

UC Santa Barbara
Ted Bergstrom Papers

Title

Love and Spaghetti, The Opportunity Cost of Virtue

Permalink

<https://escholarship.org/uc/item/60s2f9ps>

Author

Bergstrom, Ted

Publication Date

1989-03-01



Puzzles: Love and Spaghetti, The Opportunity Cost of Virtue

Ted Bergstrom

The Journal of Economic Perspectives, Volume 3, Issue 2 (Spring, 1989), 165-173.

Stable URL:

<http://links.jstor.org/sici?sici=0895-3309%28198921%293%3A2%3C165%3APLASTO%3E2.0.CO%3B2-W>

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The Journal of Economic Perspectives is published by American Economic Association. Please contact the publisher for further permissions regarding the use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/aea.html>.

The Journal of Economic Perspectives
©1989 American Economic Association

JSTOR and the JSTOR logo are trademarks of JSTOR, and are Registered in the U.S. Patent and Trademark Office. For more information on JSTOR contact jstor-info@umich.edu.

©2003 JSTOR

Puzzles

Love and Spaghetti, The Opportunity Cost of Virtue

Ted Bergstrom

In honor of St. Valentine's day, we offer a puzzle about love and a puzzle about virtue. Surely these topics are inexhaustible sources of riddles for thoughtful people in all disciplines. Readers are invited to suggest better answers to the puzzles, to pose new puzzles or to offer comments. Please send correspondence to Barry Nalebuff, "Puzzles," *Journal of Economic Perspectives*, Department of Economics, Princeton University, Princeton, N.J. 08544.

Puzzle 1: When Love and Spaghetti Intertwine

Romeo loves Juliet and Juliet loves Romeo. Besides love, they consume only one good, spaghetti. Romeo likes spaghetti, but he also wants Juliet to be happy and he knows that what makes Juliet happy is her spaghetti and his own happiness. Similarly for Juliet. Located just above each person's navel is a small gauge (called a hedonimeter) with numbers and a pointer. The reading on either person's hedonimeter is higher the happier he or she is.¹ Let S_R and S_J denote Romeo's and Juliet's consumptions of spaghetti and let U_R and U_J be the readings on their hedonimeters. Assume a warm climate so that both wear flimsy clothing and can easily read each other's hedonimeters.

¹Readers who mistrust such fanciful notions as abdominal hedonimeters may want to imagine that Romeo and Juliet can determine their lovers' happiness from smiles and the light in each other's eyes.

■ *Ted Bergstrom is Professor of Economics, University of Michigan, Ann Arbor, Michigan.*

Specifically, the readings on their hedonimeters are:

$$U_R(S_R, U_J) = \sqrt{S_R} + aU_J(S_J, U_R)$$

$$U_J(S_J, U_R) = \sqrt{S_J} + bU_R(S_R, U_J),$$

where $a > 0$ and $b > 0$.

Part 1. Disentangling love from spaghetti.

Incorrigible philistines that they are, economists will want to know how Romeo and Juliet feel about *allocations of spaghetti* rather than about mixtures of love and spaghetti. If $ab \neq 1$, then the two equations that define the hedonimeter readings will determine utility functions over allocations for Romeo and Juliet of the form:

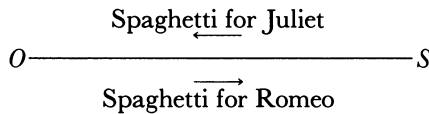
$$V_R(S_R, S_J) = \alpha_R\sqrt{S_R} + \alpha_J\sqrt{S_J}$$

$$V_J(S_J, S_R) = \beta_J\sqrt{S_J} + \beta_R\sqrt{S_R}$$

where the α 's and β 's are constants. Solve for the α 's and β 's and show that they are all positive if $ab < 1$ and that they are all negative if $ab > 1$.

Part 2. Love on the Edgeworth Line.

