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ABSTRACT

Production of K mesons has been investigated by using "hydrogen-like" $\bar{p}$ annihilations observed in a 30-in. propane bubble chamber exposed to a separate $\bar{p}$ beam of 1.07 Bev/c. The K production among annihilation events is found to be about 9% and is associated with 3.0 ± 1.6 π mesons. The average momentum in the $\bar{p}p$ center-of-mass system is 333 Mev/c for K and 344 Mev/c for π, respectively. The momentum spectrum of K and π mesons fit well with those computed from the covariant phase-space factor. The angular correlations between π-π, K-$\bar{K}$, and π-K show a characteristic backward peaking, with a backward-to-forward ratio of $B/F = 2.2 \pm 0.9$, 2.8 ± 1.6, 1.2 ± 0.3, respectively. The ratios predicted by the conservation of energy and momentum are 1.43, 3.26 and 2.10. An attempt has been made to compare the experimental data with statistical-model calculations using two parameters, Ω_K and Ω_π, corresponding to the interaction volumes of K and π meson. The best fit values are $\Omega_\pi \cong 10$ (in units of $\Omega_0 = 4\pi/3 \cdot 1/\mu^3$) and $\Omega_K/\Omega_\pi \cong 0.3$. These values fit also low-energy $\bar{p}$ annihilation data. A comparison with the isobar model excludes the assumption of an isobar state of 3 pion masses.

STATISTICAL-MODEL ANALYSIS OF K MESONS PRODUCTION BY $\bar{p}$ ANNIHILATION*

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I. INTRODUCTION

In a recent investigation of $\bar{p}$ interactions at 1.07 Bev/c carried out with a 30-in. propane chamber at Berkeley, 115 events have been observed with emission of strange particles.[1] Among these events, 35 cases are found to be "hydrogen-like" and give rise altogether to 53 charged or neutral K mesons. Although the present data are still meager, nonetheless they provide, to date, the only sizeable sample to allow a tentative analysis of K-meson production by $\bar{p}$ annihilation. An attempt is therefore made to investigate this problem in detail in terms of the statistical model.

In this paper we shall compare the momentum spectra of K mesons and associated π mesons emitted in $\bar{p}$-p annihilations, and also their angular distributions and angular correlations which we have calculated using covariant phase-space integrals. To evaluate the percentage of K-meson production, we have introduced an interaction volume Ω_K in addition to the usual π interaction volume Ω_π. The latter has been the only parameter used thus far to fit the Fermi model with π- emission data.[2] The model thus modified gives a satisfactory interpretation of K-meson production observed in our experiment as well as that at low energy.

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II. EXPERIMENTAL DATA

In order to avoid uncertainties caused by the presence of the Fermi momentum in case of $\bar{p}$ annihilations with bound nucleons, we limit ourselves only to "hydrogen-like" events. The criteria used to select such events are (a) conservation of charge and (b) absence of nuclear excitation. From annihilation cross sections on hydrogen and carbon measured by counter experiments,[3] we estimate that about 80% of these selected events are actually pure hydrogen events. We have accepted 35 cases, giving 52 measurable K-mesons. The method used to identify the K mesons will be discussed in a separate paper.[1] We list in Table I the charge states of the K mesons used in the present analysis. Regarding the neutral K mesons, we have classified them phenomenologically as either θ_1--short-lived mesons decaying via $\pi^+ + \pi^-$--or θ--those escaping observation inside the chamber (i.e., those of the θ_1 decaying into $\pi^0 + \pi^0$ and long-lived θ_2).

Table I

Charge states of K mesons produced by $\bar{p}$-p annihilations

Charge state		Number of cases
$K^+ \bar{K}^0$	$K^+ \theta_1$	5
	$K^+ \theta$	2
$K^+ \bar{K}$		6
$K^0 \bar{K}^0$	$\theta_1 \theta_1$	4
	$\theta_1 \theta$	14
	$\theta \theta$	Undetectable
$\bar{K}^- K^0$	$K^- \theta_1$	3
	$K^- \theta$	1

The multiplicity of charged pions associated with K mesons is shown in Fig. 1. The average number of charged pions is

$$n_{\pm} = 1.9 \pm 1.1 .$$

It is to be noted that, unless otherwise specified, all errors quoted in this paper are r. m. s. deviations from the mean.

In order to estimate the total number of charged and neutral π mesons involved in K-meson production, we have estimated the multiplicity of π from the missing mass computed from those events in which both K mesons are observed. The average number of π mesons thus estimated is

$$n_{\pm 0} = 3.4 \pm 1.2 .$$

Combining this result with the average number of charged pions given above, we estimate the average number of charged and neutral π mesons associated with the K mesons produced by $\bar{p}$ annihilation at our energy to be

$$n = 3.0 \pm 1.6 \; .$$

Assuming a normal distribution for n, we find for the percentage for each type of annihilation leading to K emission by the reaction

$$\bar{p} + p \longrightarrow K + \bar{K} + n\pi \; , \qquad (1)$$

the values shown in Table II.

Table II

Frequency of $p + p \longrightarrow K + \bar{K} + n\pi$

n =	0	1	2	3	4	5
%	0.5	3.5	37	46	12.5	0.5

III. MOMENTUM SPECTRA OF K MESONS AND ASSOCIATED π MESONS

The momenta of all 52 K mesons observed in 35 "hydrogen-like" events have been measured. Among the associated charged mesons we have measurements of 61 cases; the remaining three cases failed to give a good measurement (kinks, poor quality of picture, etc.). The average momenta of the K meson and the associated π meson in the $\bar{p}$-p center-of-mass (c.m.) system are:

$$\bar{P}_{K}^{*} = 333 \pm 254 \text{ Mev/c}$$

$$\bar{P}_{\pi}^{*} = 344 \pm 226 \text{ Mev/c} .$$

As is to be expected, within experimental error the average momenta of the K and associated π mesons are equal.

The momentum distributions for K and π mesons are shown in Figs. 2 and 3, together with those calculated from the covariant phase-space factor for reaction (1) with n = 1, 2, 3 and 4. Leaving aside that part of the integral over space coordinates, we have for the covariant phase space:

$$\rho_{2Kn\pi} = \int \cdots \int \prod_i \frac{m_i d^3 \vec{P}_i}{E_i} (2\pi)^4 \delta(\Sigma \vec{P}_i)\, \delta(E_0 - \Sigma E_i), \qquad (2)$$

where

$E_i = (P_i^{\,2} + m_i^{\,2})^{1/2}$ and $c = 1$. The results are plotted in the same figure of the observed distribution, with each spectrum normalized to the same area as the histogram under consideration. The case corresponding to n = 5 has not been computed because of the extreme length of time required by the computer to perform successive integrals. Since its contribution to the over-all K-meson production amounts to only less than 1% in the present experiment (see Table II), we shall leave the case n = 5 aside in the following considerations.

