Title
A STUDY OF THE REACTION 1Y*+ p-^ST+K* NEAR THRESHOLD AND AT 1170 MeV/c

Authors
Crawford, Frank S.
Grard, Fernard H.
Smith, Gerald A.

Publication Date
1962-05-11

UCRL-10243

University of California

Ernest O. Lawrence Radiation Laboratory

Berkeley, California

DISCLAIMER

Conference paper - CERN July 4-11, 1962

UCRL-10243

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

A STUDY OF THE REACTION $\pi^{+}+p \rightleftarrows \Sigma^{+}+K^{+}$ NEAR THRESHOLD AND AT 1170 MeV/c

Frank S. Crawford, Jr., Fernand H. Grard, and Gerald A. Smith

May 11, 1962

UCRL-10243

A STUDY OF THE REACTION $\pi^+ + p \rightarrow \Sigma^+ + K^+$ NEAR THRESHOLD AND AT 1170 MeV/c*

Frank S. Crawford, Jr., Fernand H. Grard,† and Gerald A. Smith

Lawrence Radiation Laboratory
University of California
Berkeley, California

May 11, 1962

(to be presented by Fernand H. Grard)

INTRODUCTION

A study of the production of $\Sigma^+ + K^+$ by $\pi^+ + p$ interaction in the threshold region and at 1170 MeV/c has been made with the 72-inch hydrogen bubble chamber. The pictures were obtained from two exposures of the chamber to π^+ beams at the Bevatron. The momentum at the center of the chamber was equal to 1050 MeV/c and 1170 MeV/c respectively. The events were measured on the Franckenstein measuring machines at the Lawrence Radiation Laboratory, and were reconstructed and analysed kinematically by means of the PANG and KICK programs.[1] The details concerning the experimental beam setup, beam contamination, and the experimental biases have been published elsewhere.[2] A total of 525 events (274 at 1050 MeV/c and 251 at 1170 MeV/c) was selected, and the results obtained for the angular distributions and the polarizations are presented in this paper. The data have been analysed in terms of production partial-wave amplitudes and have been compared with those available for Σ^0 and Σ^- production in the same energy region for a test of the charge-independence hypothesis.

---

*Work done under the auspices of the U. S. Atomic Energy Commission.

†Now at: Institut interuniversitaire des Sciences nucléaires, Laboratoire des hautes Énergies, Brussels, Belgium.

RESULTS

Because of energy loss by ionization in the liquid hydrogen of the chamber, the events of the first exposure were produced in the momentum interval extending from 1037 MeV/c through 1065 MeV/c. The momentum resolution was about ±3 MeV/c. In this energy region it has been assumed that the threshold energy dependence for the production amplitudes is still valid, e.g.,[3]

$$f_\ell^{\pm} \propto p_\Sigma^{\ell+1/2}, \qquad (1)$$

where $f_\ell^{\pm}$ represents the partial-wave amplitude for the total angular momentum $J = \ell \pm 1/2$, and p_Σ the Σ^+ momentum in the production center-of-mass system. Therefore the analysis has been made taking into account the position of the production vertex along a direction nearly parallel to the beam represented by the y-coordinate axis of the laboratory reference system. Figure 1 shows the distribution of the production cross sections as a function of the y coordinate of the production vertex. The points have been determined from the observed numbers of events corrected for experimental biases and beam attenuation due to π^+-p interactions in the hydrogen of the chamber. The distribution corresponds to the momentum interval given above, and the momentum decreases linearly at a rate of 0.24 MeV/c per cm. Figure 2 represents the production differential cross section determined with the events of the first half ($y < -7$) and the second half ($y > -7$) of the above y interval, therefore corresponding to average incident momenta of 1058 MeV/c and 1042 MeV/c respectively. These angular distributions could be fitted satisfactorily by polynomials of order 1 in $\cos\theta_\Sigma$ and slightly better by polynomials of order 2. The asymmetry coefficients calculated from the up-down distributions for the $\Sigma^+ \to p + \pi^0$ decay mode are given in Table I for the two average incident momenta given above. These results indicate that s and p waves at least are present in the production mechanism. The data showed no evidence for waves of higher order.