Let us think about how Romeo and Juliet would approach the problem of allocating a fixed amount of spaghetti between them. Two commodities are needed for an Edgeworth box. Since we only have one commodity to allocate, we construct an "Edgeworth line." The length of the Edgeworth line, OS , is equal to the length of the total supply of spaghetti, laid end to end. A point x on the line represents an allocation where Romeo gets all of the spaghetti to the left of x and where Juliet gets all of the spaghetti to the right of x .



The Edgeworth Line

Given their utility functions for allocations, it is possible to solve for each lover's favorite way to allocate a fixed quantity of spaghetti. If the choice for how to allocate the total supply of spaghetti were left to Romeo, what would be the ratio of his consumption to Juliet's? If Juliet could choose the allocation, what would be the ratio of Romeo's consumption to her own? Show that if $ab < 1$, then on the Edgeworth line, Romeo's favorite allocation lies to the right of Juliet's favorite point. Where, on the line are the Pareto optimal allocations? In what sense is it true that when $ab < 1$, Romeo and Juliet, though loving, are still selfish?

Part 3. Spaghetti and true love.

Suppose that Romeo and Juliet are really fond of each other so that whenever they disagree about the allocation of a fixed amount of spaghetti, each wants the other to have the larger share. Romeo's favorite point on the Edgeworth line would then have to lie to the left of Juliet's. We showed earlier that if $ab < 1$, then Romeo's favorite point lies to the right of Juliet's. So we will have to consider the case where $ab > 1$. But from the result of Part 1, it follows that if $ab < 1$, then the partial derivatives of $V_R(S_R, S_J)$ and $V_J(S_R, S_J)$ must all be negative. Could it be that we are about to discover the following?

Theorem? True lovers hate spaghetti.

Of course not, you say. That would be silly. In order to save economics from the embarrassment of *suspicion of possessing a silly theorem*, your task is to figure out what must be true about the readings on the two lovers' hedonimeters if both people like spaghetti and if Romeo's favorite point on the Edgeworth line is to the left of Juliet's.

Part 4. Cardinal knowledge.

In the old debates about utility theory, it was sometimes argued that one needs "cardinal utility" in order to have an adequate theory of interpersonal comparisons of utility. Let us explore this question in the case of Romeo and Juliet.

Suppose that Juliet passes through an airport metal detector and the magnetic field recalibrates her hedonimeter, but otherwise has no effect on her psychological condition. For any situation, her hedonimeter reading is now only 1/10 as high as it would have been before she encountered the metal detector. (Whereas her pleasure had previously been measured in "utils," it is now measured in "decautils," where 1 decautil equals 10 utils.) What would happen to Romeo's and Juliet's preferences over allocations if Romeo was not aware of what happened to Juliet and did not respond by readjusting his hedonimeter? How must Romeo adjust his hedonimeter so that his preferences on allocations are the same as before?

We know that a monotonic increasing transformation of a selfish consumer's utility function leaves his ordinal preferences on commodity bundles unchanged. In the case of Romeo and Juliet, describe the general class of transformations of their hedonimeter functions that leaves their preferences over allocations unchanged. In what sense does interpersonal comparability of utility between Romeo and Juliet imply "cardinal utility?"

Part 5. Of love and feedback.

One could model our story of Romeo and Juliet as a dynamic feedback system. Suppose that each person's utility level in period t depends on his or her own spaghetti consumption in period t and on the other person's hedonimeter reading in period $t - 1$. Show that the resulting system of difference equations is stable if and only if $ab < 1$.

Discussion

In the real world, we receive only imperfect indications of the mental states of our loved ones. Thus it seems natural to ask: "What happens if Romeo and Juliet live in

Minnesota and have to keep their hedonimeters covered most of the time?” Worse yet, what if each has the capacity to give misleading signals about his or her happiness?

I don't know quite what to make of the fact that the case where $ab > 1$ is unstable in the simple feedback system proposed in Part 5. Should we take this as evidence that such unselfish love is unstable and therefore unlikely to be observed, or should we be looking for different kinds of dynamic systems which would sustain unselfish love?

