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Essays in Urban Economics, Economic Geography and Water

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by

Juan Carlos Goethe Lopez

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To my parents, for all the encouragement and babysitting.

ABSTRACT OF THE DISSERTATION

Essays in Urban Economics, Economic Geography and Water

by

Juan Carlos Goethe Lopez

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2016
Dr. Richard J. Arnott, Chairperson

This study has two primary focuses. The first is on the uneven distribution of high quality land across space. Of particular interest is how the spatial heterogeneity of land affects the location of households across regions, the development of land for urban and agricultural purposes within regions and the spatial structure of urban land use patterns within a given city. A second focus is how interbasin water transfers, which ameliorate the uneven distribution of water across regions, affect inter and intraregional land-use patterns. More specifically, the question this research asks is, what are the consequences of water transfers across regions when the preferences of households and the productivity of agricultural appears to favor the land in arid locations? And when land quality varies spatially, how does that alter the development patterns of cities and regions generally?

This research develops a framework for analyzing the implications of interbasin water transfers on interregional migration and intraregional land use patterns. The model employed in Chapter 2 is a novel synthesis of the two dominant models in the urban economics literature, namely, the two region core-periphery model and the monocetric city model. However the model is modified to account for spatial disparities in the quality of land across regions. In particular, a scenario is explored where one region has a greater degree of natural amenities and thus, *ceteris paribus*, is preferred by households, as well as a comparative advantage in agricultural production. In addition, there are agglomeration

externalities in the urban labor force, which, if sufficiently strong, leads to a concentration of all households in a single region. A second modification is that the more attractive region lacks water resources and, in order to satisfy household and agricultural water demand, must import water from the other region. This setup focuses on the tension between what is termed the amenity premium for households and the productivity premium for the agricultural sector, both of which compete for land in the same region.

Chapter 3 introduces public infrastructure into the previous model. The infrastructure, which is endogenous and defined by the demand for water in each region, is financed through a flat tax. In addition, we consider that there are transport costs in the distribution of agricultural output across regions. This assumption allows for three different trade regimes. Autarky, in which each region produces agriculture solely for the local population. Incomplete specialization, in which the more productive region produces all local supply and any excess is sold to the other region to supplement local output. Complete specialization, where all agricultural production is concentrated in a single region.

Chapter 4 turns to the issue of the heterogeneity in the quality of land within cities. The monocentric city modeling framework has developed a robust specification of urban spatial structure. In particular, it has shown how commuting costs play a crucial role in the spatial variation of housing rents, building heights and household living spaces. However, the model has been less capable in explaining the structure of cities at a more local scale. For instance, in cities with a dominant Central Business District (CBD) there does tend to be a decline in both building heights and housing rents as one moves further from the CBD. However, in a given neighborhood building heights and rents may not decline monotonically. There could be a host of reasons why this would be the case: historical factors, zoning regulations, idiosyncratic development patterns. Chapter 4 proposes that one such explanation is the heterogeneity of developable land over the space of a city.

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Chapter 1

Introduction

This study has two primary focuses. The first is on the uneven distribution of high quality land across space. Of particular interest is how the spatial heterogeneity of land affects the location of households across regions, the development of land for urban and agricultural purposes within regions and the spatial structure of urban land use patterns within a given city. A second focus is how interbasin water transfers, which ameliorate the uneven distribution of water across regions, affect inter and intraregional land-use patterns. More specifically, the question this research asks is, what are the consequences of water transfers across regions when the preferences of households and the productivity of agricultural appears to favor the land in arid locations? And when land quality varies spatially, how does that alter the development patterns of cities and regions generally?

Chapter 2 develops a framework for analyzing the implications of interbasin water transfers on interregional migration and intraregional land use patterns. The model proposes a novel synthesis of the two dominant models in the urban economics literature, namely, the two region core-periphery model and the monocetric city model. However the model is modified to account for spatial disparities in the quality of land across regions. In particular, a scenario is explored where one region has a greater degree of natural amenities and thus, *ceteris paribus*, is preferred by households, as well as a comparative advantage in agricultural production. In addition, there are agglomeration externalities in the urban labor force, which, if sufficiently strong, leads to a concentration of all households in a single region. A second modification is that the more attractive region lacks water resources and, in order to satisfy household and agricultural water demand, must import water from the other region. This setup focuses on the tension between what is termed the amenity premium for households and the productivity premium for the agricultural sector, both of which compete for land in the same region.

The results show that the land in the arid region is divided between urban and agricultural use only in the case when neither the amenity premium nor the productivity premium dominates the other and the economies of scale in the manufacturing sector are low. In the case that neither the amenity or productivity premia dominates but scale economies are high all households choose to concentrate in a single region, allowing the other region to be used solely for agricultural production. Finally, if either the amenity or productivity premia dominates the other and there are strong agglomeration economies there is only a stable, concentrated equilibrium in a single region. If the amenity premium is dominant, all household's locate in the arid region, leaving the less productive region for agricultural production. In contrast, if the productivity premium all agricultural production occurs in the arid region, leaving all households in the less amenable region.

While the above results are useful in identifying the long term trend in spatial land use patterns, they omit a crucial feature of interbasin water transfers, namely, the public infrastructure required to move water from one region to another. Chapter 3 addresses this by introducing public infrastructure into the previous model. The infrastructure, which is endogenous and defined by the demand for water in each region, is financed through a flat tax. In addition, we consider that there are transport costs in the distribution of agricultural output across regions. This assumption allows for three different trade regimes. Autarky, in which each region produces agriculture solely for the local population. Incomplete specialization, in which the more productive region produces all local supply and any excess is sold to the other region to supplement local output. Complete specialization, where all agricultural production is concentrated in a single region.

A simplified model is considered that allows for a sufficiently tractable framework to explore the comparative statics of the key parameters. It is found that increases in agricultural productivity initially drive households toward the more productive, arid region. However, as trade between regions is introduced, migration switches direction, allowing for more land being devoted to agricultural production in the arid region. An increase in transport costs and amenities drive households back to the arid region.

The model is calibrated numerically to reflect some stylized facts of land and water use patterns in California. A subsidy of agricultural water is considered as an extension. It is found that the subsidy leads to excessive water use in the agricultural sector to the detriment of urban households. Additionally, the water subsidy makes the less productive region artificially competitive, generating inefficient water use by the agricultural sector in the region.

Chapter 4 turns to the issue of the heterogeneity in the quality of land within cities. The monocentric city modeling framework has developed a robust specification of urban spatial structure. In particular, it has shown how commuting costs play a crucial role in the spatial variation of housing rents, building heights and household living spaces. However, the model has been less capable in explaining the structure of cities at a more local scale. For instance, in cities with a dominant Central Business District (CBD) there does tend to be a decline in both building heights and housing rents as one moves further from the CBD. However, in a given neighborhood building heights and rents may not decline monotonically. There could be a host of reasons why this would be the case: historical factors, zoning regulations, idiosyncratic development patterns. Chapter 4 proposes that one such explanation is the heterogeneity of developable land over the space of a city. A model is developed that introduces land quality into the monocentric city model. Three different specifications are employed. The first considers the standard static model in which all housing supply is perfectly malleable. It is shown that improvements in land quality increase building heights, holding distance from the CBD fixed. Additionally, building heights increase with distance from the CBD if the decline in marginal costs of construction exceeds the loss in revenue per square foot of rent. The second specification considers a dynamic model in which developers choose not only the building height but also the date of construction. This provides the condition for leapfrog development, in which some parcels of land closer to the CBD are left undeveloped while land further from the CBD is developed. It is shown that when land quality is improving further from the CBD, if the rent gradient is relatively flat and the decline in total costs from developing on better land are high, parcels of land further from the CBD will be developed earlier. Finally, in a third extension the possibility of an uneven distribution of amenities over a city is considered. It is shown that higher local amenities raise rents. And if amenities are increasing with distance from the CBD then so do rents if the household benefit from amenities exceeds the marginal increase in commuting costs.

Chapter 5 concludes by summarizing the key results of the research and offering some prospective avenues for new research possibilities.

Chapter 2

Interbasin water transfers and the size of regions: an economic geography example

Abstract

This paper contributes to the literature on regional land use and migration, developed separately in the theory of the monocentric city and the New Economic Geography, respectively, by introducing water as a mobile factor for use in household consumption and agricultural production. We develop a two-region, spatial general equilibrium model to explore the implications of interregional water transfers on household migration and the intraregional distribution of land between urban and agricultural use. A particular example is considered where an arid region lacks water resources but has a comparative advantage in both agricultural productivity and household amenities. Greater agricultural productivity promotes migration toward the less productive region, while a higher level of natural amenities favors local urban development. When economies of scale in the production in the manufacturing sector are low, relatively even levels of both amenities and agricultural productivity generate a more even distribution of the population. However, if scale economies

are sufficiently high all households concentrate in a single region. And if either agricultural productivity or natural amenities dominates the other only one of the concentrated equilibria is stable. Numerical simulations provide a graphical example of the set of stable equilibria in the parameter space. Finally, the model is calibrated using data on household consumption and agricultural productions patterns in the US.

2.1 Introduction

The growth of the American West over the last fifty years can be thanked in no small part for its ability to draw on additional water resources beyond the local supply. The California State Water Project and the Colorado River Aqueduct both supply Southern California households and agricultural producers with water from the northern portion of the state and the Colorado River, respectively, while the Central Utah Project and the Central Arizona Project both significantly supplement their region's local water resources. More generally, a recent study found that arid regions globally are increasingly reliant on imported water for both urban use and agricultural irrigation (McDonald *et al.* (2014)).

Given the significant constraint that limited water resources place on a region's economic viability, why have such large cities and farming communities emerged in these arid locations? In regards to cities, the urban economics literature has focused on local amenities as a driver of household growth in arid regions (Roback (1982), Rappaport (2006), (2008)). As moving costs have declined, households are drawn to regions with agreeable weather or physical beauty. As for agriculture, provided that there is sufficient water for irrigation, the moderate climates and a lack of unpredictable weather create highly productive and year-round growing conditions.

The contribution of this paper is in extending the research in spatial economics to include water as a mobile factor across regions to be distributed for urban and agriculture use. As a particular example, this paper explores how interbasin water transfers affect long-term migration patterns in a two-region trade model when arid regions provide a greater degree of natural amenities and agricultural productivity. Our results show that when one region is endowed with both a higher level of amenities and agricultural productivity relative to the other region, these features push households to migrate in opposite directions. A higher degree of local amenities increases the local population, while greater agricultural productivity promotes migration toward the less productive region leaving the more productive region for agricultural use. It follows that when amenities and productivity are sufficiently close to one another there is some level of dispersion of the population between the two regions. When economies of scale are introduced in the manufacturing sector, if they are sufficiently high, the wage premium generated by the concentration of households in a single region dominates any benefit from a more even distribution of households among both regions. However, if scale economies are high and either amenities or productivity dominates the other, only one of the concentrated equilibria is stable.

There has been little theoretical research on the implications of regional water transfers. The international trade literature has focused on 'virtual water', which allows arid regions to reduce the amount of water needed for irrigation by importing goods embedded with a high degree of water content (Reimer (2012)). While in the short run this is a viable alternative to maintaining a local agricultural sector for a country facing water scarcity, this strand of the literature ignores the possible productivity benefits from locating production in highly fertile but arid regions.

In order to explore the interplay between natural amenities and agricultural productivity on household migration and land use patterns, a novel synthesis of the monocentric city framework and the two-region trade models associated with the New Economic Geography (NEG) is developed. The monocentric city model has been the workhorse model in urban economics for the last 40 years, for its capability in analyzing the tradeoffs in the scale economies in urban concentration and the diseconomies of scale in commuting in an intraregional setting. The NEG, in contrast, has provided a class of models designed to explore the impact of interregional trade costs on population migration when there are scale economies in a monopolistically competitive manufacturing sector (Krugman (1991), Fujita *et al.* (1999), Baldwin *et al.* (2002)). This paper combines these models in a relatively tractable framework while making a couple of modifications. First, the quantity of land in each region is fixed and divided between urban and agricultural use, which endogenizes the opportunity cost of urban land. This is in contrast to the monocentric city framework where the city extends to the point that urban rents meet the threshold of a parametric agricultural land rent. Second, this paper considers an uneven distribution of resources between regions in the supply of water, natural amenities and agricultural productivity.

There have been a number of papers in the NEG literature that aim to integrate land use into the core-periphery model (Helpman (1997), Tabuchi (1998), Pflüger and Südekum (2007), Pflüger and Tabuchi (2010)). However, the analysis is often restricted to the urban areas. One of the key features of this model is how variations in agricultural productivity and amenities across regions affect the price of urban land and the extent to which they promote, or curtail, the concentration of households in a single region. Recent research suggests that all land is not, in fact, equal and that variations in land quality across regions have a significant effect on urban land costs (Burchfield *et al.* (2006), Saiz (2010)). Additionally, the uneven distribution of agricultural productivity can be a key determinant in the growth of a region (Matsuyama (1992)).

The paper is presented as follows. Section 2 develops the model, providing a discussion of both the short-run and the long-run equilibrium and some results. Section 3 provides a numerical simulation of the model using US data on household consumption and agricultural productivity. Section 4 concludes and offers some suggestions for future research.

2.2 The Model

Table 1 provides an index of notation for the model. Consider two regions that are populated by a mass, $N = 1$, of identical households, with n_i the share of households in region $i = 1, 2$. Each region is assumed to be of a fixed length and width equal to 1. There is a single source of water with a finite supply, $W = 1$, located in region 2. It is costless to

For regions $i = 1, 2$			
a_i	demand for agricultural good	α	agricultural good expenditure share
h_i	demand for urban land	β_i	agricultural total factor productivity
m_i	demand for manufacturing good	γ	water expenditure share
n_i	regional population share	δ	scale economies in manufacturing sector
r_i^a	agricultural land rent	η	manufacturing good expenditure
t	per unit commuting costs	ϕ_i	regional amenity shift factor
w_i^a	agricultural water demand	τ	water transport costs
w_i^u	urban water demand	Φ	amenity premium
x_i	household commuting distance		
y_i	household wage		
A	manufacturing output with no scale economies		
B	productivity premium		
N	total population		
W	total supply of water		
V_i	indirect utility		

Table 2.1: Notational Glossary

transport water within region 2. To supply water in region 1, there are iceberg transport costs that require $\tau > 1$ units to be ordered to supply 1 unit, with $\tau - 1$ units lost in transit.

Each region contains a Central Business District (CBD), which holds an urban manufacturing sector. All local households provide an inelastic supply of labor to the local manufacturing sector and receive the wage y_i. Households have a unit demand for land, which implies that the size of the city in each region is equivalent to the share of each region's total population. In the case that all households concentrate in a single region all land is used for household consumption. In addition a household that lives the distance x_i from the city faces commuting costs tx_i, where t is the units of the numeraire good required to travel a unit of distance. It is assumed that interregional commuting is not possible and that all land rent accrues to absentee landlords. [1]

Utility is derived from household consumption of the numeraire manufactured good,

[1] While this assumption is chosen for simplification it does not qualitatively affect the results. What changes is the mechanism whereby relative net incomes vary across regions. In the model with absentee landlords the net income of households within each region varies with the local rental price. When the model is adapted so that all rental income accrues back to households, the regional variation in net income enters through changes in the transfer payment. However, in both models, changes in parameters vary relative incomes in the same direction.

m_i, and the agricultural good, a_i, which are both freely traded with respective prices 1 and p^a. Additionally households in each region consume urban water, w_i^u, and face the regional water price p_i^w, whose difference reflects the transport costs in the distribution of water and is represented by the relationship $p_1^w = \tau p_2^w$. Utility takes the Cobb-Douglas form and a household's problem in each region at location x_i is then

$$V_i = \max_{a_i, m_i, w_i^u} \quad \phi_i a_i^\alpha (w_i^u)^\gamma m_i^\eta, \quad \alpha + \gamma + \eta = 1, \tag{2.1}$$

$$s.t.$$

$$y_i - r_i(x_i) - tx_i = p^a a_i + p_i^w w_i^u + m_i.$$

Here, ϕ_i is a regional shift factor that represents the endowment of local natural amenities, such as climate or landscape, in each region. It is assumed that $\phi_1 > \phi_2$, that is, all else equal, households receive a higher utility in region 1 than in region 2.

Utility maximization yields the following demand functions

$$a_i = \alpha \frac{y_i - r_i(x_i) - tx_i}{p^a}, \quad w_i^u = \gamma \frac{y_i - r_i(x_i) - tx_i}{p_i^w}, \quad m_i = \eta(y_i - r_i(x_i) - tx_i). \tag{2.2}$$

In order for households to be indifferent across locations within a city, utility must be constant, which implies the following relationship

$$r_i'(x_i) = -t. \tag{2.3}$$

This indicates that marginal rents must decline with the distance from the city in order to compensate households for the additional commuting costs they incur from locating further from the CBD. Using the terminal condition that the rent at the boundary of the city is equivalent to the agricultural rent, r_i^a, the regional bid rent function can be written as

$$r_i(x_i) = r_i^a + t(n_i - x_i). \tag{2.4}$$

Inserting (2.4) in to the budget constraint, the indirect utility can be written as

$$V_i = \alpha^\alpha \gamma^\gamma \eta^\eta \phi_i \frac{(y_i - r_i^a - tn_i)}{(p^a)^\alpha (p_i^w)^\gamma}. \tag{2.5}$$

Notice that due to the assumption of an inelastic demand for land by households, the demand and indirect utility functions are independent of each resident's commuting distance from the CBD, x_i. Therefore the term x_i is dropped from the remainder of the paper.

2.2.1 Manufacturing

The manufacturing sector in each region contains a continuum of small firms, which produce outputs using labor with the aggregate linear production function $A_i n_i$. Firms take A_i as given, therefore perfect competition drives profits to zero, implying $A_i = y_i$. However it is assumed that there are external benefits to production from the size of the local labor force. Therefore workers are paid their average rather than their marginal product with wages taking the form

$$y_i = A_i = A(1 + n_i)^\delta, \tag{2.6}$$

where δ is a measure of the degree of scale economies due to the local population share and A is the marginal product of labor of an isolated worker. The functional form in (2.6) implies that there is a wage premium for the region with a larger population.

2.2.2 Agriculture

In each region the land available for agricultural production is the remainder not devoted to urban use, $1 - n_i$. The agricultural good is produced using water and land with the intensive form function

$$2\beta_i\sqrt{w_i^a}, \tag{2.7}$$

where w_i^a is the quantity of agricultural water used per unit of land and β_i is a shift factor that measures the agricultural productivity of the region. It is assumed that $\beta_1 > \beta_2$ to ensure that the arid region *ceteris paribus* is more productive. The agricultural sector faces the water price p_i^w and the agricultural land rent r_i^a and receives p^a for each unit of the agricultural good sold. The profit function per unit of land is then given by

$$p^a 2\beta_i\sqrt{w_i^a} - p_i^w w_i^a - r_i^a. \tag{2.8}$$

Profit maximization and perfect competition yield the agricultural water demand and land rents

$$w_i^a = (\frac{p^a\beta_i}{p_i^w})^2, \quad r_i^a = \frac{(p^a\beta_i)^2}{p_i^w}. \tag{2.9}$$

Note that the ratio of agricultural rents is constant with

$$\frac{r_1^a}{r_2^a} = B, \quad B \equiv \frac{1}{\tau}(\frac{\beta_1}{\beta_2})^2. \tag{2.10}$$

where B, which measures the marginal rate of transformation between each region's agricultural land and is assumed to be greater than 1, reflects the water weighted productivity premium of agricultural land in the arid region (hereafter referred to as simply the productivity premium).

2.2.3 Short-run equilibrium

In the short run, the population share in each region is assumed to be fixed. The equilibrium conditions for agricultural goods and water are given by

$$2(1-n_1)\beta_1\sqrt{w_1^a} + 2(1-n_2)\beta_2\sqrt{w_2^a} = n_1 a_1 + n_2 a_2, \tag{2.11}$$

$$1 = n_2 w_2^u + (1-n_2)w_2^a + \tau(n_1 w_1^u + (1-n_1)w_1^a). \tag{2.12}$$

Note that the water demand in region 1 is multiplied by τ to account for the additional water lost in transit. Inserting (2.2),(2.9) and (2.10) into (2.11) and (2.12) yields short-run prices as function of n_1

$$r_2^a = \frac{\alpha}{2}\frac{n_1(A(1+n_1)^\delta - tn_1) + (1-n_1)(A(1+(1-n_1))^\delta - t(1-n1))}{(1-n_1)(B+\frac{\alpha}{2}) + n_1(1+B\frac{\alpha}{2})}, \tag{2.13}$$

$$p_2^w = \left(\frac{n_1(A(1+n_1)^\delta - tn_1) + (1-n_1)(A(1+(1-n_1))^\delta - t(1-n1))}{(1-n_1)(B+\frac{\alpha}{2}) + n_1(1+B\frac{\alpha}{2})}\right) \tag{2.14}$$

$$\times\,(\gamma + \frac{\alpha}{2})(B(1-n_1)+n_1) \tag{2.15}$$

$$p^a = \left(\frac{n_1(A(1+n_1)^\delta - tn_1) + (1-n_1)(A(1+(1-n_1))^\delta - t(1-n1))}{(1-n_1)(B+\frac{\alpha}{2}) + n_1(1+B\frac{\alpha}{2})}\right) \tag{2.16}$$

$$\times\,\sqrt{\frac{\alpha}{2}(\gamma + \frac{\alpha}{2})(B(1-n_1)+n_1)}, \tag{2.17}$$

with $p_1^w = \tau p_2^w$ and $r_1^a = Br_2^a$.

