THE MODERN THEORY OF THE LOP AND PPP: SOME IMPLICATIONS*

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Abstract

What I call the modern theories of the law of one price and purchasing power parity extend those theories in two important ways: First by recognizing the nonlinearities caused by transaction costs and other impediments to trade and second by recognizing the importance of time in commodity arbitrage. This new approach, which has developed for the last two decades, raises questions about many widely accepted ideas. These ideas include the following: (1) The relevance of PPP. (2) Tests for cointegration and unit roots. (3) Relative versus absolute PPP. (4) Border effects. (5) Excessive volatility. (6) Large half lives for deviations from PPP. (7) The large increase in the volatility of exchange rates after the collapse of Bretton Woods.

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What I call the modern theories of the law of one price and purchasing power parity have evolved over the last two decades. Both emphasize two aspects of commodity arbitrage that traditional theories ignore: (1) Information costs, transaction costs and other trade impediments create commodity points similar to gold points.[1] (2) Commodity arbitrage takes place over time as well as across space. Both time and the nonlinearities caused by commodity points have important implications for how we think about and test the law of one price and purchasing power parity. The primary objective of this article is to point out some of those important implications.

I begin by briefly reviewing the traditional approaches to the LOP and PPP. Then I outline the modern approach to the two theories and develop the implications for the following six areas: (1) The relevance of PPP. Contrary to the conventional wisdom, PPP probably works better during normal times than during inflation. (2) Tests for unit roots and cointegration. Those tests have even less power than is generally realized even after taking into account the standard econometric effects of thresholds. (3) Relative versus absolute PPP. Contrary to the conventional wisdom, absolute PPP may be more useful than relative PPP. (4) Border effects. The effect of borders on commodity arbitrage has probably been exaggerated. (5) Excessive short-run volatility. What is generally seen as excessive short-run volatility in exchange rates may be consistent with the modern theory of PPP. (6) Long half lives for deviations from PPP. Conventional tests for those half lives are highly suspect. (7) The large increase in volatility after the collapse of Bretton Woods. That large increase in volatility may be consistent with the modern theory of PPP.

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[1] Although the idea of commodity points apparently goes back to at least 1916, see Obstfeld and Taylor (1997), the important econometric effects of commodity points have been recognized only recently.

I. Traditional LOP and PPP

The term purchasing power parity means different things to different people. The resulting confusion has plagued the analysis of the relation between exchange rates and commodity prices. I want to be clear about what I mean by ‘purchasing power parity'. By purchasing power parity I mean the idea that, in the long run, exchange rates (or relative price levels) tend to adjust so that the purchasing power of a currency is the same, or at parity, at home and abroad. In the 'long run', if $100 buys a certain bundle of goods in the United States, then, after converting that $100 into pound sterling, the $100 should be able to buy approximately the same bundle of goods in the United Kingdom. This traditional interpretation of purchasing power parity rests on a traditional interpretation of the law of one price.

A. Law of One Price

Ignoring for now various impediments to trade, let the law of one price hold for the good q.

$$(\$/q)_t = (\$/£)_t(£/q)_t \tag{1.1}$$

where $(\$/q)_t$ and $(£/q)_t$ are the domestic and foreign price of q that are relevant for international arbitrage. $(\$/£)_t$ is the relevant domestic price of foreign exchange. For individual commodities, exchange rates are exogenous.

$$(\$/q)_t/(£/q)_t = (\$/£)_t \tag{1.2}$$

B. Purchasing Power Parity

Traditional purchasing power parity is based on the law of one price. If the LOP holds for every q, then it holds for any arbitrary bundle of goods Q. [2]

$$(\$/Q)_t = (\$/£)_t(£/Q)_t \qquad (2)$$

An implicit assumption of the traditional version of PPP is that Q is a broadly based bundle of goods including both traded and nontraded goods. With Q a broadly based bundle of goods, for PPP to hold, relative prices between traded and nontraded goods must be approximately the same at home and abroad. Because of this implicit assumption, PPP is normally applied primarily to developed countries.

With P^D the domestic price of Q and P^F the foreign price of Q, equation 3 describes absolute PPP.

$$(\$/£)_t = P_t^D/P_t^F \qquad (3)$$

In most traditional interpretations of PPP, with flexible exchange rates, relative price levels are effectively exogenous. Whether or not that interpretation is correct depends on monetary policy in the two countries.[3] If exchange rates play no significant role in the formulation of monetary policy in either country, then, at least in the long run, relative price levels can be treated as exogenous. If that is not the case in either country, then exchange rates and relative price levels are interdependent even in the long run.

Absolute PPP implies relative PPP. Relative PPP explains changes in exchange rates from some base period to some other period. Because price indexes describe the change

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2 For an excellent review of the traditional approach, see Officer (1976)

3 The implicit assumption here is that, in the long run, trade determines relative prices and monetary policy, either intentionally or unintentionally, determines price levels.

in the price level relative to some base period, any empirical test of PPP that uses price indexes tests relative PPP, not absolute PPP.[4]

Tests of the traditional LOP and PPP normally account for the effects of information, transaction and shipping costs, and other impediments to arbitrage, by adding well behaved error terms to Equations 1 to 3. Modern theories of the LOP and PPP analyzes the effects of those costs and other impediments in more detail. These modern theories also emphasize the importance of the appropriate prices for empirical tests of the LOP and PPP.

II. Modern Theories of the LOP and PPP

Modern theories of the LOP and PPP extend the traditional approaches in two important ways. First, they recognize the nonlinearities created by information, transaction and transportation costs, and other trade impediments. Second, they recognize the importance of time for commodity arbitrage.

Although the idea of applying commodity points like gold points to the LOP apparently goes back as far as 1916, recent developments in the theory of arbitrage across time and space have refined and formalized this early insight. For theoretical work see Benninga and Protopapadakis (1988), and Dumas 1992. Combined theoretical and empirical work includes Michael, Nobay and Peel (1994, 1997), Obstfeld and Taylor (1997) and Taylor and Peel (2000). Early theoretical and empirical work by Protopapadakis and Stoll (1983, 1986) is particularly important. For a recent

[4] For a more complete explanation for why price indexes cannot be used to test for absolute PPP see Crownover, Pippenger and Steigerwald (1996).

contribution to, and brief review of, this literature see Sarno, Taylor and Chowdhury (2004).

A. Time

Protopapadakis and Stoll (1983, 1986) appear to be the first to point out that commodity arbitrage takes place across time as well as space. Because arbitrage takes time, the theoretical exchange rates and the commodity prices in the law of one price are *forward* or *futures* prices, not spot prices. Since the modern theory of purchasing power parity rests on the modern theory of the law of one price, the theoretical exchange rates and commodity prices in PPP are also forward or futures prices.

The analogy to gold points plays an important role in the modern theory of the LOP. The standard textbook description of gold points illustrates the importance of recognizing that commodity arbitrage takes place over time as well as over space. The textbook story about gold points between New York and London is a fairy tale. Over the entire gold standard, it took weeks, not hours, to ship gold between London and New York. Until the trans-Atlantic cable was laid, it was even impossible for someone in (London) New York to buy spot pounds in (New York) London.

As a result, it was physically impossible for movements of gold between New York and London to *directly* restrain the spot price of pound sterling. What arbitrage could restrain was forward prices. Consider the following mental experiment. In June, an importer in New York enters into a contract to pay £1,000 to an exporter in London in September. One option is to buy a sterling bill of exchange in New York that has been accepted by a 'name' bank that matures in September. (That bill was issued to pay for

exports from the United States to the United Kingdom.) If the dollar price of those pounds is too high, an alternative is to buy gold in New York, ship it to London, and sell it there in September. Gold flows can directly restrain the forward exchange rates implicit in bills of exchange. Gold flows can not directly restrain spot exchange rates.

Except for a very few commodities such as diamonds or Rembrandts, even today time is as important for arbitrage as space. Consider the following mental experiment. Because of a sudden threat of a serious crop failure in the United Kingdom and Europe, in May the spot and forward sterling prices for number 2 red wheat in Tillsbury England rise by 30 percent. There is no way that wheat exporters in New Orleans can respond to the higher spot price for wheat in Tillsbury. What exporters can do is enter into forward contracts to buy wheat in New Orleans in July and to deliver wheat to Tillsbury in August.[5] Wheat prices in those contracts are forward prices. If payment is in sterling, when this is pure arbitrage, exporters sell the sterling forward. If payment is in dollars, importers buy dollars forward. This arbitrage restrains the gap between forward prices for wheat in New Orleans and Tillsbury. This arbitrage does not *directly* restrain the gap between current spot prices. However arbitrage does reduce the differential in spot prices indirectly. First, arbitrage moderates the rise in forward prices in Tilbury which moderates the rise in spot prices by reducing the incentive to carry over current stocks of wheat. Second, arbitrage increases forward prices in New Orleans which raises spot prices by increasing the incentive to carry over current stocks of wheat.

[5] It takes time to organize and unanticipated shipment. To avoid any risk, an arbitrager must buy forward and sell farther forward.

Although the importance of their work is not yet widely recognized, Protopapadakis and Stoll (1986) show that the law of one price holds better for futures prices than for spot prices.

> The hypothesis that the Law of One Price holds in the long run is supported by the data. The support is largely due to the cases for which data are futures or forward prices. In the cases for which data are spot prices, the support is much more limited.

Unfortunately almost all other research on the law of one price and the closely related Borders literature has used sticky spot retail prices. A similar problem applies to standard empirical tests of purchasing power parity.