Douglas Bernheim and Oded Stark (1988) have a nice article which poses the question “When is more love a good thing?” and offers some surprising and stimulating answers. Miles Kimball's paper “Making Sense of Two-Sided Altruism,” treats the issue of mutual love between parents and children in an overlapping generations model of capital accumulation. The issues raised in the case of Romeo and Juliet were discussed in Bergstrom (1971). A considerably more general treatment appears in Bergstrom (1988). Further references and a clear discussion of the relation between interpersonal comparability of utility and cardinality can be found in Harsanyi (1987).

Puzzle 2: The Opportunity Cost of Virtue

According to conventional wisdom, if there is a progressive income tax, then the effect on income distribution of allowing tax-deductions for voluntary contributions to charity is regressive. The reason is that the marginal cost of giving to charity is smaller for the rich than it is for the poor. Musgrave and Musgrave (p. 362) express this nicely by saying that with the tax deductibility of charitable contributions, “A philosopher-economist might observe that the opportunity cost of virtue falls as one moves up the income scale.”

The conventional view has been challenged in an article by Russell Roberts (1987). Roberts argues that the effect of tax-deductibility of charitable contributions for the maintenance of pure public goods will be to redistribute income from the rich to the poor. Roberts' paper is a nice example of a case where general equilibrium analysis yields dramatically different conclusions from partial equilibrium. The logic behind Roberts' surprising claim should become clear from the solution of this puzzle.

Part 1. Is cheap virtue advantageous?

Aristotle and Plato each consume one private good and one shared public good. They have equal wealths and both have the same concave, twice-differentiable utility function, $U(X_i, Y)$, where X_i is Philosopher i 's consumption of private goods and Y is the amount of public goods. Public goods and private goods can both be purchased at a constant cost of \$1 per unit. We consider a Nash equilibrium in which the public good is supplied by voluntary contributions, where each philosopher assumes that the other's contributions are uninfluenced by his own. Suppose that for every dollar's worth of public good that Aristotle contributes, he receives a subsidy of $\$s$ where $0 < s < 1$. Plato, on the other hand receives no subsidy. Aristotle's subsidy is paid for

by a lump sum tax assessed equally on the two philosophers. Assuming that in equilibrium a positive amount of public good is supplied, would Plato be better off if the subsidy were available to him and not to Aristotle?

Part 2. Morality and subsidized virtue.

Suppose that Aristotle and Plato have received extensive moral training and get pleasure not only from the effects of their gifts, but also from the act of giving. Each person derives utility from private consumption, from the supply of public good and from the amount of private good that he voluntarily sacrificed to provide public goods. As before, suppose that Aristotle's contributions are subsidized and Plato's contributions are not. Would Plato be better off if he, rather than Aristotle, received the subsidy? (Hint: Notice that here we are assuming that moral satisfaction comes from one's sacrifice or net contribution. The answer will be different if a person's moral satisfaction from his own gift depends on his gross contribution.)

Part 3. Subsidized virtue and progressive taxation.

If Aristotle is richer than Plato, under what circumstances is it true that Plato would prefer to have Aristotle's donation subsidized at a higher rate than his own?

Discussion

An alternative model would have each philosopher concerned about how influential his own donations are. Thus, if he is subsidized at the rate s , his utility function might take the form $U(X_i, 1/(1-s)Y_i, Y)$ where Y_i is his net donation. In this case, it can be advantageous to have one's gifts subsidized at a higher rate. This model would be appropriate for contributions to a political party or to a university which rewards influential contributors with favors.

The results in this puzzle depend critically on the assumption that the contributors contribute toward the supply of the same public good. If Aristotle and Plato had different tastes in public goods and if each made contributions to a public good that he liked but which was of little interest to the other, the subsidized philosopher is likely to be better off. It has sometimes been suggested that in the interest of progressive taxation, "tax credits" for charitable contributions should be substituted for "tax deductions." The answers to this puzzle suggest that it might be more effective to pay attention to whether the "charities" for which tax deductions are allowed are public goods or simply disguised private consumption.

Answers

Answers to Puzzle 1.