A comparison of calculated spectra with the histograms representing the experimental results indicates that, in case of K-meson momentum distribution as well as that of the associated pion, the spectra corresponding to associated π mesons $n = 3$ are already very close to the observed experimental distribution. If we combine the computed spectra according to the frequency of each reaction as indicated in Table II, we obtain the resultant spectrum shown in Figs. 4 and 5. The agreement with experimental data is very satisfactory.

IV. INTERACTION VOLUMES Ω_π AND Ω_K

In following Fermi's approach to the statistical model, it has customarily been assumed that the matrix element of reaction (1) is a constant and is expressed in terms of a parameter Ω having the dimensions of a volume related to the coupling constant. Because the K meson and π meson may have different coupling constants, it is natural to intsoduce in the matrix element of the reaction (1) two specific parameters Ω_π and Ω_K. Therefore, for the transition probability of reaction (1) we write

$$W_{2Kn\pi} = c\ \frac{\bar{S}\,(I)}{2!\,n!} \cdot \frac{(\Omega_K)^2}{(2\pi)^6} \cdot \frac{(\Omega_\pi)^n}{(2\pi)^{2n}} \cdot \rho_{2Kn\pi} \qquad (3)$$

where $\bar{S}$ (I) designates the average value of the I-spin statistical weight for total $I = 0$ and 1 of the secondary particles, $\rho_{2Kn\pi}$ is the covariant phase space factor in the final state defined in Eq. (2), and $\hbar = c = 1$.

The π interaction volume can be estimated by adjusting Ω_π to fit the observed multiplicity of pions associated with K production. Trials with different values of Ω_π (in units of $\Omega_0 = 4\pi/3\ \mu^3$) have been made; the results of the predicted pion multiplicity are shown in Fig. 6. The best-fit value is found to be $\Omega_\pi \simeq 10$.

It is to be noted that our estimate of Ω_π is consistent with values determined by previous authors[2,4] for $\bar{p}$ annihilation without K production and at a different energy. In order to estimate Ω_K, we have to compare the transition probability of Eq. (3) to that of $\bar{p}$-p annihilations without K production. The simplest way is to consider the ratio of these two processes leading to the same number of secondary particles. We have, for instance,

$$\frac{W_{2K3\pi}}{W_{5\pi}} = \frac{5!}{2!\,3!} \cdot \frac{\overline{S}_{2K3\pi}(I)}{\overline{S}_{5\pi}(I)} \cdot \frac{(\Omega_K)^2}{(\Omega_\pi)^2} \cdot \frac{\rho_{2K3\pi}}{\rho_{5\pi}} \; . \qquad (4)$$

At the energy of our experiment we find that K production by $\bar{p}$ annihilation is about 5.0 ± 1.5 mb; the total cross section for annihilation is 58 ± 18 mb; the percentage of the 2K3π reaction is 46% (Table II); and the percentage of 5π annihilation is 29%. This leads to

$$\frac{\Omega_K}{\Omega_\pi} \simeq 0.3 \; .$$

That this ratio is less than one reflects the fact that the K-meson coupling constant must be smaller than that for the π meson. This is consistent with the current view that the K-meson interaction is weaker than that of the π meson.

It should be pointed out that if the parameter Ω_K here introduced had also been taken into account in the previous estimation of K production by $\bar{p}$ annihilation, one should have obtained a smaller value for the K production than previously estimated. This point is discussed in detail in Section VII.

V. ANGULAR CORRELATION BETWEEN π-π, K-$\bar{K}$ and π-K PAIRS

In Fig. 7 we have plotted the distributions of angles in the $\bar{p}$-p c.m. system of the pairs of secondary particles such as π-π, K-$\bar{K}$, and π-K. Because of small statistics available in the present investigation, no attempt has been made to distinguish between the charge states of the assorted pairs. All three distributions show a characteristic feature of a tendency toward large angles. This property, as pointed out by Kalogeropoulos,[5] reflects a consequence of the constraint imposed by the energy-momentum conservation on the particles emitted by the $\bar{p}$ annihilation. The observed backward-to-forward ratios are:

$$(B/F)_{exp} = 2.2 \pm 0.9 \text{ for } \pi\text{-}\pi$$
$$= 2.8 \pm 1.6 \text{ for } K\text{-}\bar{K}$$
$$= 1.2 \pm 0.3 \text{ for } \pi\text{-}K .$$

The errors here quoted are statistical. It is interesting to compare these ratios to those predicted by conservation of energy-momentum, for if eventually there is any significant departure of the observed values from those predicted, then this would indicate the presence of some other effect due to the mutual interaction of the particles. Calculations have been made for the three distributions under consideration using the covariant phase-space factor of Eq. (2) for K-meson production involving $n = 2, 3$ and 4 π mesons. The results are shown in Fig. 8. We note that the predicted shape of the distributions changes little with the number of associated π mesons; however, the predicted backward-to-forward ratio is rather sensitive to the number of associated π mesons. If we combine, for each case of π-π, K-$\bar{K}$, the computed curves according to the percentage of the corresponding reaction, we obtain the resultant curves shown in Fig. 7 and predicted backward-to-forward ratios as follows:

$$(B/F)_{\text{predicted}} = 1.43 \text{ for } \pi\text{-}\pi$$
$$= 3.26 \text{ for } K\text{-}\overline{K}$$
$$= 2.10 \text{ for } K\text{-}\pi \ .$$

A comparison with the experimental data indicates that the observed ratios for π-π, K-$\overline{K}$, and K-π deviate respectively about 1, 1/3, and 3 standard deviations from the predicted values. Since the statistics of samples available for the present analysis are rather small, and in addition there is some uncertainty in the evaluation of the predicted ratio due mainly to the uncertainties of the weighting factors (Table II) used to combine different reactions of the K-meson production, therefore, from the statistical point of view, these deviations are probably not significant. We are unable to decide whether there is any definite effect among the pairs of particles other than that predicted by conservation of energy and momentum.

VI. ANGULAR DISTRIBUTIONS OF K AND π MESONS

The histograms in Fig. 9 present the angular distributions of K mesons and associated charged π mesons, the emission angle θ^* being referred to the $\overline{p}$-p c.m. system with respect to the impinging $\overline{p}$-p direction. Both distributions are fairly symmetric with respect to the $\theta^* = \pi/2$ direction and are expected to be isotropic on the basis of the Fermi model. If we fold these distributions about $\theta^* = \pi/2$ and compute the second-order and third-order moments for these distributions, we find for the K distribution $\mu_2 = 0.24$ and $\mu_3 = 0.12$, and for the π distribution $\mu_2 = 0.14$ and $\mu_3 = 0.05$. For an isotropic distribution, we should expect $\mu_2^0 = 0.31$ and $\mu_3^0 = 0.11$. Since the number of samples are rather small, we can hardly tell if both distributions are actually different from the expected isotropic distribution.