2.2.4 Long-run equilibrium

In the long run households locate in the region where they receive the higher utility. The ad hoc equation of motion is given by

$$\dot{V} = (V_1 - V_2)n_1(1-n_1). \tag{2.18}$$

There are solutions at $n_1 = 0, n_1 = 1$, and $n_1 \in (0,1)$ such that $V_1 = V_2$. A concentrated equilibrium is stable if $\dot{V}|_{n_1=0} < 0$, $\dot{V}|_{n_1=1} > 0$, while an interior equilibrium is stable if $V|_{n_1^* \in (0,1)} = 0$ and $\frac{\partial \dot{V}}{\partial n_1}|_{n_1^* \in (0,1)} < 0$, where n_1^* denotes an interior equilibrium. Inserting (2.13) into (2.5) and noting the fixed ratio between each region's agricultural land rents and water prices, an interior solution implies

$$V_1 - V_2 = \phi_1 \frac{(A(1+n_1)^{\delta} - r_1^a - tn_1)}{(p^a)^{\alpha}(p_1^w)^{\gamma}} - \phi_2 \frac{(A(1+(1-n_1))^{\delta} - r_2^a - t(1-n_1))}{(p^a)^{\alpha}(p_2^w)^{\gamma}} \tag{2.19}$$

$$= A\left(\Phi(1+n1)^{\delta} - (1+(1-n_1)^{\delta}\right) - t\left(\Phi n_1 - (1-n_1)\right)$$

$$- (B\Phi - 1)\frac{\alpha}{2}\left(\frac{n_1(A(1+n_1)^{\delta} - tn_1) + (1-n_1)(A(1+(1-n_1))^{\delta} - t(1-n1))}{(1-n_1)(B+\frac{\alpha}{2}) + n_1(1+B\frac{\alpha}{2})}\right) = 0.$$

Here $\Phi = \frac{\phi_1}{\phi_2}\frac{1}{\tau^{\gamma}}$ measures the ratio of the marginal utility of net income between region 1 and 2 and is assumed to be greater than 1. This reflects the water cost weighted amenity premium for locating in the arid region (hereafter simply the amenity premium). There is an incentive for concentration through the wage premium, which favors region 1 over region 2 due to the additional amenity benefit. However, concentration in region 1 raises local urban rents by increasing both commuting costs and the agricultural land rent. The proceeding section will focus on the interplay of these competing features of the model.

2.2.5 Results

In order to understand the competing roles of the productivity and amenity premia it is useful to consider the model with no agglomeration economies, i.e. $\delta = 0$.

Proposition 1. *When there are no scale economies in the urban manufacturing sector, there is a unique interior equilibrium if*

$$\frac{A(1+\frac{\alpha}{2}(B-1)) + \frac{\alpha}{2}t}{(A-t)} > \Phi > \frac{(A-t)B}{A(B-\frac{\alpha}{2}(B-1)) + \frac{\alpha}{2}t}. \tag{2.20}$$

Proof. See Appendix A

The intuition is that if the amenity premium is sufficiently high households receive a greater benefit from concentrating in region 1, leaving the less fertile land in region 2 for agricultural use. Conversely, if the productivity premium dominates, all households locate in region 2 to allow for the more productive land in region 1 to be used for agricultural production. Therefore, in order for the population to be divided between the two regions the amenity and productivity premia have to be relatively close to one another.

Suppose that the parameters are such that an interior equilibrium exists. Consider how changes in B and Φ affect the equilibrium prices, utility and population share. In the short run, an increase in B raises agricultural output reducing the agricultural price and increasing the price of water. In region 2, in order for the zero-profit condition to hold, agricultural rents fall to accommodate the loss in revenue, while the agricultural rents rise in region 1 due to the increase in productivity. The overall effect is to raise net income, and thus utility, in region 2 relative to region 1, leading to a reduction in n_1.

An increase in Φ has no short-run effects. In the long run, increasing amenities in region 1 raises the local utility level, generating migration toward region 1. This increases the demand for urban land in region 1 which drives up r_1^a and raises p^a. The increase in the agricultural price raises revenue for the agricultural sector in region 2, increasing r_2^a. In addition, an increase in p^a lowers the demand for agricultural goods and thus the water inputs needed for agricultural production, reducing p_i^w.

Introducing urban agglomeration economies we have the following result.

Proposition 2. *For the given functional forms the parameter space can be divided into four cases:*

Case 1: If

$$\frac{B(A2^{\delta}-t)}{A(B-\frac{\alpha}{2}(B-2^{\delta}))+tB\frac{\alpha}{2}} < \Phi < \frac{A(1+\frac{\alpha}{2}(B-2^{\delta}))+\frac{\alpha}{2}t}{(A2^{\delta}-t)}, \tag{2.21}$$

there is a stable interior equilibrium and no stable concentrated equilibria.

Case 2: If

$$\frac{B(A2^{\delta}-t)}{A(B-\frac{\alpha}{2}(B-2^{\delta}))+tB\frac{\alpha}{2}} > \Phi > \frac{A(1+\frac{\alpha}{2}(B-2^{\delta}))+\frac{\alpha}{2}t}{(A2^{\delta}-t)}, \tag{2.22}$$

both concentrated equilibria are stable with an unstable interior equilibrium.

Case 3: If

$$\Phi > \max\left\{\frac{B(A2^{\delta}-t)}{A(B-\frac{\alpha}{2}(B-2^{\delta}))+tB\frac{\alpha}{2}}, \quad \frac{A(1+\frac{\alpha}{2}(B-2^{\delta}))+\frac{\alpha}{2}t}{(A2^{\delta}-t)}\right\}, \tag{2.23}$$

$n_1 = 1$ *is a stable equilibrium,* $n_1 = 0$ *is unstable. Any interior equilibria come in pairs and alternate between stable and unstable.*

Case 4: If

$$\Phi < \min\left\{\frac{B(A2^{\delta}-t)}{A(B-\frac{\alpha}{2}(B-2^{\delta}))+tB\frac{\alpha}{2}}, \quad \frac{A(1+\frac{\alpha}{2}(B-2^{\delta}))+\frac{\alpha}{2}t}{(A2^{\delta}-t)}\right\}, \tag{2.24}$$

$n_1 = 0$ *is a stable equilibrium,* $n_1 = 1$ *is unstable. Any interior equilibria come in pairs and alternate between stable and unstable.*

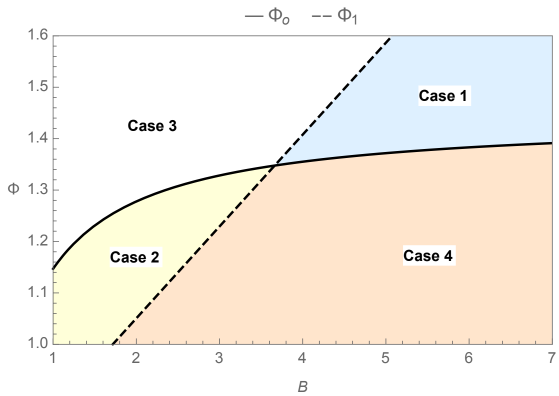
Proof. See Appendix C

Figure 2.1 provides graphical examples of Proposition 2 in $\{\Phi, B\}$, $\{\Phi, \delta\}$ and $\{\Phi, B, \delta\}$ space, respectively.

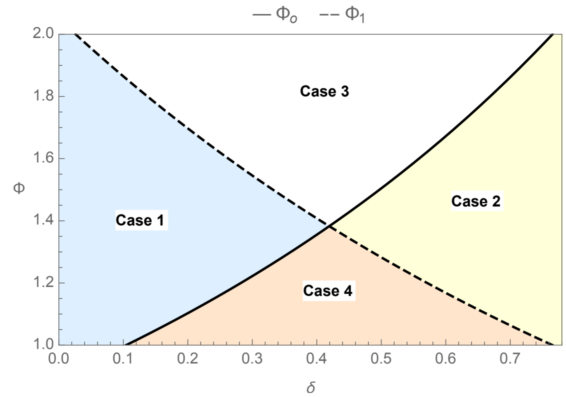
Case 1: When neither the amenity nor productivity premium dominates and scale economies are low there is a stable dispersed equilibrium. This is due to two factors. The first is that when scale economies are low, the wage premium generated by household concentration in a single region is not sufficient to offset the additional commuting costs from a single large city. Second, the moderate levels of both Φ and B, relative to one another, ensure that land in each region is devoted to both urban and agricultural use.

Case 2: In the case that the amenity and productivity premia are relatively close to one another, and scale economies are high, the wage premium is dominant, leading to concentration in either region. The competition for land between the urban and agricultural sector and the higher urban costs from agglomeration are not sufficiently strong to counter the wage premium generated from household concentration in a single region.

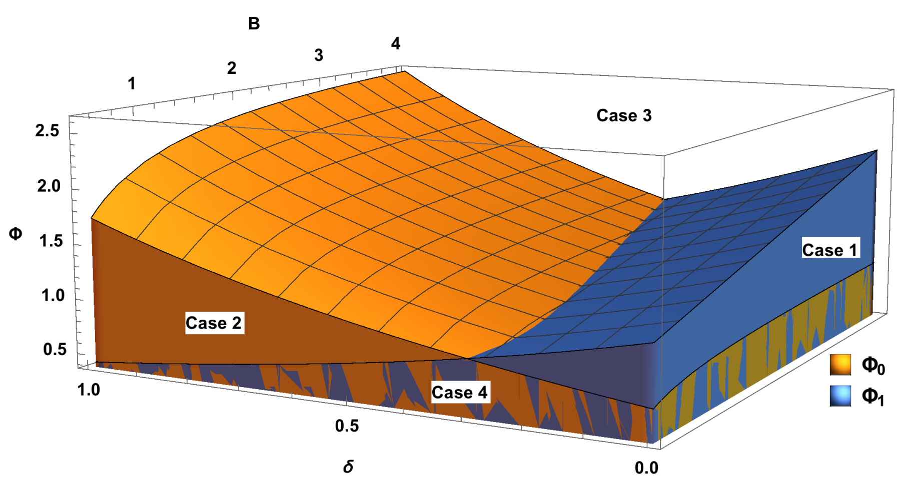
Case 3: There is a tendency toward concentration in region 1 driven by both the wage



(a) $\{\Phi, B\}$ space with $\delta = .4$



(b) $\{\Phi, \delta\}$ space with B=4



(c) $\{\Phi, B, \delta\}$ space

Figure 2.1: Equilibrium regions in parameter space.
Note: A=10, t=2, $\alpha = .4$,

$$\Phi_0 \equiv \frac{B(A2^{\delta}-t)}{A(B-\frac{\alpha}{2}(B-2^{\delta}))+tB\frac{\alpha}{2}}, \quad \Phi_1 \equiv \frac{A(1+\frac{\alpha}{2}(B-2^{\delta}))+\frac{\alpha}{2}t}{(A2^{\delta}-t)}$$

premium and the amenity premium when Φ is sufficiently larger than B and there are moderate to high scale economies.

Case 4: In contrast to Case 3, when the productivity premium dominates and there are strong economies of scale, all households locate in region 2 collecting the the full agglomeration wage, $A2^{\delta}$, leaving the more fertile land in region 1 for agricultural production.

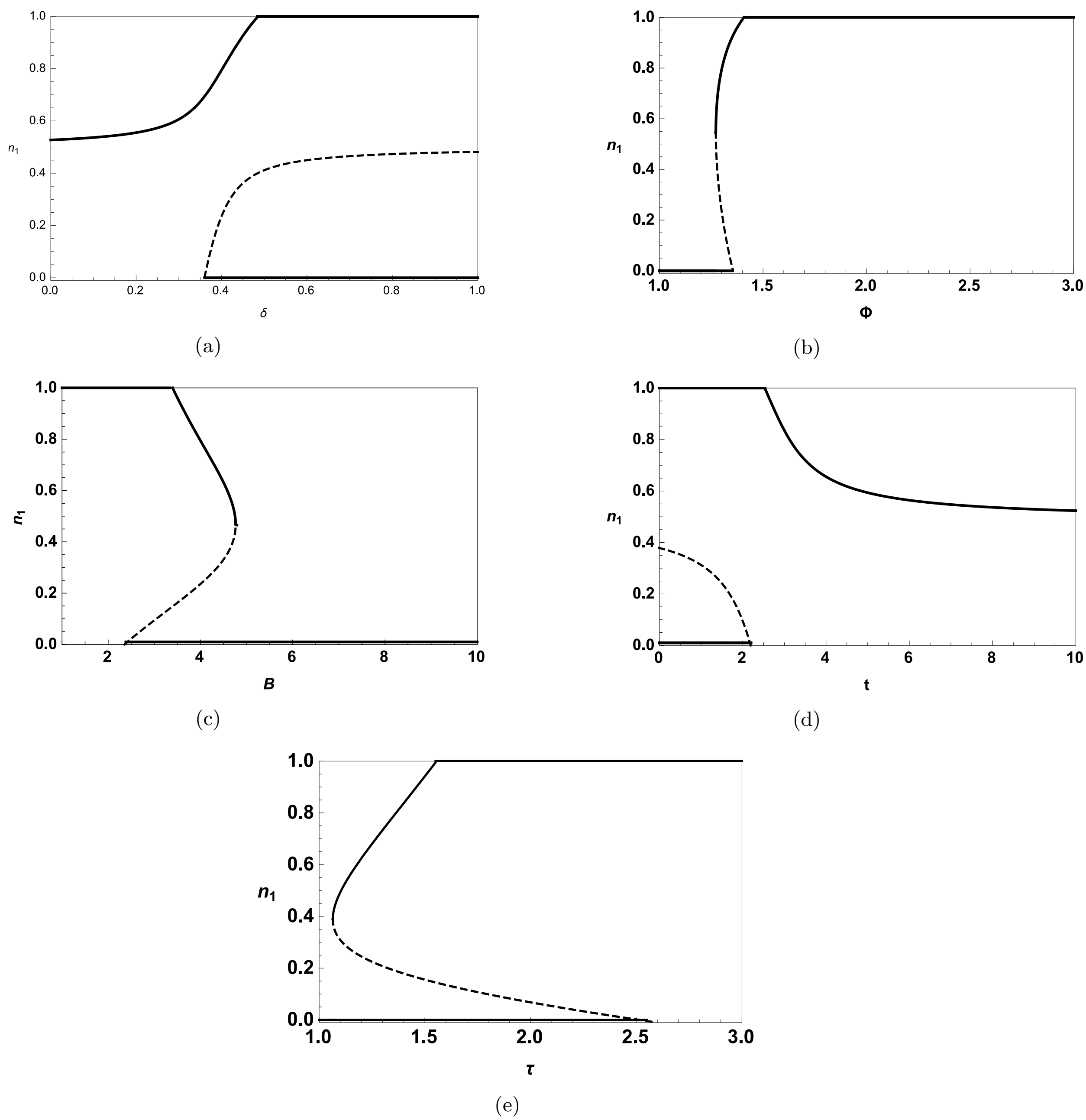


Figure 2.2: Equilibrium population shares due to changes in δ, Φ, B, t and τ. Note: Solid lines and dashed lines represent stable and unstable long-run equilibria, respectively. A=10, $\delta = .4$, $B = 4$, t=2, $\Phi = 1.3$, α=.4, $\gamma = .05$

Up to this point in the analysis, the focus has been on the determinants of the equilibrium. In order to shed some light on how the population distribution changes with the parameters, Figure 2 provides bifurcation diagrams for δ, Φ, B, t, and τ. In Figure 2.2a, for the given parameter values when there are no scale economies, i.e $\delta = 0$, the population share, and thus the wage, are higher in region 1. Therefore, as δ increases the wage differential between region 1 and 2 grows, leading to further increases in n_1 up to a stable concentrated equilibrium at 1 for $\delta = .49$. A second unstable arm emerges at $\delta = .36$ along with a stable concentrated equilibrium at $n_1 = 0$ as the wage premium from household concentration becomes dominant.

In Figure 2.2b, for relatively low levels of Φ, B dominates and the population is concentrated in region 1. As Φ increases, both a stable and an unstable equilibrium emerge in the interior, up to a critical level (1.41) after which households are concentrated in region 1. Similarly, in Figure 2.2c at low values of B, the amenity premium dominates and all households locate in region 1. As the land becomes more productive in region 1 through an increase in B, households migrate toward region 2 to allow the more fertile land to be employed in agricultural production, until the whole of the population is concentrated in region 2.

In Figure 2.2d, when commuting costs are low there exist multiple equilibria, with both concentrated equilibrium stable and an unstable interior equilibrium. As commuting costs increase, a stable interior equilibrium emerges from $n_1 = 1$ and tends to an even division of the population between regions. Intuitively, as commuting costs take up a larger portion of income households move to minimize those costs by distributing themselves into two smaller cities.

Finally, in Figure 2.2e when water transport costs are low, the productivity premium dominates as all land is devoted to agriculture in region 1. As τ increases both B and Φ decline, however, the effect on Φ is less significant as it is dampened by the term γ. Therefore increases in transport costs reduce the benefit of agricultural production to a greater degree than urban development in the arid region. In response, all households locate in region 1 at the critical point τ =2.57.

2.3 Numerical Example

The above model has provided a broad view of the tension between amenity and productivity premia on regional land use patterns and household location choice. This section provides a numerical example of the model that is calibrated using stylized facts from US data on household consumption and agricultural production. Table 2.2 gives the parameter values employed in the simulation. In the case of the productivity premium, an index of total factor productivity (TFP) by state in 2004 ranges from a low of 0.5712 for Wyoming to a high of 1.7979 for California, implying an upper bound of $(\frac{\beta_1}{\beta_2})^2 = 9.95$. Given that in this model what is relevant is the relative agricultural productivity between regions, B takes on the values of 2, 4 and 6, respectively. Note that, holding β_2 fixed, a change in B may reflect either a change in the productivity in region 1 or the change in the costs of water transport. In fact, estimates of interregional unit water transport costs show a great deal of variation between regions, largely due to differences in distance and topography.

The model is considered both with and without scale economies in production. The positive value of δ chosen reflects an elasticity of output to population size of 1.07, consistent with empirical estimates. Recall that t reflects not only the commuting cost per unit of distance, but also the cost of commuting from the boundary of the region. Therefore a worker commuting from the boundary spends roughly 20% of gross income on commuting. Finally, the value of Φ chosen represents an empirically modest, though as we will see not insignificant, amenity premium for region 1.

Table 2.3 provides the results of the simulation. There are two features to note regarding the distribution of the population, n_1. First, when the productivity premium is relatively low and there are scale economies, all households concentrate in region 1. And

$t = \$10,000$	(AASHTO, 2013)
$A = \$50,000$	(FRED, 2014)
$B = \{2, 4, 6\}$	(USDA ERS, 2014)
$\alpha = .1$	(USDA ERS, 2016)
$\delta = .21$	(Ciccone and Hall, 1996)
$\gamma = .02$	(BLS, 2013)
$\tau = 1.2$	(Zhou and Tol, 2004), (Hodges *et al.*, 2014)
$\Phi = 1.1$	(Rappaport, 2008)

Table 2.2: Parameter values

as B increases the majority of households continue to remain in region 1. This is due to the particularly low expenditure share of household income on food in the US, the lowest in the world, which therefore puts a greater emphasis on the amenity premium. Second, the effect of scale economies on the equilibrium population share decreases as B increases, with n_1 increasing by 60% when $B = 2$ to only 13% when $B = 6$. This follows from the fact that the wage premium declines as the population becomes more evenly distributed between regions. Therefore, as B increases the tendency of δ to concentrate a larger share of the population in the larger region is reduced. Additionally, all prices uniformly increase with δ, holding B fixed, as aggregate nominal income rises from the external benefits of local population size.

$\Phi = 1.1$									
	$B = 2$			$B = 4$			$B = 6$		
	$\delta = 0$	$\delta = .12$	$\%\Delta$	$\delta = 0$	$\delta = .12$	$\%\Delta$	$\delta = 0$	$\delta = .12$	$\%\Delta$
n_1	0.63	1.00	59.85	0.57	0.69	21.82	0.54	0.61	12.75
p_1^w	3543.29	3651.35	3.05	3564.37	3829.73	7.44	3575.56	3879.30	8.50
p_2^w	2952.74	3042.79	3.05	2970.31	3191.44	7.44	2979.63	3232.75	8.50
p^a	2129.38	2573.58	20.86	1653.73	1939.37	17.27	1388.17	1591.98	14.68
r_1^a	3071.22	4353.44	41.75	3682.86	4714.05	28.00	3880.38	4703.88	21.22
r_2^a	1535.61	2176.72	41.75	920.72	1178.51	28.00	646.73	783.98	21.22
W_1^a	0.388	0	-100.00	0.539	0.460	-14.67	0.597	0.566	-5.12
W_2^a	0.326	0.716	119.88	0.175	0.254	45.13	0.118	0.148	25.98
W_1^u	0.173	0.286	65.84	0.155	0.191	23.28	0.148	0.168	13.50
W_2^u	0.113	0	-100.00	0.131	0.095	-27.51	0.138	0.118	-14.52
V_i	11628.70	12192.10	4.84	11925.50	12743.30	6.86	12146.20	13063.80	7.55

Table 2.3: Simulation results for an increase in the productivity premium relative to the amenity premium with and without scale economies

W_i^a and W_i^u represent aggregate agricultural and urban water use, respectively, in region i. Holding δ fixed agricultural water use rises in region 1 and falls in region 2 when the productivity premium increases, as the population rises in region 2 and more land is devoted to agriculture in region 1. Meanwhile, when B is held fixed, an increase in δ leads to a rise in n_1 raising urban water demand in region 1 and reducing agricultural water demand as less land is left available for agricultural production. It follows that in region 2 there is a decline in water use from the urban sector and increase in the agricultural sector, with an increase in δ and the opposite effect with an increase in B. Finally, as would be expected, both B and δ raise the overall utility level.

2.3.1 A discussion of the numerical results in light of US migration patterns

Can the results in Table 2.3 be used to describe migratory patterns in the US? Of course a linear two-region model cannot fully account for the migration patterns across households in fifty states. We nevertheless feel that it is capable of reproducing qualitatively many features in the data. Consider a relatively small region such as Southern California where the productivity of the land is relatively homogenous over the region, while the area closer to the coast has more amenable weather compared to the much warmer regions further inland. In the early 1900s the land in Southern California was largely devoted to agriculture. Presently, the region is completely urbanized from Ventura County to the Mexican border and well inland through Los Angeles County, Riverside County, Orange County and San Diego County. The eastern portion of Southern California that buttresses Arizona and Nevada remains largely agricultural containing two of the most productive regions in the country, the Imperial Valley and Coachella Valley. This result would be consistent with the first set of columns in Table 2 where the productivity premium is low and the amenity premium dominates. Similarly, we can compare Florida, which ranked 2nd in US agricultural TFP in 2004 and grew from the 33rd most populous state to the 4th between the years 1900 and 2000 (Hobbs and Stoops, 2002), to Iowa which ranked

3rd in TFP and fell in population rank from 10th to 30th between 1900 to 2000. This is consistent with a specification where B is relatively low as both regions have similar productive capabilities, while Φ would favor Florida with its warmer climates and coastal amenities.

Finally, consider California in relation to Texas. Both offer warm weather and water supply concerns yet California ranked 1st in agricultural TFP in 2004 while Texas was ranked 43rd. In 1960 California farm output was 50% greater than that of Texas and by 2004 it was 100% larger, while over the same time period both Houston and the Dallas-Fort Worth areas entered into the top 10 of the most populous US cities. This is consistent with a more dominant productivity premium relative to the amenity premium. Both California and Texas have significant urban sectors while the agricultural sector is larger and growing at a faster pace in California.