The differential cross section $d\sigma/d\Omega$ and the polarization P_Σ can be written in terms of the non-spin-flip amplitude $g(\theta_\Sigma)$ and the spin-flip amplitude $h(\theta_\Sigma)$ by[4]

$$\frac{d\sigma}{d\Omega} = g(\theta_\Sigma)^2 + h(\theta_\Sigma)^2 ,$$

and

$$P_\Sigma \cdot \frac{d\sigma}{d\Omega} = 2 \,\mathrm{Im}[h^*(\theta_\Sigma)\, g(\theta_\Sigma)] . \qquad (2)$$

where

$$g(\theta_\Sigma) = \sum_{\ell=0}^{\infty} \left[(\ell + 1) f_\ell^+ + \ell f_\ell^-\right] P_\ell (\cos\theta_\Sigma) ,$$

$$h(\theta_\Sigma) = \sin\theta_\Sigma \sum_{\ell=1}^{\infty} \left[f_\ell^+ - f_\ell^-\right] \frac{dP_\ell (\cos\theta_\Sigma)}{d\cos\theta_\Sigma} . \qquad (3)$$

Then the probability for the observation of an event characterized by the production angle θ_Σ and the decay angle ϕ between the decay pion and the normal to the production plane (see Fig. 4) is given by

$$W = \frac{d\sigma}{d\Omega}(\theta_\Sigma) \left[1 - a\, P_\Sigma(\theta_\Sigma) \cos\phi\right] , \qquad (4)$$

where a is the decay asymmetry parameter of the sigma decay (a^0 for π^0 - and a^+ for π^+- decay mode). For the analysis of the events at 1050 MeV/c, only s and p waves have been considered in the expansions (3), and the energy dependence represented by Eq. (1) has been used. Near threshold, the Σ c.m. momentum p_Σ is related to the incident laboratory-system pion momentum p_π by

$$p_\Sigma = 19.63 \times (p_\pi - 1020)^{1/2} \text{ MeV/c}, \qquad (5)$$

and therefore can be simply related to the y coordinate of the production vertex in the chamber. After the momentum-resolution curve for the incident beam has been folded in, the expression for the probability that an individual event will be observed is a function of angles θ_{Σ} and ϕ and the y coordinate of the production vertex. The s and p amplitudes have been determined by the maximum-likelihood method, using this probability function. The results, combined with the knowledge of the average production cross section, are given in Table II. Since in the present analysis only relative phases can be determined, the s amplitude has been assumed real. An alternative set of amplitudes is obtained by reversing the sign of the non-spin-flip phase angle χ_b, and by replacing the spin-flip phase angle χ_c by $(\pi-\chi_c)$, which also fits the data. The amplitudes of Table II have been determined by using for the decay asymmetry parameters α^0 and α^+ the values 1 and 0 respectively. This is in good agreement with the currently available data on Σ decay.[5]

In order to evaluate the degree of confidence that can be given to the maximum-likelihood solution, the differential cross sections, average polarizations, and production cross section as a function of the incident momentum have been calculated from the amplitudes of Table II and compared with the data. The solid curves in Figs. 1 and 2 represent the results of this calculation. The results obtained for the polarization are quoted in Table I. It is obvious that the fitted amplitudes are fully consistent with the experimental data. Finally, the asymmetry coefficient $\alpha^+ \bar{P}_{\Sigma}$ averaged over 1037 through 1065 MeV/c for the decay mode $\Sigma^+ \rightarrow n + \pi^+$ has been found experimentally equal to 0.02±0.13, and therefore in agreement with the zero value assigned to α^+ for the maximum-likelihood fit.

A similar analysis was carried on with the events at 1170 MeV/c, except that the absence of an a priori energy-dependent representation of the amplitudes led to a determination of the amplitudes averaged over the energy interval investigated (1170±15 MeV/c). The angular distribution obtained with the selected events corrected for experimental biases is shown in Fig. 3, along with the results of fits to polynomials of order 4 (solid curve) and of order 5 (dashed curve) in $\cos\theta_\Sigma$. Order 3 fitted the data equally as well as order 4, with $\chi^2 = 30.3$ for 16 degrees of freedom. The fit to a polynomial of order 5 was better, with $\chi^2 = 17.5$ for 15 degrees of freedom. Baltay et al. have studied the same reaction at a somewhat higher incident momentum: 1220 MeV/c.[6] The production angular distribution they obtained can be fitted satisfactorily by a polynomials of order 3 in $\cos\theta_\Sigma$ with coefficients in very close agreement with those we obtained for order 3. Therefore it may be reasonable to assume that the presence of f waves in the production mechanism suggested by our data on the basis of the χ^2 test may have been simulated by a statistical fluctuation. The polarization distribution determined with the events in the mode $\Sigma^+ \rightarrow p + \pi^0$, as represented in Fig. 4, does not permit further comment on the possible presence of f waves.