However, it is of interest to bear in mind that the angular distribution provides a means of test for the validity of the basic assumption on the matrix element of the Fermi model, and that it is worthwhile to examine other possibilities. For instance, in a recent approach by Cook et al., attempts have been made to include in the matrix element the angular-momentum states of the secondary particles.[6] According to their results, the momentum spectrum remains essentially the same as that predicted by the Fermi model, while the angular distribution may assume a different shape. In this case it requires a better statistical data to discriminate the details of the distribution in order to settle this point.

VII. COMPARISON WITH LOW-ENERGY DATA

In this section we shall devote ourselves to an analysis of data on $\bar{p}$ annihilations at low energy. We shall attempt to investigate whether it is possible to interpret in a consistent way these data in terms of the parameters $\Omega_\pi = 10$ and $\Omega_K/\Omega_\pi = 0.3$ which we have determined from our present experiment.

For this purpose we must determine first if the value $\Omega_\pi = 10$ gives also a good fit to the π multiplicity observed in $\bar{p}+p \rightarrow n\pi$ at rest. We shall use the hydrogen-bubble-chamber data obtained by Horwitz et al.[4] According to their results, the average π multiplicity is $n = 4.9 \pm 0.6$. The value $\Omega_\pi = 10$ we have obtained from $\bar{p}$ annihilation with K production at our energy is comparable to the value obtained by those authors using calculations based on the classical phase-space factor. It is further to be noted that a recent analysis of the low-energy hydrogen-bubble-chamber data by Bepin Desai[7] using the covariant phase-space factor leads essentially to the same result.

Once we have fixed the parameter Ω_π, we can compute the pion multiplicity associated with K production by $\bar{p}$ annihilation at rest, assuming $\Omega_K/\Omega_\pi = 0.3$. The results are listed in Table III.

Table III

Computed pion multiplicity associated with K mesons in $\bar{p}$ annihilation at rest				
Pion multiplicity n	1	2	3	4
Percentage	2.3	31.7	65.0	1.0

We then compute the percentage of K production among $\bar{p}$ annihilations and find about 5% which is to be compared with experimental data.

Regarding the low-energy K-production data, we content ourselves with the results obtained by Agnew et al. in a 30-in. propane chamber.[8] The data consist of 17 K mesons emitted by $\bar{p}$ annihilations at energies ranging from 26 to 200 Mev. Figure 10 shows the K momentum spectrum; a comparison with the computed spectra according to Eq. (2) indicates that the average number of pions associated with K mesons is probably between two and three. This seems smaller than the number we have observed (see Section 2). The difference is quite conceivable since the energy released by $\bar{p}$ annihilation differs possibly by some 150 Mev between the two experiments. The percentage of K-meson production estimated by these authors is about $4.0 \pm 1.2\%$, which is consistent with the value we have estimated according to the statistical model using the same parameters determined from our experimental data. However, the agreement is only qualitative, since we have not singled out the proton events and since most of the annihilations are not at rest.

VIII. COMPARISON WITH ISOBAR MODEL

In an attempt to overcome the difficulty arising from the large values for the interaction volume found in $\bar{p}$ annihilation experiments, several authors have proposed to modify the Fermi model by assuming a π-π resonant state.[20] It is of interest to compare the present data on K production with predictions of such a model.

Assuming an isobar σ composed of three π mesons, we substitute the following for reaction (1) for n = 3 and 4:

$$\bar{p} + p \rightarrow K + \bar{K} + \sigma$$

$$K + \bar{K} + \pi + \sigma$$

The momentum spectrum for the K meson normalized to the same area as in Fig. 2, is shown in Fig. 11. If we combine those spectra together with the two corresponding to n = 1 and 2 associated π mesons of previous calculations (see Section 3), we obtain the resultant K momentum spectrum shown in Fig. 12. A comparison with the histogram indicates that there is a significant departure of the experimental data from the isobaric-model prediction. Consequently the assumption of an isobaric state corresponding to three pion masses is ruled out by our present data.

Nevertheless, the case of an isobar of four π mesons is still to be considered. Because of small percentages of K production associated with n = 4 π mesons, the difference between the predictions of the Fermi and isobar models is small. Consequently our statistics are not sufficient to investigate further this point.

IX. CONCLUSION

From our analysis of $\bar{p}$ annihilation we conclude that the experimental data of $\bar{p}$ annihilations at our energy, i.e. ~ 480 Mev, and those at low energy are both consistent with the predictions of the Fermi statistical model in terms of two parameters, $\Omega_\pi \simeq 10$ and $\Omega_K / \Omega_\pi \simeq 0.3$. This is mainly because of the large phase-space volume which constitutes the dominant factor in the expression of transition probability. The fact that the K-meson interaction volume is estimated to be smaller than that of the π meson suggests that the K-meson coupling constant must be weaker than that of the π meson. However, it is still difficult to understand the physical meaning of the large value of the interaction volume Ω_π.

Our data on K production rule out the isobar model if we assume an isobar of mass equal to three π mesons.

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FOOTNOTES

1. G. Goldhaber and S. Goldhaber, Lawrence Radiation Laboratory, unpublished work.
2. For details of discussion on this subject, see the following articles: Barkas, Birge, Chupp, Ekspong, Goldhaber, Goldhaber, Heckman, Perkins, Sandweiss, Segré, Smith, Stork, van Rossum, Amaldi, Baroni, Castagnoli, Franzinetti, and Manfredini, Phys. Rev. 105, 1037 (1957); Chamberlain, Goldhaber, Jauneau, Kalogeropoulos, Segré, and Silberberg, Phys. Rev. 113, 1615 (1959).
3. Elioff, Agnew, Chamberlain, Steiner, Wiegand, and Ypsilantis, Phys. Rev. Letters 3, 285 (1959).
4. Horwitz, Miller, Murray, and Tripp, Phys. Rev. 115, 472 (1959).
5. Theodore E. Kalogeropoulos, Study of the Antiproton Annihilation Process in Complex Nuclei (Thesis), UCRL-8677, March 6, 1959.
6. L. F. Cook, Jr. and J. V. Lepore, Multiple Meson Production in Nucleon-Antinucleon Annihilations, UCRL-8841 Rev, December 1959.
7. Bipin R. Desai, Pion Multiplicity in Nucleon-Antinucleon Annihilation, UCRL-9024, January 1960.
8. Agnew, Elioff, Fowler, Gilly, Lander, Oswald, Powell, Segré, Steiner, White, Wiegand, and Ypsilantis, Phys. Rev. 110, 994; Lewis E. Agnew, Jr., Antiproton Interactions in Hydrogen Below 200 Mev (Thesis), UCRL-8785, July 23, 1959.
9. E. Ebelle, Nuovo cimento 8, 619 (1958); T. Goto, Nuovo cimento 8, 625 (1958); F. Cerrulus, Nuovo cimento 14, 827 (1959).