2.4 Conclusion and Future Research

This paper has developed a simple two-region economic geography model to explore the interplay of agricultural productivity and amenity premia in arid regions when water is a mobile factor and there are economies of scale in the urban manufacturing sector. When scale economies are sufficiently low, it was shown that amenity and productivity premia drive land-use patterns in opposite directions. Amenity premia encourage the development of land for urban use while productivity premia support land use for agricultural production. For moderate levels of both there is a stable and dispersed equilibrium. If economies of scale in the manufacturing sector are high and neither the productivity or amenity premia dominates the other, the wage premium generated from all households concentrating in a single region overwhelms the benefits of a more even distribution of the population. When either the amenity premium or the productivity premium dominates the other and there is a high degree of scale economies, only one of the concentrated equilibria is stable. The parameter space was explored in order to define the conditions for stable and unstable equilibrium configurations and bifurcation diagrams were shown numerically for key parameters. Finally, the model was calibrated to reflect US data on household consumption and agricultural production patterns.

While this paper has developed a coherent framework to explore how competing urban and agricultural interests vie for water and land, in the interests of tractability a number of realistic features have been excluded. Iceberg transport costs provide a convenient way to conceptually model freight costs; however, they focus solely on the marginal cost of distribution. In practice, water distribution networks, both intra- and inter-regionally, have significant fixed costs, which require financing from local, state and federal governments. The economies of scale and public financing of interregional water transfers should be further explored in order to gain a more robust understanding of the effect of water transfers on migration and land use.

Additionally, this model has provided a competitive framework where water is allocated to its best use through the price system. However, water transfers are often dominated by a Byzantine set of rules, where water prices vary not only by city or region, but by consumers within cities as well. A portion of agricultural land in certain regions may be allocated water rights that are not available to the remaining shares of land. Therefore, an understanding of the institutional factors that define water-use patterns, often over-and-above the market structure, are crucial in gauging the future development of arid regions and the ability to sustain future growth in population and agricultural production.

Finally, high amenity cities tend to have higher rents. Households respond to these higher prices by reducing the quantity of living space they consume, presumably offsetting the loss through the additional benefits they receive from the local amenities. Relaxing the assumption of inelastic demand for land by households can provide a more realistic portrait of land distribution in a region with both high productivity and amenities. In particular, it would allow for the possibility of a large share of the population residing in a single region on a relatively small share of the land.

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Appendix A

A.1 Proof of proposition 1

Setting $\delta = 0$ in (2.17) implies that there is a quadratic solution in n_1 for $\dot{V} = 0$. This further implies that the slope of $\dot{V}$ changes signs at most once. It follows that solving for Φ in $\dot{V}|_{n_1=0} > 0$ and $\dot{V}|_{n_1=1} < 0$, ensures a single crossing through the x-axis.

A.2 Proof of proposition 2

The conditions for each case are the following: case 1 requires $V|_{n_1=0} > 0$ and $V|_{n_1=1} < 0$, case 2 requires $V|_{n_1=0} < 0$ and $V|_{n_1=1} > 0$, case 3 requires $V|_{n_1=0} > 0$ and $V|_{n_1=1} > 0$ and case 4 requires $V|_{n_1=0} < 0$ and $V|_{n_1=1} < 0$. Solving for Φ in each case yields the result.

Chapter 3

Water for Arid Regions: An Economic Geography Approach

3.1 Abstract

This paper develops a two-region trade model to investigate how the uneven allotment of water resources and the availability of interbasin water transfers affect the intraregional distribution of land between urban and agricultural use and the interregional distribution of the population when there is heterogeneity between regions in natural amenities, agricultural productivity and urban agglomeration economies. The model consists of a small country with two regions, one of which is endowed with water while the other has more productive agricultural land and provides a greater degree of natural amenities to households. A public infrastructure network links the two regions, providing water to both households and an agricultural sector. The agricultural good faces transport costs between regions. This model is unique in combining a core-periphery model with a monocentric city and a fully realized agricultural sector, which must compete for a fixed quantity of land. In addition, the water infrastructure is explicitly modeled and its size endogenously determined. A simplified example is presented which allows for comparative static analysis. The model is solved numerically for a general case. Finally, a subsidy on agricultural water is considered.

3.2 Introduction

Historically, a local supply of water has been a crucial component for the location of cities and agriculture. Indeed, an alternate history of westward expansion of the United States in the 19th century focuses on the Bureau of Land Reclamation and the Army Corps of Engineers struggling to reconfigure existing water resources to accomodate the idiosyncratic land use patterns of the early Western settlers (Reisner, 1986). Increasing urbanization in recent years, particularly in arid regions, has placed considerable stress on their existing water resources. Many regions, in response, have turned to imported water via interbasin transfers to supplement existing resources. Recent research suggests that water transportation infrastructure has greatly reduced the number of water-stressed cities globally (McDonald et al. (2014)). Such infrastructure projects can be attractive for regional governments looking to promote growth. For instance, in the 1960's, California governor Pat Brown, inaugurated the State Water Project, which was developed to supply Southern California cities and agriculture with water from the northern part of the state. His stated intention was to "correct an accident of people and geography" (Bourne, 2010). In light of the fact that cities are becoming less reliant on local water sources, this paper presents a novel approach to understanding urban and agricultural water needs across space when water is a mobile factor.

We develop a model to investigate water use patterns across regions when the physical endowments of land varies across space. The model consists of two regions of equal size each of which devotes land to either agricultural production or to housing for residents who work in an urban manufacturing sector. The regions are separated by an uninhabitable valley. There is a fixed number of households who gain utility from land, agricultural goods, a manufactured good, water and region-specific natural amenities. In addition, there is a fixed quantity of water that is located solely in one region. A publicly financed water distribution network is developed to transport water across both regions for urban and agricultural use. Each city produces the manufactured good which is freely traded across regions, and the manufacturing sector in each region shows increasing returns to scale dependent on the size of the regional labor force. Agricultural land in each region differs in productivity, and there are iceberg transport costs associated with the trade of the agricultural good. Households distribute themselves until a spatial equilibrium is found when utility equalizes across regions.

This paper looks at a specific subset of possibilities for this framework, namely, the case where one region is endowed with both a more productive agricultural sector and higher level of natural amenities yet is devoid of water resources. Three trade regimes are considered which are dependent on the level of the agricultural productivity differential between both regions: (1) an autarkic regime where each region produces the agricultural good solely for the local population; (2) incomplete specialization, where local agriculture in the less productive region competes with the agricultural imports of the more productive region; and (3) complete agricultural specialization, where the more productive region is the sole producer of agriculture. A simplified example is considered, which allows for comparative static analysis. The model is then solved numerically to quantify the effects of agglomeration economies and natural amenities on the equilibrium. Results are compared to a social planner's problem. Additionally, a policy experiment analyzes a subsidy on the price of agricultural water.

It is found that agricultural productivity acts as an agglomerative force, as households benefit from allowing the more productive land to be used for agriculture, leading to concentration of households in the less productive region. However, increases in transport costs or in the natural amenities of the more productive region reduces the agglomerative effect of agricultural productivity. Economies of scale are found to have little effect when the population is more evenly dispersed; however, increases in agglomeration economies when the population share differential is high increases concentration towards the more populated region. The numerical results show that when agricultural production is concentrated in one region, agricultural water subsidies lead to a significant decline in household income and an increase in regional price indices.

The objective of this paper is to explore how the introduction of water transportation infrastructure in a two-region model with regional asymmetries in agricultural and manufacturing production, resources and natural amenities, will affect the population share and water use interregionally and the distribution of land between agricultural and urban use intraregionally. From a policy perspective, this research develops a novel framework for conceptualizing the allocation of water resources across space. In addition, it allows for the cost-benefit analysis of water distribution infrastructure and the comparison of various pricing and tax financing schemes. This paper continues in a long tradition of using computable general equilibrium (CGE) models of the monocentric city to explore the effects of policy changes including transportation costs on land rents and congestion (Arnott and Mackinnon (1977a), Arnott and Mackinnon (1977b), Arnott and Mackinnon (1978), Tikoudis et al. (2015)), the development of urban subcenters (Sullivan (1986), Helsley and Sullivan (1991)) and urban environmental policy and land use (Nijkamp and Verhoef (2002), Bento et. al. (2006)).

From a theoretical point of view the model combines the closed monocentric city model (Pines and Sadka (1984)) and the two-region core-periphery model developed by Krugman (1991) and Fujita et. al. (1999),which has since provided the basis for the New Economic Geography (NEG). The model is closed in the sense that the total population is exogenous, household utility is endogenous and rental income is redistributed back to households. This allows for welfare analysis under various trade and policy regimes. In addition, the model fixes the quantity of land reflecting the fact that land use is limited by physical or political boundaries. Tabuchi (1998) integrated the Alonso-Muth-Mills model into the NEG framework; however, his model retained a central tenet of the monocentric city, that agricultural land rent is exogenous. In contrast, this model, by holding constant the quantity of land available for agricultural or urban use in each region, endogenizes the land rent at the boundary of the city, creating a tension between urban agglomerative processes and increasing agricultural productivity, reinforcing Pflüger and Tabuchi's statement of "the long standing wisdom in spatial economics that ultimately there is only one immobile resource, land" (Pfĺuger and Tabuchi, 2010). Other authors have explored the effect of limits to developable land on urban growth (Helpman (1998), Saiz (2010) , Chatterjee and Eyigungor (2012)), however, the effect of heterogeneity in agricultural productivity as a factor in the urban supply of land is not treated.

Picard and Zheng (2005) extend the model of Ottaviano et al. (2002) to integrate more explicitly an agricultural sector with transport costs, which competes with the manufacturing sector for labor. This model, on the contrary, assumes that agriculture competes with the urban households for land and water. Matsuyama (1992) proposed an endogenous growth model that considered both a closed and a small open economy. A positive link was found between agricultural productivity and growth in a closed economy and a negative link in an open setting. Our analysis confirms this result. Under autarky, the more productive region has a larger share of the population and thus a larger manufacturing sector, as well as more abundance in the agricultural good. In contrast, when trade is possible, the region with less productive agricultural land has a larger manufacturing sector.

A recent set of papers has focused on how a limited land supply can dampen agglomerative forces (Pflüger and Südekum (2007), Pflüger and Tabuchi (2010)). In our model, this result occurs due to the tension between households' preferences for natural amenities and agricultural goods. A rise in agricultural productivity has two effects. It increases the opportunity cost of urban land at the boundary of the city, while increasing output and thus reducing the price of the agricultural good. However, when a region is abundant in natural amenities, households are willing to pay a higher rent to locate there, which reduces the opportunity cost of urban land. Yet, the household demand for the agricultural good requires that land be used for agricultural production, and is utilized best in the most productive region. The cumulative effect is to drive up rents at the urban boundary in the region with the more productive agricultural sector.

However, when natural amenities are low the agricultural productivity effect dominates, driving households to the less productive region. This effect is compounded by agglomeration economies as one region becomes relatively more populous than the other, which increases relative wages in favor of the larger region. These results are consistent with the literature on quality of life and urban amenities (Roback (1982), Rappaport (2006), Rappaport (2008)). As in Pflüger and Südekum (2007), an intuitively appealing outcome of this model is that unlike the standard NEG model, where at a critical value the whole population goes instantaneously from dispersion to agglomeration, this model shows gradual shifts in the population with changes in productivity. Finally, the model is novel in introducing interregional water transportation infrastructure into the monocentric city and NEG models.

The remainder of the paper is presented as follows. Section 2 outlines the model and describes the equilibrium under various trade regimes. Section 3 describes the functional forms and parameter values that were used to calibrate the numerical simulation. In addition an analytical exercise is conducted to allow for a graphical analysis of the comparative statics. Section 4 presents and discusses the numerical results. Section 5 proposes topics and extensions for future research. Section 6 concludes.

3.3 The Model

3.3.1 Model Overview

The theoretical analysis combines a standard linear monocentric city model with a two-region new economic geography model *a la* Helpman (1998). The model consists of a small country, comprised of two regions, whose borders are fixed either by natural or political boundaries. There is a fixed number of identical households that are split into monocentric cities, one located in each region. Each household supplies labor to an urban manufacturing sector. Urban land is devoted to residential housing. The remainder of the land in each region is left for agricultural production, which is produced using water and land. Water is located solely in one region, but is made available across the country through a water distribution network that links agriculture and households in both regions to the water source. Table 1 provides a notational glossary.

Table 3.1: Notational Glossary

a_i	household demand for agricultural good in region i
i	regional subscript
m_i	household demand for manufacturing good
p_i^a	regional agricultural good price
p_i^m	regional manufacturing price, numeraire
p^w	common water price
q	marginal cost of water infrastructure
r_i^a	regional agricultural land rent
$r_i(x_i)$	regional bid-rent function
$\bar{r}$	rental transfer
t	household commuting cost
w_i^a	agricultural water demand per unit of land
w_i^u	household water demand
$\bar{w}$	per-capita supply of water
x_i	distance from the cbd
y_i	regional wage
W_i^a	total regional agricultural water demand
L_i^a	total regional agricultural land
A	shift factor on marginal product of labor
A_i	regional marginal product of labor
I_i	regional net income
L	common length of each region
L_s	distance between regions
N	total population
P_i	regional price indices
T	ratio of net incomes in supplemental and specialized regimes
U_i	regional utility level
W	available supply of water
α	water share of agricultural production costs
β_i	regional agricultural productivity
γ	budget share of water
δ	degree of economies of scale in manufacturing sector
η	budget share of manufacturing goods
θ	agricultural water subsidy
λ	share of households in region 1
μ	budget share of agricultural goods
ρ	defines the elasticity of substitution in agricultural production
σ	elasticity of substitution between land and water in agricultural production
τ	agricultural transport costs across regions
ϕ_i	shift parameter denoting household preferences for regions
Θ	functional abbreviation
Λ	functional abbreviation
Φ	ratio of regional utility derived from natural amenities in autarkic equilibrium

3.3.2 Formal Presentation

The model consists of a small country populated by N identical households. The country is divided into two regions, 1 and 2, respectively, with λN households in region 1 and $(1-\lambda)N$ in region 2. The space of the country is a line of length $2L + L_s$ and width 1, where L is the size of region $i = 1, 2$ and L_s is a length separating the two regions. The land and water supply of the country are commonly owned by all residents. Each region contains a monocentric city with an urban manufacturing sector which employs a share of the population to produce a manufactured good in the central business district (CBD). A share of each region's land is used for housing the local population. The remainder of the land is devoted to agricultural production. Demand for land by households is fixed at a single unit, which is chosen such that $L = N$. This implies that the size of the city in region 1 is λN and in region 2 is $(1-\lambda)N$, while, symmetrically, the land devoted to agriculture in region 1 is $(1-\lambda)N$ and in region 2 is λN. The country contains a fixed supply water, W, located at the CBD of region 2 and used for irrigation by the agricultural sector and by households for personal consumption. The supply of water is assumed to be fully allocated. A publicly financed infrastructure network transports water from the source to households and the agricultural sector in each region.

Figure 3.1 gives a visual description of the space of the model. The top line gives the land distribution. In the center is the length L_s that separates the two regions. At the boundary of L_s and region 1 and 2 is the local CBD. Along the distance x_i is the length of the city, which ends at λN for region 1 and $(1-\lambda)N$ for region 2. The remaining land up to the length N in each region is devoted to agriculture and is denoted in the figure by $Ag1$ and $Ag2$, respectively. The bottom line describes the infrastructure, denoted as "Pipe" in the figure. The water supply is located at the CBD of region 2. It travels across the length L of region 2 to the right, while to the left it travels the length $L + L_s$, in order to supply region 1.

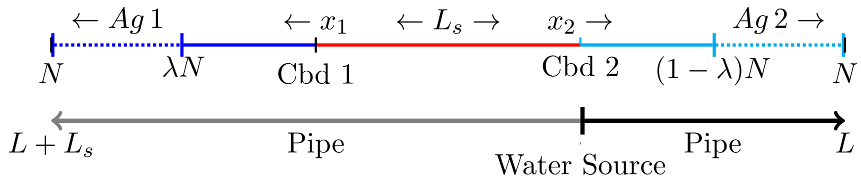


Figure 3.1: Regional Space

3.3.3 Demand

Households

Each household supplies labor inelastically to the manufacturing sector and receives the wage y_i. All households work at the CBD. For a household who commutes the distance x_i to the CBD from their home, faces land rents $r_i(x_i)$ and commuting costs $t(x_i)$. It is assumed that it is costless to migrate between regions but is prohibitively costly to commute between regions, ensuring that all households work where they live. In addition to wage income, households receive a transfer $\bar{r}$, which is the household share of aggregate land rents, and face the flat tax f, which is used to finance the water distribution infrastructure. Households have preferences over the numeraire manufacturing good, m_i, urban water, w_i^u, and the agricultural good, a_i, for which they face the agricultural price p_i^a and the common water price, p^w, and the price of the manufactured good, p^m, is the numeraire and equal to 1. The utility maximization problem is then given by

$$\max_{m_i, a_i, w_i^u} U^i(m_i, a_i, w_i^u; \phi_i) \tag{3.1}$$

$$s.t.$$

$$y_i + \bar{r} - f - r_i(x_i) - t(x_i) = m_i + p_i^a a_i + p^w w_i^u, \quad i = 1, 2,$$

where ϕ_i is a shift factor that measures region-specific natural amenities such as weather or attractive landscape. Solving for m_i from the budget constraint and inserting it into the utility function, the first-order condition yields

$$p^w = \frac{U^i_{w^u_i}}{U^i_{m_i}}, \quad p^a_i = \frac{U^i_{a_i}}{U^i_{m_i}}, \tag{3.2}$$

where the second subscript denotes partial derivatives. Combining (3.2) with the budget constraint yields the following household uncompensated demand functions.

$$m_i = m_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w; \phi_i), \tag{3.3}$$

$$a_i = a_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w; \phi_i), \tag{3.4}$$

$$w^u_i = w^u_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w; \phi_i). \tag{3.5}$$

The indirect utility function is then given by

$$V_i(y_i + \bar{r} - f - r_i(x_i) - t(x_i), p^a_i, p^w; \phi_i). \tag{3.6}$$

For households to be indifferent across locations in the city implies that the derivative of the indirect utility function with respect to x_i be zero, which yields

$$r'_i(x_i) = -t'(x_i). \tag{3.7}$$

Integrating over x_i gives

$$r_i(x_i) = -t(x_i) + k, \tag{3.8}$$

where k is a constant of integration. Using the terminal condition that the rent at the boundary of the city equals the agricultural rent, r_i^a, gives each region's bid-rent function

$$r_1(x_1) = r_1^a + t(\lambda N) - t(x_1), \quad x_1 \in [0, \lambda N], \tag{3.9}$$

$$r_2(x_2) = r_2^a + t((1-\lambda)N) - t(x_2), \quad x_2 \in [0, (1-\lambda)N], \tag{3.10}$$

where use is made of the fact that the boundary of the city in region 1 is λN and $(1-\lambda)N$ in region 2.

3.3.4 Supply

Manufacturing

The manufacturing good is produced by a continuum of small firms, with a linear technology utilizing solely labor. Producers face the wage cost y_i. The aggregate profit function for manufacturing firms in each region is given by

$$A_1 \lambda N - y_1 \lambda N, \tag{3.11}$$

$$A_2(1-\lambda)N - y_2(1-\lambda)N, \tag{3.12}$$

where A_i is the marginal private product of labor and is taken as given by firms. The industry is assumed to exhibit increasing returns from regional population size due to agglomeration economies at the aggregate level, which are captured in each region by the term $A_1 = A_1(\lambda; \delta),\ A_2 = A_2((1-\lambda); \delta)$. At the firm level, perfect competition drives profit to zero yielding,

$$A_1(\lambda; \delta) = y_1, \tag{3.13}$$

$$A_2((1-\lambda); \delta) = y_2, \tag{3.14}$$

where δ is a parameter relating the elasticity of the marginal product of labor to local population size.

Agricultural Production

Agriculture is organized competitively. It is produced using water, W_i^a, and land L_i^a, with the linearly homogeneous production function $F_i(W_i^a, L_i^a; \beta_i)$, where β_i is a region-specific shift factor capturing the productivity of agriculture. It is assumed that $\beta_1 \geq \beta_2$. Given that in equilibrium the land devoted to agriculture in each region is simply the share not used by households, with $L_1^a = (1-\lambda)N$ and $L_2^a = \lambda N$, it is useful to write the production function in intensive form as

$$F_i(W_i^a, L_i^a; \beta_i) = L_i^a F_i(\frac{W_i^a}{L_i^a}, 1; \beta_i) = L_i^a f_i(w_i^a; \beta_i). \tag{3.15}$$

Producers face the prices of water, p^w, and land rent r_i^a, and charge the price p_i^a. The profit function for unit of land is then

$$p_i^a f_i(w_i^a; \beta_i) - p^w w_i^a - r_i^a. \tag{3.16}$$

The first-order condition is given by

$$p_i^a f_i'(w_i^a; \beta_i) - p^w = 0, \tag{3.17}$$

which yields the agricultural water demand function per unit of land

$$w_i^a(p_i^a, p^w; \beta_i). \tag{3.18}$$

Finally, agricultural rents adjust until profits are equal to zero:

$$p_i^a f(w_i^a(p_i^a, p^w; \beta_i); \beta_i) - p^w w_i^a(p_i^a, p^w; \beta_i) = r_i^a. \tag{3.19}$$

3.3.5 Government

The government plays two roles. First, it collects land rents and redistributes the proceeds back to residents as the lump sum transfer $\bar{r}$. Second, it oversees the construction of the water transportation infrastructure and the pricing of water, and levies a tax on households for any additional costs not covered by the sale of water, f. Note that the assumption of a common flat tax for residents of both regions ensures that there is no migration by households looking to benefit from preferential tax rates.