Part 1. Disentangling love from spaghetti. Substitute from Juliet's hedonimeter equation into Romeo's to get: $U_R = \sqrt{S_R} + a\sqrt{S_J} + abU_r$. This equation holds if and only if

$$U_R = \frac{a}{1-ab}\sqrt{S_R} + \frac{a}{1-ab}\sqrt{S_J}.$$

So Romeo's preferences over allocations will be consistent with the readings on the two lovers' hedonimeters if and only if these preferences are represented by the utility function:

$$V_R(S_R, S_J) = \frac{1}{1-ab}\sqrt{S_R} + \frac{a}{1-ab}\sqrt{S_J}.$$

Similar reasoning shows that Juliet's utility function over allocations will be consistent with the hedonimeters if and only if

$$V_J(S_J, S_R) = \frac{1}{1-ab}\sqrt{S_J} + \frac{b}{1-ab}\sqrt{S_R}.$$

Therefore,

$$\alpha_R = \frac{1}{1-ab}, \quad \alpha_J = \frac{a}{1-ab}, \quad \beta_J = \frac{1}{1-ab}, \quad \text{and} \quad \beta_R = \frac{b}{1-ab}.$$

It follows that the α 's and β 's are all positive if $ab < 1$ and negative if $ab > 1$.

Part 2. Love on the Edgeworth Line. Romeo's favorite allocation of spaghetti is found by maximizing $V_R(S_R, S_J)$ subject to the budget $S_R + S_J = S$. At this allocation, his marginal rate of substitution between his own spaghetti and Juliet's must be 1. This happens when

$$\frac{\alpha_R\sqrt{S_J}}{\alpha_J\sqrt{S_R}} = 1.$$

Since $\alpha_R/\alpha_J = 1/a$, this implies $S_R/S_J = 1/a^2$. Similar reasoning finds that Juliet's favorite allocation has $S_R/S_J = b^2$. Therefore Romeo's preferred ratio of S_R to S_J exceeds Juliet's if and only if $1/a^2 > b^2$, or equivalently $ab < 1$. Therefore if $ab < 1$, then Romeo's favorite point lies to the right of Juliet's favorite point on the Edgeworth line.

At allocations to the left of Juliet's favorite point on the Edgeworth line, both lovers favor small reallocations of spaghetti from Juliet to Romeo. To the right of Romeo's favorite point, both lovers prefer small reallocations from Romeo to Juliet. The Pareto optimal allocations are the allocations located between their favorite points. At these points, there is "selfish" conflict of interest between the lovers. Each would prefer a reallocation in which he or she got a little more spaghetti at the other's expense.

Part 3. Must true lovers hate spaghetti? If Romeo likes spaghetti, then holding Juliet's spaghetti consumption constant, he would prefer an increase in his own spaghetti consumption. But if Romeo and Juliet love each other intensely, it may be that, holding Juliet's utility constant, he would not want an increase in his own spaghetti. If Juliet's utility is to be held constant when Romeo gets more spaghetti, then she must be given less spaghetti. But Romeo loves Juliet and wants her to have more spaghetti. Indeed in the case we are considering, Romeo's marginal rate of substitution between spaghetti for Juliet and spaghetti for Romeo exceeds Juliet's marginal rate of

substitution between spaghetti for Juliet and spaghetti for Romeo. Therefore, when Romeo gets an extra inch of spaghetti, taking enough spaghetti away from Juliet to leave her no better off than she was before will make Romeo worse off than he was before he got the extra spaghetti.

This means that if Romeo and Juliet both like spaghetti and Romeo's favorite point on the Edgeworth line is to the left of Juliet's, then the partial derivatives of $U_R(S_R, U_J)$ and $U_J(S_J, U_R)$ with respect to S_R and S_J , respectively must be *negative*. Here is an example of hedonimeter readings that imply that both like spaghetti and that Romeo's favorite allocation lies to the left of Juliet's. Let $a > 0$, $b > 0$ and $ab > 1$ and let the hedonimeter readings be as follows.