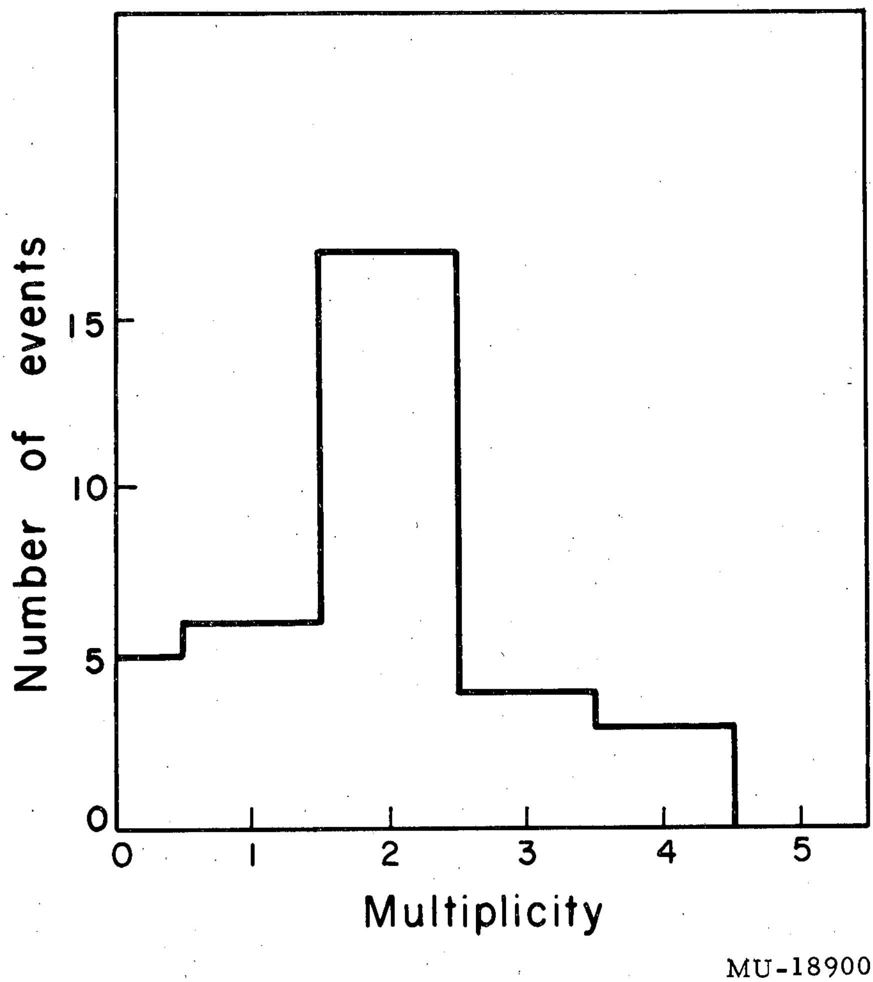

Fig. 1. Histogram of the multiplicity of charged π mesons associated with K-meson production.

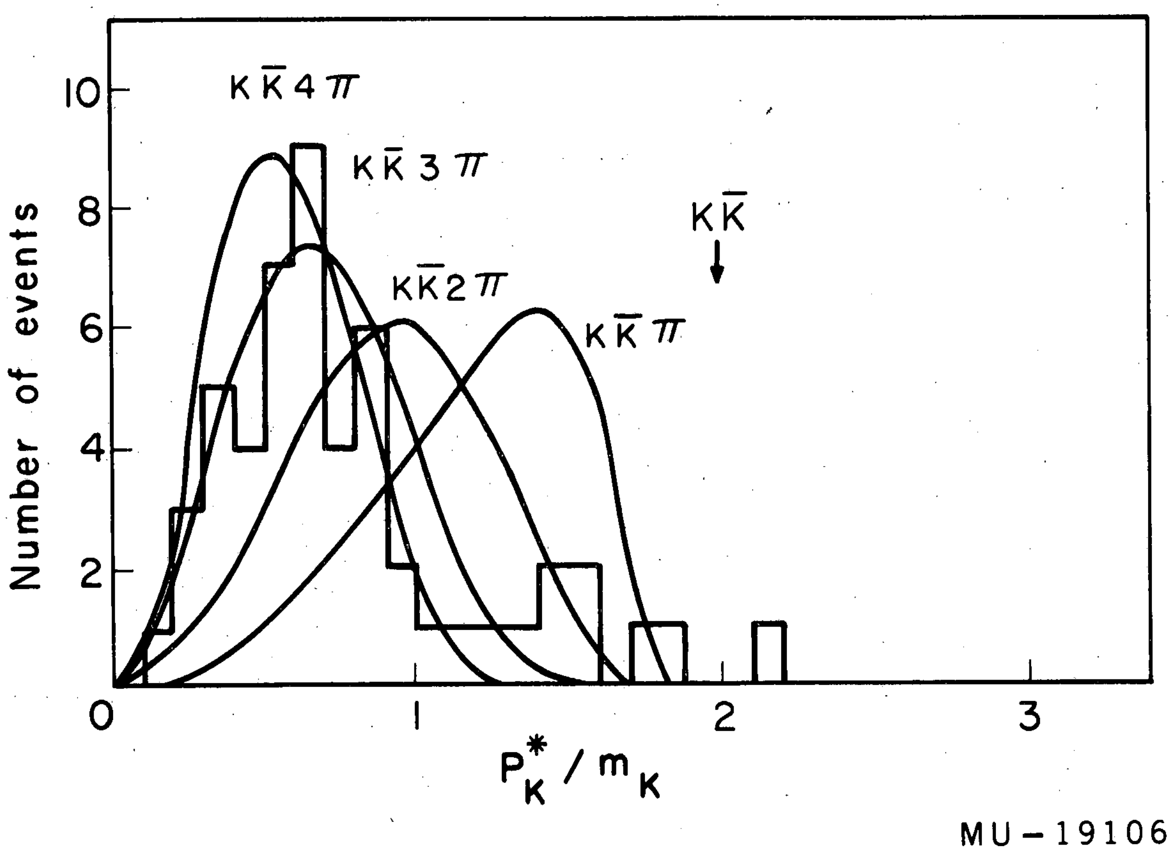

Fig. 2. Momentum spectrum for K mesons in the $\bar{p}$-p c.m. system. The curves represent spectra computed according to the covariant phase-space factor and are normalized to cover the same area as the histogram.