Rental Transfer

Integrating over the household bid-rent functions yields

$$\int_0^{\lambda N} r_1(x_1)dx_1 = \lambda N r_1^a + \int_0^{\lambda N} t(x_1)dx_1, \quad \int_0^{(1-\lambda)N} r_2(x_2)dx_2 = (1-\lambda)N r_2^a + \int_0^{(1-\lambda)N} t(x_2)dx_2. \tag{3.20}$$

The rental revenue from agriculture in regions 1 and 2, respectively, is $(1-\lambda)Nr_1^a$, and $\lambda N r_2^a$. It follows that the rental transfer to each household is given by

$$\bar{r} = r_1^a + r_2^a + \frac{\int_0^{\lambda N} t(x_1)dx_1 + \int_0^{(1-\lambda)N} t(x_2)dx_2}{N}. \tag{3.21}$$

Infrastructure Tax

The water transportation infrastructure uses q units per mile of the manufacturing good to transport one acre foot of water one mile . The size of the infrastructure is modeled as proportional to the share of the total water supply going in each direction from the source to region 1 or 2, multiplied by the distance the water must travel. The infrastructure needed to supply each region with water is then

$$\text{Region 1} : qW(L+L_s) \times [\frac{(1-\lambda)Nw_1^a + \lambda N w_1^u}{W}], \quad \text{Region 2} : qWL \times [\frac{\lambda N w_2^a + (1-\lambda)Nw_2^u}{W}]. \tag{3.22}$$

By assumption, the water is fully allocated so the total infrastructure can be rewritten as

$$qWL + qWL_s[\frac{(1-\lambda)Nw_1^a + \lambda Nw_1^u}{W}]. \tag{3.23}$$

The total revenue from the sale of water is simply $p^w W$, thus the per-capita infrastructure tax [1] can be written as

$$f = qW\left(1 + \frac{L_s}{N}[\frac{(1-\lambda)Nw_1^a + \lambda Nw_1^u}{W}]\right) - p^w\bar{w}, \tag{3.24}$$

where use is made of the fact that $L = N$ and $\bar{w} = \frac{W}{N}$ represents the share of available water per person. The central term on the right is the region 1 water share of the total, which must be transported the additional length L_s. It is useful, in order to simplify notation, to define household net income as

$$I_1 \equiv y_1 + r_2^a + p^w\bar{w} + \frac{\int_0^{\lambda N} t(x_1)dx_1 + \int_0^{(1-\lambda)N} t(x_2)dx_2}{N} - t(\lambda N) \tag{3.25}$$
$$- qW[1 + L_s[\frac{(1-\lambda)Nw_1^a + \lambda Nw_1^u}{W}]],$$

$$I_2 \equiv y_2 + r_1^a + p^w\bar{w} + \frac{\int_0^{\lambda N} t(x_1)dx_1 + \int_0^{(1-\lambda)N} t(x_2)dx_2}{N} - t(1-\lambda)N \tag{3.26}$$
$$- qW[1 + L_s[\frac{(1-\lambda)Nw_1^a + \lambda Nw_1^u}{W}]].$$

3.3.6 Equilibrium Market Clearing

An additional feature of the model are transport costs for the agricultural good, which take the iceberg form and are represented by the parameter $\tau \geq 1$. The assumption is that in transit a share of the transported good is lost, so in order to receive one unit of the good,

[1] Note that there are no fixed costs with regard to the development of the infrastructure so the Mohring Effect will hold. That is if each agent (agriculture and households in each region) faced a price equal to their marginal cost , the infrastructure costs would be fully recovered. The assumption here is that the government is unable to levy such differentiated prices.

τ units must be ordered, with the share $\tau - 1$ vanishing in transit. Therefore, in order for an agricultural producer in region 1 to sell to a consumer in region 2, she must set a price $p_2^a = \tau p_1^a$. Given the asymmetries in the location of water and agricultural productivity, it is of interest how the population and thus manufacturing and agricultural production will be distributed across the two regions. *A priori* it is not possible to know in which direction trade will flow. However, given the assumption that the agricultural sector in region 1 is more productive, the analysis will focus on trade from region 1 to region 2 as productivity increases. We consider three possible regimes: autarky, incomplete agricultural specialization and complete agricultural specialization.

Autarky

An autarkic equilibrium will occur when τ is sufficiently high such that there is no trade between regions. Each region then produces agriculture solely for the local population. The regional agricultural goods equilibrium is then

$$(1-\lambda)Nf_1(w_1^a(p_1^a, p^w; \beta_1) = \lambda N a_1(I_1, p_1^a, p^w; \phi_1), \tag{3.27}$$

$$\lambda N f_2(w_2^a(p_2^a, p^w; \beta_2) = (1-\lambda)Na_2(I_2, p_2^a, p^w; \phi_2). \tag{3.28}$$

These equations will yield the equilibrium agricultural price for each region. The government sets the water price to clear the market. The equilibrium condition is

$$W = \lambda N w_1^u(I_1, p_1^a, p^w; \phi_1) + (1-\lambda)Nw_2^u(I_2, p_2^a, p^w; \phi_2) + (1-\lambda)Nw_1^a(p_1^a, p^w; \beta_1) + \lambda N w_2^a(p_2^a, p^w; \beta_2). \tag{3.29}$$

Finally, the manufacturing equilibrium is given by,

$$N(\lambda A_1(\lambda;\delta) + A_2((1-\lambda);\delta)) = N(\lambda m_1(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) m_2(I_2, p_2^a, p^w; \phi_2))$$
$$+ \int_0^{\lambda N} t(x_1)dx_1 + \int_0^{(1-\lambda)N} t(x_2)dx_2 + N\left(\frac{W + L_s(\lambda w_1^u(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) w_1^a(p_1^a, p^w; \beta_1))}{W}\right). \tag{3.30}$$

The left-hand side is the aggregate manufacturing output while the right-hand side is the sum of the aggregate demand for the manufacturing good, aggregate commuting resources and the water transport infrastructure. Provided that these two markets are in equilibrium, by Walras Law the manufacturing sector will be as well.

Incomplete Agricultural Specialization

Under incomplete agricultural specialization an equilibrium occurs when both regions are agricultural producers, but one region produces in excess of local demand and trades the remaining share to supplement demand in the other region. In order for trade to occur, the imported price must be no higher than the local price. If both regions are producing, this implies that $p_2^a = \tau p_1^a$. Therefore, market clearing in agriculture is simply that aggregate supply equal aggregate demand

$$\begin{aligned}(1-\lambda)Nf_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) + \lambda N f_2(w_2^a(p_2^a, p^w; \beta_2); \beta_2) \\ - (\tau - 1)((1-\lambda)Nf_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) - \lambda N a_1(I_1, p_1^a, p^w; \phi_1)) \\ = \lambda N a_1(I_1, p_1^a, p^w; \phi_1) + (1-\lambda) N a_2(I_2, p_2^a, p^w; \phi_2),\end{aligned} \tag{3.31}$$

where the second line in (3.30) denotes the share of the exported agricultural good lost in transit. The water equilibrium remains as in (3.29).

Complete Agricultural Specialization

Complete specialization occurs when β_1 is sufficiently high such that region 1 is the sole agricultural producer, however both regions may continue to produce the manufacturing good, i.e., there may not be complete concentration of the population in one region. As in the supplemental equilibrium, the agricultural price relationship is given by, $p_2^a = \tau p_1^a$. Given that no agriculture is produced, the region 2 agricultural rent is 0 and no agricultural water is used. The agricultural goods equilibrium is

$$(1-\lambda)Nf_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) - (\tau - 1)((1-\lambda)Nf_1(w_1^a(p_1^a, p^w; \beta_1); \beta_1) - \lambda N a_1(I_1, p_1^a, p^w; \phi_1)) \tag{3.32}$$

$$= \lambda N a_1(I_1, p_1^a, p^w; \phi_1) + (1-\lambda)N a_2(I_2, p_2^a, p^w; \phi_2),$$

while the water use equilibrium is given by

$$W = \lambda N w_1^u(I_1, p_1^a, p^w; \phi_1) + (1-\lambda)N w_2^u(I_2, p_2^a, p^w; \phi_2) + (1-\lambda)N w_1^a(p_1^a, p^w; \beta_1), \tag{3.33}$$

where the region 2 agricultural water use is omitted from (3.29).

3.3.7 Spatial Equilibrium

In the long run, households locate where they can achieve the highest utility. Therefore, a spatial equilibrium occurs when utility equalizes across regions yielding

$$V_1(I_1, p_1^a, p^w; \phi_1) = V_2(I_2, p_2^a, p^w; \phi_2). \tag{3.34}$$

Eq. (3.34) closes the model by defining the equilibrium population share λ as a function of model parameters. The equilibrium number of markets is as follows. In autarky, there are (not including the numeraire, p^m) seven equilibrium prices $\{p^w,\, p_i^a,\, r_i^a,\, y_i\}$, eight allocations $\{a_i,\, m_i,\, w_i^u,\, w_i^a\}$ two government taxes and transfers, $\{\bar{r},\, f\}$, the equilibrium population share, λ, and the common regional utility level U, which yields nineteen endogenous variables. Under incomplete specialization the number of endogenous variables falls to eighteen with the introduction of trade, which reduces the agricultural goods equilibrium from two equations to one, and the assumption of iceberg transport costs leaves $p_2^a = \tau p_1^a$. Finally, under complete specialization, the absence of an agricultural sector in region 2 discards r_2^a and w_2^a, reducing the number of equilibrium variables to sixteen.

3.4 Calibration, Policy Evaluations and Analytical Exercise

3.4.1 Description of Functional Forms, Parameters, and Policy Evaluations

Household utility is given by

$$U_i(a_i, m_i, w_i^u) = \phi_i m_i^\eta a_i^\mu (w_i^u)^\gamma. \tag{3.35}$$

The share parameters are set at $\eta = .75$, $\mu = 0.2$ and $\gamma = 0.05$ such that the household share of net income devoted to water and agriculture is 25%. The agricultural production function takes the constant elasticity of substitution (CES) form,

$$F_i(L_i^a, W_i^a; \beta_i) = \beta_i(\alpha(L_i^a)^\rho + (1-\alpha)(W_i^a)^\rho)^{1/\rho}, \quad -\infty \leq \rho \leq 1, \tag{3.36}$$

where ρ defines the elasticity of substitution, σ, between land and water, with $\sigma = \frac{1}{1-\rho}$. Consistent with empirical estimates (Luckman et al. (2014), Graveline and Merel (2014)), σ is set at 0.2, which implies $\rho = -4$. α denotes the share of agricultural production costs devoted to land and is set at 0.5.

The commuting cost function is given as

$$t(x_i) = tx_i, \tag{3.37}$$

where t represents the quantity of the numeraire good needed to commute a unit of distance. t is set such that in the equilibrium where households are evenly dispersed across regions, households who live at the boundary of each city spend 10% of their gross income on transportation.

The model is calibrated to exhibit a number of stylized facts. We abstract from a city with a population density of $10,000$ people per square mile, which is consistent with urban population densities for smaller cities in Los Angeles County and the San Francisco/ Bay Area (representative cities with this population density are Berkeley, Santa Monica, East Palo Alto and Redondo Beach). Therefore each mile is assumed to hold 100 lots. The length of each region is assumed to be 200 miles long, which implies that the region can hold up to 20,000 individuals. The length of land separating the two regions is assumed to be 60 miles, which makes the length of the country 460 miles. The United States Geological Survey (USGS, 2014) estimates that roughly one acre foot of water is used per household in the state of California, therefore the total population is set equal to the available water supply in acre feet. The urban agglomeration parameter δ is set at 0.075 (Nijkamp and Verhoef (2002), Helsley and Sullivan(1991)) while the threshold transport cost τ is set at 1.2 (Volpe, et. al. (2013)). The regional preference parameter ϕ_1 is set at 1.02 while ϕ_2 will be fixed at unity. Due to how ϕ_i enters the utility function, large increases can quickly lead to corner solutions. Therefore ϕ_1 is chosen to ensure interior solutions across all regimes. The agricultural productivity parameter will be fixed at 1 for region 2 and vary between 1 and 2 for region 1, consistent with the United States Department of Agriculture (USDA) data on regional agricultural total factor productivity (TFP). Table 2 presents the parameters values.

The base case will be compared to a benchmark model where the only asymmetry is in the regional agricultural productivity i.e., $\delta = 0$, $\phi_1 = \phi_2$. The benchmark acts as a proxy to quantify how increasing returns in the agricultural sector and the uneven distribution of natural amenities across regions affects the equilibrium outcomes.

Table 3.2: Parameter Values

Benchmark	Base Case	Technology	Free Parameters
$\tau = 1.2$	$\tau = 1.2$	$\eta = 0.75$	$A = 50,000$
ϕ_1 =1	$\phi_1 = 1.02$	$\mu = 0.2$	W=20,000 acre feet
$\delta = 0$	$\delta = .075$	$\gamma = 0.05$	L=200 miles
		$t = 0.5$	$N = 20,000$ households
		$\alpha = 0.5$	$L_s = 60$ miles
		$\theta = 0.6$	
		$\phi_2 = 1$	
		$\beta_2 = 1$	
		$\rho = -4$	

Additionally, we will consider the social planner's problem. In this case a social planner chooses the quantity of water and land to devote to agricultural production, the size of the city in each region, and the allocation of final goods to households in order to maximize utility. The problem is formally given as,

$$\max_{\lambda, W_i^a, w_i^u, a_i, m_i} U_1(m_1, a_1, w_1^u) \quad s.t. \tag{3.38}$$

$$U_1(m_1, a_1, w_1^u) = U_2(m_2, a_2, w_2^u), \tag{3.39}$$

$$F_1(L_1^a, W_1^a; \beta_1) + F_2(L_2^a, W_2^a; \beta_2) - (\tau - 1)ES_1 - (\tau - 1)ES_2 \geq \lambda N a_1 + (1 - \lambda) N a_2, \tag{3.40}$$

$$W \geq W_1^a + W_2^a + \lambda N w_1^u + (1 - \lambda) N w_2^u, \tag{3.41}$$

$$\begin{aligned} \lambda N(A(1+\lambda)^\delta + (1-\lambda)N(1+(1-\lambda))^\delta &\geq \lambda N m_1 + (1-\lambda) N m_2 \\ &+ qWN\left(1 + \frac{\lambda N w_1^u + (1-\lambda) N w_1^a}{W}\right) + \frac{tN}{2}(\lambda^2 N + (1-\lambda)^2 N), \end{aligned} \tag{3.42}$$

$$ES_i \geq 0, \quad i = 1, 2. \tag{3.43}$$

Where ES_i are excess supply functions for agricultural output in each region,

$$ES_1 = F_1(L_1^a, W_1^a; \beta_1) - \lambda N a_1, \quad ES_2 = F_2(L_2^a, W_2^a; \beta_2) - (1 - \lambda) N a_2.$$

Any excess supply that is exported uses the transport technology such that $(\tau - 1)ES_i$ is lost in transit. Eq. (3.41) is the manufacturing equilibrium. The left-hand side is aggregate output, while the right-hand side is the sum of household demand for manufacturing goods, the water distribution infrastructure and the household commuting infrastructure.

Finally, we will examine the effect of subsidizing agricultural water by assuming the agricultural price is a constant share of the urban water price, $p_a^w = \theta p^w$. In this case the infrastructure tax is given by

$$f = qW\left(1 + L_s\frac{(\lambda N w_1^u + (1-\lambda)N w_1^a)}{W}\right) - p^w + (1-\theta)p^w((1-\lambda)w_1^a + \lambda w_2^a), \tag{3.44}$$

where the last term on the right is each household's share of the additional revenue needed to cover the water subsidy.

The next section will focus on an analytical derivation of the model given the functional forms described here.

3.4.2 A Simplified Graphical Example

A common criticism of CGE simulations is that the solution procedure is a "black box". One can counter that the theoretical underpinnings of a standard general equilibrium model with constant returns to scale are sufficiently sound, to make credible the numerical results of the CGE model, which extend that framework. However, in the case of increasing returns, the criticism becomes more relevant. As shown in the NEG literature, the nonconvexities generated by increasing returns to scale, can lead to multiple equilibria, as parameters, in particular the transport costs, reach critical levels. In addition, the nonconvexities not only vary the number of equilibria but also the stability of each equilibrium. In order to dispel concerns about the robustness of the numerical results, a special case of the model is considered where the production function is Cobb-Douglas, which is simply a special case of the CES production with $\sigma = 1$. The model is then sufficiently tractable to allow for graphical comparative static analysis of the general equilibrium effects using parameters consistent with the calibration shown in section 3.1. [2]

To avoid excessive notation we assume that $W = N$ and that $L_s = L$. This implies that there is one unit of water per person and that the length separating the regions is equivalent to the size of the regions themselves. Productivity in the manufacturing sector, is assumed to be given by,

$$A_1(\lambda;\delta) = A(1+\lambda)^{\delta}, \tag{3.45}$$

$$A_2((1-\lambda);\delta) = A(1+(1-\lambda))^{\delta}, \tag{3.46}$$

where A is a positive constant. Eqs. (3.45) and (3.46) then give the regional household wage. The commuting costs are $t(x_i) = tx_i$, which implies that the rental transfer is given by,

$$\bar{r} = r_1^a + r_2^a + \frac{tN}{2}(\lambda^2 + (1-\lambda)^2).$$

Household preferences are given by

$$U_i(m_i, a_i, w_i^u; \phi_i) = \phi_i m_i^{\eta} a_i^{\mu} (w_i^u)^{\gamma}. \tag{3.47}$$

Demand functions are then

$$m_i = \eta I_i, \quad a_i = \mu \frac{I_i}{p_i^a}, \quad w_i^u = \gamma \frac{I_i}{p^w}, \tag{3.48}$$

and the indirect utility function is then

$$V_i(I_i, p_i^a, p^w; \phi_i)) = \phi_i \eta^{\eta} \mu^{\mu} \gamma^{\gamma} \frac{I_i}{(p_i^a)^{\mu}(p^w)^{\gamma}}. \tag{3.49}$$

[2] A fully tractable solution is possible if we make the more restrictive assumption that household utility is quasi-linear, which generates no income effects. However, empirical work has shown that income effects matter with regard to household demand for water and agricultural goods. Given that this paper aims to address policy concerns, it seems appropriate to reduce the number of assumptions that are simply made for tractability.

The production function per unit of land is assumed to be

$$F(W_i^a, L_; \beta_i) = 2\beta_i \sqrt{W_i^a L_i}. \tag{3.50}$$

The intensive form of the production function can then be written as

$$f(w_i^a; \beta_i) = 2\beta_i \sqrt{w_i^a}. \tag{3.51}$$

Profit maximization and the zero profit condition yield the following water demand and land rent functions,

$$w_i^a = \left(\frac{\beta_i p_i^a}{p^w}\right)^2, \quad r_i^a = \frac{(\beta_i p_i^a)^2}{p^w}. \tag{3.52}$$

The infrastructure tax, f, can be written as

$$\begin{aligned} f =& \frac{qW}{N}\left(N\left(\frac{W + N(\lambda w_1^u + (1-\lambda))w_1^a}{W}\right) - p^w W\right) \\ =& qW\left(1 + \lambda w_1^u + (1-\lambda)w_1^a\right) - p^w \\ =& qW(1 + \frac{\gamma\lambda I_1}{p^w} + \frac{(1-\lambda)r_1^a}{p^w}) - p^w, \end{aligned} \tag{3.53}$$

where use is made of the assumption that $W = N$, and the household and agricultural water demand functions for region 1 are substituted. Finally, the regional net incomes are

$$I_1 = A(1+\lambda)^\delta + r_2^a + \frac{tN}{2}((1-\lambda)^2 - \frac{1}{2}) + p^w - qW(1 + \frac{\gamma\lambda I_1 + (1-\lambda)r_1^a}{p^w}), \tag{3.54}$$

$$I_2 = A(1+(1-\lambda))^\delta + r_1^a + tN(\lambda^2 - \frac{1}{2}) + p^w - qW(1 + \frac{\gamma\lambda I_1 + (1-\lambda)r_1^a}{p^w}). \tag{3.55}$$

It is straightforward to verify that there is a symmetric equilibrium with $\lambda = 1/2$ when $\phi_1 = \phi_2$, $\beta_1 = \beta_2$, $\tau = 1$ and $\delta = 0$.

Autarky

Under autarky, using (3.48) and (3.52) the agricultural goods and water equilibrium imply the following:

$$r_1^a = \frac{\mu}{2}\frac{\lambda}{1-\lambda}I_1, \quad r_2^a = \frac{\mu}{2}\frac{1-\lambda}{\lambda}I_2, \quad p^w = (\gamma + \frac{\mu}{2})(\lambda I_1 + (1-\lambda)I_2). \tag{3.56}$$

Finally, the spatial equilibrium is given by,

$$\Phi\frac{P_2}{P_1}I_1 = I_2, \quad \text{with} \quad \Phi \equiv \frac{\phi_1}{\phi_2}, \quad \frac{P_2}{P_1} = \left(\frac{\phi_1}{\phi_2}\left(\frac{(1-\lambda)\beta_1}{\lambda\beta_2}\right)^2\right)^{\frac{\mu}{(2-\mu)}}. \tag{3.57}$$

Φ measures the ratio of natural amenities, between regions 1 and 2, while $\frac{P_2}{P_1}$ is the ratio of regional price indices between regions 2 and 1. Intuitively, $\Phi\frac{P_2}{P_1}$ is an income premium that households in region 2 must receive to compensate them for higher prices and a lower level of amenities. Using the manufacturing equilibrium condition we can show that

$$I_1^A(\lambda) = \frac{\lambda A(1+\lambda)^\delta + (1-\lambda)A(1+(1-\lambda))^\delta - \frac{tN}{2}(\lambda^2+(1-\lambda)^2) - qW(1+\frac{\lambda}{\lambda+\Phi\frac{P_2}{P_1}(1-\lambda)})}{\lambda+\Phi\frac{P_2}{P_1}(1-\lambda)}, \tag{3.58}$$

where the A subscript denotes the autarkic region 1 income. Substituting (3.58) into (3.56) and (3.57) allows for the remaining endogenous variables to be written as functions of λ. Combining (3.58) with (3.56), solving for I_1 and equating with (3.59) provides an implicit solution for λ,

$$\begin{aligned}&\frac{\lambda A(1+\lambda)^\delta + (1-\lambda)A(1+(1-\lambda))^\delta - \frac{tN}{2}(\lambda^2+(1-\lambda)^2) - qW(1+\frac{\lambda}{\lambda+\Phi\frac{P_2}{P_1}(1-\lambda)})}{\lambda+\Phi\frac{P_2}{P_1}(1-\lambda)} \\ &\qquad = \frac{A(1+(1-\lambda))^\delta + tN(\lambda^2-\frac{1}{2}) - qW(1+\frac{\lambda}{\lambda+\Phi\frac{P_2}{P_1}(1-\lambda)})}{\Phi\frac{P_2}{P_1} - \frac{\mu}{2}\frac{\lambda}{(1-\lambda)} - (\gamma+\frac{\mu}{2})(\lambda+\Phi\frac{P_2}{P_1}(1-\lambda))}.\end{aligned} \tag{3.59}$$

Figure 3.2 plots the effects of changes in β_1, ϕ_1 and δ from the symmetric equilibrium as functions of the population share. Each column represents the general equilibrium changes for a single parameter. The top row provides a plot of the spatial equilibrium with U_1 as the downward sloping curve, and U_2 upward sloping. The remaining rows plot the equilibrium prices as functions of λ. The intersection of the two utility curves pins down the equilibrium λ. That value is then traced down to identify the equilibrium prices. The solid vertical line gives the initial equilibrium while the dashed vertical line traces out the equilibrium after the change in parameter values.