B. Commodity Points

Like gold points, commodity points introduce nonlinearities. Within the commodity points, arbitrage does not reverse deviations from the law of one price.[6] Beyond the commodity points, commodity arbitrage reduces deviations from the law of one price. Equation 4 provides a simple description of this modern view of the LOP where F_t is the logarithm of an appropriate forward exchange rate and X_t is the logarithm of $(\$/q)_t/(\pounds/q)_t$ where these prices are now the corresponding forward prices.

At and within the commodity points,

$$\Delta X_t = e_t, \qquad |F_t - X_t| \leq cp \qquad (4.1)$$

where e_t is a stationary error term that is likely to be approximately white noise and cp is the logarithm of the commodity points. For simplicity, the commodity points are symmetric.

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6 See Dumas (1992) for a detailed discussion of the behavior within the commodity points.

Outside the commodity points, profitable arbitrage produces an error correction mechanism.

$$\Delta X_t = -\gamma(S_t - X_t) + e_t, \qquad |F_t - X_t| > cp \qquad (4.2)$$

Information and transaction costs, and other impediments to trade such as tariffs produce a threshold autoregressive model or TAR.[7]

C. Modern Theory of PPP

Like the traditional theory of PPP, the modern theory of PPP uses the LOP. But the modern theory of PPP uses the modern theory of the LOP. As a result, there are commodity points within which commodity arbitrage does not affect exchange rates. Outside the commodity points, arbitrage produces an error correction mechanism. Aizenman (1981) was among the first to describe how commodity points affect PPP. Dumas (1992) develops a more sophisticated approach. Davutyan and Pippenger (1990) were among the first to show empirically that the related idea of economic distance is important for PPP.

Let F_t be the logarithm of the forward exchange rate at time t. Let P_t be the logarithm of the ratio of price levels at time t, $(\$/Q)_t/(£/Q)_t$. An important difference between the traditional and modern theories of PPP is that now the prices in $(\$/Q)_t/(£/Q)_t$ are forward prices that match the forward exchange rate F_t.

[7] Note that there is no delay in equation 4. Most of the research on the LOP uses monthly, quarterly or yearly data. It seems unlikely that highly competitive firms would wait that long to respond to potential profits from arbitrage.

Equation 5 describes the modern theory of purchasing power parity where CP captures the 'commodity points'.[8] At and within the commodity points there is no error correction mechanism.

$$\Delta F_t = \varepsilon_t, \qquad |P_t - F_t| \leq CP \qquad (5.1)$$

where ε_t is a stationary process that captures the effects of information about a variety of real and monetary shocks. Outside the commodity points arbitrage restricts the movement in exchange rates.

$$\Delta F_t = \alpha(P_t - F_t) + \varepsilon_t, \qquad |P_t - F_t| > CP \qquad (5.2)$$

This threshold model again assumes that, at least in the long run, relative price levels are exogenous. If that is not the case, there will be a different error correction mechanism. But the basic point of the model remains. The relevant real exchange rate depends on futures or forward prices and it is a nonlinear process. Those two ideas are the foundation of the modern theory of the law of one price and purchasing power parity.

In Dumas (1992), exchange rates are not a random walk within the commodity points. But the evidence is very strong that, in the absence of intervention by central banks, when flexible, spot exchange rates are almost impossible to distinguish from a random walk.[9] See for example Pippenger (1973, 2003). Because spot exchange rates act like random walks, for simplicity I assume that ε_t is approximately a random walk.

The use of a threshold model for the modern theory of PPP requires some justification. Given the large number of commodities traded between developed countries, one might

[8] All the literature assumes that the error correction process is proportional as in equation 5.2. But there is no reason that the error correction could not be integral, that is depend on the accumulated errors, or be a combination of integral and proportional.

[9] Like prices in other organized markets, strictly speaking, exchange rates are martingales. But the literature has ignored the distinction and so I do so also.

expect commodity arbitrage to begin with only very small deviations from PPP. In that case, as the exchange rate deviated more and more from PPP, more and more commodities would move outside their commodity points and the error correction mechanism would become stronger and stronger. That view produces the smooth transition autoregressive (STAR) model used in Michael, Nobay and Peel (1997), and Sarantis (1999) or the exponential smooth transition autoregressive (ESTAR) model used in Baum, Barkloular and Caglayan (2001).

Although the empirical evidence may in time show that a STAR or ESTAR is better, here I have stated the modern theory of PPP in terms of a TAR. There are two reasons for that choice. The first is practical. A TAR is simpler and, with respect to most of the important issues, produces results similar to STAR or ESTAR. The second reason is based on the modern theory of the LOP that underlies the modern theory of PPP.

The LOP recognizes that arbitrage in individual commodities does not affect the exchange rate. As a result, for commodity arbitrage to affect exchange rates there must be a 'critical mass' of commodity arbitrage before it affects exchange rates. A critical mass suggests a threshold.[10]

Almost all tests of purchasing power parity use consumer price indexes or CPIs. A few use wholesale or producer price indexes. Prices in CPIs are sticky retail prices. Prices in wholesale or producer price indexes are often sticky posted prices.[11] In either case, those prices are not the futures or forward prices relevant for commodity arbitrage

[10] Beyond the thresholds, a STAR or ESTAR may be more appropriate than a simple proportional error correction.

[11] There is another problem with using wholesale or producer price indexes to test PPP. They are dominated by traded commodities while PPP implicitly refers to a broad bundle of goods including what are normally thought of as nontraded goods such as hair cuts.

and PPP. Except for exchange rates and a few commodities such as wheat and oil, forward or futures prices are not easily available. At least for now, for broadly based price levels or price indexes, we must use sticky retail prices. One of the important contributions of the modern theory of PPP is to point out that we must take into account the effect of using sticky retail prices when we test PPP. To do that, we must specify a relationship between the ratios of price levels or price indexes that we can observe and the theoretical ratios that we cannot observe.

There are a variety of reasons for why retail prices are so sticky. Some suggest that the relationship between retail and the auction prices is a threshold process like equations 4 and 5. Others suggest a STAR or ESTAR. For simplicity, I use a simple model of proportional adjustment. The current change in the ratio of CPIs depends on the difference between that ratio and the ratio of appropriate price indexes that we do not observe. Let p_t be the logarithm of a ratio of conventional CPIs. As before, P_t is the logarithm of the ratio of price indexes using futures prices.

$$\Delta p_t = \lambda(P_t - p_{t-1}) + \eta_t \qquad (6)$$

where σ_η is small relative to $\sigma_{\Delta P}$. A one unit change in P_t at time t causes p_t to change by λ at time t. The steady state or long-run response of p_t to a one unit change in P_t is one. In the long run, p_t equals P_t. In the short run, p_t is 'sticky'. Since forward exchange rates and prices in commodity futures markets are approximately random walks, at least for now I assume that ΔP_t is approximately white noise. For most of the implications of the modern theory of PPP that I discuss below, equation 6 is adequate.

The frequent failure to recognize the problems created by commodity points and the general failure to recognize the importance of time in commodity arbitrage produces potentially serious problems for almost every aspect of the LOP and PPP. Some of these problems have been discussed in the literature. This paper points out some new important implications of commodity points and the role of time in commodity arbitrage.

III. New Implications

A. Relevance of PPP

The conventional wisdom about PPP is that it works well during inflation, but not in the absence of inflation. Rogoff (1996, 654) states the conventional wisdom as follows:

> Jacob A. Frenkel (1978) does find some support for PPP on hyperinflation data, which is not surprising given the over whelming predominance of monetary shocks in such environments. But test after test has rejected purchasing power parity for more stable monetary environments; see for example Frenkel (1981) or Krugman (1978).

But as the nonlinear model developed above suggests, and Sercu, Uppal and Van Hulle (1995) show, standard econometric tests of PPP are deceiving. With no change in the threshold model, inflation tends to improve the econometric results from standard regressions of exchange rates on purchasing power parity. As a result, on the one hand, even when PPP works relatively well, standard econometric test are likely to reject PPP in a stable monetary environment. On the other hand, even when PPP works relatively poorly, standard econometric tests are likely to support PPP in an inflationary environment.

The influential 1981 article by Frankel that Rogoff cites is titled "The collapse of purchasing power parities during the 1970's". That article convinced most people that PPP

worked well in the inflationary 1920's, but failed in the more stable 1970's. However, without relying on the modern theory of PPP, Davutyan and Pippenger (1985) show that Frankel and others misinterpreted the evidence. PPP worked at least as well in the 1970s as in the 1920s.

One of the advantages of the modern theory of PPP is that it provides a more precise way to think about PPP working better or worse. PPP works better when commodity points are closer together or the error correction process outside the commodity points is stronger. Other things equal, the narrower are the thresholds and the larger is α, the smaller are the deviations from PPP. But, given CP, α and the variance in e_t, econometric results will depend on the variance in ΔP_t. Those other things equal, the smaller the variance in ΔP_t, the more standard econometric test will reject PPP. But these apparent rejections are the result of an inherent bias in the test and do not imply that the modern theory of PPP has failed.

Among many others, Frankel (1981) and Krugman (1981) fell into this econometric trap. Using data from the 1920's and the 1970's, Frankel (1981) reports the results of estimating what was a standard test equation for PPP.

$$S_t = a + bp_t + u_t \qquad (7)$$

Using monthly data and wholesale indexes, Frankel concludes that PPP worked in the 1920's, but failed in the 1970's.

Table 1 is based on Tables 1 and 2 in Davutyan and Pippenger (1985). Like Frankel, they also use monthly wholesale indexes and GLS to estimate equation 7 for the 1920's. Unlike Frankel, they also show the results of estimating equation 7 for the 1920's between pairs of countries where neither country suffered from serious monetary instability. The

countries with serious monetary instability are Germany and France. During the early 1920's, Germany suffered from hyperinflation while France suffered from substantial inflation and deflation. The United Kingdom experienced moderate instability. The United Kingdom suffered from persistent deflation, much of it deliberate. The countries with only mild monetary instability are Canada, Japan and the United States.