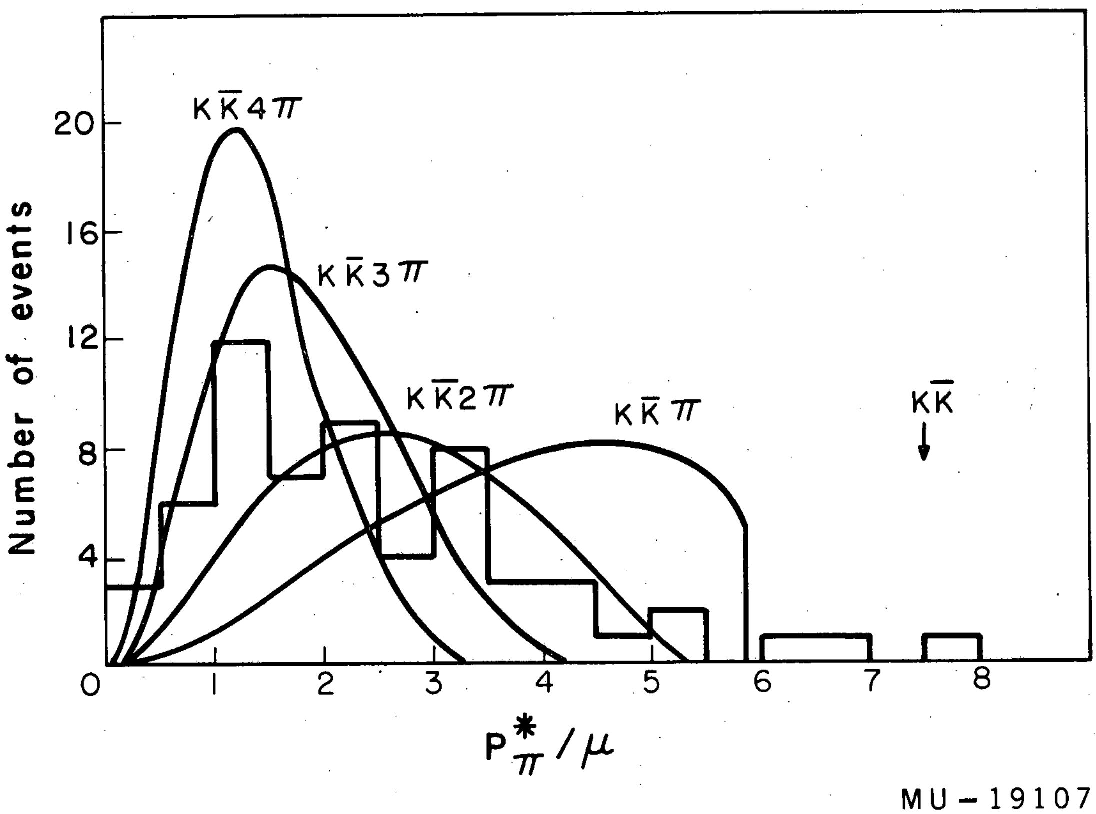

Fig. 3. Momentum spectrum for π mesons in the $\bar{p}$-p c.m. system. The curves represent spectra computed according to the covariant phase-space factor and are normalized to cover the same area as the histogram.

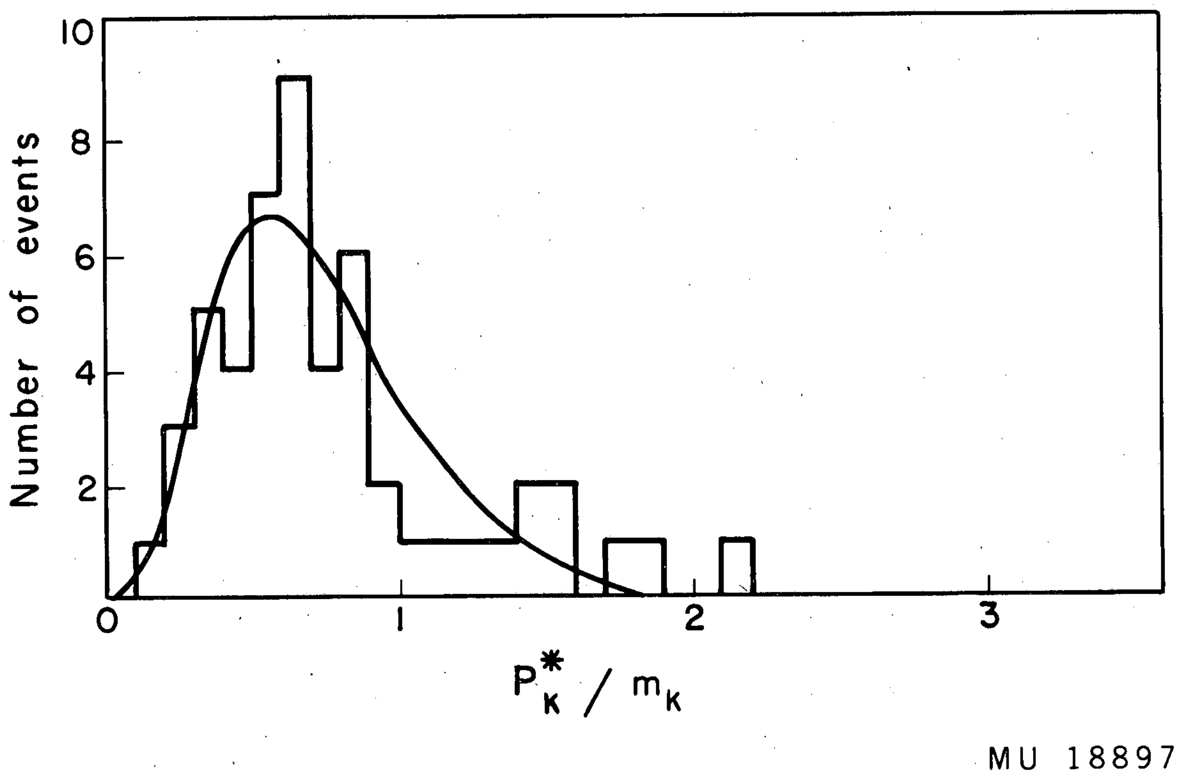

Fig. 4. Comparison of the K-meson momentum spectrum with that predicted by the covariant phase-space factor. The curve is the combination of those in Fig. 2. according to the percentage of reaction (1) with n = 1, 2, 3, and 4.