$$U_R(S_R, U_J) = -\sqrt{S_R} + aU_J(S_J, U_R)$$

$$U_J(S_J, U_R) = -\sqrt{S_J} + bU_R(S_R, U_J).$$

Substituting and rearranging as we did in Part 1, we find utility functions, $V_R(S_R, S_J) = \alpha_R\sqrt{S_R} + \alpha_J\sqrt{S_J}$ and $V_J(S_J, S_R) = \beta_J\sqrt{S_J} + \beta_R\sqrt{S_R}$ where

$$\alpha_R = \frac{-1}{1 - ab}, \quad \alpha_J = \frac{-a}{1 - ab}, \quad \beta_J = \frac{-1}{1 - ab}, \quad \text{and} \quad \beta_R = \frac{-b}{1 - ab}.$$

Since $a > 0$, $b > 0$, and $ab > 1$, the α 's and β 's are all positive, so we are assured that both lovers like spaghetti. But in this case Romeo's favorite point on the Edgeworth line lies to the left of Juliet's.

Part 4. Cardinal knowledge. After she passes through the metal detector, Juliet's hedonimeter reading is described by the function $U_J^*(S_R, U_J) = \frac{1}{10}U_J(S_R, U_J)$. If Romeo is unaware of the recalibration and does not recalibrate his hedonimeter, his hedonimeter reading will be $U_R = \sqrt{S_R} + aU_J^* = \sqrt{S_R} + \frac{1}{10}aU_J$. Solving this system to express Romeo's utility as a function of his spaghetti and Juliet's, we find that for any given allocation of spaghetti, Romeo's marginal rate of substitution between spaghetti for himself and spaghetti for Juliet is only one tenth as large as it was before the recalibration. If Romeo is to have the same preferences over allocations as he did before, he should recalibrate his hedonimeter so that its reading is described by the equation $U_R^* = \sqrt{S_R} + 10aU_J^*$.

Generally, suppose that Romeo's and Juliet's hedonimeter readings are determined by the functions $U_R(S_R, U_J)$ and $U_J(S_J, U_R)$. Let F and G be any two strictly monotonically increasing functions and let F^{-1} and G^{-1} be their inverses. Then the functions $U_R^*(S_R, U_J^*) = G(U_R(S_R, F^{-1}(U_J^*)))$ and $U_J^*(S_J, U_R^*) = F(U_J(S_J, G^{-1}(U_R^*)))$ determine exactly the same preferences over allocations as the original functions $U_R(S_R, U_J)$ and $U_J(S_J, U_R)$. To see this, substitute the equation for U_J^* into the equation for U_R^* and vice versa. The new set of hedonimeter readings are obtained from the old ones by rescaling Romeo's and Juliet's hedonimeters respectively with the possibly nonlinear, but monotone increasing functions G and F and by making the corresponding adjustment to each person's hedonimeter to take into account the rescaling of the other person's utility.

From this example one sees that the “cardinality” of an individual’s utility matters in the sense that if you change the units of measurement for Juliet, you must make corresponding adjustments in the way that Romeo evaluates her utility. On the other hand, even though each person makes interpersonal comparisons of utility, the theory is “ordinalist” in the sense that the same preferences over allocations can be represented by different systems of hedonimeter readings which are nonlinearly related to each other.

Part 5. Of love and feedback. This is a routine exercise with difference equations. See any standard text.

Answers to Puzzle 2.

Part 1. Is cheap virtue advantageous? No, Plato is better off if Aristotle gets the subsidy. By assumption, the amount of public good supplied is not zero, so one or both of the philosophers must be making positive contributions. There are three cases to consider.

Case 1. Aristotle donates a positive amount of public good and Plato donates none. Since they both consume the same amount of public good and have the same preferences, the one who consumes the greater amount of private good in equilibrium will be better off. Since both have the same income originally and both pay the same lump sum tax, Plato’s private consumption must exceed Aristotle’s by the amount of Aristotle’s net contribution.

Case 2. Both persons donate positive amounts of public good in equilibrium. Since both are contributing, Aristotle’s marginal rate of substitution between public goods and private goods will be $1 - s$ and Plato’s marginal rate of substitution between public and private goods will be 1. But since both are consuming the same amount of public good and since we have assumed diminishing marginal rate of substitution, the only way that this can happen is that Plato has more private goods than Aristotle.