Figure 3.2 column (a) considers the case of a 30% increase in β_1 from the equilibrium in which each region is equally productive. When $\beta_1 = \beta_2$ the population and agricultural production are evenly dispersed between regions, with each region producing solely for the local population. The first-order effect of a rise in productivity is an increase in agricultural output in region 1, which lowers the regional agricultural price. This raises the utility of households in region 1 relative to region 2, inducing migration and leading to an increase in λ. In response to the shifts in the population, rents rise in region 1 and fall in region 2. The reduction in agricultural production costs in region 2 generates a fall in the regional agricultural price. While for the agricultural sector in region 1, the increase in the local population moves the rental price upwards along the curve increasing production costs. The cumulative effect is a fall in the region 1 agricultural price.

Notice that there are two opposing effects with respect to the price of water. The productivity increase in the agricultural sector in region 1 reduces the amount of water per unit of land necessary to produce the same quantity of output, while the reduction in the agricultural price and the increase in the population in region 1, increase the demand for the agricultural good. Figure 3.3 provides a plot of each region's share of the total water devoted to agriculture, which shows that an increase in β_1 leads to a downward shift in agricultural water use in region 1 and an upward shift in region 2, with the change in each region largely cancelling out the other at the equilibrium point. We see that the water price remains largely unaffected.

Columns (b) and (c) of Figure 3.3 provide the comparative statics with respect to ϕ_1

and δ. An increase in natural amenities raises the overall utility level in region 1, inducing migration. However, this leads to an increase in rents and the price index in region 1, while in region 2 there is a fall. The cumulative effect is a modest increase in λ. Notice that an increase in δ from the symmetric equilibrium has no effect on the equilibrium values besides a rise in utility from an increase in manufacturing output. This is due to the fact that when the population is equally dispersed, an increase in δ raises wages equally.

Incomplete Agricultural Specialization

Recall that under incomplete specialization the agricultural price relationship is given by, $p_2^a = \tau p_1^a$. The equilibrium conditions then yield the following equations:

$$r_1^a = Br_2^a, \quad r_2^a = \frac{\frac{\mu}{2}((2-\tau)\lambda I_1 + \frac{(1-\lambda)}{\tau} I_2)}{\frac{\lambda}{\tau} + (2-\tau)B(1-\lambda)}, \tag{3.60}$$

$$p^w = \gamma(\lambda I_1 + (1-\lambda)I_2) + ((1-\lambda)B + \lambda)r_2^a,$$

where

$$B \equiv \left(\frac{\beta_1}{\tau\beta_2}\right)^2$$

reflects the relative productivity between regions when transport costs are present. τ in essence allows region 2's agricultural sector to be more competitive by increasing the productivity threshold that the agricultural sector in region 1 must surpass in order to compete in the foreign market. The spatial equilibrium is then given by,

$$TI_1 = I_2, \tag{3.61}$$

where

$$T \equiv \frac{\phi_1 \tau^\mu}{\phi_2}$$

measures the ratio of relative regional natural amenities to relative regional price indices between region 1 and 2. Solving for I_1 yields,

$$I_1^I(\lambda) = \frac{[A(1+\lambda)^\delta + tN((1-\lambda)^2 - \frac{1}{2})] - qW\left(1 + \frac{(B\lambda(1-\lambda)(2-\tau)(\gamma+\frac{\mu}{2}) + \frac{\gamma\lambda^2}{\tau} + B\frac{\mu T}{2\tau}(1-\lambda)^2)}{\Lambda}\right)}{1 - \frac{\Lambda}{(B(1-\lambda)(2-\tau)+\frac{\lambda}{\tau})} - \frac{\frac{\mu}{2}((2-\tau)\lambda + \frac{T}{\tau}(1-\lambda))}{(B(1-\lambda)(2-\tau)+\frac{\lambda}{\tau})}}, \tag{3.62}$$

where

$$\Lambda = \gamma(\lambda + T(1-\lambda))((1-\lambda)(2-\tau)B + \frac{\lambda}{\tau}) + \frac{\mu}{2}((1-\lambda)B + \lambda))((2-\tau)\lambda + \frac{T}{\tau}(1-\lambda)),$$

and the superscript I refers to the income under incomplete specialization. The first term on the right in Λ is a convex combination of the household unit expenditure on water in each region multiplied by the ratio of the value of output to rent per unit of land. The second term on the right is a convex combination of the value of agricultural water demand times the average value of agricultural output per unit of land.

As in the autarkic case, equation (3.62) can be used to derive the implicit spatial equilibrium condition, given by,

$$\frac{[A(1+\lambda)^\delta + tN((1-\lambda)^2 - \frac{1}{2}) - qW\left(1 + \frac{(B\lambda(1-\lambda)(2-\tau)(\gamma+\frac{\mu}{2}) + \frac{\gamma\lambda^2}{\tau} + B\frac{\mu T}{2\tau}(1-\lambda)^2)}{\Lambda}\right)}{1 - \frac{\Lambda}{(B(1-\lambda)(2-\tau)+\frac{\lambda}{\tau})} - \frac{\frac{\mu}{2}((2-\tau)\lambda + \frac{T}{\tau}(1-\lambda))}{(B(1-\lambda)(2-\tau)+\frac{\lambda}{\tau})}} = \tag{3.63}$$

$$\frac{[A(1+(1-\lambda))^\delta + tN(\lambda^2 - \frac{1}{2}) - qW\left(1 + \frac{(B\lambda(1-\lambda)(2-\tau)(\gamma+\frac{\mu}{2}) + \frac{\gamma\lambda^2}{\tau} + B\frac{\mu T}{2\tau}(1-\lambda)^2)}{\Lambda}\right)}{T - \frac{\Lambda}{(B(1-\lambda)+\frac{\lambda}{\tau})} - B\left(\frac{\frac{\mu}{2}(\lambda + \frac{T}{\tau}(1-\lambda))}{(B(1-\lambda)+\frac{\lambda}{\tau})}\right)},$$

Figures 3.4 and 3.5 trace out the equilibrium for changes in β_1, ϕ_1, τ and δ from the benchmark parameter values.Under incomplete specialization, an increase in the agricultural productivity generates more variation in λ than under autarky. The introduction of trade lowers agricultural rents and prices for region 2, which leads migration to shift in the other direction towards the *less* productive region as β_1 increases. There is an unambiguous decline in p^w, p_1^a, p_2^a and r_2^a with productivity as the curves shift down and λ falls. With an increase in β_1 there is an upward shift in in the region 1 land rent curve, which is offset by the fall in the local population, leading to no significant change in r_1^a. The cumulative effect is that households in region 2 benefit more with the fall in rent and agricultural prices, leading to a significant shift in the population toward region 2, and an increase in overall utility.

As in autarky, a rise in the natural amenities in region 1 leads to an increase in the local utility level, inducing migration towards region 1. All prices rise as agricultural rents increase in region 1 to accommodate the larger population, while in region 2 agricultural rents increase due to a rise in the agricultural price. The increase in land costs leads to the agricultural sectors substituting away from land towards water, raising the water price.

An increase in τ reduces demand for the agricultural good and increases the price index in region 2, inducing migration towards region 1. The cumulative effect is that agricultural prices are largely unaffected while region 1 rents fall due to the drop in foreign demand. Additionally, an increase in τ raises the agricultural price for region 2 agriculture, increasing the agricultural rent and the demand for water in the region.

Finally, an increase in δ leads to a rise in all prices as incomes increase. However, given that in the initial equilibrium, nearly two-thirds of the population were in region 1, the introduction of agglomeration economies favors the more populous region, increasing λ.

Complete Agricultural Specialization

In the case that only region 1 produces agriculture, $r_2^a = 0$. The agricultural and water equilibria are then given by,

$$r_1^a = \frac{\mu}{2}(\frac{\lambda}{1-\lambda}I_1 + \frac{I_2}{(2-\tau)\tau}), \quad p^w = (\gamma + \frac{\mu}{2})\lambda I_1 + (\gamma + \frac{\mu}{(2-\tau)2\tau})(1-\lambda)I_2. \tag{3.64}$$

The spatial equilibrium again is given by,

$$TI_1 = I_2, \tag{3.65}$$

where T is defined as in (61). Following a similar algorithm as above, region 1 income under specialization is given as,

$$I_1^C(\lambda) = \frac{A(1+\lambda)^\delta + tN((1-\lambda)^2 - \frac{1}{2}) - qW\left(1 + \frac{(\gamma+\frac{\mu}{2})\lambda + (1-\lambda)\frac{\mu T}{(2-\tau)2\tau}}{\Theta}\right)}{1-\Theta}, \tag{3.66}$$

where the superscript C denotes complete specialization and

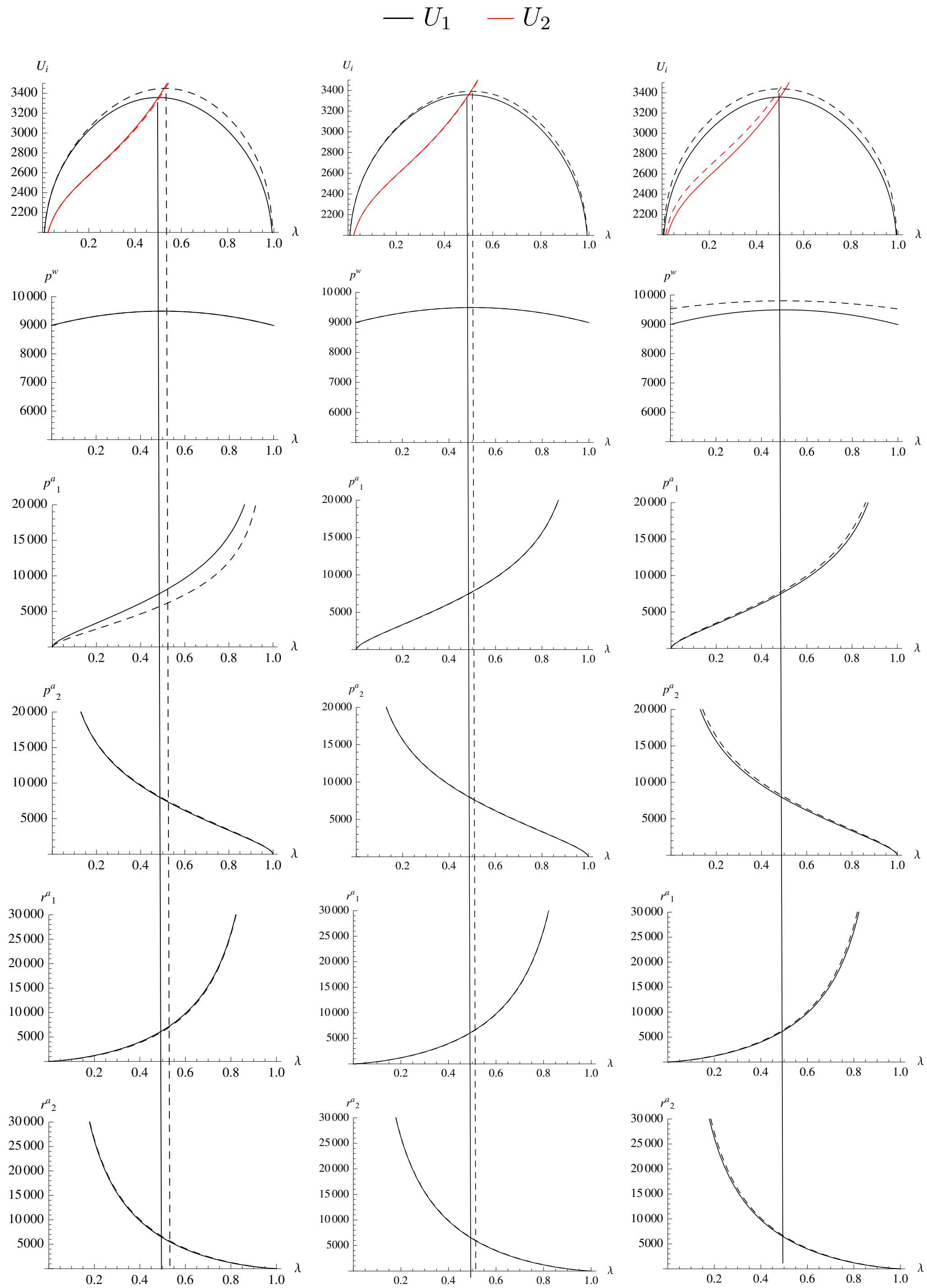
$$\Theta \equiv (\lambda(\gamma + \frac{\mu}{2}) + (1-\lambda)T(\gamma + \frac{\mu}{(2-\tau)2\tau})$$

reflects a convex combination of a unit of expenditure in each region spent on the consumption of water. This is seen explicitly through γ, and implicitly through the water share of each unit expenditure devoted to the agricultural good, reflected in the term $\frac{\mu}{2}$. The spatial equilibrium condition is then given by,

$$\begin{aligned} &\frac{A(1+\lambda)^\delta + tN((1-\lambda)^2 - \frac{1}{2}) - qW\left(1 + \frac{(\gamma+\frac{\mu}{2})\lambda + (1-\lambda)\frac{\mu T}{(2-\tau)2\tau}}{\Theta}\right)}{1-\Theta} = \\ &\frac{A(1+(1-\lambda))^\delta + tN(\lambda^2 - \frac{1}{2}) - qW\left(1 + \frac{(\gamma+\frac{\mu}{2})\lambda + (1-\lambda)\frac{\mu T}{(2-\tau)2\tau}}{\Theta}\right)}{T - \Theta - \frac{\mu}{2}(\frac{\lambda}{1-\lambda} + \frac{T}{(2-\tau)\tau})}. \end{aligned} \tag{3.67}$$

Figures, 3.6 and 3.7 plot the effects of β_1, ϕ_1, τ and δ on the equilibrium. Under complete specialization, a majority of households reside in region 2, allowing for region 1 to be primarily used for agricultural production. Notice that while an increase in productivity generates a substantial increase in utility there is no effect on migration. This follows from (3.67), where the spatial equilibrium is independent of β_1. However the increase in productivity reduces the agricultural price, raising overall utility. The effects of ϕ_1 are similar to those under autarky and incomplete specialization. The rise in utility generates an increase in λ, which raises agricultural rents and thus the price for agricultural goods. In reaction, region 1 agriculture substitutes away from land toward water, increasing p^w.

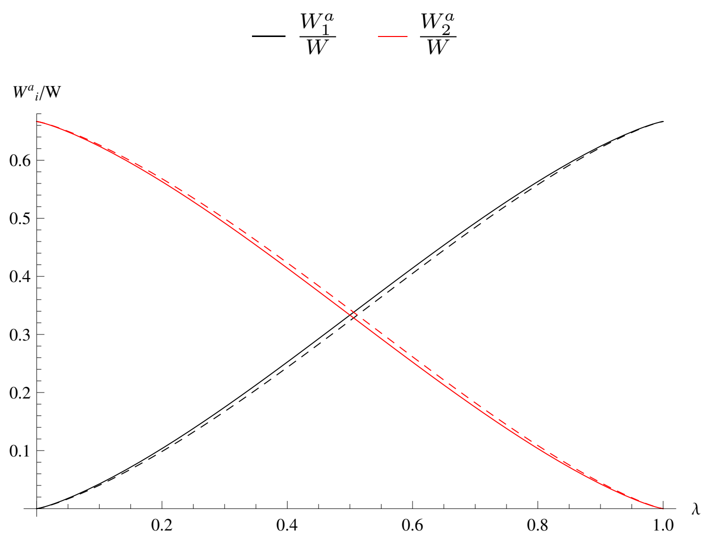
An increase in τ under specialization is largely identical to the case under incomplete specialization. A rise in economies of scale, δ, as above, increases all prices. However, as the largest share of the population is in region 2 the wage increase favors those households, leading to further migration towards region 2. The following section presents the results of the numerical simulation.



(a) $-\ \beta_1 = 1$, - - $\beta_1 = 1.3$ (b) $-\ \phi_1 = 1$, - - $\phi_1 = 1.02$ (c) $-\ \delta = 0$, - - $\delta = .075$

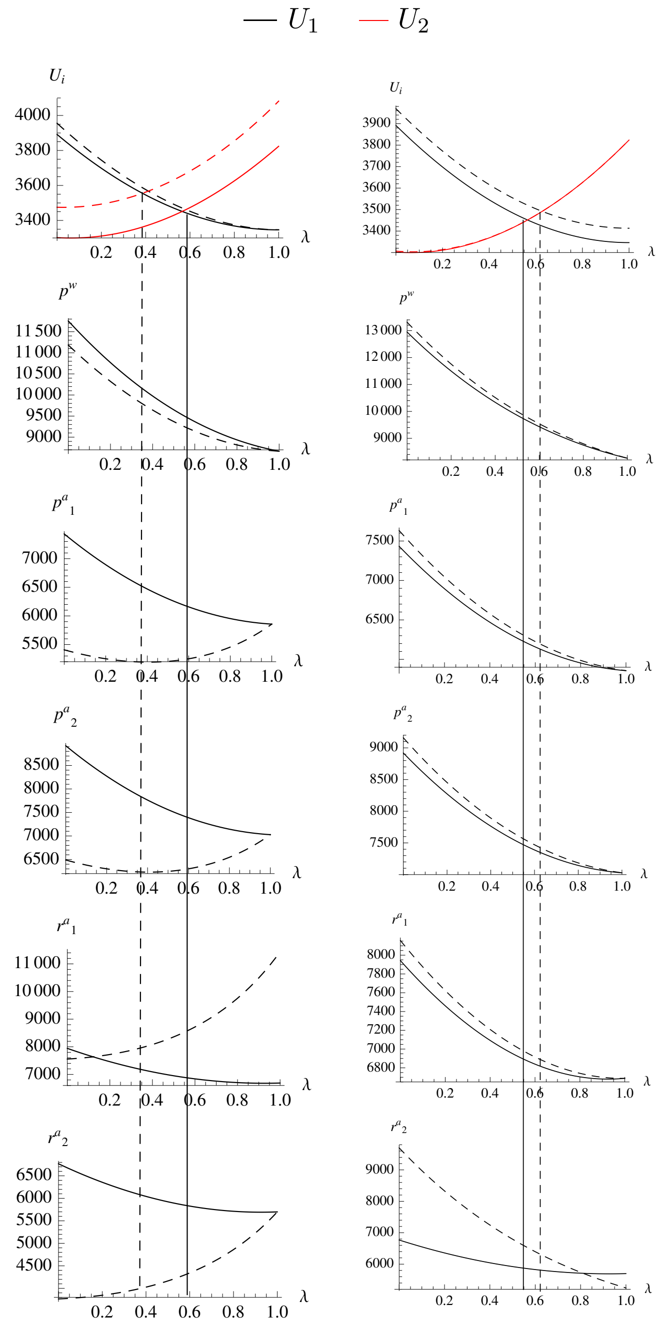
Figure 3.2: Comparative statics of equilibrium utility and prices in autarky with respect to (a) β_1, (b) ϕ_1 and (c) δ from the symmetric equilibrium at the benchmark parameter values

Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000



(a) $-\ \beta_1 = 1$, - -$\beta_1 = 1.3$

Figure 3.3: The effect of an increase in region 1 agricultural productivity on the share of total water devoted to agriculture in each region
Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000

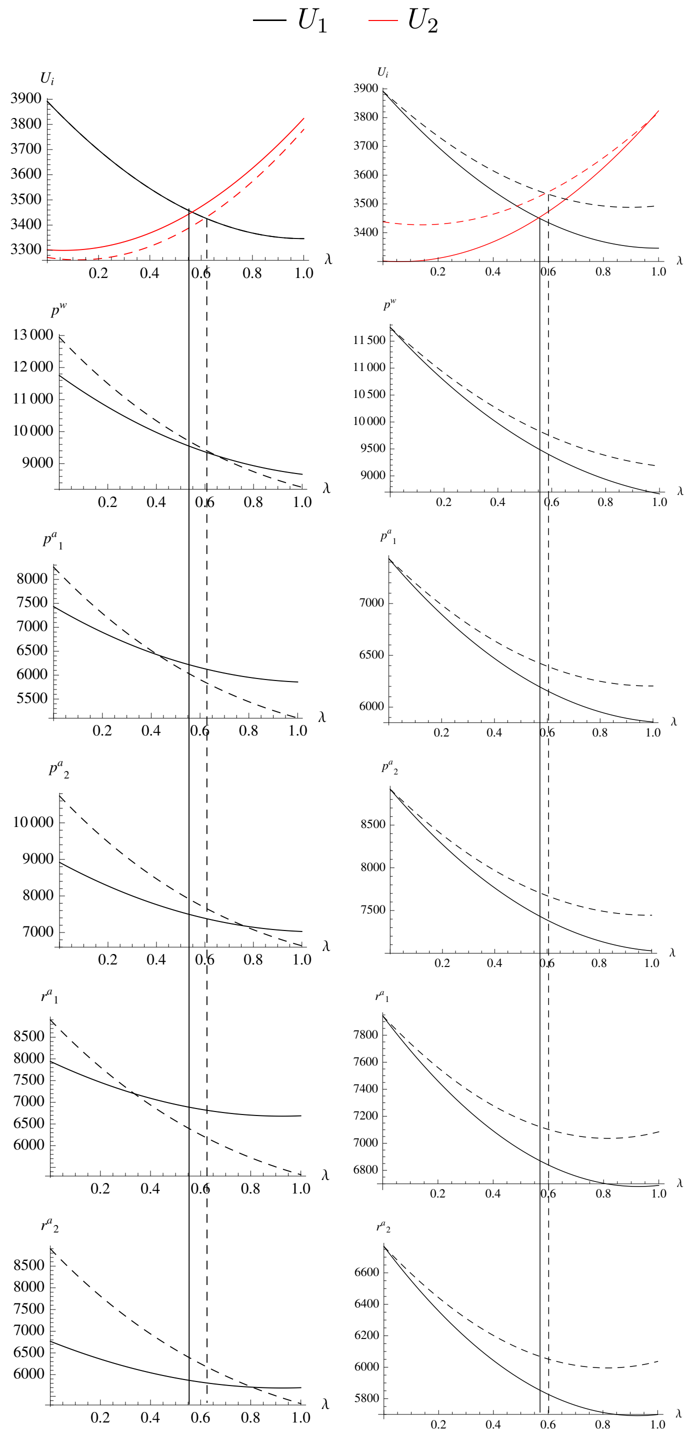


(a) $-\ \beta_1 = 1.3$, - - $\beta_1 = 1.7$

(b) $-\ \phi_1 = 1$, - - $\phi_1 = 1.02$

Figure 3.4: Comparative statics of equilibrium utility and prices under incomplete agricultural specialization with respect to (a) β_1 from autarkic value and (b) ϕ_1 from the benchmark

Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000



(a) $-\tau = 1.2$, - - $\tau = 1.3$ (b) $-\ \delta = 0$, - - $\delta = .075$

Figure 3.5: Comparative statics of equilibrium utility and prices under incomplete agricultural specialization with respect to (a) τ and (b) δ from the benchmark
Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000

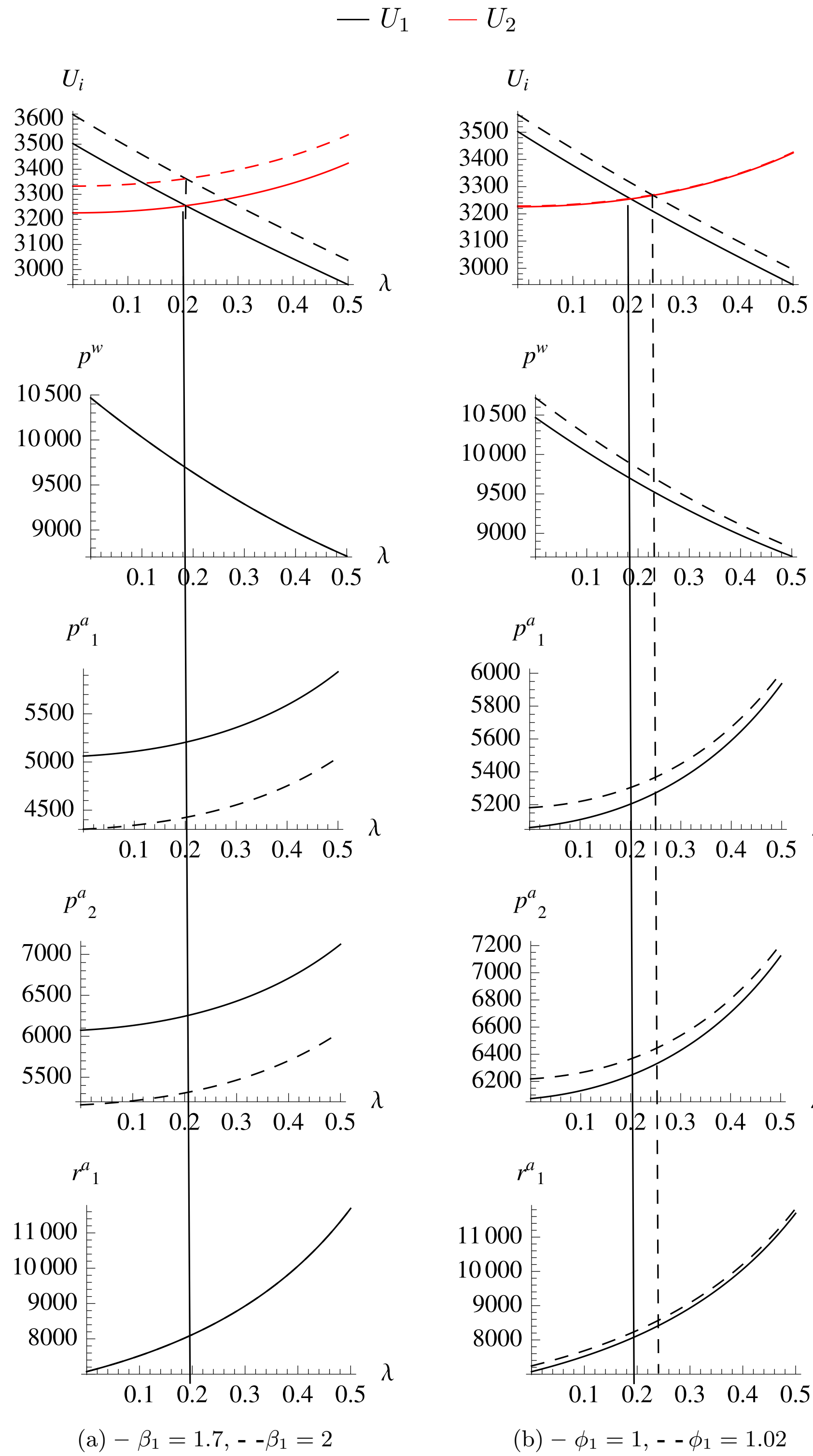


Figure 3.6: Comparative statics of equilibrium utility and prices under complete agricultural specialization with respect to (a) β_1 and (b) ϕ_1 from the benchmark. Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000

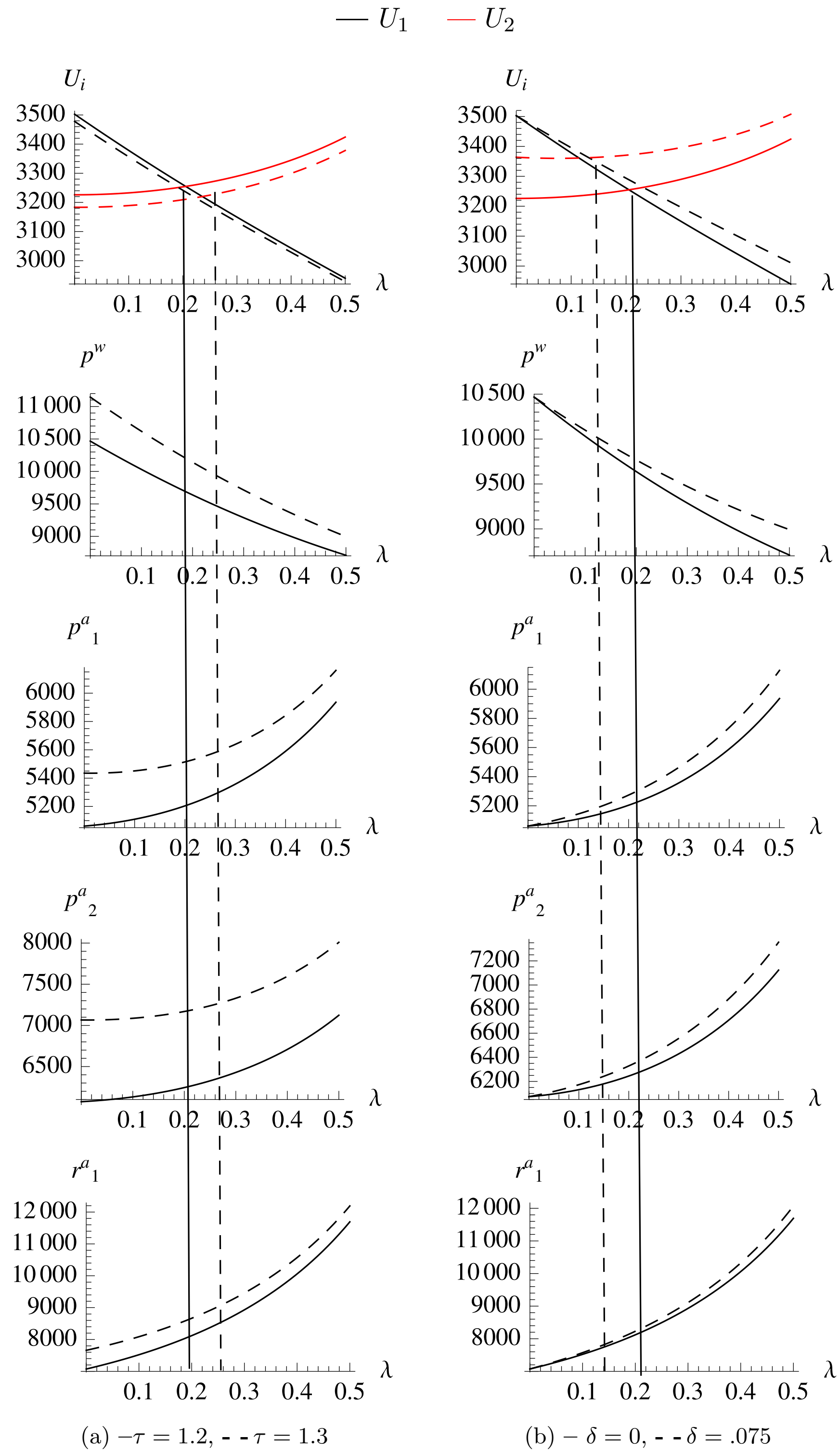


Figure 3.7: Comparative statics of equilibrium utility and prices under complete agricultural specialization with respect to (a) τ and (b) δ from the benchmark. Note: $\beta_2 = 1$, ϕ_2=1, $t = .5$ $\mu = 0.2$, $\gamma = 0.05$, N=20,000, A=50,000

3.5 Numerical Results

Tables 3.3-3.11 in Appendix B present the results of the numerical simulation. Agricultural productivity is varied to analyze how increasing asymmetry in agricultural productivity affects relative prices, water allocation, population shares and utility. In contrast, one could, of course, hold the productivity of land in each region fixed while varying the transport cost τ. Conceptually, the results would be similar. For two regions of different productivity, at high enough transport costs autarky will hold. As τ is lowered beyond the threshold price ratio, trade will occur. As trade costs become sufficiently low, region 2's agricultural rents will fall to zero, leading to all agricultural production being concentrated in region 1.

However, since this paper is focused on the heterogeneity between different regions, rather than the costs of transport, we have chosen agricultural productivity as the parameter of variation. The values of β_1 are chosen to ensure consistency of the results across different regimes and policy experiments. For the autarkic case, $\beta_1 = 1.3$ ensures that the agricultural price ratio between region 2 and region 1 is below the threshold transport cost and no trade will occur, i.e. $\frac{p_2^a}{p_1^a} < \tau$. $\beta_1 = 1.7$ is sufficiently high to allow for trade while both regions continue to produce agriculture. Finally, in the case of complete specialization, a value of $\beta_1 = 2$ guarantees that there is no agricultural production in region 2.

3.5.1 Base Case versus Benchmark and Social Planner

Table 3.3 provides the numerical results for the base case in autarky relative to the benchmark and the social planner. As would be expected, regional incomes and utility rise in the base case relative to the benchmark with the introduction of agglomeration economies, which increases aggregate output and thus regional wages. In the autarkic case, the addition of natural amenities, raises the costs of agricultural production in region 1, as agricultural rents rise by nearly 14%, leading to a 6% increase in the agricultural price, and a nearly 3% decline in household consumption of the agricultural good. In addition, the rise in rents and agricultural prices leads to an increase in the intensity of water use per unit of land. In region 2, agricultural rents fall by 6.65% and agricultural water use falls by roughly 2% per unit of land. However, there is only a modest shift in the population share toward region 1.

As agricultural productivity increases, trade becomes possible between regions as the ratio of agricultural prices rises above the trade barrier τ. As shown in Table 3.4, in contrast to the autarkic case, relative agricultural rents fall in the base case relative to the benchmark. Introducing natural amenities and agglomeration economies leads to a rise in the agricultural price thus increasing the residual from net revenues that can be consumed by the agricultural rent in region 2. Trade pushes households towards region 2, with 59.61% of land in region 1 in the benchmark and 55.3% in the base case devoted to agriculture, as the higher level of natural amenities in region 1 slows migration. Given the relatively small population share differential, the amenity effect dominates the urban economies of scale, as relative wages remain roughly the same.

Under complete specialization, as shown in Table 3.5, in contrast to the above cases, agglomeration economies dominate the effect of natural amenities. Given that the population is disproportionately concentrated in region 2, increasing returns in the manufacturing sector leads to a 4% higher wage in region 2, inducing further migration.

From the social planner's problem, p_s^w and p_s^a are the normalized shadow prices for water and the agricultural good, respectively. The base case water price is 8% below, and the region 1 and region 2 agricultural prices are 11.76% and 2.15% below the social planner's shadow prices in autarky, respectively. The social planner places a significantly lower share of residents in region 1, with nearly two-thirds of the land devoted to agriculture. In addition, each unit of land is used more intensively. In contrast, the base case devotes insufficient resources to the agricultural sector in region 1, while allowing for excessive irrigation in the less productive region 2.

As productivity increases, the social planner continues to devote a larger share of region 1 to agriculture, with less than a quarter of land devoted to housing, in contrast to the 44.7% which is used under the base case in incomplete specialization. While the water devoted to agricultural production in region 1 is only 2% below the social planner, that devoted to region 2 is over 60% above, as the social planner drastically reduces the water allocation per unit of land.

Finally, as β_1 reaches 2, the social planner's equilibrium consists of complete concentration of households in region 2, with all land in region 1 devoted to agriculture. Agricultural water per unit of land is higher in the base case under complete specialization, yet total water devoted to agriculture is higher under the social planner.

3.5.2 Agricultural Water Subsidy versus. Base Case and Social Planner

There are three notable effects of agricultural water subsidies relative to the base case. The first is household water consumption declines dramatically, as the subsidized water price increases the agricultural sector's demand for water, leaving a smaller share of the fixed resource available for households. In both autarky and incomplete specialization, household consumption is roughly 30% below the base case and under compete specialization is 20% below. Second, urban water prices show large increases, with the greatest increase being under incomplete specialization with the urban water price 45.62% above the base case. In this case, the water price paid by the agricultural sector is only 12% below the base case, once subsidies are taken into account. Third, agricultural rents increase substantially, in particular in the autarkic case, where r_1^a increases by 21% and r_2^a by 38%. This result is in line with policy work, suggesting that the effects of agricultural subsidies are often overstated as the value of the subsidies ultimately accrues to the land rent (see Hanak et al. (2009)). In addition, in the autarkic and incomplete specialization cases, relative agricultural rents between regions 1 and 2 *fall*, as the reduction in costs allows for the less productive sector to become relatively more competitive.

As agricultural productivity in region 1 increases, the decline in utility also increases. In autarky, the utility level with water subsidies is only 0.38% below that of the base case, however once region 1 completely specializes in agricultural production, utility is over 3% below the base case. This follows from the fact that in autarky and incomplete specialization, the agricultural water subsidy slightly increases regional price indices, as the increase in the urban water price dominates the effect of a reduction in the price of the agricultural good. However, net income in these two regimes remains largely unaffected by the subsidy, leading to a modest reduction in utility. Under complete specialization the rise in the regional price indices is coupled with a nearly 2.5% reduction in net income, leading to a significant fall in overall utility. CV_i gives the compensating variation for each region, which is the additional income that would have to be given to households in each region in order to be as well off as under the base case. We see that under autarky and incomplete specialization households in each region would need to be compensated between roughly $ 250 and $350. However, under complete specialization the compensating variation increases to roughly $2000 per household.

In comparison to the social planner, the agricultural water subsidies lead to excessive agricultural production in region 2, with water use per unit of land 19.19% higher than the social planner under autarky and 85.36% under incomplete specialization. In addition, the subsidies lead to an overuse of water per unit of land by region 1, which becomes most pronounced when the region is most productive, at which point agricultural water use exceeds the social planner's allocation by 24%.

3.6 Conclusion and Future Research

This paper has developed a spatial two-region general equilibrium trade model with water as a mobile factor of production and heterogeneity between regions in consumption amenities, agricultural productivity and initial endowments of water. The model was solved analytically for a special case. A numerical simulation was then done to allow for a comparison across various policy scenarios. The analysis suggests that when trade cannot occur, a greater share of the population lives in the more agriculturally productive region. When the same region has the additional benefit of natural amenities, the effect is compounded. As trade is introduced, migration tends toward the less productive region. However, this effect is dampened if the more productive region has a higher level of natural amenities. In addition, economies of scale play a significant role in migration patterns if the population share differential between the two regions is sufficiently high. The numerical analysis showed that subsidizing agricultural water led to insufficient water being allocated to households while the less productive region was over irrigated.

This research dealt with a very specific problem, namely, how will the uneven distribution of water, agricultural productivity and natural amenities, affect the size of cities and agricultural production in each region. However, there are a number of other factors that play an important role in interbasin water transfers. One is the energy needed to pump water through the network, particularly uphill over mountain ranges. The model could be adapted to take into account topographical irregularities, which would vary the marginal and fixed costs of distribution over space. In addition, one could consider the possibility of electricity generation from the water flow in order to measure net energy use.

A timely extension would be to add the possibility of water desalination into the model. This could be done by introducing a water production technology that can add to the existing supply. Crucial questions include the scale and location of water production. Explicit dynamics could be introduced to solve for the optimal time to introduce the water desalination technology. In addition, variability in seasonal or annual water supply could be integrated. Additionally, this model has not considered fixed costs, which are a crucial component to interbasin water transfers as well as in the construction of desalination plants.

There are environmental and ecological concerns related to interbasin water transfers, which may limit the extent to which they can be carried out. Integrating these constraints, in addition to increasing the level of realism, can also highlight alternative conservation methods to stretch existing water resources in the absence of substantial water transfer options.

Finally, the model is well equipped to answer the extent that regions that are water scarce can benefit from imported goods that are water intensive to produce (see Reimer

(2012)). In addition, as water resources in many regions are becoming increasingly scarce, it will be necessary to identify in what location is the water put to best use given the possibility of transport .

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Appendix B

B.1 Derivation of Spatial Equilibrium in 3.4.2

B.1.1 Autarky

From equation (3.57) we have

$$\Phi \frac{P_2}{P_1} I_1 = I_2, \tag{B.1}$$

where

$$\frac{P_2}{P_1} = \left(\frac{p_2^a}{p_1^a}\right)^{\mu}. \tag{B.2}$$

From (3.56) we have

$$\frac{r_1^a}{r_2^a} = (\frac{\lambda}{1-\lambda})^2 \frac{I_1}{I_2} = (\frac{\lambda}{1-\lambda})^2 (\frac{p_1^a}{p_2^a})^{\mu} \frac{1}{\Phi}. \tag{B.3}$$

From (3.52) we have

$$\frac{r_1^a}{r_2^a} = (\frac{\beta_1 p_1^a}{\beta_2 p_2^a})^2. \tag{B.4}$$

Combining them gives,

$$\frac{P_2}{P_1} = (\frac{\phi_1}{\phi_2}(\frac{(1-\lambda)\beta_1}{\lambda\beta_2})^2)^{\frac{\mu}{2-\mu}}. \tag{B.5}$$

Plugging (3.58) into (3.56) and inserting into (3.54) and (3.55), and solving for I_i, gives (3.59).

From the manufacturing equilibrium we have

$$(\lambda+(1-\lambda)\Phi\frac{P_2}{P_1})I_1 = A(1+\lambda)^\delta + A(1+(1-\lambda))^\delta - \frac{tN}{2}(\lambda^2+(1-\lambda)^2) - (1+\frac{\lambda}{(\lambda+(1-\lambda)\Phi\frac{P_2}{P_1})}). \tag{B.6}$$

Solving for I_1 and inserting into I_2 gives,

$$I_2 = A(1+(1-\lambda)^\delta + tN(\lambda^2 - \frac{1}{2}) - (1+\frac{\lambda}{(\lambda+(1-\lambda)\Phi\frac{P_2}{P_1})}) + (\frac{\mu}{2}\frac{\lambda}{1-\lambda} + (\gamma+\frac{\mu}{2})((\lambda+(1-\lambda)\Phi\frac{P_2}{P_1}). \tag{B.7}$$

Inserting (3.57) and solving for I_1 and setting equal to (3.59) provides the equilibrium condition in (3.60)

B.1.2 Incomplete and Complete Specialization

The algorithm to solve for the spatial equilibrium under incomplete and complete specialization is more straightforward than above. Since $p_2^a = \tau p_1^a$, from (3.52) the spatial equilibrium given by,

$$\frac{I_2}{I_1} = \frac{\phi_1 \tau^\mu}{\phi_2} \equiv T, \tag{B.8}$$

and relative rents are given by,

$$\frac{r_1^a}{r_2^a} = (\frac{\beta_1}{\beta_2 \tau})^2 \equiv B. \tag{B.9}$$

The agricultural goods equilibrium is given by,

$$2((1-\lambda)N\beta_1(\frac{\beta_1 p_1^a}{p^w}) + \lambda N\beta_2(\frac{\beta_2 p_2^a}{p^w})) - (\tau-1)(2((1-\lambda)N\beta_1(\frac{\beta_1 p_1^a}{p^w}) - \mu\lambda N\frac{I_1}{p_1^a})$$
$$= \mu(\lambda N\frac{I_1}{p_1^a} + (1-\lambda)N\frac{I_2}{p_2^a}. \tag{B.10}$$

Multiplying both sides by p_1^a and using (3.74) and (3.52) gives (3.60). This expression can then be inserted into (3.54) and (3.55) to yield I_i.

Under complete specialization, there is no agricultural rent in region 2, therefore the agricultural goods equilibrium is then given by,

$$2((1-\lambda)N\beta_1(\frac{\beta_1 p_1^a}{p^w})) - (\tau-1)(2(1-\lambda)N\beta_1(\frac{\beta_1 p_1^a}{p^w})) - \mu\lambda N\frac{I_1}{p_1^a}) = \mu(\lambda N\frac{I_1}{p_1^a} + (1-\lambda)N\frac{I_2}{p_2^a}, \tag{B.11}$$

or

$$2((1-\lambda)Nr_1^a - (\tau-1)(2(1-\lambda)Nr_1^a - \mu\lambda NI_1) = \mu(\lambda NI_1 + (1-\lambda)N\frac{I_2}{\tau}) \tag{B.12}$$

which gives (3.64).

B.1.3 Complete Specialization

The equilibrium conditions for complete specialization mirror those of incomplete specialization except there is agricultural output in region 2.