To show how the regression results can be misinterpreted, in their Table 2 Davutyan and Pippenger include annual estimates of the deviations from absolute PPP using prewar mint par as the base period exchange rate. Table 1 repeats their results for the 1920's and includes averages of their annual averages. The average real exchange rate for each country is a measure of the average predictive error for purchasing power parity.

TABLE 1
GLS Estimates of Equation 7 for the 1920's

	a	b	R^2/SE	DW/ρ	Real Exch. Rate
Germany	-0.75	0.98	0.997	1.57	0.57
1919-1924	(0.08)	(0.01)	0.37	0.81	
France	0.64	1.36	0.536	2.02	0.79
1920-1925	(0.14)	(0.15)	0.06	0.81	
United Kingdom	-0.04	0.71	0.489	1.39	1.00
1920-Mar. 1925	(0.02)	(0.09)	0.03	0.03	
Canada	0.05	0.29	0.267	1.52	0.93
1919-1925	(0.07)	(0.05)	0.01	0.82	
Japan	0.04	0.20	0.204	1.23	1.08
1919-1925	(0.04)	(0.04)	0.02	0.46	

From Tables 1 and 2 in Davutyan and Pippenger (1985).
Standard errors in parentheses.

With an R^2 of 0.997, the regression results for Germany provide the best support for PPP. France is next with an R^2 of 0.536. Following those two inflationary countries come the

other countries: the United Kingdom with 0.489, Canada with 0.267 and Japan with 0.204. Estimates of *b* follow a similar pattern. For Germany and France, the estimates are not statistically different from one at the 1 percent level. The estimate for the United Kingdom is significantly above zero, but also significantly below one. Estimates of *b* for Canada and Japan are also significantly above one, but well below the estimate for the United Kingdom.

Based on deviations from purchasing power parity, PPP works the worst for Germany and France and the best for the United Kingdom, Canada and Japan. The average real exchange rate is only 0.57 for Germany and 0.79 for France. For the United Kingdom, Canada and Japan, those averages are respectively 1.00, 0.93 and 1.08. During the 1920's, purchasing power parity worked the best when there was relative monetary stability and the regression results were the worst. In terms of the modern theory of PPP, apparently the CPs are smaller or α larger in relatively stable monetary environments than when there is substantial inflation.

Commodity points are certainly an important part of the explanation for why the econometric results can improve with inflation even as the commodity points become larger and the error correction mechanism outside the commodity points becomes weaker. But measurement error caused by using p_t rather than P_t probably also plays a role. With serious inflation, retail prices become less sticky. With prices doubling over a weekend as they did in the German hyperinflation, retail firms must change their prices very quickly or they will sell at prices below what it will cost them to restock. The resulting reduction in measurement error as λ in equation 6 increases should improve the econometric results with

inflation. But that improvement has nothing to do with CPs getting smaller or α getting larger.

B. Absolute versus Relative PPP

The conventional wisdom is that, while relative PPP may be useful, absolute PPP is not. This conventional wisdom is routinely repeated in textbooks. See for example Salvatore (2001, 509).

> As a result, the absolute PPP theory cannot be taken too seriously..... Whenever the purchasing-power-parity theory is used, it is usually used in its relative formulation.

In addition to the usual *a priori* arguments against absolute PPP, as Rogoff (1996) points out, there is the practical problem of data to test absolute PPP. With respect to the problem of data, the general rejection of absolute PPP has been something of a self-fulfilling prophesy. The lack of interest in absolute PPP has discouraged the search for data. As a result, for years the profession has ignored an important source for such information, the Statistisches Bundesamt. Since 1927, the German Statistical Office has published information about the cost of living in Germany relative to the cost of living in the capitals of a wide range of countries. The Statistisches Bundesamt tries to use the same Q for all countries.[12] Using fully-modified OLS estimators to account for the presence of unit roots and the possible joint endogeneity of exchange rates and relative price levels, Crownover, Pippenger and Steigerwald (1996) use this German data to test for absolute PPP. They find some support. Of course all the econometric problems associated with the modern theory of

[12] What the Statistical Office publishes is the DM/$ exchange rate implied by (DM/Q)/($/Q). Unfortunately it does not publish the DM/Q and $/Q separately. For more details, see Crownover, Pippenger and Steigerwald (1996)

PPP affect their tests just as they affect similar tests for relative PPP, so it is not surprising that they found only modest support for absolute PPP.

As is well known from Meese and Rogoff (1983a, 1983b), out of sample, relative PPP normally does not beat a random walk. This result is not consistent with PPP and, using ESTAR, Lutz and Taylor (2003) find that relative PPP beats a random walk at horizons of two to three years, but not at shorter horizons.

The modern theory of PPP suggests that absolute PPP should beat a random walk at shorter horizons. Consider the following mental experiment. Try to use relative PPP to predict the change in exchange rates between the United States and the United Kingdom from June 2003 to July 2003. Without some knowledge of absolute PPP in June and July, equation 5 suggests that the best estimate of the exchange rate for July is probably the exchange rate for June.

But if one knows the absolute PPP exchange rates for June and July, the commodity points and α, then it should be possible to make a more accurate prediction. How the exchange rate is likely to change from June to July depends on where the exchange rate for June is relative to absolute PPP for June and July. For example, suppose the exchange rate in June is at the lower commodity point and the absolute PPP rates for June and July are the same. In that case, the best prediction for the exchange rate is not a random walk because the exchange rate is more likely to rise than to fall. In this mental experiment, absolute beats relative PPP. If this mental experiment is supported empirically, then we may have to reverse the conventional wisdom regarding relative versus absolute PPP.

C. Borders

The effect of borders on commodity arbitrage is currently a hot topic. From the perspective of the modern theory of the LOP, this research is an attempt to see how borders affect the width of the commodity points and the error correction mechanism outside the commodity points. For two recent studies of border effects, see Engel and Rogers (2001) and Parsley and Wei (2001). For a current bibliography of the relevant literature, see their references.

Although Parsley and Wei recognize some of the effects of commodity points, neither Engel and Rogers, nor Parsley and Wei, nor any of the other Borders' research, has considered the potential problems caused by using sticky retail prices. For that reason, here I concentrate on the potential effects of sticky retail prices.

The Borders literature claims that borders between countries produce huge increases in the economic distance between cities. For example, Parsley and Wei (2001, 87) summarize their results as follows:

> Focusing on dispersion in prices between city pairs, we confirm previous findings that crossing national borders adds significantly to price dispersion. Using our point estimates, crossing the U.S. Japan 'Border' is equivalent to adding as much as 43,000 trillion miles to the cross-country volatility of relative prices.

Because Parsley and Wei and other similar research uses sticky retail prices, the modern theory of the LOP suggests that their estimate of 43,000,000,000,000 miles is a substantial exaggeration.

To illustrate the econometric problem created by sticky retail prices, consider the following simple example: π_t^{NY} is the logarithm of the retail price for shoes in New York,

π^{SF}_t is the logarithm of the retail price for shoes in San Francisco and π^{L}_t is the logarithm of the retail price for shoes in London. Let $\sigma^2_{\Delta\pi}$ (NY,SF) be the variance of $\Delta(\pi^{NY}_t - \pi^{SF}_t)$ and let $\sigma^2_{\Delta\pi}$ (NY,L) be the variance of $\Delta[\pi^{NY}_t - (\pi^{L}_t + S_t)]$. For this example, S_t is the logarithm of the spot dollar price of pound sterling. To estimate the border effect, Border studies typically regress variances like these against various explanatory variables including a dummy for the city, a dummy for whether or not there is a border and the variance for ΔS_t, $\sigma^2_{\Delta S}$. For a variety of products, Parsley and Wei (2001) compare the dispersion of $\pi^{NY}_t - \pi^{SF}_t$ within the United States and Japan to the dispersion of $\pi^{NY}_t - (\pi^{L}_t + S_t)$ between the United States and Japan.

For each city, let a corresponding Π_t represent the logarithm of the *forward* price of shoes relevant for commodity arbitrage. For simplicity, using Π_t the LOP holds perfectly between all cities. Arbitrage works as well between New York and London as between New York and San Francisco. $\Pi^{NY}_t - \Pi^{SF}_t$ always equals zero. $\Pi^{NY}_t - (\Pi^{L}_t + S_t)$ always equals zero. There is no dispersion in either case.

In this simple example, Π_t and π_t are related as follows:

$$\Delta\pi_t = \upsilon(\Pi_t - \pi_{t-1})$$

With this simple proportional adjustment mechanism,

$$\pi_t = \{\upsilon/[1.0-(1.0-\upsilon)L]\}\Pi_t,$$

where υ is less than one and L is the lag operator. Lx_t equals x_{t-1}. In the long run, a one percent change in Π_t generates a one percent change in π_t. But in the short run π_t is 'sticky'. A one percent change in Π_t only changes π_t by υ.

Using retail prices does not create a problem for estimating the effectiveness of arbitrage between New York and San Francisco.

$$\pi_t^{NY} - \pi_t^{SF} = \{\upsilon/[1.0-(1.0-\upsilon)L]\}[\Pi_t^{NY} - \Pi_t^{SF}] = 0$$

Retail shoe prices between San Francisco and New York never diverge. The dispersion between domestic cities is zero.

