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Janus and RG-flow interfaces in gauged supergravity

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Janus and RG-flow interfaces in gauged supergravity

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Doctor of Philosophy in Physics

by

Charlie Hultgreen-Mena

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ABSTRACT OF THE DISSERTATION

Janus and RG-flow interfaces in gauged supergravity

by

Charlie Hultgreen-Mena
Doctor of Philosophy in Physics
University of California, Los Angeles, 2025
Professor Michael Gutperle, Chair

In this dissertation, we construct Janus-type solutions of three-dimensional gauged supergravity. We find solutions that correspond to RG-flow interfaces between CFTs with different central charges by solving the BPS flow equations or the equations of motion. Quantities such as transmission coefficients and holographic quantities such as the entanglement entropy are calculated, as well as plots of the RG-flows to determine whether some supersymmetry is preserved.

In chapter 1, we review the AdS/CFT correspondence, supergravity and an introduction to transmission in conformal interfaces. In chapter 2, we construct Janus-type solutions of three-dimensional gauged supergravity with sixteen supersymmetries, as well as the RG-flow interfaces. In chapter 3, we generalize the previous work with a general α for the embedding tensor of the gauged supergravity. In chapter 4, we discuss the Janus and RG-flow solutions of $N = 2$, d = 3 gauged supergravity.

The dissertation of Charlie Hultgreen-Mena is approved.

Per Kraus

Eric D'Hoker

Zvi Bern

Michael Gutperle, Committee Chair

University of California, Los Angeles

2025

To my mom . . .

who has been there unconditionally

giving her best since the day I was born

Gracias por todo mami.

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CONTRIBUTION OF AUTHORS

Chapter 2 is based on [1] in collaboration with Kevin Chen and Michael Gutperle. Chapter 3 is based on [2] in collaboration with Michael Gutperle. Chapter 4 is based on [3] in collaboration with Michael Gutperle.

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PUBLICATIONS

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CHAPTER 1

Introduction

The construction of a quantum theory of gravity and descriptions of strongly-coupled gauge theories have been among the greatest challenges of modern theoretical physics. Treating gravity as a classical field theory and directly quantizing gives a non-renormalizable theory. The development of string theory as a framework and the discovery of the dualities might be the path to success through gravity/gauge holographic duality. In one of the earliest examples of weakly coupled string theory/gauge field theory, 't Hooft studied [4] the case of large N limit gauge theories where the gauge theory simplifies and matches the perturbation theory of the Feynman diagrams, which are organized in graphs in terms of a genus expansion, with the perturbation theory of a weakly coupled string theory hinting at a gauge/gravity duality.

Treating black holes as quantum systems revealed a proportionality between the entropy and the area of the horizon [5] contrasting the typical relation where for local quantum field theories the entropy is an extensive variable scaling with the volume. The implications of this discovery led to the holographic principle according to which the states of any quantum gravity theory are contained in a theory without gravity defined at the boundary of the space [6–8]. An independent result by Brown and Henneaux studying the asymptotic symmetry group of 3D Anti-de Sitter space (AdS) leading to a 2D Virasoro algebra which is the symmetry group of 2-dimensional Conformal Field Theory (CFT), [9] suggested that theories of gravity with AdS asymptotic are connected with lower dimensional conformal field theories.

The discovery of D-branes [10–12] in string theory and the study of open strings on them raised the possibility that lower-dimensional gauge theories living in the near-horizon limit could be the holographic dual of the gravity theory. In 1997, Maldacena's conjecture [13] shows an equivalence between certain theories of closed strings in AdS spaces and conformally invariant gauge theories in fewer dimensions. These gauge theories were proved to live at the AdS boundary allowing us to compute string theory observables in the bulk from the boundary theory [14, 15].