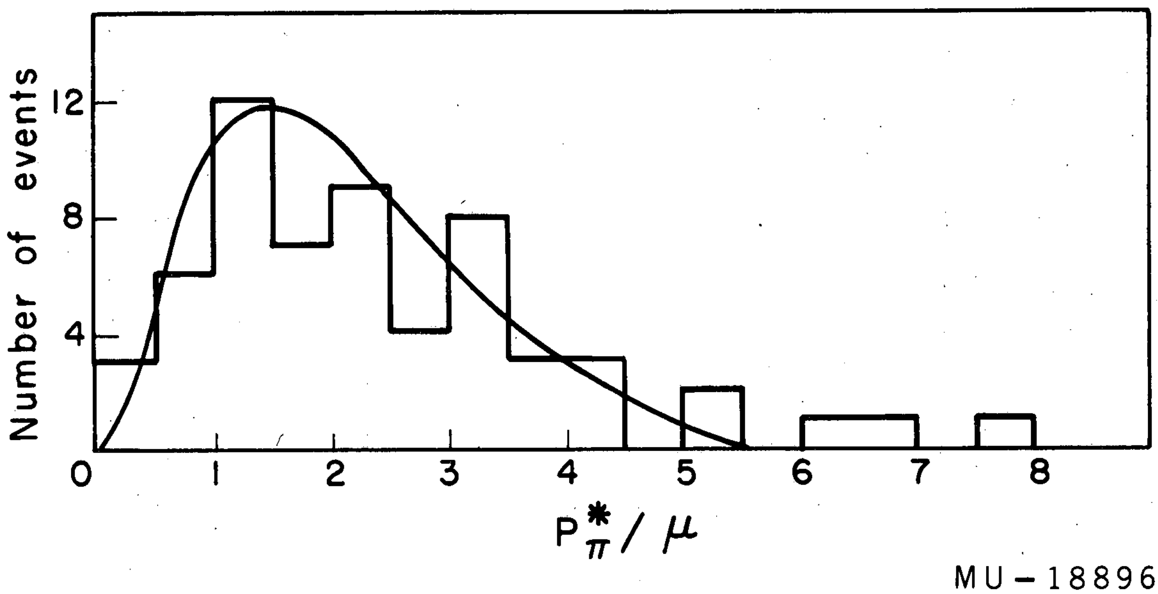

Fig. 5. Comparison of the π-meson spectrum with that predicted by the covariant phase-space factor. The curve is the combination of those in Fig. 3 according to the percentage of reaction (1) with n = 1, 2, 3, and 4.

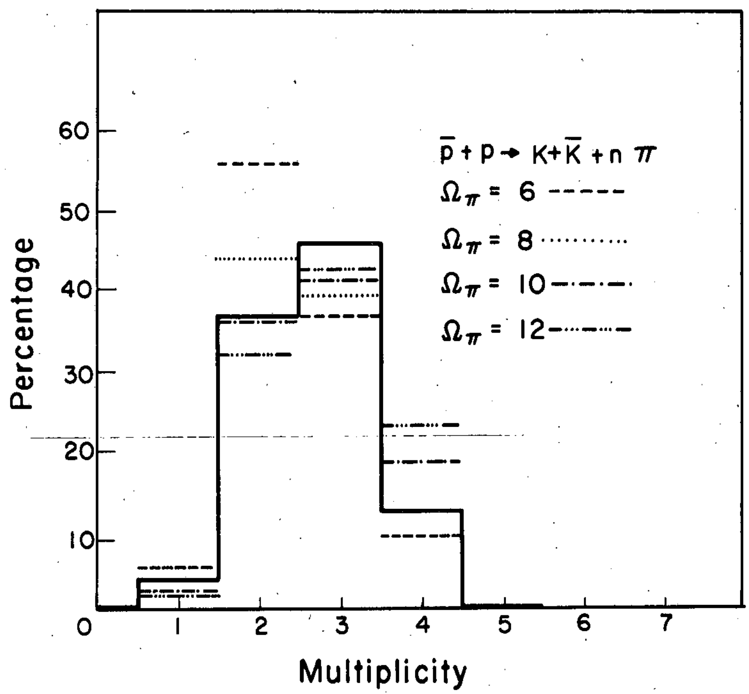

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Fig. 6. Estimate of Ω_π by best fit to the observed pion multiplicity associated with K production.

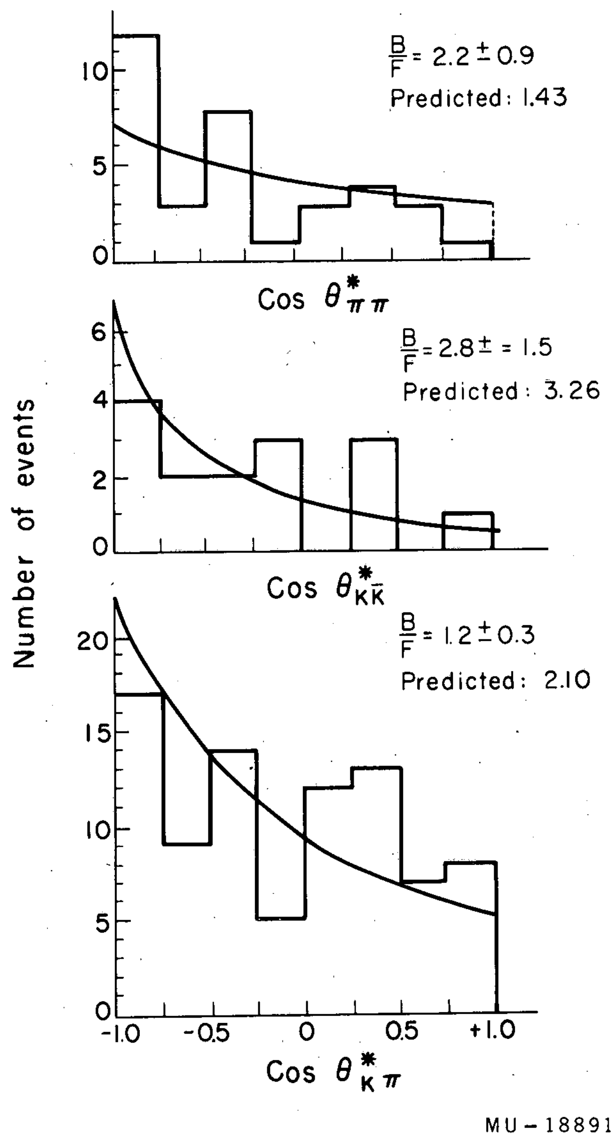

Fig. 7. Angular correlations of π-π, K-$\overline{K}$, and π-K pairs. Each curve represents the combination of computed results shown in Fig. 8.