B.2 Tables of Numerical Results

Table B.1: Base Case versus. Benchmark and Social Planner

Autarky

	Base Case	Benchmark	Δ Base Case from Benchmark (%)	Social Planner	Δ Base Case from Social Planner (%)
a_1	0.98	1.01	-2.95	0.92	6.97
a_2	0.92	0.89	2.92	0.94	-1.52
m_1	48071.19	46763.04	2.80	50927.89	-5.61
m_2	50056.53	48206.76	3.84	51946.39	-3.64
w_1^u	0.24	0.25	-0.12	0.24	2.90
w_2^u	0.26	0.25	0.91	0.24	5.07
w_1^a	0.80	0.79	2.03	0.82	-2.51
w_2^a	0.70	0.72	-1.93	0.65	7.63
p_1^a	13073.01	12342.07	5.92	14814.88	-11.76
p_2^a	14496.20	14368.58	0.89	14814.88	-2.15
r_1^a	4372.00	3840.98	13.83		
r_2^a	2277.10	2439.25	-6.65		
p_1^w	13083.38	12712.27	2.92	14264.90	-8.28
$\bar{r}$	9162.94	8787.08	4.28		
u	3074.42	2970.32	3.50	3162.94	-2.80
λ	0.54	0.53	2.09	0.38	40.15
I_1	64094.92	62350.72	2.80		
I_2	66742.04	64275.68	3.84		
f	-13037.34	-12666.24	2.93		
p_s^w				14264.904	
p_s^a				14814.877	

Note: $\beta_1 = 1.3$

Table B.2: Base Case versus. Benchmark and Social Planner

Incomplete Specialization

	Base Case	Benchmark	Δ Base Case from Benchmark (%)	Social Planner	Δ Base Case from Social Planner (%)
a_1	1.18	1.21	-2.11	1.11	7.17
a_2	1.04	1.05	-0.14	1.13	-7.37
m_1	47619.96	46529.10	2.34	51033.96	-6.69
m_2	50376.21	48257.06	4.39	52054.59	-3.22
w_1^u	0.25	0.25	-0.76	0.24	3.18
w_2^u	0.26	0.26	1.24	0.24	7.02
w_1^a	0.85	0.84	0.81	0.87	-2.15
w_2^a	0.62	0.61	2.19	0.39	60.23
p_1^a	10717.84	10251.82	4.55	12310.60	-12.94
p_2^a	12861.41	12302.19	4.55		4.47
r_1^a	5634.11	5248.02	7.36		
r_2^a	1176.33	1022.71	15.02		
p_1^w	12879.58	12486.73	3.15	14240.61	-9.56
$\bar{r}$	9338.26	8863.02	5.36		
u	3171.48	3069.95	3.31	3289.39	-3.58
λ	0.45	0.40	10.75	0.23	97.05
I_1	63493.28	62038.80	2.34		
I_2	67168.27	64342.75	4.39		
f	-12856.11	-12463.12	3.15		
p_s^w				14240.606	
p_s^a				12310.596	

Note: $\beta_1 = 1.7$

Table B.3: Base Case versus. Benchmark and Social Planner

Complete Specialization

	Base Case	Benchmark	Δ Base Case from Benchmark (%)	Social Planner	Δ Base Case from Social Planner (%)
a_1	1.44	1.45	-0.69	0	
a_2	1.27	1.25	1.29	1.38	-7.91
m_1	48320.23	47110.14	2.57	0.00	
m_2	51117.00	48859.68	4.62	52618.39	-2.85
w_1^u	0.25	0.27	-4.86	0.00	
w_2^u	0.27	0.28	-2.98	0.22	23.61
w_1^a	0.84	0.87	-2.45	0.78	7.63
p_1^a	8959.31	8674.61	3.28	10191.09	-12.09
p_2^a	10751.17	10409.53	3.28		5.50
r_1^a	5458.72	5730.64	-4.75		
p_1^w	12763.39	11836.49	7.83	16238.18	-21.40
$\bar{r}$	7371.05	7557.93	-2.47		
u	3337.07	3222.50	3.56	3430.49	-2.72
λ	0.14	0.16	-15.11	0	
I_1	64426.97	62813.52	2.57		
I_2	68156.00	65146.24	4.62		
f	-12714.23	-11787.27	7.86		
p_s^w				16238.180	
p_s^a				10191.087	

Note: $\beta_1 = 2$

Table B.4: Subsidized Water Pricing versus. Base Case and Social Planner

Autarky			
	Agricultural Water Subsidy	Δ Agr. Subsidy from Base Case (%)	Δ Agr. Subsidy from Social Planner(%)
a_1	1.05	6.53	13.95
a_2	0.98	6.71	5.09
m_1	48061.85	-0.02	-5.63
m_2	50026.28	-0.06	-3.70
w_1^u	0.18	-27.85	-25.76
w_2^u	0.18	-27.91	-24.26
w_1^a	0.87	7.71	5.00
w_2^a	0.78	10.74	19.19
p_1^a	12268.70	-6.15	-17.19
p_2^a	13576.46	-6.34	-8.36
r_1^a	5269.25	20.52	
r_2^a	3153.24	38.48	
p_1^w	18133.74	38.60	27.12
$\bar{r}$	10933.99	19.33	
u	3062.71	-0.38	-3.17
λ	0.53	-0.61	39.29
I_1	64082.46	-0.02	
I_2	66701.70	-0.06	
f	-12126.41	-6.99	
CV_1	245.02		
CV_2	255.04		

Note: $\beta_1 = 1.3$

Table B.5: Subsidized Water Pricing versus. Base Case and Social Planner

Incomplete Specialization

	Agricultural Water Subsidy	Δ Agr. Subsidy from Base Case (%)	Δ Agr. Subsidy from Social Planner(%)
a_1	1.22	2.60	9.96
a_2	1.07	2.59	-4.97
m_1	48159.14	1.13	-5.63
m_2	50946.59	1.13	-2.13
w_1^u	0.17	-30.55	-28.34
w_2^u	0.18	-30.52	-25.65
w_1^a	0.90	6.75	4.46
w_2^a	0.72	15.69	85.36
p_1^a	10564.84	-1.43	-14.18
p_2^a	12677.81	-1.43	2.98
r_1^a	6820.95	21.07	
r_2^a	2128.67	80.96	
p_1^w	18748.68	45.57	31.66
$\bar{r}$	11492.83	23.07	
u	3156.80	-0.46	-4.03
λ	0.43	-2.91	91.32
I_1	64212.18	1.13	
I_2	67928.79	1.13	
f	-12512.05	-2.68	
CV_1	298.71		
CV_2	315.89		

Note: $\beta_1 = 1.7$

Table B.6: Subsidized Water Pricing versus. Base Case and Social Planner

Complete Specialization

	Agricultural Water Subsidy	Δ Agr. Subsidy from Base Case (%)	Δ Agr. Subsidy from Social Planner(%)
a_1	1.42	-0.93	
a_2	1.26	-0.93	-8.77
m_1	47177.98	-2.36	
m_2	49908.65	-2.36	-5.15
w_1^u	0.20	-20.64	
w_2^u	0.21	-20.64	-1.90
w_1^a	0.97	15.39	24.20
p_1^a	8829.69	-1.45	-13.36
p_2^a	10595.63	-1.45	3.97
r_1^a	8246.04	51.06	
p_1^w	15701.95	23.02	-3.30
$\bar{r}$	11717.14	58.96	
u	3234.01	-3.09	-5.73
λ	0.19	38.53	
I_1	62903.98	-2.36	
I_2	66544.86	-2.36	
f	-10665.23	-16.12	
CV_1	2004.49		
CV_2	2120.51		

Note: $\beta_1 = 2$

Chapter 4

The Monocentric City with Heterogeneous Land

Abstract

The purpose of this paper is to integrate heterogeneous land quality into the monocentric city model. Three distinct models are employed. The first adds variable land quality to the developer's cost function in a static, land use model, in which all development is perfectly malleable. Structural density is found to be increasing with land quality. A second, dynamic model analyzes how land quality affects development timing and structural density decisions. Sufficient conditions are found for buildings of higher structural density to be constructed earlier and for leapfrog development to occur. A third model includes demand over spatial amenities in the household's residential location choice. This gives rise to local variation in the rental price, which affects the developers choice of structural density. These models allow for greater flexibility in the structural density, development timing and bid-rent functions at a finer spatial scale.

4.1 Introduction

> The location of almost all U.S. cities can be understood in terms of land heterogeneity...
>
> *Edwin S. Mills (1967)*

In the monocentric city model all land is assumed to be homogenous. This paper extends the model to take into account heterogenous land quality. For over fifty years the monocentric city model has been the bedrock of modern urban economics. Its popularity is due in no small part to its elegant simplicity. In addition, the model's predictive richness has provided a useful tool for generating a broad outline of urban development patterns. The model has been calibrated and estimated, and its comparative statics have been tested, in a variety of ways for different metropolitan areas, in many contexts. For instance, the conclusions of the model provide the theoretical basis for estimating metropolitan population density gradients, and help explain the secular decline in this gradient for almost every major metropolitan area (see Mills and Tan (1980) and Angel *et. al.* (2005)).

However, by construction the model fails to explain the well documented increase in employment decentralization, dispersion and subcentering. At a finer spatial scale, the model is unable to account for scattered development as well as local variations in building height and floor-area ratios.There have been a variety of explanations for these factors: zoning regulations, a desire for privacy, economies of scale in production.

The focus of this paper is on the effect of land quality on urban spatial structure. Recent empirical work (Burchfield et al. (2006), Saiz (2010)) has noted that land quality plays a significant role in both the supply and spatial distribution of residences. Furthermore, casual empiricism points to the importance of land quality in the spatial development pattern, over time as structure rents and values in metropolitan areas increase, higher quality land appears to be developed earlier and proceeds to less desirable sites. Indeed, the importance of land quality as a determinant of urban spatial structure is well recognized in the theoretical literature. In introductory classes on land use and land rent theory, the von Thünen model of agricultural land use, which is based on differences in accessibility to a market center, is contrasted with the Ricardian model, which focuses on differences in agricultural land quality or fertility. Furthermore, in extending those models to the metropolitan setting, it is noted that Ricardian differences in land, as well as differences in accessibility, play an important role in metropolitan spatial structure. It is surprising, therefore, that the monocentric city model has not been formally extended to include differences in land quality.

Brueckner *et. al.* (1999) have adapted the monocentric city model to take into account spatial amenities. However, their analysis does not take into account the development costs associated with exploiting those amenities. For instance, urban housing near the ocean often commands a higher rent. However, such locations may require more secure structural supports, which increase with building heights. This may lead to lower structural density in relation to locations with greater land quality, limiting the amount of floor-area the landlord-developer can make available for rent, regardless of the higher rental price that can be charged.

The monocentric city model is a competitive general equilibrium model in which land at different locations defines a continuum of commodities, and competition between consumers and producers of land generates a set of market clearing land rents and an equilibrium pattern of land-use. In keeping with this conceptualization it is necessary to differentiate between land *in production* which enter through the development costs, and land *in consumption* which relates to the household utility function. While this paper focuses primarily on the former, a method to integrate the latter is developed in the fifth section.

The basic monocentric city model, the Muth-Mills model (Brueckner, 1987) is static. One interpretation is that it describes a steady state. A second is that structures are sufficiently malleable such that each outcome can be described as an instantaneous equilibrium, which is independent of past or future conditions. Yet given the durability of structures, urban spatial structure does exhibit some history dependence (and future dependence, if expectations are forward looking). There is a second strand of literature that treats metropolitan structures as infinitely durable and unalterable. The first case describes a city that is *perfectly malleable*. The second is defined by *complete immutability*. Reality lies somewhere in between. The question asked is this: Given two locations equidistant from the central city, which ensures equal transport costs, and are distinguishable only in land quality, how do these locations differ with respect to structural density and development time?

There will be six additional sections. The first will discuss land quality in the production of housing. The second section is devoted to the analysis of the model under perfect malleability. The third section extends the model to take into account dynamics under complete immutability. The fourth incorporates the amenity value of land quality into the consumers utility function. The fifth sections discusses areas for future research. The sixth section will conclude.

4.2 Land Quality in Production

For the landlord-developer, land quality enters through the structure cost function. In order to simplify the analysis, maintenance costs and depreciation are treated implicitly. Given that the static monocentric city model treats flow or amortized construction costs, while dynamic models treat stock construction costs it is necessary to relate the two. Under the assumption of zero depreciation and maintenance costs, and in the absence of taxes, the cost of capital is simply the interest rate r and amortized construction costs are the interest rate multiplied by the stock construction costs function. This framework allows us to use a common construction cost in both the perfectly malleable and completely immutable structures. To further simplify, differences in structural quality are ignored.

Define $C(s, q)$ as the cost of constructing floor-area of structural density s on land of quality q. It is assumed that $C_s > 0$, $C_{ss} > 0$ and $C_q < 0$. Marginal costs are weakly positive and weakly increasing in s, and developing on higher quality land is less costly.

Motivated by the comparative statics, some attention will later be given to specific functional forms for the cost function.

4.3 Perfect Malleability

In the steady-state case in which all housing is perfectly malleable at each instance the landlord-developer decides on the level of structural density.

Initially, the focus of the analysis is on a unit of land some distance from the city center at any given point in time. The following assumptions are made in the analysis:

i Amortized construction costs for a unit of density s on land of quality q are independent of time and given by $rC(s, q)$.

ii The rental price per unit of floor area p is determined by the general equilibrium of the economy.

iii The rentable floor area as a function of structural density is given by $h(s)$, which is increasing and concave in s.

iv The amortized land value of quality q is given by $rV(q)$.

v Perfect competition in the housing market drive profits to zero.

The profit function for the developer is

$$\pi(s, q) = ph(s) - rC(s, q) - rV(q). \tag{4.1}$$

Profit maximization with respect to the structural density yields the first-order condition

$$ph'(s) - rC_s(s, q) = 0, \tag{4.2}$$

which states that structural density should be increased to the point that an extra unit of floor space just covers its marginal cost. The second-order condition for an interior maximum is given by

$$ph''(s) - rC_{ss} < 0, \tag{4.3}$$

which are assumed to hold for any q and at all locations. The first-order condition defines the optimal choice of structural density given land quality, $s = s(q)$. From assumption (v) the value of land is then

$$rV(q) = ph(s(q)) - rC(s(q), q). \tag{4.4}$$

Of interest is the effect of land quality on the choice of structural density at a given point in time on land that is equally accessible to the central city. This yields the following proposition.

Proposition 3. *Structural density is nondecreasing in land quality.*

Proof. Total differentiation of the first- order condition with respect to q gives

$$\frac{ds}{dq} = \frac{rC_{sq}}{ph''(s) - rC_{ss}}. \tag{4.5}$$

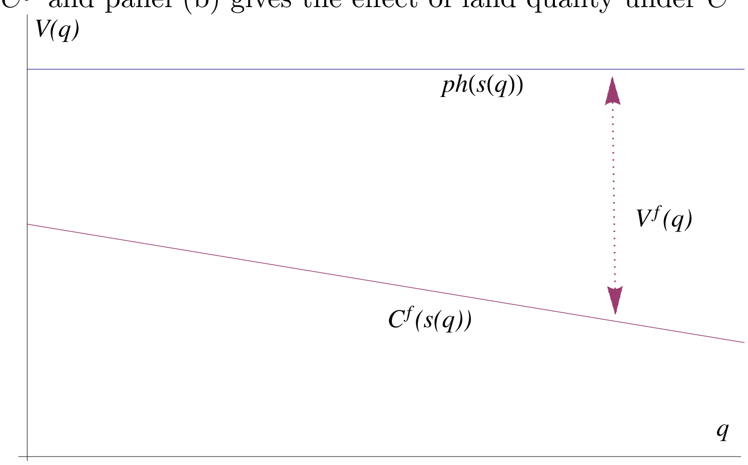
It is straightforward to show that developing on inferior land leads to lower structural density. This leaves the following cases: $C_{sq} = 0$ and $C_{sq} < 0$. The former case describes a situation in which land quality enters only as a fixed cost, such as grading a site, but has no bearing on the marginal cost. This leaves the choice of structural density unchanged but increases the value of land. In the latter case the marginal cost, and perhaps the fixed cost, is decreasing in land quality. This leads to an unequivocal increase in both structural density and the land value.

It has been assumed in the above analysis that land quality is a scalar. However it is more appropriate to view land quality as a vector of attributes. Furthermore, it is an ordinal concept which in the context of this paper requires a cardinalization. Structure quality has obvious cardinalizations such as construction cost per square foot. However, in the case of land quality it is necessary to account for the manner in which it affects development cost. The following cardinalizations are employed for different forms of the cost function:

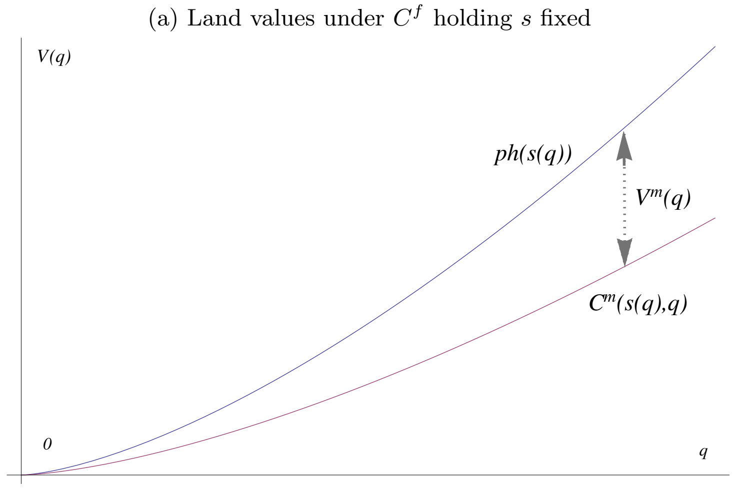
Cardinalization I In the additively separable case, where land quality enters through a fixed cost, define the cost function $C^f(s,q) = c(s) - q$.

Cardinalization II Where land quality plays a role in marginal cost, assume divisional separability and the cost function takes the form $C^m(s,q) = c(s)/q$.

Figure 4.1 displays the results from (4.4) and (4.5), with panel (a) describing the developers problem under C^f and panel (b) gives the effect of land quality under C^m.



(a) Land values under C^f holding s fixed



(b) Land values under C^m holding s fixed.

Figure 4.1: The effect of land quality on revenue, costs and land values

Define $V^f(q)$ and $V^m(q)$ as land values where land quality enters as a fixed or marginal cost, respectively. In panel (a), since land quality has no effect on structural density, the revenue function remains unchanged. However costs decline as q increases leading to higher land values, $V^f(q)$. In panel (b), under C^m, both revenue and costs are increasing in q, but costs rise at a slower rate leading to higher land values, $V^m(q)$.

Using C^m we can rewrite (4.5) as

$$\frac{ds}{dq} = \frac{-rc_s/q}{q(ph''(s) - rc_{ss}/q)}. \tag{4.6}$$

This can be rewritten in elasticity form as

$$\eta_{s,q} = -\frac{1}{\eta_{c_s,s} - \eta_{h',s}}, \tag{4.7}$$

which reads that the elasticity of structural density with respect to land quality is equal to the negative of the reciprocal of the second order conditions for profit maximization.

Given these results it is natural to consider how they affect the variation in structural density over space. It is assumed that the rent gradient follows the Muth condition, $p'(x) = -t/H < 0$, which says that rents decline with distance in order to offset the increase in the households marginal transportation costs. Suppose that for a radial distance x from the CBD, the land quality is defined by $q(x)$. We then have the following result.

Proposition 4. *Structural density is locally increasing with distance from the CBD when the reduction in the marginal cost of construction due to an increase in quality exceeds the decline in the rental price.*

Proof. Total differentiation of (4.2) with respect to x yields:

$$\eta_{s,x} \stackrel{s}{=} \eta_{p,x} - \eta_{C_s,x}, \tag{4.8}$$

where $\eta_{s,x} = \frac{q}{s}\frac{\partial s}{\partial q}\frac{x}{q}\frac{\partial q}{\partial x}$, $\eta_{p,x} = \frac{x}{p}\frac{\partial p}{\partial x}$ and $\eta_{C_s,x} = \frac{s}{C_s}\frac{\partial C_s}{s}\frac{q}{s}\frac{\partial s}{\partial q}\frac{x}{q}\frac{\partial q}{\partial x}$ are the elasticity of structural density, housing rents and marginal costs, respectively, to distance from the CBD. From the Muth condition $\eta_{p,x} < 0$ and $\eta_{C_s,x} < 0$ from the decline in the marginal cost due to an increase in the land quality. The result follows immediately.

This formalizes the point that structural density is increasing locally along the radius from the CBD if q is increasing sufficiently rapidly to offset the decline in rental income.

4.4 Complete Immutability

In the case where there is no flexibility to alter a building once constructed, the landlord-developer must decide not only on the levels of structural density but also on the development time, T. In this section it is shown that improvements in land quality may lead to higher densities at earlier or later development dates, or earlier development at lower densities. The conditions are derived such that improvements in land quality lead to earlier development at higher densities. Finally, the possibility of lower density development at later dates is ruled out.

Under complete immutability, once constructed a structure cannot be altered and remains at that density forever. To simplify the analysis it is assumed that the interest rate, $r > 0$, the growth rate, $g > 0$ and that $r > g$, and the construction technology are constant over time. Denote p as the housing rent for unit of floor area at $t = 0$. Under these assumptions the developer's present value of profit is:

$$\pi(s, T, q) = \frac{ph(s)}{r-g} e^{-(r-g)T} - C(s, q)e^{-rT}. \tag{4.9}$$

The first-order conditions for profit maximization with respect to s and T are given by

$$s : \ \frac{ph'(s)}{r-g} e^{-(r-g)T} - C_s e^{-rT} = 0, \tag{4.10}$$

$$T : \ -ph(s)e^{-(r-g)T} + rC(s, q)e^{-rT} = 0. \tag{4.11}$$

(4.10) states that structural density is chosen such that discounted marginal revenue just equals the marginal construction costs. (4.11) states that development time is postponed until amortized construction costs equal the foregone rent. The following are the second-order derivatives of the profit function

$$\pi_{ss} =[\frac{ph''(s)e^{gT}}{r-g} - C_{ss}]e^{-rT} < 0, \tag{4.12}$$

$$\pi_{sT} = \pi_{Ts} =[-ph'(s)e^{gT} + rC_s]e^{-rT} = gC_s e^{-rT} > 0, \tag{4.13}$$

$$\pi_{TT} =[-gph(s)e^{gT}]e^{-rT} < 0. \tag{4.14}$$

The signs of these terms follow directly from the first-order conditions and the assumption of the concavity of $h(s)$ and the convexity of $C(s,q)$ with respect to s. The additional requirements which ensure that the first-order conditions define a local maximum are set out in appendix (C.1) and are assumed to hold.