Although f waves cannot be excluded conclusively, the production amplitudes have been determined by assuming that all partial waves except s, p and d waves could be neglected. This determination was done by the maximum-likelihood method, using the probability function (4) and considering for each event the production angle θ_Σ and the decay angle ϕ. The amplitudes normalized to the observed total cross section of 0.205±0.014 mb, are given in Table III. The production angular distribution calculated from the fitted amplitudes is identical with the solid curve in Fig. 3. The calculated polarization distribution, (solid curve) is represented in Fig. 4, and is consistent with the experimental points. Here again the zero value assigned to α^+ is justified by the fact that the asymmetry coefficient $\alpha^+ \overline{P}_\Sigma$ was found equal to -0.18±0.13 from the up-down distribution of events in the $\Sigma^+ \to n + \pi^+$ decay mode.

In the analysis at 1170 MeV/c the ambiguity in phase angle also exists, and an alternative solution can be obtained by making the same simultaneous transformation on the non-spin-flip and spin-flip phase angles of the Table III solution. An additional ambiguity exists because the set of amplitudes obtained by a Minami transformation followed by the complex conjugation operation gives the same angular distribution and polarization.[47] In an attempt to remove this second kind of ambiguity the s and p amplitudes determined near threshold have been extrapolated to 1170 MeV/c by using the energy dependence given by Eq. (1). It should be emphasized that no Minami ambiguity exists for these amplitudes, since the energy dependence was taken into account in the previous analysis. Therefore, at 1170 MeV/c, the true solution should deviate less from the extrapolated s and p amplitudes than the ambiguous solutions do. Unfortunately, owing to the large estimated errors, it was not possible to decide between the possible sets of amplitudes. The amplitudes resulting from the Minami transformation and the complex conjugation operation are given in Table IV.

With the data at 1050 and 1170 MeV/c the amplitudes were also determined by using different values of α^0. The results showed that the amplitudes are a slowly varying function of α^0. The amplitudes given in Tables II and III would therefore not differ appreciably, within the assigned errors, from those corresponding to the real value of α^0, expected to be close to unity.

The results obtained in other laboratories for the Σ^+K^+ production cross section are plotted in Fig. 5 and are found in good agreement with the values determined in this experiment. The solid curve in Fig. 5 represents the variation of the cross section assuming that the s and p waves are dominant and follow the energy dependence of Eq. (1). This assumption seems to hold at least up to 1090 MeV/c, where data on Σ^0 and Σ^- production exists.[8] The results obtained from the analysis near threshold have been extrapolated up to 1090 MeV/c and found consistent with the Σ^0 and Σ^- data on the basis of the charge-independence hypothesis.

Other data on Σ^0 and Σ^- production in the threshold region have been obtained at 1220 MeV/c by Crawford et al.[9] from which minimum values and errors for the Σ^+ differential cross section allowed by the charge-independence hypothesis have been calculated. These have been represented with their errors as hatched areas in Fig. 3 for comparison with the differential cross section obtained at 1170 MeV/c. If one neglects the effect that could result from the small difference between the incident momenta, the data are not inconsistent with the charge-independence hypothesis.

ACKNOWLEDGMENTS

The authors acknowledge the encouragement and support of Professor Luis W. Alvarez. We also wish to thank Jared Anderson; Dr. Hugo Camerini, Dr. Jack Fry, and Dr. Myron L. Good; and Lester J. Lloyd for their assistance in obtaining the data.

Table I. Values of the asymmetry coefficient $\alpha^{0}\overline{P}_{\Sigma}$

Average incident momentum (MeV/c)	Determined experimentally	Computed with the fitted amplitudes
1042	0.55±0.23	0.66
1058	0.76±0.18	0.70
1170	0.62±0.19	0.55

Table II. Results for the s and p amplitudes near threshold (1050 MeV/c). (Amplitudes are given in $(mb/sr)^{\frac{1}{2}}$, and the Σ c.m. momentum p_Σ in MeV/c.)

$$a = +(5.75 \pm 0.76) \cdot p_\Sigma^{1/2} \cdot 10^{-3}$$

$$b = +(4.31 \pm 1.62) \cdot p_\Sigma^{3/2} \cdot 10^{-5}$$

$$\chi_b = +52^\circ.1 \pm 8^\circ.0$$

$$c = -(3.57 \pm 1.41) \cdot p_\Sigma^{3/2} \cdot 10^{-5}$$

$$\chi_c = +80^\circ.8 \pm 27^\circ.2$$

$f_0^+ \equiv a$, $2f_1^+ + f_1^- \equiv b \exp[i\chi_b]$, $f_1^+ - f_1^- \equiv c \exp[i\chi_c]$.