Mixing sticky retail prices with exchange rates that are auction prices makes it appear as though arbitrage is less effective internationally than intra-nationally. Internationally, the law of one price appears to fail when it actually holds. The apparent dispersion across borders is positive. Consider New York and London in this simple example.

$$\pi_t^{NY} - (\pi_t^{L} + S_t) = \{\upsilon/[1.0-(1.0-\upsilon)L]\}[\Pi_{t-1}^{NY} - \Pi_{t-1}^{L}] - S_t = \{(1.0-\upsilon)/[1.0-(1.0-\upsilon)L]\}\Delta S_t$$

The larger the current change in the exchange rate, the larger the apparent current violation of the law of one price. But this violation is spurious. It is solely the result of measurement error.[13]

Under these conditions, if one regresses variances like $\sigma^2_{\Delta p}(NY,SF)$ and $\sigma^2_{\Delta p}(NY,L)$ against a dummy for a border and $\sigma^2_{\Delta S}$, both explanatory variables will be significant. The coefficient for the dummy will be significant because it will capture the average effect of

[13] Correspondence rules relate theoretical terms such as Π_t to things we can observe. In that context, associating observable retail prices with Π_t represents an inappropriate correspondence rule. An inappropriate correspondence rule makes it impossible to reject the theory being tested.

$\sigma^2_{\Delta S}$. The coefficient for $\sigma^2_{\Delta S}$ will be significant because it will capture the effect of the differences in $\sigma^2_{\Delta S}$ between countries. Both results would be spurious.

Of course, in a more 'realistic' model, the effect of mixing retail and auction prices will be more complicated and perhaps less serious. But my objective is not to prove that such a mixture introduces a serious bias. My objective is only to show that the modern theory of the law of one price implies that using sticky retail prices can exaggerate border effects.

D. Unit Roots and Cointegration

The literature on unit roots and cointegration quickly recognized the econometric problems caused by nonlinearities. See for example Tong (1983) and Priestly (1988). At a practical level, M. Pippenger and Goering (1993) show how thresholds seriously reduce the already low power of tests for unit roots.

Responding to these econometric problems, a number of empirical studies of PPP have estimated nonlinear models for evidence of cointegration. They include Michael, Nobay and Peel (1997), Sarantis (1999), Taylor and Peel (2000), Baum, Barkoular and Caglayan (2001) and Sarno, Taylor and Chowdhury (2004). But none of this work recognizes the additional potential problems caused by ignoring the important role of time in commodity arbitrage.

Time affects these tests through two channels. First there is the link between the sticky retail prices in price indexes and the forward or futures prices relevant for arbitrage. Although it might affect short samples, the measurement error in equation 6 should not affect long samples because in the long run p_t converges to P_t. But replacing equation 6

with a TAR relationship should further reduce the already very low power of tests for unit roots and cointegration in a nonlinear framework.

Second there is the link between forward and spot exchange rates. Covered interest rate arbitrage provides a link between forward and spot exchange rates. Because this arbitrage involves information and transaction costs, covered interest rate arbitrage also involves a threshold. Some net covered yield must emerge before covered interest rate arbitrage takes place.

Let i_t represent the domestic interest rate that matches the forward exchange rate F_t. Let i^*_t represent the corresponding foreign interest rate. As before, S_t is the logarithm of the spot exchange rate. Inside the arbitrage boundaries AB, the covered interest rate differential has a unit root.

$$\Delta(F_t - S_t + i^*_t - i_t) = u_t \qquad |F_t - S_t + i^*_t - i_t| \leq AB \qquad (8.1)$$

where u_t is a stationary error. Outside the arbitrage boundary, covered arbitrage creates an error correction mechanism.

$$\Delta(F_t - S_t + i^*_t - i_t) = \beta(AB - F_t - S_t + i^*_t - i_t) + u_t \qquad F_t - S_t + i^*_t - i_t > AB \qquad (8.2)$$

$$\Delta(F_t - S_t + i^*_t - i_t) = \beta(-AB - F_t - S_t + i^*_t - i_t) + u_t \qquad F_t - S_t + i^*_t - i_t < -AB \qquad (8.3)$$

Probably the best way to see what the modern theory of PPP implies about the behavior of the logarithm of the observed real exchange rate $S_t - p_t$ is to compare that real exchange rate to the real exchange rate that the modern theory says is the appropriate way to measure real exchange rates, namely $F_t - P_t$. That relationship also involves a threshold.

Combining equations 6 and 8 and using the lag operator L, equation 9 shows the relationship between $S_t - p_t$ and $F_t - P_t$. Inside the arbitrage boundaries, $(S_t-p_t)-(F_t-P_t)$ has a unit root because $[1/(1-L)]u_t$ has a unit root.

$$(S_t-p_t)-(F_t-P_t) = \{[1-\lambda)(1-L)/[1-(1-L)]\}P_t-i_t+i^*_t-[1/(1-L)]u_t \qquad F_t-S_t+i^*_t- i_t \leq AB \qquad (9.1)$$

With $(1-L)P_t$ stationary, as long as $i_t - i^*_t$ is stationary, outside the arbitrage boundaries $(S_t-p_t)-(F_t-P_t)$ is stationary.

$$(S_t-p_t)-(F_t-P_t)=\{[1-\lambda)(1-L)/[1-(1-L)]\}P_t-i_t+i^*_t-[1/1+\beta-L)](u_t+\beta AB)] \quad F_t-S_t+i^*_t-i_t>AB \quad (9.2)$$

$$(S_t-p_t)-(F_t-P_t)=\{[1-\lambda)(1-L)/[1-(1-L)]\}P_t-i_t+i^*_t-[1/1+\beta-L)](u_t-\beta AB)] \quad F_t-S_t+i^*_t-i_t<-AB \quad (9.3)$$

From the point of view of the modern theory of PPP, it is hardly surprising that the real exchange rates used to test purchasing power parity since the collapse of Bretton Woods appear to have unit roots and very long half lives. Not only is $F_t - P_t$ a TAR, but the link between $S_t - p_t$ and $F_t - P_t$ is also a TAR. This second threshold and the sticky prices in p_t further reduce the already very low power of unit root and cointegration tests.

E. 'Excessive' Short-Run Volatility?

Most economists, central bankers and politicians believe that the short-run volatility of exchange rates is excessive.[14] To reach such a conclusion, there must be a benchmark for the appropriate level of volatility. The most common benchmark is the short-run volatility consistent with PPP. The primary evidence supporting the belief that the short-run volatility

[14] Not everyone believes that the volatility of exchange rates is excessive. See for example Bergstrand (1983), Frenkel and Goldstein (1988), Bui and Pippenger (1990) and Bartolini and Bodnar (1996).

of exchange rates is excessive is that the variance of monthly changes in exchange rates is much larger than the variance of monthly changes in ratios of price indexes.[15]

Although it is now widely recognized that prices in price indexes are sticky and that commodity points are important for evaluating purchasing power parity, it is not widely recognized how the combination of commodity points and sticky retail prices exaggerate conventional measures of relative volatility. Let $\sigma^2_{\Delta F}$ and $\sigma^2_{\Delta S}$ be the variances for monthly ΔF_t and ΔS_t. Let $\sigma^2_{\Delta P}$ and $\sigma^2_{\Delta p}$ be the variances for monthly ΔP_t and Δp_t. Variance ratios like $\sigma^2_{\Delta S} / \sigma^2_{\Delta p}$ are widely used as a measure of the 'excessive' short-run volatility of exchange rates. But commodity points and the use of ΔS_t and Δp_t rather than ΔF_t and ΔP_t biases these variance ratios upwards. The more stable the monetary environment, the greater the bias.

Consider first the effects of nonlinearities. It should be clear from equations 5.1 and 5.2, that, because of the commodity points, $\sigma^2_{\Delta F} / \sigma^2_{\Delta P}$ will go to infinity as $\sigma^2_{\Delta P}$ goes to zero. In other words, even if we could observe $\sigma^2_{\Delta F} / \sigma^2_{\Delta P}$, flexible exchange rates would appear to work the worst when countries were the most successful in coordinating their monetary policies and reducing $\sigma^2_{\Delta P}$.

Using sticky consumer prices rather than the prices appropriate for commodity arbitrage increases the bias in these variance ratios. Spot and forward exchange rates are both volatile auction prices, but sticky retail prices reduce $\sigma^2_{\Delta p}$ relative to $\sigma^2_{\Delta P}$. As a result, $\sigma^2_{\Delta S} / \sigma^2_{\Delta p}$ is almost certainly larger than $\sigma^2_{\Delta F} / \sigma^2_{\Delta P}$.

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[15] See for example De Grauwe and Verfaille (1988), Bleaney (1992) and Isard (1995).

The next subsection describes the monthly data I use and the following section provides estimates for $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$, $\sigma^2_{\Delta S}$, $\sigma^2_{\Delta p}$, and $\sigma^2_{\Delta S} - \sigma^2_{\Delta p}$. First the United States is the home country and then the United Kingdom is the home country.

Data: Exchange rates and CPIs are from the CD ROM for *International Financial Statistics*, September 2000, produced by the International Monetary Fund. Exchange rates are end of period. To allow for a transition from generally pegged exchange rates to more flexible exchange rates after the collapse of Bretton Woods, the data start in 1975. The data end in 1998 with the initial stages in the adoption of the euro.

Observations cover 17 developed countries. In addition to the United Kingdom and the United States they are Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, The Netherlands, Norway, Portugal, Spain, Sweden and Switzerland. Australia, Ireland and New Zealand are not included because only quarterly CPIs are available for those countries. Luxembourg is not included because it has a common currency with Belgium and the results for the two countries are identical. Changes in logarithms of exchange rates, ΔS_t, and changes in logarithms of relative CPIs, Δp_t, are both multiplied by 100. Multiplying by 100 converts these changes into approximately percentage changes and helps reduce any errors due to rounding very small numbers.

Variance Ratios and Variances: The most common way to measure relative volatility is to use a variance ratio. As Tables 2 and 3 illustrate, such ratios hide at least as much as they reveal. Using the United States as the home county, Table 2 shows the following

information for each country: $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$, $\sigma^2_{\Delta S}$, $\sigma^2_{\Delta p}$ and $\sigma^2_{\Delta S} - \sigma^2_{\Delta p}$. Table 3 shows the same information with Great Britain as the home country.