Totally differentiating (4.10) and (4.11) with respect to q yields the effects of the improvement in land quality on structural density and development time.

$$\frac{ds}{dq} = [\pi_{sT}\pi_{Tq} - \pi_{TT}\pi_{sq}]J^{-1}, \tag{4.15}$$

$$\frac{dT}{dq} = [\pi_{Ts}\pi_{sq} - \pi_{ss}\pi_{Tq}]J^{-1}, \tag{4.16}$$

where

$$\pi_{sq} = -C_{sq}e^{-rT} > 0, \ \ \pi_{Tq} = rC_q e^{-rT} < 0. \tag{4.17}$$

Under cardinalization C^f, $\pi_{sq} = 0$ and (4.15) and (4.16) reduce to,

$$\frac{ds}{dq} = \pi_{sT}\pi_{Tq} < 0, \tag{4.18}$$

$$\frac{dT}{dq} = -\pi_{ss}\pi_{Tq} < 0. \tag{4.19}$$

Figure 4.2 displays the results. When land quality enters into the developer's problem only as a fixed cost, there is no effect on the first-order condition with respect to s, shown as

$\pi_s = 0$. However, an improvement in land quality reduces the marginal benefit of postponing development. This shifts the first-order condition with respect to T to the left, from π_{T_0} to π_{T_1}, leading to an unambiguous decline in both structural density and development time from the initial equilibrium defined at A to a new equilibrium at B.

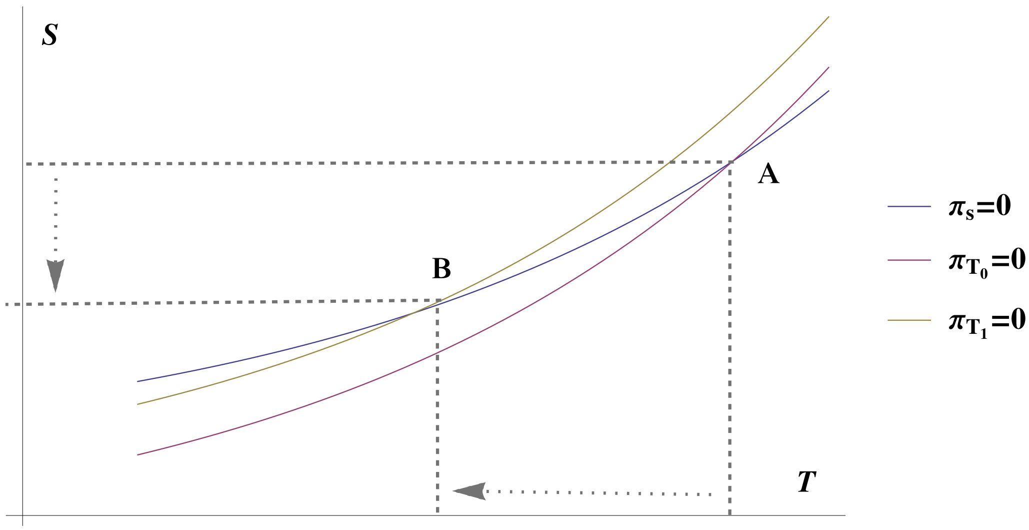


Figure 4.2: Changes in structural density and development time when land quality enters as a fixed cost

Proposition 5. *Structural density is increasing in land quality if marginal cost is more elastic than total cost with respect to*

Proof. It is shown in appendix (C.2) that a sufficient condition to sign the changes in structural density with respect to land quality is given by

$$\eta_{C,q} \gtrless \eta_{C_s,q} \implies \frac{ds}{dq} \gtrless 0, \tag{4.20}$$

where $\eta_{C,q}$ and $\eta_{C_s,q}$ are the elasticity of total and marginal cost with respect to q, respectively. From the assumptions on the cost function, both terms are nonpositive, which implies for structural density to be increasing (decreasing), marginal cost must be less (more) elastic than total cost with respect to q.

The intuition follows from the first-order conditions. Marginal costs affect the choice of density while the total cost affects the choice of development time. An increase in q will lead to higher density if the fall in the marginal cost of an extra unit of s exceeds the decline in the marginal benefit of postponing T.

Appendix (C.3) demonstrates that a sufficient condition to sign changes in development time with respect to land quality is given by

$$\frac{g}{r}\frac{\eta_{C_s,q}}{\eta_{C,q}} \gtrless -\frac{\eta_{C_s,s} - \eta_{h',s}}{\eta_{C,s}} \implies \frac{dT}{dq} \gtrless 0. \tag{4.21}$$

Both terms on each side of the inequality to the left are positive by assumption. Inspection of the ratio to the far left in (4.21) makes it clear that the development date is likely to be brought forward when improvements in land quality lowers the marginal benefit of postponing construction considerably more than lowering the marginal costs, *i.e.* $\frac{\eta_{C_s,q}}{\eta_{C,q}} \sim 0$.

Combining (4.20) and (4.21) yields a set of conditions for the development of taller structures to be built at an earlier date:

$$|\eta_{C_s,q}| > |\eta_{C,q}| \implies \frac{ds}{dq} > 0, \tag{4.22}$$

$$\frac{g}{r}\frac{\eta_{C_s,q}}{\eta_{C,q}} < -\frac{\eta_{C_s,s} - \eta_{h',s}}{\eta_{C,s}} \implies \frac{dT}{dq} < 0. \tag{4.23}$$

(4.20) and (4.21) provide a set of comparative statics with regard to land quality improvements on choices of structural density and development times at any location in the city. As shown, density may be increasing at earlier dates, decreasing at earlier dates or increasing at later dates. The following proposition rules out the possibility of density increasing and development time decreasing.

Proposition 6. *Improvements in land quality never leads to lower structural density at later dates.*

Proof. See Appendix C.4

This follows from the fact that an increase q lowers the marginal cost of an extra unit of density and reduces the marginal benefit of postponing the development date by reducing the necessary outlay for new construction. Therefore, the effect is either to increase structural density at earlier or later dates, or bring development forward at higher or lower densities.

This model can be extended to yield conditions for leapfrog development, where parcels of land further from the CBD are developed earlier. As in the static problem, assume that $q = q(x)$ and $p = p(x)$. Differentiating the first order conditions with respect to x gives:

$$\frac{ds}{dx} = [\pi_{sT}\pi_{Tx} - \pi_{TT}\pi_{sx}]J^{-1}, \tag{4.24}$$

$$\frac{dT}{dx} = [\pi_{Ts}\pi_{sx} - \pi_{ss}\pi_{Tx}]J^{-1}, \tag{4.25}$$

where

$$\pi_{sx} = \frac{p'(x)h'(s)}{r-g}e^{-(r-g)T} - C_{sq}q'(x)e^{-rT}, \tag{4.26}$$

$$\pi_{Tx} = -p'(x)h(s)e^{-(r-g)T} + rC_q q'(x)e^{-rT}. \tag{4.27}$$

Proposition 7. *When total cost is more elastic than marginal cost with respect to q, leapfrog development will occur when the ratio of the elasticity of price and land quality with respect to x lies between the two cost elasticities, i.e.*

$$\eta_{c,q} < \frac{\eta_{p,x}}{\eta_{q,x}} < \eta_{C_s,q} \implies \frac{dT}{dx} < 0. \tag{4.28}$$

Proof. See Appendix (C.5)

Here, the term $\frac{dT}{dx}$ determines the slope of the development timing decision with distance from the CBD. Once again, this result emphasizes that development time is brought forward when improvements in land quality primarily affect the total cost of development. Eq. (4.28) makes the case that leapfrog development will likely occur if the household price gradient is relatively flat and land quality plays a large role in reducing total costs while having little effect on marginal costs.

Under cardinalization C^f, the signs of (4.26) and (4.27) reduce to,

$$\pi_{sx} \overset{S}{=} \eta_{px} < 0, \tag{4.29}$$

$$\pi_{Tx} \overset{S}{=} -[\eta_{px} + \eta_{qx}], \tag{4.30}$$

where η_{px} and η_{qx} are the elasticity of price and land quality with respect to x. This reaffirms the analysis in (4.8), from the static model, which relies crucially on the relative changes of rental prices and land quality. In the case that $\eta_{qx} > -\eta_{px}$, both development times and structural density are decreasing in x.

4.5 Land Quality in Demand

In the above analysis it is assumed that land quality enters only through the landlord-developers cost function. This section focuses on extending the analysis of variable land quality to the household's location decision. In the standard Muth-Mills model the demand for housing is measured in floor area. However, households are generally searching for a vector of amenities, floor-area being just one. If additional amenities are substitutable, to some degree, with floor-area the renter may choose to forego some space in exchange for those amenities. Furthermore, land that provides a higher level of amenities will, *ceteris paribus*, command a higher rent. Developers will exploit the effect as long as the increased cost of provision does not exceed the rental premium. For instance, urban homes on cliffs overlooking the ocean are choice property. However the costs of fortifying the homes with stronger framing materials or concrete reinforcements, to protect against erosion or natural disasters, may limit the development potential of those locations.

A model is derived where households benefit from local amenities. All households are identical and their preferences are given by the utility function, $U(z, H, a)$, where z is the numeraire Hicksian composite good, H is the household consumption of floor area and a is the demand for amenities. It is assumed that utility is increasing and concave in both z and H. All households receive an income y and face commuting costs tx, where t is the marginal cost of traveling distance x. The budget constraint is given by

$$y - tx = p(x)h + z, \tag{4.31}$$

where $p(x)$ is the housing rental price as a function of distance from the city center. Utility maximization yields the following condition

$$U_H = p(x)U_z. \tag{4.32}$$

Given that all households are identical a uniform utility level, u, must hold everywhere. It follows that

$$u = U(z, H, a). \tag{4.33}$$

Totally differentiating (4.31) and (4.33) with respect to a and eliminating $\frac{dz}{da}$ and $\frac{dH}{da}$, yields the marginal rate of substitution between a and H,

$$\frac{U_a}{U_H} = \frac{dp}{da}\frac{H}{P}. \tag{4.34}$$

Given the assumption of the concavity of $U(\cdot)$ in each of its arguments it follows that higher amenities lead to an increase in the bid-rent function.

It is natural to consider how amenities vary over the space of the city. Given that in this model amenities are linked to the land, define $a(x)$ as the amenities at each location.

Proposition 8. *Housing rents increase with distance from the CBD if the value of an increase in amenities exceeds additional transportation costs.*

Proof. Eliminating z in (4.33) by inserting (4.31) and differentiating with respect to

x yields the effect of an increase in distance from the CBD on the household rental price,

$$\frac{dp}{dx} = \frac{-t}{H} + \frac{dp}{da}a'(x). \tag{4.35}$$

This is simply the Muth condition plus the effect of changes in amenities on the household bid-rent function. Rewriting (4.37) in elasticity form yields

$$\eta_{p,x} = -\frac{tx}{pH} + \eta_{p,a}\eta_{a,x} \tag{4.36}$$

The sign of (4.38) is ambiguous, however, if the amenities are increasing sufficiently fast to offset additional transportation costs then rental prices will be increasing in x.

Note that the distance elasticity of price will likely see more variation closer to the CBD where the household travel costs are relatively small. However moving further from the CBD transportation costs assume a larger role and may dominate the effect.

4.5.1 Structural Development and Amenities

As in the previous sections, landlord-developers produce housing taking the rental price as given. Units of floor area, $h(s)$, is a function of structural density, s, and is assumed to be increasing and concave. Developer's face present value costs, $rC(s)$, which are assumed to be convex in s, and pay a land cost $V(a)$ which is the value of land with amenity level a. The developer's profits are given by

$$\max_s ph(s) - rC(s) - V(a) \tag{4.37}$$

Profit maximization yields the first-order condition

$$ph'(s) - rC'(s) = 0. \tag{4.38}$$

Perfect competition drives profits to zero which gives the value of land as the difference between the developer's revenue and construction costs,

$$ph(s) - rC(s) = V(a). \tag{4.39}$$

Proposition 9. *Locations with greater amenities are more densely developed.*

Proof. Totally differentiating (4.39) with respect to a and manipulating yields

$$\eta_{s,a} = -\frac{\eta_{p,a}}{\eta_{h',s} - \eta_{C',s}} > 0, \tag{4.40}$$

where $\eta_{s,a}$ is the elasticity of structural density to an increase in amenities and $\eta_{h',s}, \eta_{C',s}$ are the elasticity of floor-area and costs with respect to changes in s. Intuitively, locations that provide more amenities increase the household's willingness to pay and thus increases the marginal benefit of an extra unit of density to the landlord.

Total differentiation of (4.40) with respect to a gives

$$\frac{dV(a)}{da} = \frac{dp}{da}h(s) > 0. \tag{4.41}$$

In elasticity form this becomes,

$$\eta_{V,a} = \eta_{p,a}\frac{ph(s)}{V(a)}. \tag{4.42}$$

It follows that land values are more elastic than housing rents with respect to amenities since $\frac{ph(s)}{V(a)} > 1$.

In choosing the level of structural density in construction further from the CBD the developer takes into account changes in the household's bid-rent function with distance.

Proposition 10. *Marginal changes in structural density and land values move in the same direction as marginal changes in rents with respect to distance from the CBD.*

Proof. Total differentiation of (4.40) with respect to x, after manipulation yields,

$$\frac{ds}{dx} \stackrel{s}{=} \frac{dp}{dx}. \tag{4.43}$$

Similarly, differentiation of (4.41) with respect to x yields

$$\frac{dV}{dx} \stackrel{s}{=} \frac{dp}{dx}. \tag{4.44}$$

The sign of (4.45) is contingent on the slope of the bid-rent function. If rental prices are increasing in distance then density will be as well.

Note that these are local measures. Significant variation in amenities can lead to variationss around the Muth condition in both the bid-rent function and structural density over the city. A decline in amenities will reinforce and hasten the fall in p and s with respect to x, predicted by the standard model. While an increase in amenities in x can locally offset the rise in transportation costs, leading to locally higher rents and density.

4.6 Future Research

This paper has focused on the extent to which land quality affects the supply of and demand for housing in a monocentric city. A more complete approach could take into account how land quality adjusts other urban spatial decisions. Two factors are particularly pertinent: provision of public services and household transportation costs. With respect to the latter, the general assumption of the monocentric city model is that two households equidistant from the city center face the same commuting costs. However, in a model that incorporates a heterogenous landscape, variations in road conditions will lead to asymmetric monetary and time costs. This will, in turn, lead to a divergence in the rental price and both the supply and demand for housing at equidistant locations.

With regards to public services, two issues that stand out are the provision of public infrastructure for roads and water distribution, and the availability of public transportation.

In the case of roads, inferior land quality will lead to increased costs in both construction and maintenance. In the case of water distribution, poor land quality and topographical irregularities increase infrastructure and distribution costs. In practice, infrastructure for new development is often financed through impact fees. The distribution between developers and households of extra costs arising from construction on low quality land may lead to both a decline in structural density and an increase in the rental price.

Regarding public transportation, one feature of the analysis in sections 3 and 4 is that low quality land leads to the development of housing with lower structural density. Given that household demand for floor area is unchanged, this implies lower population density at those locations. In the provision of public transportation, a threshold level of population density is used as a measure for the economic viability of providing local access to the transportation network, e.g. a bus stop or subway station, given the costs. In areas that fail to meet that threshold, households living in those locations face an additional travel cost to access the transportation network. This would further reduce the rental price for those locations.

4.7 Conclusion

The purpose of this paper has been to explore the effect of heterogenous land quality in a monocentric city model. Three models were developed. The first took into account variable land quality into the developers cost function in a static, urban spatial model where all structures are perfectly malleable. The second integrated land quality into a dynamic model where developers choose both structural density and construction dates where all development is completely immutable. Finally, a model was derived in which land quality provides amenities to households, affecting the equilibrium household rent gradient and the developer's choice of structural density. The results provide a more flexible specification for the structural density, development time and bid-rent functions at a finer spatial scale than the standard monocentric city model.

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Appendix C

C.1

For the first-order conditions to define a local maximum requires $\pi_{ss} < 0$, $\pi_{TT} < 0$ and $J \equiv \pi_{ss}\pi_{TT} - \pi_{sT}\pi_{Ts} > 0$. Totally differentiating π_s yields $\frac{ds}{dt}_{FOC_s} = -\pi_{sT}/\pi_{ss} > 0$. Similarly, differentiating π_T yields $\frac{ds}{dt}_{FOC_T} = -\pi_{TT}/\pi_{Ts} > 0$. It follows that the slope of the first order condition with respect to T be steeper than with respect to s for the second-order conditions to hold. Using (4.12)-(4.14) and plugging into J yields the following condition for a maximum:

$$-[\eta_{h',s} - \eta_{c_s,s}] > \frac{g}{r}\eta_{c,s}. \tag{C-1}$$

C.2

This provides the derivation of (4.20). From (4.15) we have

$$\frac{ds}{dq} \stackrel{s}{=} \pi_{sT}\pi_{Tq} - \pi_{TT}\pi_{sq}, \tag{C-2}$$

$$\stackrel{s}{=} [[gC_s][rC_q] - [-gph(s)e^{gT}][-C_{sq}] \tag{C-3}$$

From [11] $ph(s)e^{gT} = rC(s,q)$. It follows that:

$$\frac{ds}{dq} \overset{s}{=} \frac{C_q}{C} - \frac{C_{sq}}{C_s} \tag{C-4}$$

$$\overset{s}{=} \frac{qC_q}{C} - \frac{qC_{sq}}{C_s} \tag{C-5}$$

$$\overset{s}{=} \eta_{C,q} - \eta_{C_s,q} \tag{C-6}$$

C.3

This appendix derives (4.21). From (4.16):

$$\frac{dT}{dq} \overset{s}{=} \pi_{Ts}\pi_{sq} - \pi_{ss}\pi_{Tq} \tag{C-7}$$

$$\overset{s}{=} [-C_{sq]}[gC_s] - [\frac{ph''(s)e^{gT}}{r-g} - C_{ss}]][rC_q] \tag{C-8}$$

From (4.10) we have $C_s = \frac{ph'(s)e^{gT}}{r-g}$, which reduces (4.54) to:

$$\frac{dT}{dq} \overset{s}{=} \frac{C_{sq}}{C_s}\frac{C_s}{C}g - [\frac{h''(s)}{h'(s)} - \frac{C_{ss}}{C_s}]r\frac{C_q}{C} \tag{C-9}$$

Multiplying and dividing by q and s yields

$$\frac{dT}{dq} \overset{s}{=} g\eta_{C_s,q}\eta_{C,s} - r[\eta_{h',s} - \eta_{C_s,s}]\eta_{C,q} \tag{C-10}$$

C.4

This section provides the proof that a developer does not choose lower structural density at a later date. The second-order condition requires $J \equiv \pi_{ss}\pi_{TT} - \pi_{sT}\pi_{Ts} > 0$ or $\frac{-\pi_{TT}}{\pi_{sT}} > \frac{-\pi_{Ts}}{\pi_{ss}}$. In the case where $\frac{ds}{dq} < 0$, $\frac{dT}{dq} > 0$, combining (4.15) and (4.16) and $\pi_{Ts} = \pi_{sT}$, gives the result $\frac{-\pi_{Ts}}{\pi_{ss}} > \frac{-\pi_{Tq}}{\pi_{sq}} > \frac{-\pi_{TT}}{\pi_{sT}}$, which is a contradiction.

C.5

This section gives the conditions for leapfrog development at higher structural density. (4.24) and (4.25) are:

$$\frac{ds}{dx} = [\pi_{sT}\pi_{Tx} - \pi_{TT}\pi_{sx}]J^{-1}, \ \frac{dT}{dx} = [\pi_{Ts}\pi_{sx} - \pi_{ss}\pi_{Tx}]J^{-1}.$$

Using the fact that $\pi_{sT} > 0, \ \pi_{Ts} > 0$ and $\pi_{ss}, \ \pi_{TT} < 0$, it follows that

$$\pi_{sx} < 0, \ \pi_{tx} < 0 \implies \frac{ds}{dx} > 0 \ , \frac{dt}{dx} < 0.$$

Manipulating (4.26) and (4.27) and plugging into (4.24) and (4.25) yields

$$\frac{ds}{dx} = p'(x)\frac{C_s}{p(x)} - C_{s,q}q'(x) > 0, \ \frac{dt}{dx} = p'(x)\frac{C}{p(x)} - C_q q'(x) < 0$$

Combining these two effects yields the result.

C.6

Totally differentiating (4.38) with respect to x yields,

$$\frac{ds}{dx} = -\frac{h_s}{ph_{ss} - rC_{ss}}\frac{dp}{dx} \tag{C-11}$$

By assumption, the second order condition holds so the term on the right in front of $\frac{dp}{dx} > 0$. It follows immediately that structural density moves in the same direction as rents with distance from the CBD.

Chapter 5

Conclusion

The contribution of this research has been to extend the literature on urban and regional growth by exploring the spatial heterogeneity of land and the availability of water. We begin with the premise that some parcels of land are inherently more valuable than others. That value may be derived from characteristics important to households such as location in a more comfortable climate, or containing attractive physical characteristics. In contrast, a parcel of land may generate value through agricultural productivity or characteristics that are more suitable for structural development. And, of course, it may be the case that some land can be embedded with all of these characteristics.

Chapter 2 considered a scenario where one region is relatively more valuable in amenities to households and in productivity to the agricultural sector relative to another region. A novel model combining the core-periphery and the monocentric city models was developed to explore how amenities and agricultural productivity fuel the competition for land between competing interests. In addition, the model considers a case where the more attractive region is devoid of water resources and must import water for urban residents and agricultural producers from the other region. Conditions were derived for the land to be divided for dual uses (urban residential use and farmland) in the more attractive region. Additionally, we consider the equilibrium where all households concentrate in a single region and explore the stability of such equilibria.

Chapter 3 extended the model from Chapter 2 to integrate endogenous public infrastructure in the interregional distribution of water resources and transport costs in the agricultural good. A simplified example was considered that allowed for comparative static analysis of the key parameters. The model was then calibrated to represent some stylized facts from the California economy. A policy experiment was considered where water for the agricultural sector was subsidized

Chapter 4 introduced the spatial heterogeneity of land quality into the monocentric city model. Three separate models were considered. The first focus on the standard, static model in which all structure are permanently malleable. It was shown that improvements in land quality lead to higher structural density at any given point, and that structural density increases with distance from the city if the decline in the marginal costs are large enough to offset the fall in revenue per square foot. The second model focuses on a dynamic setting where developers choose both structural density and construction dates. Conditions for leapfrog development to occur were derived. Finally, the model allows for variation in spatial amenities across a city. A higher level of amenities raises rents at a given location and rents are increasing from the CBD if the increase in amenities exceeds the additional commuting costs.