An alternative set of solutions is obtained by the transformations $\chi_b \to -\chi_b$, $\chi_c \to \pi - \chi_c$. The α^0 and α^+ have been taken as +1 and 0 respectively.

Table III. Results for the s, p, and d amplitudes at 1170 MeV/c. (The amplitudes are given in $(mb/sr)^{\frac{1}{2}}$.)

$$a = +\,0.061 \pm 0.016$$

$$b = +\,0.131 \pm 0.077$$

$$\chi_b = +\,34^0.7 \pm 29^0.2$$

$$c = -\,0.089 \pm 0.110$$

$$\chi_c = +\,20^0.5 \pm 26^0.1$$

$$d = -\,0.062 \pm 0.063$$

$$\chi_d = -\,7^0.4 \pm 60^0.2$$

$$e = -\,0.044 \pm 0.040$$

$$\chi_e = +\,91^0.9 \pm 25^0.8$$

The meaning of the symbols used for the representation of the s and p amplitudes is the same as in Table II. The d amplitudes are: $3f_2^+ + 2f_2^- = d \exp[i\chi_d]$, $f_2^+ - f_2^- = e \exp[i\chi_e]$. An alternative set of solutions exists for the transformations $\chi_b \to -\chi_b$, $\chi_c \to \pi - \chi_c$, $\chi_d \to -\chi_d$, $\chi_e \to \pi - \chi_e$.

Table IV. Possible solutions for the s and p amplitudes at 1170 MeV/c.

	Sol. I	Sol. II	Minami I	Minami II	Sol. A	Sol. B
a =	0.061±0.016		0.102±0.094	0.036	0.083±0.014	
b =	0.131±0.077		0.066±0.066	0.062	0.156±0.060	
χ_b =	34°.7±29°.2	325°.3±29°.2	328°.8±40°.0	301°.3	52°.1±8°.0	307°.9±8°.0
c =	0.089±0.110		0.077		0.129±0.053	
χ_c =	200°.5±26°.1	339°.5±26°.1	227°.5	192°.4	260°.8±27°.2	279°.2±27°.2

Solution I corresponds to the amplitudes given in Table III. Solution II has been obtained from Solution I by the transformation $\chi_b \rightarrow -\chi_b$ and $\chi_c \rightarrow \pi - \chi_c$. The corresponding Minami transform solutions are called Minami I and Minami II respectively. The phases of the Minami solutions have been rotated in order to set the s phase angle to zero, according to the previously adopted convention. Solutions A and B are the result of the extrapolation of ~~the two so~~lutions obtained near threshold and given in Table II, and have been included for comparison. Errors shown on the Minami I solution are typical of errors on all Minami solutions. The amplitudes are given in $(mb/sr)^{\frac{1}{2}}$.

LIST OF REFERENCES

1. A. H. Rosenfeld, in Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959, W. E. Humphrey, "A Description of the PANG Program", Alvarez Group Memos Nos. 111 and 115, Lawrence Radiation Laboratory, Berkeley (unpublished); J. P. Berge, F. T. Solmitz, and H. Taft, Rev. Sci. Instr. 32, 538 (1961).
2. F. Grard and G. A. Smith (to be published in Phys. Rev.); F. S. Crawford, Jr., F. Grard, and G. A. Smith, (to be published in Phys. Rev.).
3. E. P. Wigner, Phys. Rev. 73, 1002, (1948); T. D. Lee, J. Steinberger, G. Feinberg, P. K. Kabir, and C. N. Yang, Phys. Rev. 106, 1367 (1957).
4. M. Nauenberg and A. Pais, Phys. Rev. 123, 1058 (1961).
5. R. L. Cool, B. Cork, J. W. Cronin, and W. A. Wenzel, Phys. Rev. 114, 912 (1959); B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. 120, 1000 (1960); E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters 7, 285 (1961); see also the discussion by M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957).
6. C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. I. Stonehill, and H. D. Taft, Revs. Modern Phys. 33, 374 (1961).
7. S. Minami, Progr. Theoret Phys. (Kyoto) 11, 213 (1954); S. Hayakawa, M. Kawaguchi, and S. Minami Progr. Theoret. Phys. (Kyoto) 11, 332, 1954, and 12, 355 (1954).
8. F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, H. Tanaka, P. Waloschek, V. Zoboli, M. Conversi, P. Franzini, I. Mannelli, R. Santangelo, and V. Silvestrini, Nuovo cimento 10, 468 (1958); F. S. Crawford, Jr.,

(Lawrence Radiation Laboratory, Berkeley), private communication.