TABLE 2
Measures of Relative Volatility Using Variance: United States as Base Country

Country	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$	$\sigma^2_{\Delta S}$	$\sigma^2_{\Delta p}$	$\sigma^2_{\Delta S} - \sigma^2_{\Delta p}$
Canada	14.8	1.9	0.13	1.8
Norway	35.5	8.4	0.24	8.2
Finland	46.2	9.5	0.21	9.3
Sweden	29.3	9.7	0.33	9.4
Italy	41.4	9.7	0.23	9.5
France	103.0	10.3	0.10	10.2
United Kingdom	26.4	10.4	0.40	10.0
Denmark	29.5	10.4	0.35	10.0
Spain	25.1	10.6	0.42	10.2
Portugal	8.1	10.6	1.31	9.3
Germany	80.0	10.8	0.14	10.7
Austria	47.8	10.8	0.22	10.5
Netherlands	54.1	11.1	0.21	10.9
Belgium	76.5	11.2	0.14	11.0
Japan	34.8	12.0	0.34	11.6
Switzerland	53.5	13.8	0.26	13.5
Average	32.0	10.0	0.31	9.7
Average without Canada	32.5	10.6	0.33	10.3
Average without Canada & Japan	32.3	10.5	0.32	10.2

The first column after the name of the country shows the variance ratio. In Table 2, these ratios range from a high of 103 for France to a low of 8.1 for Portugal. The average ratio is 32. These ratios are deceptive and hide important information. To help illustrate that problem, in both tables countries are ranked according to the variance for their exchange rate, which is in the second column to the right of the country names.

This ranking reveals several interesting patterns. First, in Table 2 Canada is an outlier. Although $\sigma^2_{\Delta p}$ for Canada is the same as for Germany, 0.13, and less than that for Norway,

$\sigma^2_{\Delta S}$ for Canada is only about one fourth as large as the $\sigma^2_{\Delta S}$ for Norway, which has the lowest $\sigma^2_{\Delta S}$ of all the other countries. This result is consistent with the idea behind the Borders literature that economic distance is important for the LOP and PPP. Economic integration between Canada and the United States is certainly greater than between the United States and any other country in Table 2. In terms of the model for purchasing power parity developed earlier, this pattern is consistent with the idea that the commodity points are closer together and α is larger between the United States and Canada than between the United States and the other countries in Table 2.

There are other suggestions of the effects of economic distance in Table 2. Of all the countries in Table 2 Japan is certainly the farthest from the United States, and Japan has the second largest $\sigma^2_{\Delta S}$ even though Japan's $\sigma^2_{\Delta p}$ is just slightly above the average for all the countries excluding Canada.

Another interesting feature of Table 2 is how similar $\sigma^2_{\Delta S}$ is for all the countries other than Canada. Although variance ratios for countries other than Canada range from 103 to 8.1, $\sigma^2_{\Delta S}$ only ranges from 8.4 to 13.8, with an average of 10.1. From the perspective of the modern theory of PPP, a stable monetary environment breaks the direct link between ΔP_t and ΔF_t created by arbitrage. Breaking that link also breaks the indirect link between ΔS_t and Δp_t created by arbitrage. In that context, an unusually small $\sigma^2_{\Delta p}$ can cause an unusually large $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$. But the unusually large $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is not evidence of excessive volatility. Consider the two extremes for $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ in Table 2, France and Portugal. Exchange rate volatility is almost the same for the two countries, 10.3 versus 10.6. The

reason $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is much larger for France than for Portugal is that $\sigma^2_{\Delta p}$ is much smaller for France, 0.10 versus 1.31.

Table 3 reinforces the results from Table 2. Both $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ and $\sigma^2_{\Delta S}$ are largest for countries across an ocean from the United Kingdom. For Canada, Japan and the United States, the minimum $\sigma^2_{\Delta S}$ is 10.4 and the minimum $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is 21.4. For countries on the same side of the Atlantic as the U.K., the maximum $\sigma^2_{\Delta S}$ is 9.4 and the maximum $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is only 19.0. If one excludes Switzerland, which is not in the Common market, the maximum $\sigma^2_{\Delta S}$ is only 7.5.

TABLE 3

Measures of Relative Volatility Using Variance: Great Britain as Base Country

Country	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$	$\sigma^2_{\Delta S}$	$\sigma^2_{\Delta p}$	$\sigma^2_{\Delta s} - \sigma^2_{\Delta p}$
Finland	13.8	5.7	0.41	5.3
Norway	9.5	5.9	0.62	5.3
France	19.0	6.6	0.35	6.2
Netherlands	15.3	6.6	0.43	6.2
Italy	13.9	6.8	0.49	6.3
Belgium	15.0	6.9	0.46	6.4
Denmark	12.4	6.9	0.56	6.4
Sweden	10.8	7.0	0.65	6.4
Austria	11.8	7.1	0.60	6.5
Germany	15.7	7.1	0.45	6.6
Portugal	4.8	7.3	1.51	5.8
Spain	11.1	7.5	0.68	6.8
Switzerland	15.4	9.4	0.61	8.8
United States	26.4	10.4	0.40	10.0
Canada	21.4	10.8	0.51	10.3
Japan	25.4	12.1	0.48	11.7
Average	13.5	7.7	0.57	7.1
Average without U.S., Canada and Japan	11.6	7.0	0.60	6.4

The results in Table 3 are also consistent with the idea that economic distance increases CP or reduces α. Smaller CPs and a larger α for countries on the same side of the Atlantic explains why the three countries with the highest $\sigma^2_{\Delta S}$ in Table 3 are the United States, Canada and Japan, although none of those countries has a $\sigma^2_{\Delta p}$ that is above the average of 0.57.

Comparing the countries on opposite sides of the Atlantic from the United States in Table 2 with the countries in Table 3 that are on the same side of the Atlantic as the United Kingdom reinforces the impression that economic distance is important. For countries on the opposite side of the Atlantic from the United States, the average $\sigma^2_{\Delta S}$ is 10.5. For countries on the same side of the Atlantic as the United Kingdom, $\sigma^2_{\Delta S}$ is 6.4. This is true even though the average $\sigma^2_{\Delta p}$ is smaller when the United States is the home country than when the United Kingdom is the home country.[16]

While the results in Tables 2 and 3 are in general agreement with the modern theory of PPP, those tables do not answer the crucial question: why is $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ so large? The modern theory of PPP suggests an answer.

Simulated Variances and Variance Ratios: Until we have reliable estimates of the commodity points and error correction mechanisms between countries, we will not be able to come to any definitive conclusion as to whether or not $\sigma^2_{\Delta S}$ is larger than is consistent with the modern theory of PPP. The objective of these simulations is a more modest one; to

[16] More effective arbitrage for countries on the same side of the Atlantic suggests that cointegration tests should reject the null more often for country pairs such as Switzerland and Germany than for the United States and Germany. For some evidence that this is the case, see M. Pippenger (1993).

see whether or not conservative values for the parameters CP, AB, α, λ and β introduced earlier can explain the average variance ratios and variances in Tables 2 and 3 as well as the average half lives in the next section.

All simulations use the same number of 'monthly' observations, 288, as Tables 2 and 3. To avoid possible contamination from initial conditions, each simulation is run for 300 'months' before any observations are used. To account for random variations from one realization to another, all results are averages of 100 simulations. The following values are used for the parameters both here and later: CP = 5%, AB = 2%, $\sigma_\varepsilon = 4.0$, $\sigma_u = 3.2$, $\sigma_{\Delta P} = 1.0$, $\sigma_\eta = 0.23$, $\alpha = 0.02$, $\lambda = 0.4$ and $\beta = 0.1$. The interest rate differential is assumed to be a stationary process generated as follows:

$$i_t - i^*_t = 0.9(i_{t-1} - i^*_{t-1}) + z_t,$$

where $\sigma_z = 2.0$. All random inputs are normally distributed white noise.

TABLE 4
Simulated versus Actual Variance Ratios and Variances

Averages	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$	$\sigma^2_{\Delta S}$	$\sigma^2_{\Delta p}$	$\sigma^2_{\Delta S} - \sigma^2_{\Delta p}$	$\sigma^2_{\Delta F}$	$\sigma^2_{\Delta P}$
U.K.	13.5	7.7	0.57	7.1	NA	NA
U.K. without U.S., Canada & Japan	11.6	7.0	0.60	6.4	NA	NA
U.S. without Canada	32.5	10.6	0.33	10.3	NA	NA
U.S averages	32.0	10.0	0.31	9.7	NA	NA
100 simulations	31.5	9.8	0.31	9.4	15.5	1.0

These values for the relevant parameters produce a variance ratio and variances similar to the averages in Tables 2 and 3. The first two rows in Table 4 repeat the averages for the

United Kingdom from Table 3, first for all the countries and then excluding the United States, Canada and Japan. The next two rows repeat those averages for the United States, first excluding Canada and then including Canada. The last row shows the simulated averages.

Although purchasing power parity is never violated in these simulations, the simulated averages approximate the actual average variance ratios and variances, particularly when the United States is the base country. From the perspective of the modern theory of PPP, the variance ratios in Tables 2 and 3 do not support the claim that the short-run volatility of exchange rates is excessive. These ratios could be consistent with the modern theory of PPP. Before we can have any confidence about this issue, where possible, we will need to discover the relevant parameters for individual countries.