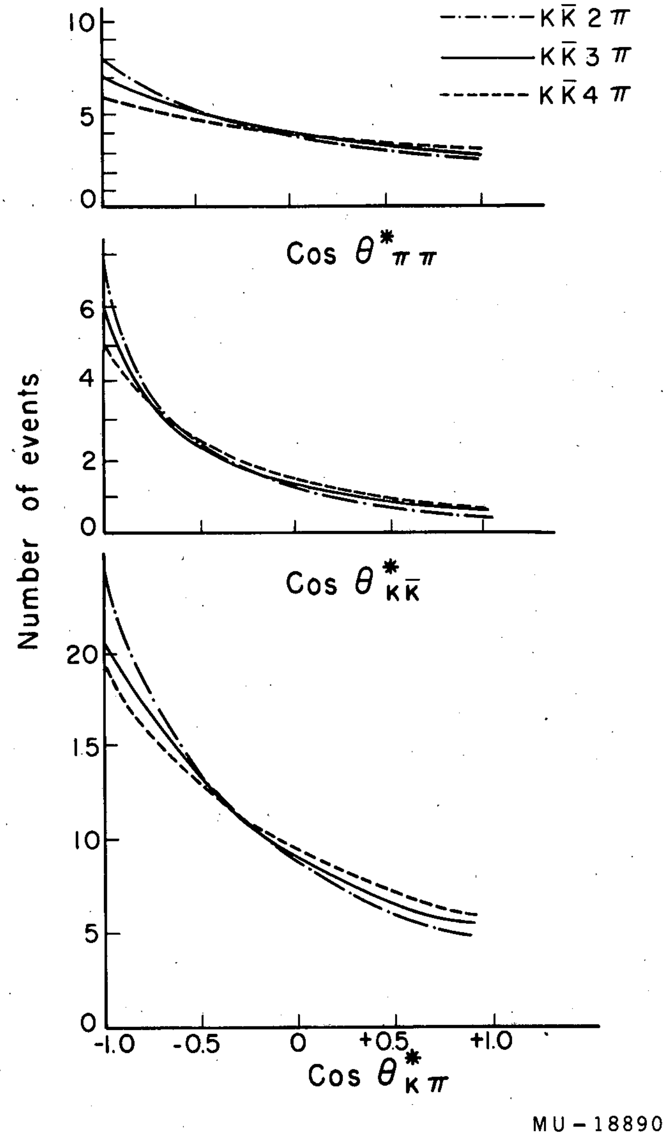

Fig. 8. Computed angular correlations according to conservation of energy and momentum.

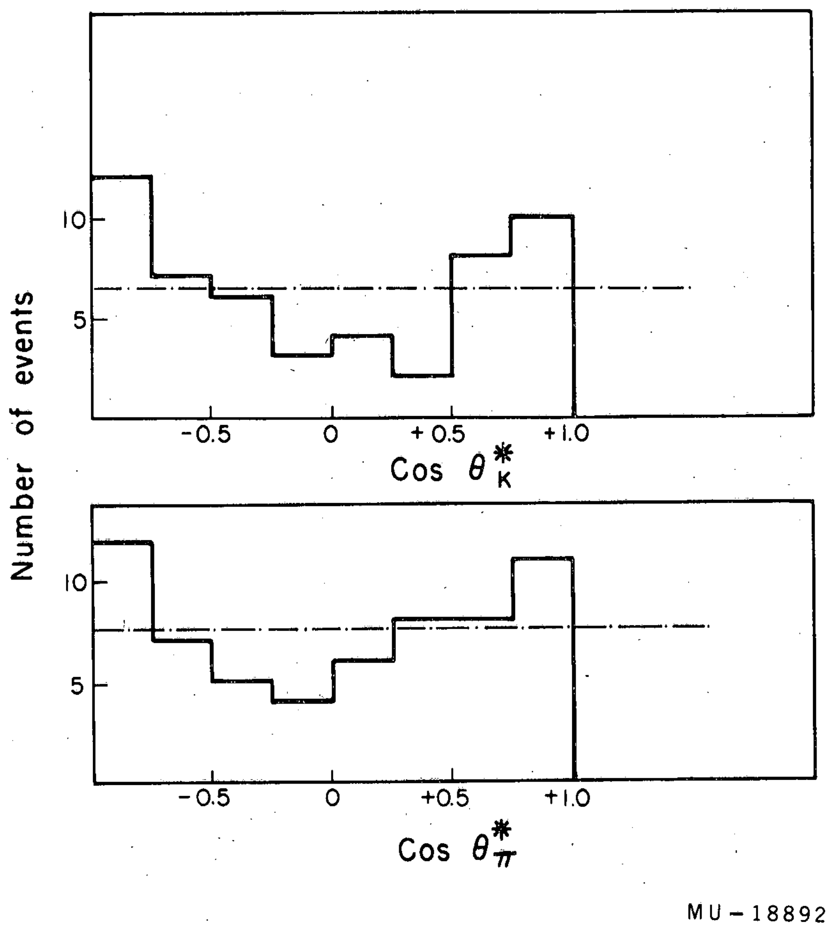

Fig. 9. Angular distributions of K and π mesons in the p-p c.m. system.

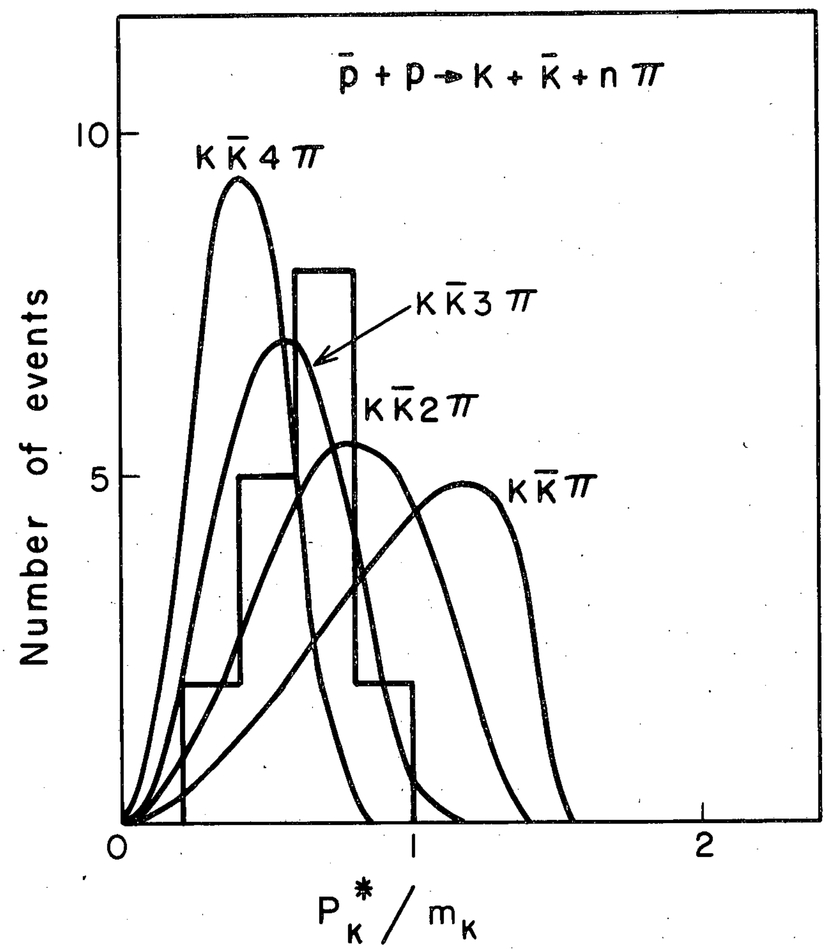

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Fig. 10. Comparison of a low-energy K-meson momentum spectrum (Agnew et al.) with computed spectra.