9. F. S. Crawford. Jr., R. Douglas, M. Good, G. Kalbfleisch, M. Stevenson, and H. Ticho, Phys. Rev. Letters 3, 394 (1959).

10. A. Berthelot, A. Daudin, O. Goussu, F. Grard, M.-A. Jabiol, F. Levy, C. Lewin, A. Rogozinski, J. Laberrigue-Frolow, C. Ouannes, and L. Vigneron, Nuovo cimento 21, 693 (1961).

11. J. L. Brown, D. A. Glaser, D. I. Meyer, M. L. Perl, and J. Van der Velde, Phys. Rev. 107, 906 (1957).

12. A. R. Erwin, Jr., J. K. Kopp, and A. M. Shapiro, Phys. Rev. 115, 669 (1959).

FIGURE LEGENDS

Fig. 1. Production cross section as a function of position in the chamber, for 274 events in the interval 1037 through 1065 MeV/c. Here $y = -65$ corresponds to an incident pion momentum of 1065±3 MeV/c.

Fig. 2. Angular distributions for the momentum intervals (a) $1050 \leq P \leq 1065$ MeV/c and (b) $1037 \leq P \leq 1050$ MeV/c. The points have been determined from the observed number of events, corrected for experimental biases and beam attenuation. The solid curves represent the angular distribution as given by the s and p amplitudes determined in this experiment.

Fig. 3. The corrected absolute differential cross section of this experiment (1170 MeV/c). The dashed curve represents a fit in powers of $\cos\theta_\Sigma$ through order 5 with a quality of fit given by $\chi^2 = 17.5$ for 15 degrees of freedom. The solid curve is the result of the s, p, and d amplitudes of this experiment with $\chi^2 = 30.3$ for 16 degrees of freedom. The cross-hatched areas represent minimum Σ^+K^+ cross sections and their errors, as allowed by charge independence and the data of Crawford et al. at 1220 MeV/c (see reference 9).

Fig. 4. Decay asymmetry with respect to $\hat{n}$ for $\Sigma^+ \rightarrow p + \pi^0$, based on 95 observed events at 1170 MeV/c. The solid curve represents the prediction of the s, p, and d amplitudes of this experiment with $\chi^2 = 1.6$ for one degree of freedom.

Fig. 5. The total cross sections of this experiment compared with other experimental results. The solid curve has been determined from Eq. (1) by using the s and p amplitudes determined at 1050 MeV/c. The dashed curves represent one standard deviation error, where proper account has been taken of correlations between errors. References to the data, cited from this article, are: ( ) this experiment; (O) Baltay et al., (reference 6); ( ) Berthelot et al., (reference 10); (▲) Brown et al., (reference 11); and (▼) Erwin et al., (reference 12).