Frequency Domain: The frequency domain can provide some insight into the bias created by using sticky retail prices. The spectrum decomposes the variance in a stationary time series by frequency. Consider some continuous stationary process x_t that is sampled each month. Equation 10 describes the relation between the variance for the sampled x_t, σ^2_x, and the spectrum for the sampled x_t, $\Gamma_{x,x}(f)$, where f is frequency measured in cycles per month.[17]

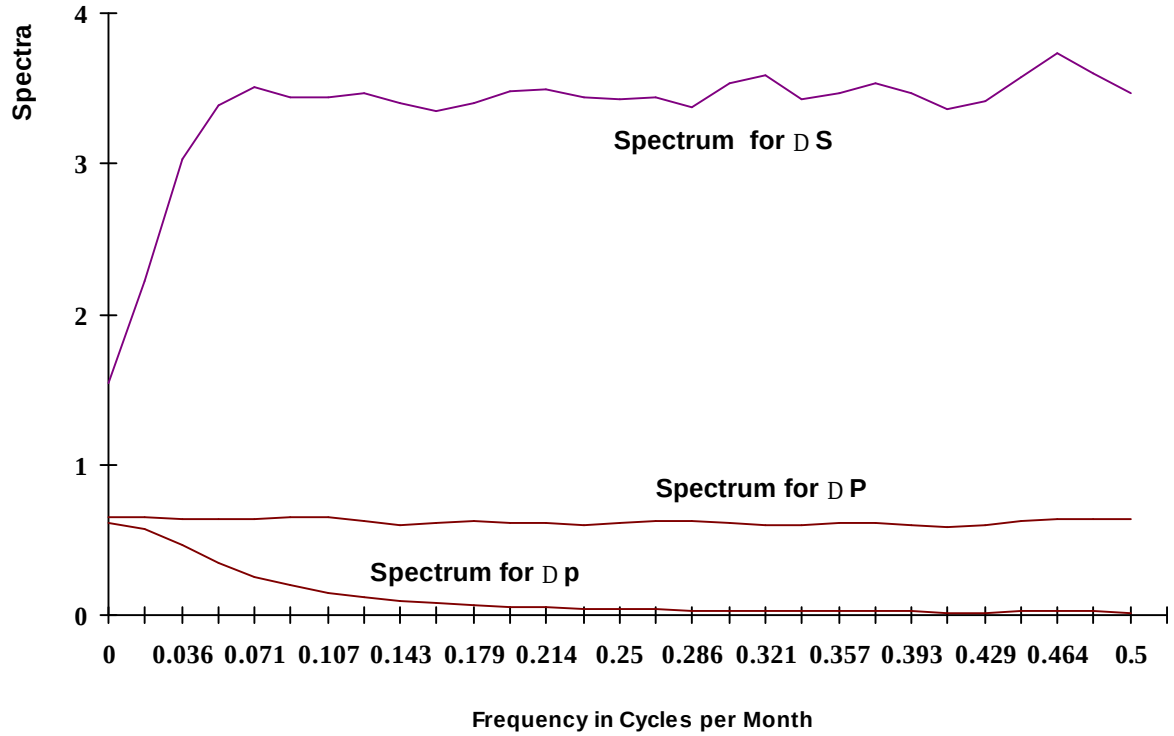
$$\sigma^2_x = \int_0^{0.5} \Gamma_{x.x}(f)df \qquad (10)$$

When x_t is white noise, $\Gamma_{x,x}(f)$, is a constant.

[17] See for example, Peña, Tiao and Tsay (2001,43).

By decomposing the variances in Table 4 by frequency, spectra provide a natural way of describing the dynamics implied by the modern theory of PPP. Equation 6 describes the relation between P_t and p_t in the time domain. Figure 1 uses average spectra for $\sigma^2_{\Delta S}$, $\sigma^2_{\Delta p}$ and $\sigma^2_{\Delta P}$ from the simulated results in Table 4 to describe that relationship in the frequency domain. For each series, the area under the spectrum represents the variance for that series. In Figure 1, the variances for $\sigma^2_{\Delta S}$, $\sigma^2_{\Delta P}$ and $\sigma^2_{\Delta p}$ are, respectively, 10.0, 1.0 and 0.31. Figure 1 shows how sticky retail prices exaggerate $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$.

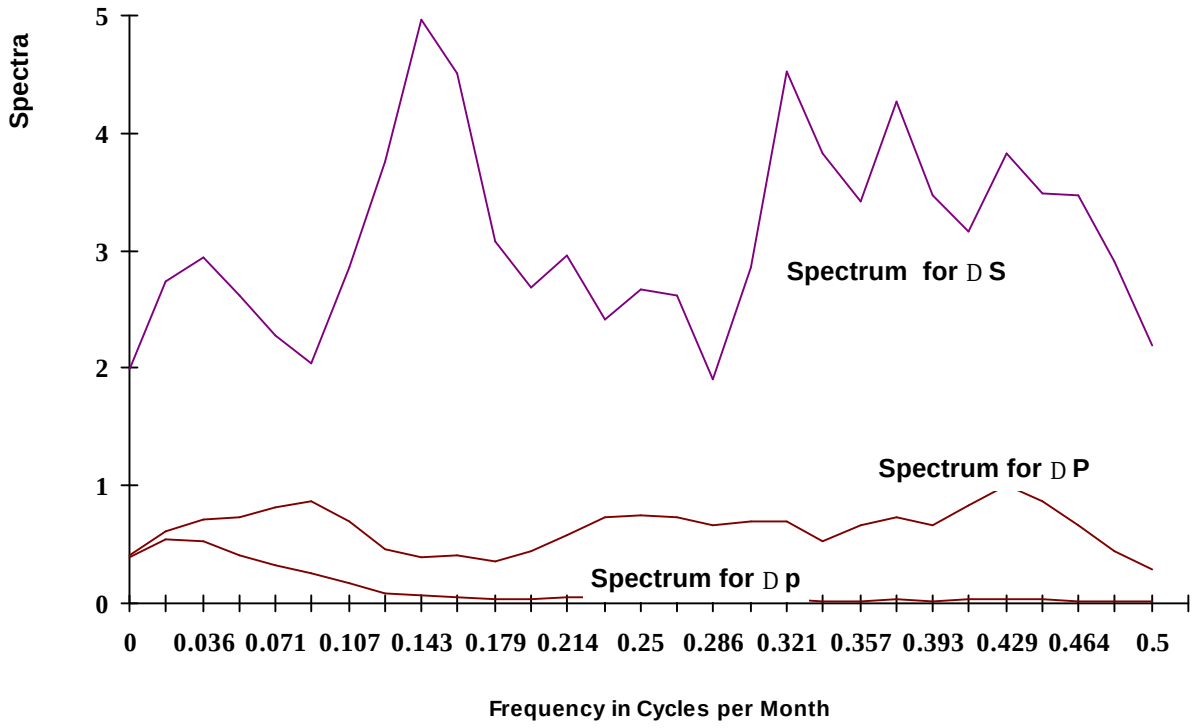
FIGURE 1
Spectra for Averages of 100 Simulations



The area between the spectra for ΔS and ΔP represents $\sigma^2_{\Delta S}$ minus $\sigma^2_{\Delta P}$. That area is consistent with the modern theory of PPP and does not represent 'excessive' volatility. The modern theory of PPP implies that, in the long run, relative price levels effectively constrain

exchange rates to move within the commodity points. That implication appears in Figure 1 as a tendency for the spectrum for ΔS_t to fall toward the spectra for ΔP_t and Δp_t as frequency approaches zero. There are two reasons the spectrum for ΔS_t does not fall all the way to the spectra for ΔP_t and Δp_t. First, with only 28 years of data it is impossible to get an accurate estimate of the 'long run'. Second, because of the threshold, even in the long run ΔS_t can vary by more than ΔP_t and Δp_t.

FIGURE 2
Spectra for First of 100 Simulations



The tendency for the spectrum for ΔS_t to decline at the lowest frequencies is obvious in Figure 1 only because the spectra are averaged over 100 simulations. As Figure 2 shows, the noise in individual realizations hides that tendency. Figure 2 shows the spectra for the first of the 100 simulations. In Figure 2 it is impossible to distinguish the spectrum for ΔS_t

from a spectrum for white noise. The spectrum for ΔS_t looks like the spectrum for ΔP_t, which is white noise by construction.

Returning to Figure 1, the area between the spectra for ΔP_t and Δp_t represents the effect of using sticky retail prices rather than the more auction like prices relevant for international trade. As frequency goes to zero, the gap between those two spectra goes to zero. Given equation 6, in the long run ΔP_t equals Δp_t. In the frequency domain, equation 6 implies that $\Gamma_{\Delta p,\Delta p}(f)$ approaches $\Gamma_{\Delta P,\Delta P}(f)$ as frequency goes to zero.

The fact that $\Gamma_{\Delta p,\Delta p}(f)$ approaches $\Gamma_{\Delta P,\Delta P}(f)$ as frequency goes to zero suggests a way of using the frequency domain to at least partly attenuate the measurement error in Tables 2 and 3 caused by using sticky retail prices. This partial solution is to replace $\sigma^2_{\Delta S}$ with $\Gamma_{\Delta S,\Delta S}(0)$ and $\sigma^2_{\Delta p}$ with $\Gamma_{\Delta p,\Delta p}(0)$. $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$ should be a reasonable proxy for the long-run component of $\sigma^2_{\Delta S}$ relative to the long-run component of $\sigma^2_{\Delta P}$. The modern theory of PPP implies that $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$ will be greater than one, but that it should be a less biased estimate of relative volatility than $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$.

$\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$ describes the relative volatility at the lowest frequency, which corresponds to the long run. The conventional objection to flexible exchange rates is that the short-run volatility is excessive. By using $\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$, it is possible to obtain some information about short-run volatility. With monthly data, the shortest observable 'run' is a two month cycle, which is 0.5 cycles per month. Since the spectrum shows how

variance is distributed by frequency, $\Gamma_{\Delta S,\Delta S}(0.5)$ is the most short-run component of $\sigma^2_{\Delta S}$ that we can observe with monthly data.

