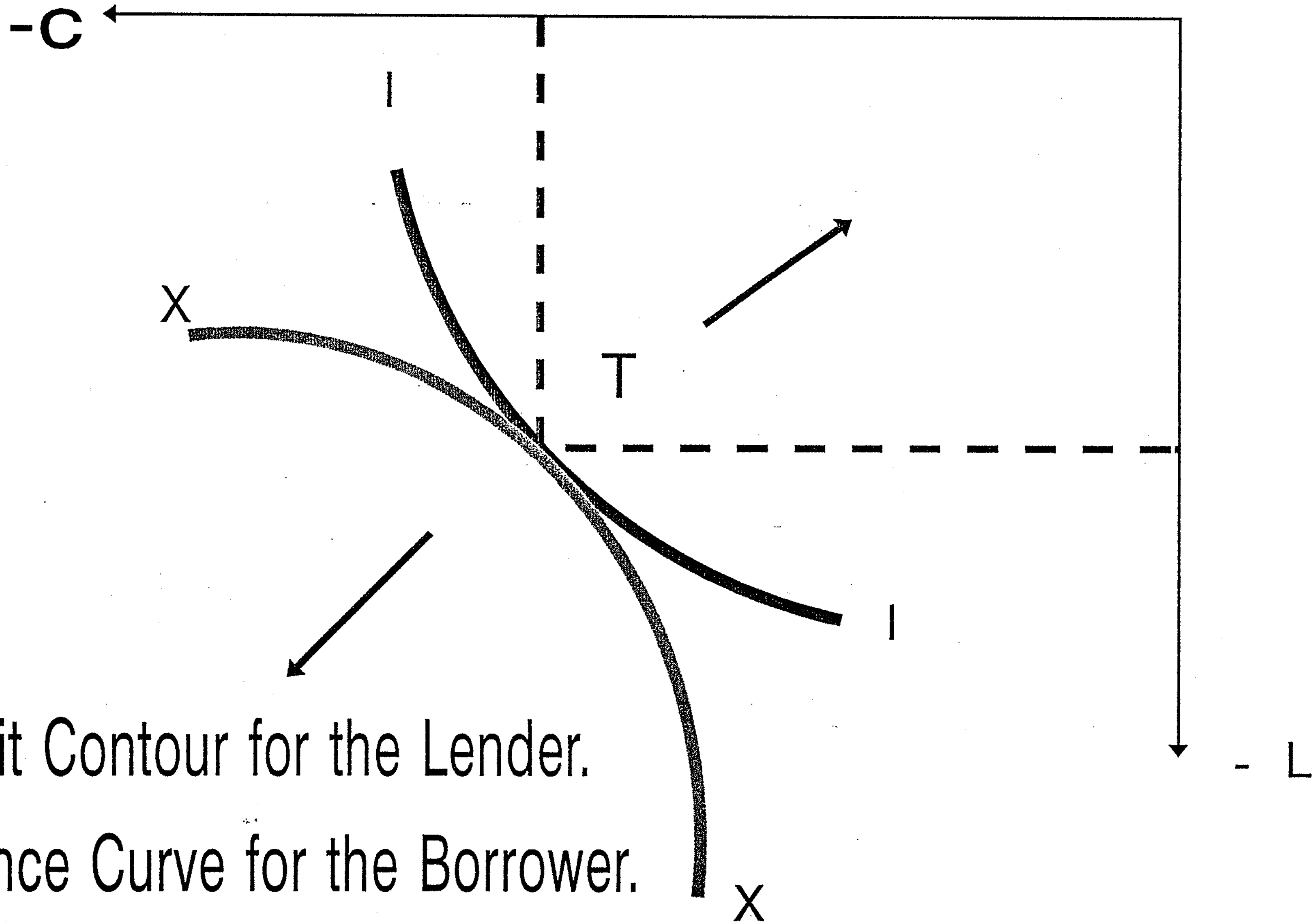


Figure 2: Borrower Expected Utility Maximization



XX: Isoprofit Contour for the Lender.

II: Indifference Curve for the Borrower.

Figure 3: Payments -- Ceiling-Floor versus Ceiling Only Loans