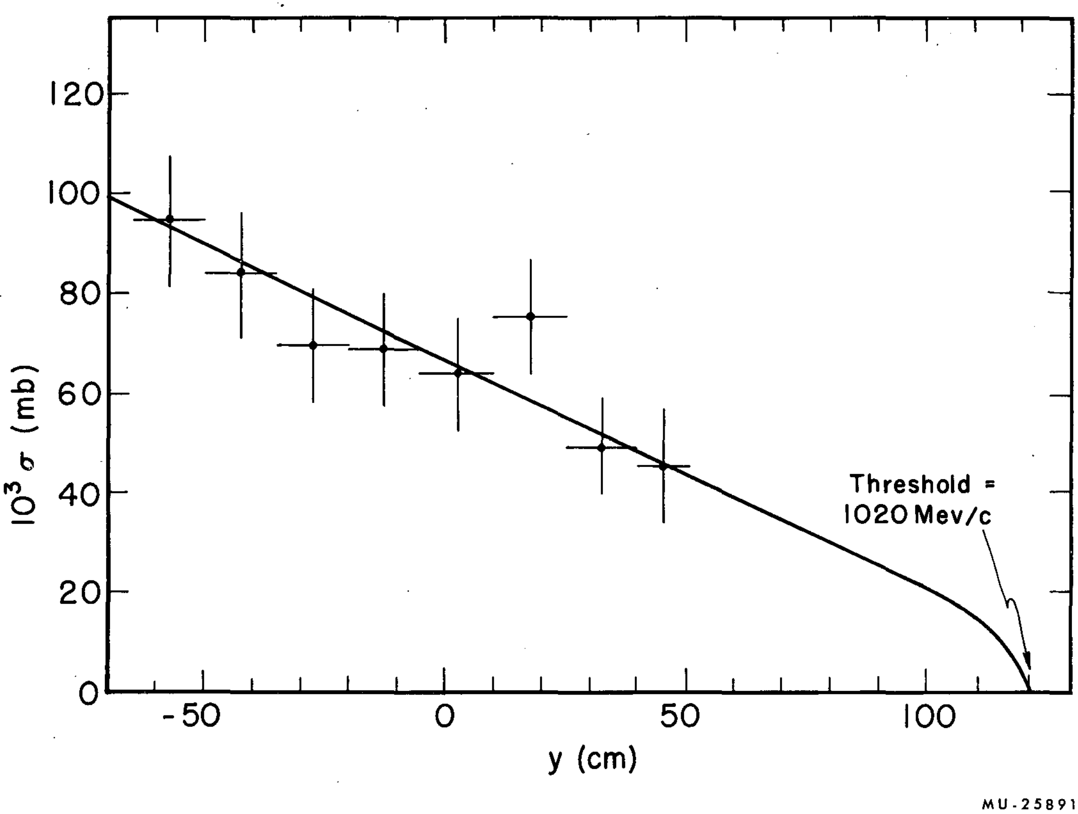
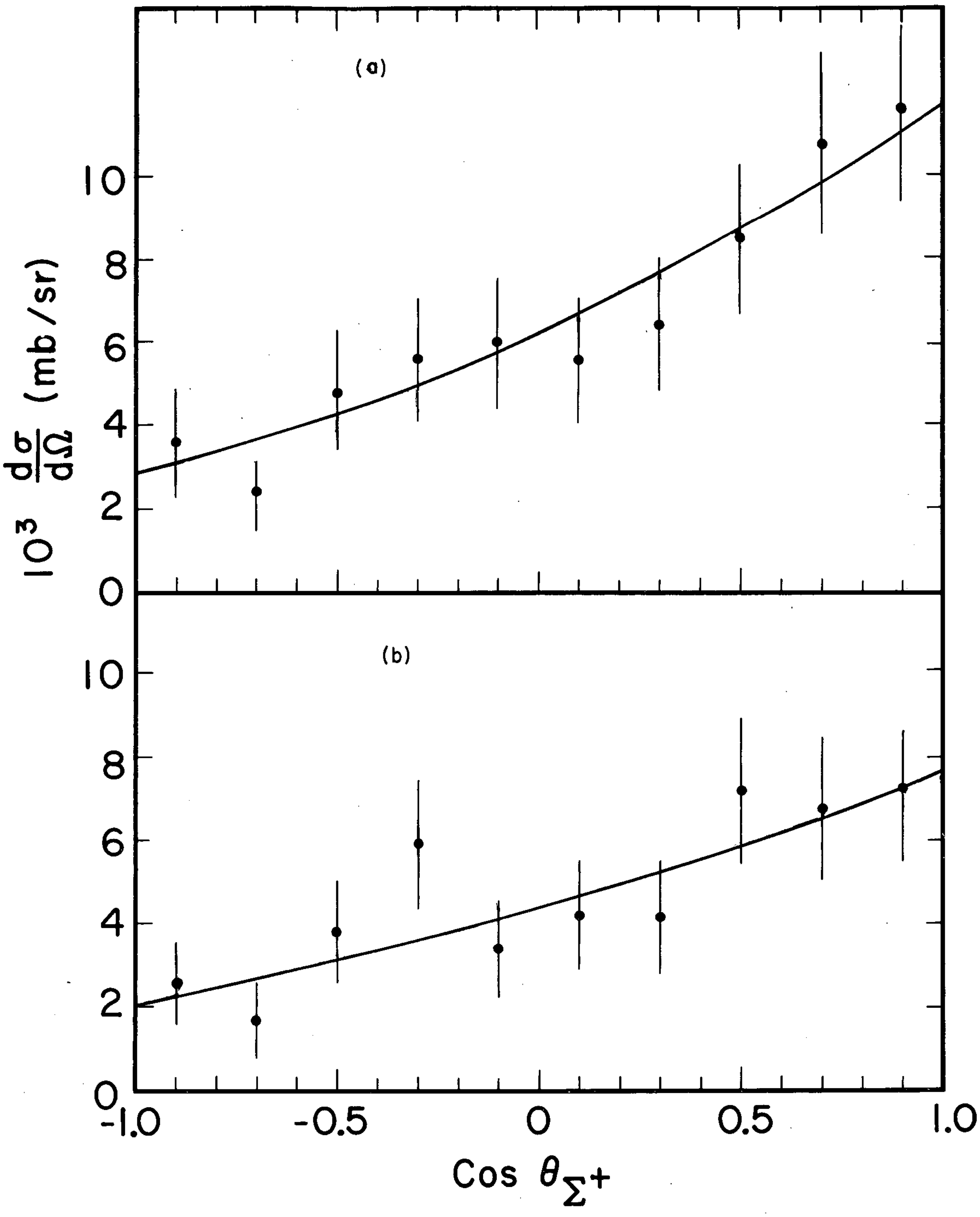