$\Gamma_{\Delta p,\Delta p}(0)$ is the best available measure of the long-run volatility in relative price indexes. In efficient markets, expectations about the future dominate the present. Therefore, if the foreign exchange market is efficient, then $\Gamma_{\Delta p,\Delta p}(0)$ should be a reasonable proxy for the short-run volatility in exchange rates consistent with PPP. These is a big 'if', but this is probably the best we can do with the available data.

Table 6 is the frequency domain analog of Table 2. Table 7 below is the frequency domain analog of Table 3.

TABLE 6
Frequency Domain Analog of Table 2: U.S as Home Country

Country	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$	$\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$	$\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$
Canada	14.85	5.89	5.04
Norway	35.54	7.08	3.59
Finland	46.16	11.24	4.84
Sweden	29.28	16.67	4.69
Italy	41.42	8.54	2.00
France	105.05	19.75	7.39
United Kingdom	26.39	9.41	2.82
Denmark	29.50	29.07	11.56
Spain	25.07	7.66	2.92
Portugal	8.07	3.62	0.77
Germany	79.86	19.68	10.31
Austria	47.82	19.13	9.68
Netherlands	54.08	19.04	12.19
Belgium	77.51	19.39	8.52
Japan	34.82	23.31	14.66
Switzerland	53.47	14.38	5.79
Average	32.04	10.66	4.37
Average without Canada	32.1	10.75	4.36
Average without Canada and Japan	31.8	10.37	4.37

In Table 6, this way of adjusting for the effects of using sticky retail prices reduces 'excess' volatility. Except for Denmark, in Table 6 every $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is much larger than the corresponding $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$. For the averages in Table 6, $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is about three times larger than $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$. In Table 7, where the estimates for $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ are generally lower, every $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is also larger than the corresponding $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$. For the averages in Table 7, $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ is about twice as large as $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$.

TABLE 7
Frequency Domain Analog of Table 3: U.K. as Home Country

Country	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$	$\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$	$\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$
Finland	13.79	10.59	6.38
Norway	9.49	2.43	2.86
France	19.00	4.51	3.35
Netherlands	15.26	5.11	3.87
Italy	13.90	2.44	2.39
Belgium	14.98	4.76	2.85
Denmark	12.38	6.72	4.36
Sweden	10.85	4.06	3.65
Austria	11.79	3.77	2.64
Germany	15.66	3.16	2.32
Portugal	4.85	2.10	1.12
Spain	11.09	3.30	3.74
Switzerland	15.42	3.20	2.76
United States	26.38	9.41	2.82
Canada	21.38	10.96	3.56
Japan	25.43	17.70	7.72
Average	12.37	5.58	3.42
Average without U.S., Canada and Japan	11.6	3.58	2.65

Using $\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$ as a measure of the short-run volatility in exchange rates relative to the volatility consistent with PPP, produces no evidence in either table that

$\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$ is larger than $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$. Just the opposite. $\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$ tends to be smaller than $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$, particularly in Table 6.

This pattern is probably the result of central banks leaning against the wind. The vast majority of the research on the effects of intervention suffers from simultaneity bias. 'Incorrectly' signed coefficients are common. When simultaneity is dealt with effectively, the evidence shows that leaning against the wind reduces the short-run volatility of exchange rates.[18] Since intervention is almost always against the dollar, intervention should affect Table 6 more than Table 7. A comparison of Tables 6 and 7 supports the idea that intervention is reducing $\Gamma_{\Delta S,\Delta S}(0.5)$ relative to $\Gamma_{\Delta S,\Delta S}(0.0)$. The difference between $\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$ and $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$ is much smaller in Table 7 than in Table 6.

The evidence from the frequency domain is consistent with the idea that using sticky retail prices exaggerates variance ratios. Excluding U.S.-Denmark, estimates of $\Gamma_{\Delta S,\Delta S}(0)/\Gamma_{\Delta p,\Delta p}(0)$ are always well below estimates of $\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$. Using $\Gamma_{\Delta S,\Delta S}(0.5)/\Gamma_{\Delta p,\Delta p}(0)$ to estimate the short-run volatility in exchange rates relative to that consistent with purchasing power parity suggests that something, probably leaning against the wind by central banks, is moderating the short-run volatility in exchange rates. From the perspective of the modern theory of purchasing power parity, the short-run volatility of exchange rates does not appear to be excessive.

F. Half Life

As Rogoff (1996) points out, the consensus of 3-5 year half lives for deviations from PPP is an important part of the 'purchasing power parity puzzle', a puzzle that I believe the

[18] See Pippenger and Phillips (1973), Phillips and Pippenger (1993) and Pippenger (2003).

modern theory of PPP can help explain. There are two parts to that explanation: nonlinearities and measurement error. Nonlinearities create serious econometric problems for unit root tests. Since those tests are closely related to tests for half lives, we should expect nonlinearities to create problems for estimating half lives for real exchange rates. The explanation based on measurement error has two parts. First all estimates of half lives use S_t-p_t which is only a proxy for F_t-P_t. Second, the prices typically used in p_t are sticky retail prices. Equation 9 shows how thresholds and sticky retail prices are likely to produce longer half lives for S_t-p_t than for F_t-P_t. Equation 9 implies that S_t-p_t will be more persistent than F_t-P_t.

TABLE 8
Estimated Half Lives in Months: Monthly data

US Versus	$S_t - p_t$	S_t	p_t	UK Versus	$S_t - p_t$	S_t	p_t
Austria	37.12	55.06	115.45	Austria	29.33	51.75	57.13
Belgium	41.91	48.62	1551.41	Belgium	40.57	38.21	64.57
Canada	219.27	77.19	69.54	Canada	45.00	20.40	40.45
Denmark	37.45	44.19	48.61	Denmark	27.19	38.47	44.26
Finland	31.79	30.17	37.24	Finland	45.51	20.57	73.10
France	31.81	44.89	72.44	France	25.70	31.03	73.67
Germany	32.89	54.73	108.24	Germany	29.33	50.80	61.42
Italy	33.21	50.04	93.75	Italy	19.13	53.38	236.12
Japan	46.66	101.42	768.65	Japan	38.33	74.06	107.77
Netherlands	30.90	50.92	275.51	Netherlands	33.91	51.49	83.15
Norway	24.83	34.52	136.29	Norway	27.99	13.43	23.00
Portugal	46.00	74.37	129.36	Portugal	32.10	96.61	206.38
Spain	45.19	55.77	73.78	Spain	22.03	58.64	128.92
Sweden	41.78	58.73	107.53	Sweden	40.46	47.39	20.86
Switzerland	28.34	44.59	65.80	Switzerland	23.16	43.01	48.43
U.K.- U.S.	24.08	21.77	35.47	U.K.- U.S.	24.08	21.77	35.47
Average	47.09	52.94	230.57	Average	31.49	44.45	81.42
Exc. Canada	35.60	51.32	241.30	Exc. Canada, US & Japan	30.49	45.77	86.08

Using the same data as used in earlier tables, Table 8 shows individual and average half lives with both the United States and the United Kingdom as the home country. These half

lives are obtained by regressing S_t-p_t against a constant and S_{t-1}-p_{t-1}, and then using log(0.5)/log(b) to calculate the half life where b is the coefficient for S_{t-1}-p_{t-1}. Estimates with the United States as the home country are on the left hand side of Table 8. Estimates with the United Kingdom as the home country are on the right hand side.

This way of calculating the half life assumes that the error in the regression is close to white noise. If not, then one should use a more appropriate regression and calculate the half life using a step or impulse response.[19] I use log(0.5)/log(β) for three reasons. First, it is easy to calculate. Second, it has been widely used to measure half lives. Third, regardless of how one calculates them, in the context of the modern theory of PPP, the idea of a half life for S_t-p_t is not well defined. For one thing, the appropriate half life is for F_t-P_t not S_t-p_t. In addition, for finite samples, there is no unique half life for either F_t-P_t or S_t-p_t. Outside the relevant thresholds, the half life for both F_t-P_t and S_t-p_t is finite and perhaps short, but within the relevant thresholds, the half life for both F_t-P_t and S_t-p_t is infinite. The absence of a unique half life for finite samples helps explain why Murray and Papell (2002) find that measures of half lives are so imprecise. For long-horizon annual data they conclude the following:

> It is worth emphasizing that the least squares point estimates of half-lives from DF regressions provide the evidence that is the basis of Rogoff's 3-5 year consensus. Once we correct the bias and calculate confidence intervals, however, we cannot even claim to have evidence that the half-lives fall within a 2-20 year range.

They conclude that there is even less information in quarterly data after the collapse of Bretton Woods.

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[19] For a discussion of how to deal with most of these problems see Murray and Papell (2002).

> For the DF regressions, while most of the lower bounds of the confidence intervals remain under 1.5 years, every upper bound is infinite. The picture does not change much with the ADF regressions. When the half-lives are calculated from the impulse response function, all but one of the lower bounds are below 1.5 years. But 19 of 20 upper bounds are above 10 years, with 16 of the 20 infinite. These confidence intervals are so wide that they provide absolutely no information regarding the speed of convergence of PPP deviations.