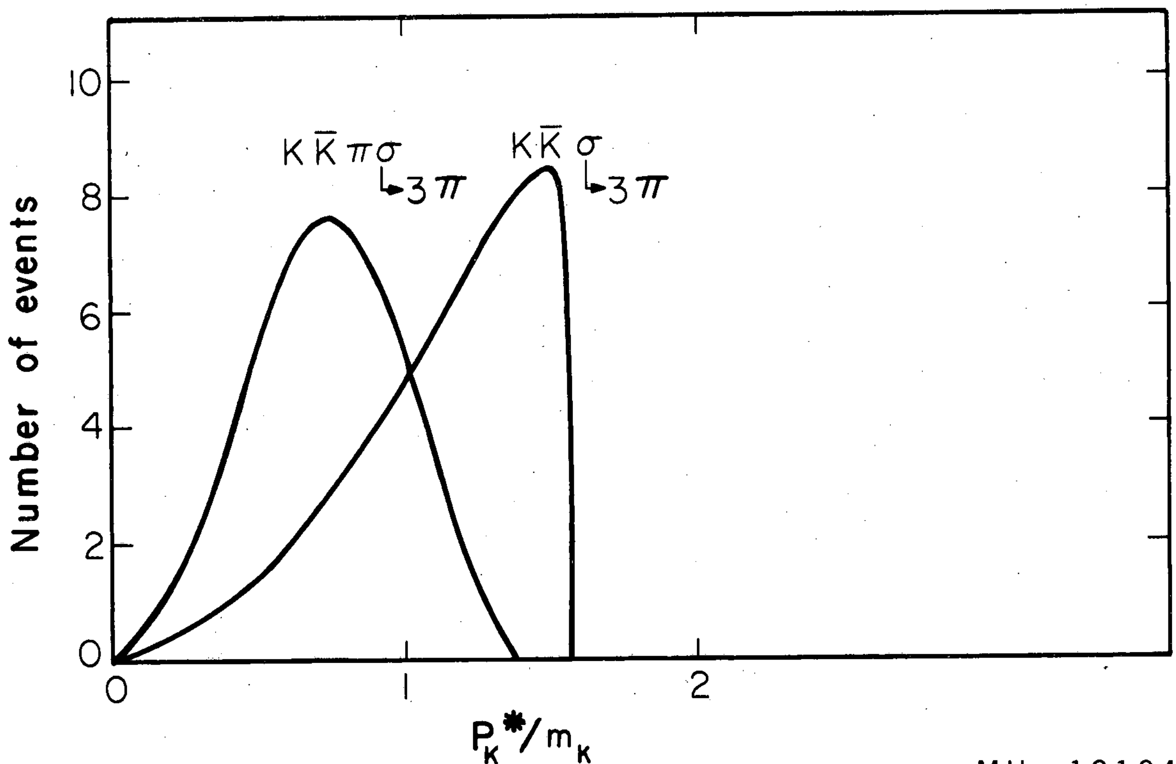

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Fig. 11. The K-meson momentum spectrum in the $\bar{p}$-p c. m. system for an isobar state $\sigma = 3\pi$.

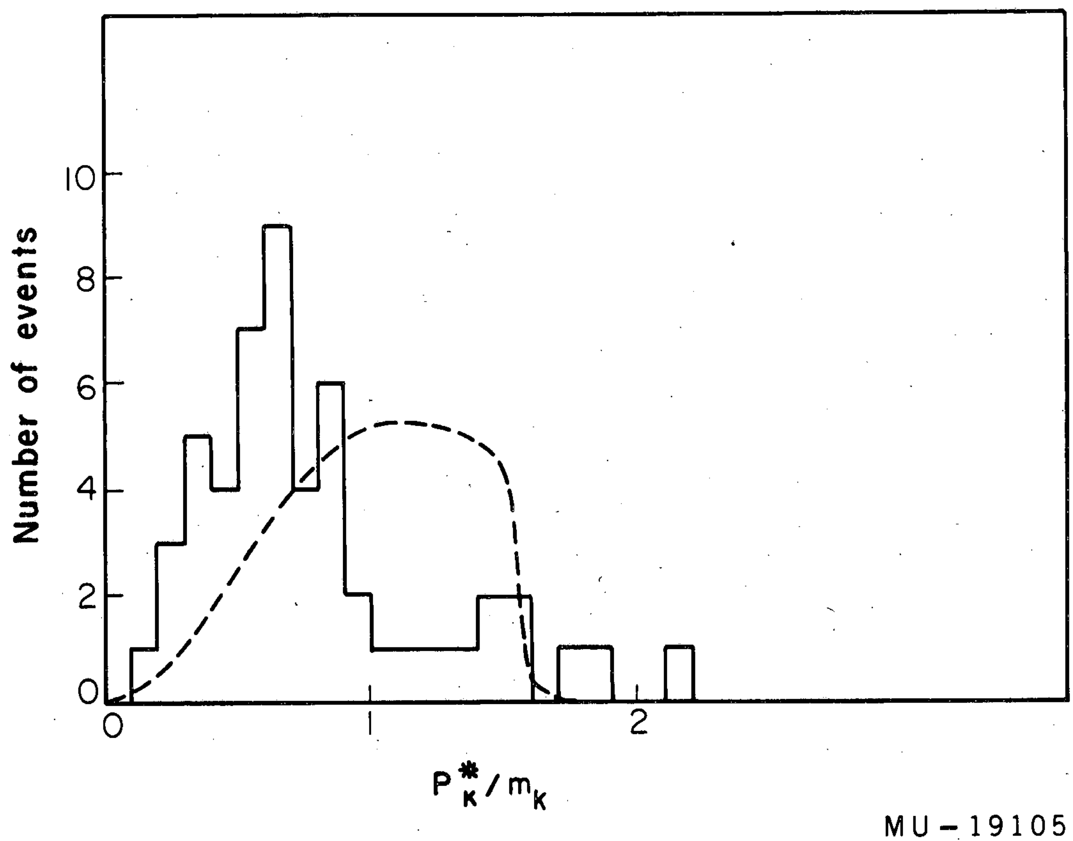

Fig. 12. Comparison of the K-meson momentum spectrum predicted by the isobar model with experimental data.