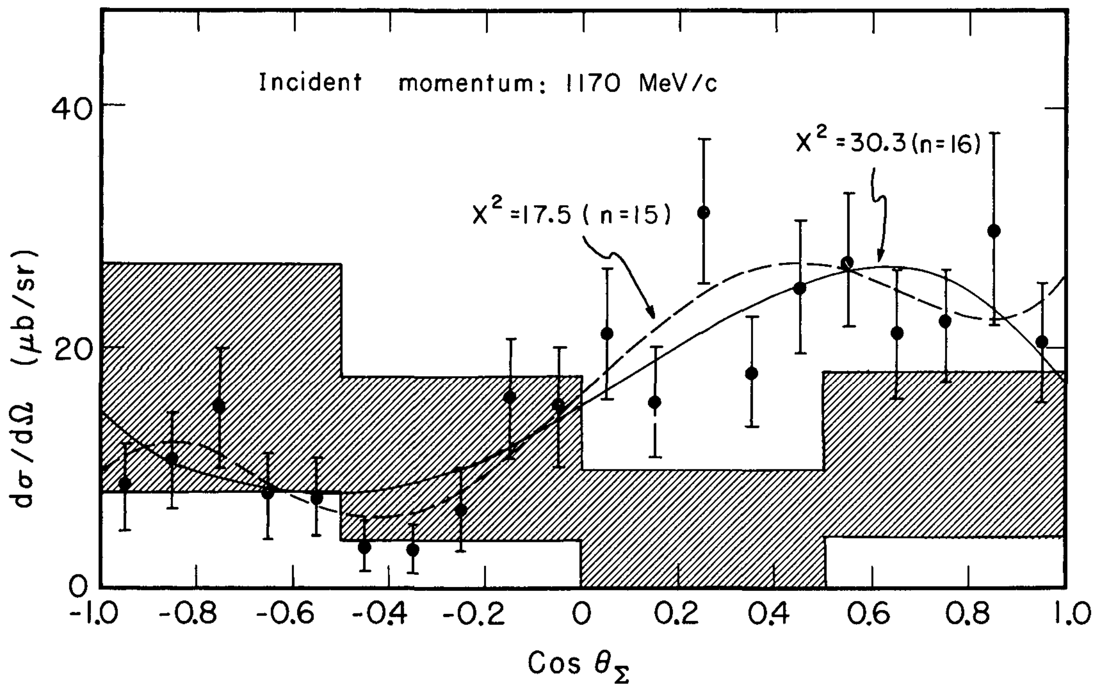