Case 3. Plato donates a positive amount of public good and Aristotle donates none. This is impossible in equilibrium. If this happened, Plato’s marginal rate of substitution between public and private goods would have to exceed Aristotle’s, while Aristotle would have more private goods and the same amount of public goods as Plato. This is inconsistent with diminishing marginal rates of substitution.

In both of the possible cases, the philosopher who does not get the subsidy is better off than the one who does. This seems crazy, because we know from consumer theory that it is always better to face lower prices than higher prices. The reason for the surprise is that each philosopher’s welfare is affected not only by his income and the prices that he pays, but also by the size of the other philosopher’s donation. Here the *opportunity cost of virtue* is lower for Aristotle than for Plato. But on reflection, one realizes that it often may be that, taking into account the full equilibrium effects, a person may prefer to have the opportunity cost of virtue lowered for his neighbors rather than for himself.²

²One is reminded of the long cold day that the little Dutch boy spent with his finger in the dike. He was probably no more virtuous than his fellow townspeople; he simply had the misfortune to be hanging around at a time when the opportunity cost of virtue was much lower for him than for anybody else.

Part 2. Morality and subsidized virtue. Again the answer is no. Let Y_i be philosopher i 's net voluntary contribution. Let the utility function of philosopher i be $U(X_i, Y_i, Y)$ where X_i is i 's private consumption, Y_i is his net donation to the public good and Y is the amount of public good supplied. If Aristotle is subsidized at the rate s for his contributions to the public good, then the amount of public good supplied is $Y = (1/1 - s)Y_A + Y_P$. After his lump sum taxes are paid, each philosopher has an income W to divide between private consumption and net contributions to the public good. Recall that we set up the problem so that the variable Y_i in i 's utility function is i 's net contribution. Therefore both philosophers face the same budget constraint, $X_i + Y_i = W$. The difference caused by the differential subsidy is that Aristotle's contributions have a greater influence on the supply of public goods than do Plato's. The same reasoning used for Part 1 shows that in equilibrium, Aristotle will make a larger net contribution and hence consume less private goods than Plato. As before, both consume the same public goods.

If the subsidy were switched from Aristotle to Plato, then since the two philosophers have the same tastes and income, the new equilibrium would have the same amount of public goods as before. But in the original Nash equilibrium, Plato rejected the opportunity to make a net contribution as large as Aristotle's. This means that he preferred the original Nash equilibrium to a situation where his net contribution was as large as Aristotle's and the amount of public good was larger than the original Nash equilibrium quantity. Therefore he must prefer the original equilibrium to a situation in which his net contribution is as large as Aristotle's original contribution and the quantity of public good is the same as in the original equilibrium. It follows that Plato would be worse off if he got the subsidy than if Aristotle got it.

Part 3. Subsidized virtue and progressive taxation. If giving Aristotle the higher subsidy rate does not decrease the equilibrium quantity of public goods, then apply the same kind of argument used previously to show that Plato will prefer that Aristotle receive the higher subsidy rate.

References

- Bergstrom, Ted**, "Systems of Benevolent Utility Interdependence," University of Michigan working paper, 1988.
- Bergstrom, Ted**, "Interrelated Consumer Preference and Voluntary Exchange." In Zarley, A., *Papers in Quantitative Economics*, Lawrence: Kansas University Press, 1971, pp. 79–94.
- Bernheim, B. Douglas, and Oded Stark**, "Altruism within the Family Reconsidered: Do Nice Guys Finish Last?" *American Economic Review*, December 1988, 78, 1034–1045.
- Harsanyi, John C.**, "Interpersonal Utility Comparisons." In *The New Palgrave*, Vol. 3. New York: The Stockton, 1987, pp. 955–958.
- Kimball, Miles**, "Making Sense of Two-Sided Altruism," *Journal of Monetary Economics*, 1987, 20, 301–326.
- Musgrave, Richard, and Peggy B. Musgrave**, *Public Finance in Theory and Practice*, New York: McGraw-Hill, 1980.
- Roberts, Russell**, "Financing Public Goods," *Journal of Political Economy*, 1987, 95, 420–437.