Average half lives in Table 8 fall in Rogoff's range of 2 to 5 years. As in Table 2, the United States versus Canada is an outlier. In Table 2, Canada is an outlier because $\sigma^2_{\Delta S}$ is so small. In Table 8, U.S.-Canada is an outlier because the half life is so large. Earlier a small $\sigma^2_{\Delta S}$ for U.S.-Canada was interpreted as being consistent with PPP working better between Canada and the United States than between any other country and the United States. That is a natural interpretation because we would expect the economic distance to be smaller between the United States and Canada than between the United States and any other country. Here the half life for the real exchange rate between Canada and the United States is over 5 times the mean of all the other countries versus the United States, 219 months or over 18 years. What makes this half life particularly odd is that it is much larger than the half life for the nominal exchange rate, 219 months versus 77. For every other country in Table 8, the half life for S_t-p_t is smaller than the half life for p_t. This bizarre result for Canada suggests that half lives for S_t-p_t may tell us very little about how well purchasing power parity works.

Comparing the estimates when the United States is the home country to the estimates when the United Kingdom is the home country reinforces this conclusion. Estimated half lives are only slightly longer with the United States as the home country. Without Canada,

for the United States the average half life for S_t-p_t is 35 months while with the United Kingdom as the home country the half life including all countries is 31 months. Excluding Canada, Japan and the United States so that the half lives only apply to countries relatively close to the United Kingdom only reduces the average half life to 30 months. Economic distance seems to have very little effect on half lives for S_t-p_t.

Table 9 shows the half lives for the simulated monthly data used earlier. The simulated data from Table 4 produces average half lives for S_t-p_t, S_t and p_t similar to those in Table 8. Table 9 also shows the simulated half lives for F_t-P_t, F_t and P_t. Half lives for F_t-P_t, F_t and P_t are all shorter than for their counterparts S_t-p_t, S_t and p_t. For F_t-P_t the half life is 24.4 months while the half life for S_t-p_t is 40.2 months.

TABLE 9
Average Simulated Half Lives: 'Monthly' Data

100 Simulations	$S_t - p_t$	S_t	p_t	$F_t - P_t$	F_t	P_t
Average	40.2	44.9	295.5	24.4	28.2	111.9

Of course, as before, all these simulations demonstrate is that it may be possible to resolve some of the PPP puzzle using the modern theory of purchasing power parity. While the parameters used here seem plausible to me, whether or not they are consistent with the actual behavior of covered interest rate arbitrage and exchange rates is far beyond the objectives of this article. My objective here is only to show how the modern theory of PPP might be consistent with a variety of stylized facts that have been widely interpreted as inconsistent with PPP. Another of those stylized facts is the apparently inexplicable increase in the volatility of exchange rates after the collapse of Bretton Woods.

G. Bretton-Woods and Exchange Rate Volatility

The large increase in the volatility of exchange rates after the collapse of the Bretton-Woods was unanticipated by both supporters and opponents of flexible exchange rates. The size of that increase started the idea that the volatility of flexible exchange rates is 'excessive' and led directly to the idea that the exchange rate is an asset price.[20] The modern theory of PPP raises serious doubts about that interpretation of the collapse of Bretton Woods. A modern approach to PPP suggests that the large increase in volatility after the collapse of pegged exchange rates was consistent with purchasing power parity. To show how this could be the case, I compare simulating a model for pegged exchange rates to the earlier simulations for flexible rates. In the absence of a pegged exchange rate, variance ratios, variances and half lives would be the same as in Tables 4 and 9.

I use the following threshold model to capture the effects of pegged exchange rates. Inside the intervention points IP, the exchange rate behaves exactly as it does with flexible exchange rates.

$$\Delta s_t = \Delta S_t, \qquad |s_t - PAR| \leq IP \qquad (11.1)$$

where S_t is the logarithm of the spot exchange rate without a peg, s_t is the logarithm of the pegged exchange rate, PAR is the logarithm of the official or par rate of exchange and IP is the intervention point

When exchange rates exceed an intervention point, the central bank intervenes. Equation 11.2 describes the effect of that intervention.

[20] To the best of my knowledge, there has been only one direct test of the asset approach versus an alternative flow approach, Pippenger (2003). Those results reject a pure asset approach in favor of a model where international trade, international investment and portfolio adjustment interact to determine daily exchange rates

$$\Delta s_t = \gamma(PAR - s_t) + e_t, \qquad |s_t - PAR| > IP \qquad (11.2)$$

At the Smithsonian Meetings in 1971, near the end of Bretton Woods, support points were increased from 1 percent to 2.5 percent. Since most of Bretton Woods operated with official support points of ± 1 percent, I assume support points of ± 1 percent. To keep exchange rates within those support points, central banks had to intervene well before a support point was reached. I therefore use 0.5 and 0.75 percent as the intervention points IP. When the intervention points are ±0.5 percent, I use 13 for γ. When the intervention points are ±0.75 percent, I use 14 for γ. These value for γ may appear large as compared to the value for the commodity points used earlier, but γ's this large are needed to keep the exchange rate within the support points.

TABLE 10
Simulating Pegged Exchange Rates and Flexible Exchange Rates Consistent with PPP

Country or Parameters	$\sigma^2_{\Delta S}/\sigma^2_{\Delta p}$ (Flexible)	$\sigma^2_{\Delta s}/\sigma^2_{\Delta p}$ (Pegged)	$\sigma^2_{\Delta S}$ (Flexible)	$\sigma^2_{\Delta s}$ (Pegged)	$\sigma^2_{\Delta P}$ (Both)
IP = 0.05, γ = 13	31.5	0.32	9.8	0.10	0.31
IP = 0.75, γ = 14	31.5	0.28	9.8	0.09	0.31

Table 10 suggests that the large increase in the short-run volatility of exchange rates after the collapse of Bretton Woods is compatible with the modern theory of purchasing power parity. Although 'economic fundamentals' with flexible and pegged exchange rates are exactly the same in these simulations, and the simulations never violate PPP, moving from pegged to flexible exchange rates increases the short-run volatility of exchange rates by over 90 times. Moving from pegged to flexible exchange rates also increases the variance ratio by over 90 times. Someone seeing such large increases might easily interpret them as

'excessive', but they are, by construction, consistent with the modern theory of purchasing power parity.

In these simulations, the reason for the large increase in volatility after a move to flexible rates is not the 'excessive' volatility of flexible rates. The large increase in volatility is the result of the artificial suppression of volatility under pegged rates. I call this suppression 'artificial' because it is not the result of a reduction in the monetary shocks that a truly fixed exchange rate like a gold standard would produce.

IV. Summary and Conclusions

Empirical and theoretical work over the last two decades on the law of one price and purchasing power parity has changed the way we need to think about both theories. I try to summarize these changes in what I call the modern theory of the law of one price and purchasing power parity. These new approaches to the LOP and PPP cast new or additional light on several important issues relating to the LOP and PPP. With respect to the issues covered here, these are my preliminary conclusions:

(1) The conventional wisdom about the relevance of PPP is probably wrong. Purchasing power parity probably works at least as well in a stable monetary environment as with hyper-inflation. Measurement errors and nonlinearities have caused the profession to misinterpret the econometric evidence.

(2) Although it is widely recognized that the nonlinearities in the modern approaches to the LOP and PPP reduce the already low power of tests for unit roots and cointegration, the reduction in power is probably even greater than is generally recognized. Using $S_t - p_t$

rather than the appropriate log of the real exchange rate $F_t - P_t$, probably reduces the power of these tests by even more than generally realized.

(3) The conventional wisdom regarding absolute versus relative PPP is suspect. Although at the moment it is purely conjectural, because of the nonlinearities inherent in the modern theory of PPP, there is a possibility that absolute PPP may yield better predictions about changes in exchange rates than relative PPP.

(4) Modern theories of the LOP and PPP recognize that commodity arbitrage involves time as well as space. Because commodity arbitrage takes time, the relevant prices are forward or futures prices. Since estimates of border effects use sticky retail prices rather than forward or futures prices, estimates of border effects probably exaggerate those effects.

(5) The conventional wisdom that the short-run volatility of exchange rates is excessive is suspect. The primary evidence for excessive variability is that the variance for monthly changes in exchange rates is much larger than the variance for monthly changes in relative CPIs. But there are two serious problems with these variance ratios. First, the thresholds implicit in the modern theory of PPP imply that such ratios should be larger than one. Indeed, the more successful countries are in coordinating their monetary policies, the larger will be these variance ratios. As a result, excess volatility will appear to be the largest when flexible exchange rates are working their best. Second, using consumer price indexes based on sticky retail prices artificially increases variance ratios.

(6) The large half life for deviations from PPP remains a major puzzle. It is possible that the modern theory of purchasing power parity may help resolve that puzzle. That theory suggests that the long half lives, and the imprecision of the estimates for those half lives, are

at least partially the result of the thresholds implied by the modern theory of PPP and the use of spot exchange rates and sticky commodity prices to measure real exchange rates rather than the more appropriate forward exchange rates and prices.

(7) The large increase in the short run volatility of exchange rates after the collapse of Bretton Woods fundamentally changed the way the profession thought about flexible exchange rates. The idea that exchange rates are asset prices and the idea that the volatility of exchange rates is 'excessive' are both a direct result of what happened to exchange rates after the collapse of Bretton Woods. But the modern theory of PPP suggests a very different interpretation of the effect of that collapse on volatility. The modern theory of PPP suggests that the volatility of exchange rates increased dramatically after the collapse, not because the volatility became excessive after the move to flexible rates, but because pegged exchange rates artificially restricted the movement in exchange rates as compared to the movement that would have been consistent with purchasing power parity. Even with no change in economic fundamentals, the modern theory of purchasing power parity may be able to explain the large increase in volatility after the collapse of Bretton Woods.

These implications of the modern theories of the law of one price and purchasing power parity suggest a large agenda for empirical research. At the very top of that agenda is the evaluation of the law of one price using forward or futures prices. If that research confirms that international arbitrage across time and space is effective, then we will have to change how we think about the law of one price and possibly how we think about purchasing power parity.

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