120
100
80
60
40
20
0
$10^3\sigma$ (mb)
-50
0
50
100
y (cm)
Threshold =
1020 Mev/c
MU-25891

Fig. 1



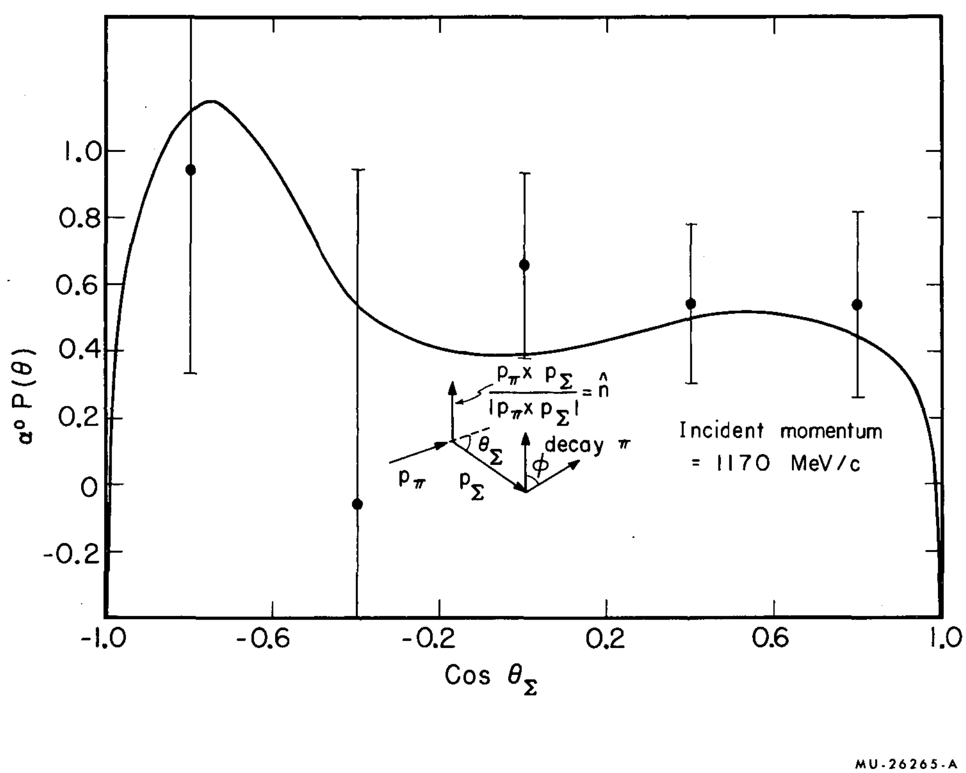
MUB-945-A

Fig. 2



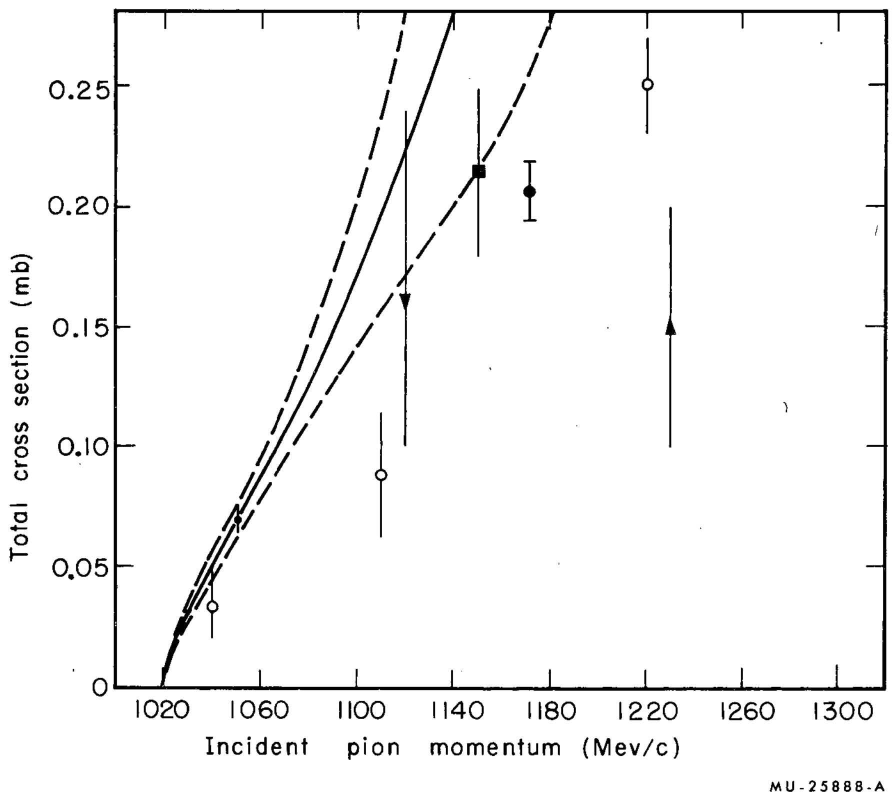
MU-26266-A

Fig. 3



1.0
0.8
0.6
0.4
0.2
0
-0.2
$\alpha^0 P(\theta)$
$\frac{p_\pi \times p_\Sigma}{|p_\pi \times p_\Sigma|} = \hat{n}$
θ_Σ
decay π
ϕ
p_π
p_Σ
Incident momentum
= 1170 MeV/c
-1.0
-0.6
-0.2
0.2
0.6
1.0
Cos θ_Σ
MU-26265-A

Fig. 4



Total cross section (mb)
0.25
0.20
0.15
0.10
0.05
0
1020
1060
1100
1140
1180
1220
1260
1300
Incident pion momentum (Mev/c)
MU-25888-A

Fig. 5