In this dissertation, chapter 1 reviews the necessary background material of the AdS/CFT correspondence and supergravity and the basics of interface conformal field theories (ICFTs), as well as some examples and applications of these theories. Chapter 2 is based on [1], where we construct Janus-type solutions of three-dimensional gauged supergravity with sixteen supersymmetries, as well as the RG-flow interfaces between CFTs with different numbers of supersymmetries and central charges. Chapter 3 is based on [2], where we continue the work with general α for the embedding tensor of the gauged supergravity, as well as the computation of holographic quantities such as the masses for the fluctuations of the scalar fields around different vacua of the theory, as well as the entanglement entropy around the defect for both sides of the interface. Chapter 4 is based on [3], where we find solutions of minimal $d = 3, N = 2$ gauged supergravity corresponding to Janus and RG-flow interfaces and use holography to calculate symmetric and interface entanglement entropy as well as reflection coefficients and bounds between these quantities.

1.1 AdS/CFT Correspondence

The AdS/CFT correspondence states duality between certain theories including gravity in (d+1)-dimensional asymptotically anti-de Sitter spaces and, conformal field theories (CFT) that live on their d-dimensional boundaries. In almost all cases, the two sides of the correspondence are related by a strong/weak coupling duality making the comparison on both sides difficult. Still, there are a few cases where the comparison can be performed like BPS states, which are protected from quantum corrections by supersymmetry, and can therefore be continued from strong to weak coupling. For a selection of reviews and lecture notes on the AdS/CFT correspondence, see [16–19].

1.1.1 Conformal Field Theory

In a quantum theory, conformal invariance is broken by introducing a renormalization scale from the regularization of ultraviolet divergences (UV) due to loops in perturbation theory. The conserved current from scale transformations relates the trace of the stress-energy tensor to the beta functions of the theory. Therefore, scale-invariant quantum field theories must have vanishing beta functions and they are important as possible fixed points of renormalization group flows. New quantum field theories can be obtained from relevant deformations along the RG flows from the UV to the Infrared (IR) of these CFTs. An introduction to CFT is given by [20].

The conformal group is the group of transformations that preserve the form of the metric up to an arbitrary scale factor, $g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x)$ under coordinate transformation. It includes the Poincaré group as well as dilation transformation $x \to ax$ and inversion transformation $x^\mu \to x^\mu / x^2$.

In Minkowski space, an infinitesimal coordinate transformation ($x_\mu \to x_\mu + \varepsilon_\mu$) to the metric conformal transformation gives us the conformal killing equation $\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu = \frac{2}{d} \partial \cdot \varepsilon \eta_{\mu\nu}$. For $d > 2$, the most general solution is

$$x^\mu \to x^\mu + a^\mu + \omega^{\mu\nu} x_\nu + \lambda x^\mu + x^2 b^\mu - 2(x \cdot b) x^\mu, \tag{1.1}$$

Using the infinitesimal conformal transformations and the well-known structure of the Poincare group on the spacetime coordinates, one can derive the conformal algebra of the group

$$
\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= i\left(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\sigma} M_{\mu\rho}\right), \\
[M_{\mu\nu}, P_{\rho}] &= i\left(\eta_{\nu\rho} P_{\mu} - \eta_{\mu\rho} P_{\nu}\right), \\
[M_{\mu\nu}, K_{\rho}] &= i\left(\eta_{\nu\rho} K_{\mu} - \eta_{\mu\rho} K_{\nu}\right), \\
[K_{\mu}, P_{\nu}] &= 2i\left(\eta_{\mu\nu} D - M_{\mu\nu}\right), \\
[D, P_{\mu}] &= iP_{\mu}, \quad [D, K_{\mu}] = -iK_{\mu},
\end{aligned} \tag{1.2}
$$

where D denotes the generator for dilations, P_μ for translations, $M_{\mu\nu}$ for Lorentz transformations, and K_μ for special conformal transformations. This algebra is isomorphic to the algebra of $SO(d,2)$ in the $\eta = (-,+,+,...,+,-)$ signature under transformations.

$$
J_{\mu\nu} = M_{\mu\nu}; \quad J_{\mu d} = \frac{1}{2}\left(K_\mu - P_\mu\right); \quad J_{\mu(d+1)} = \frac{1}{2}\left(K_\mu + P_\mu\right); \quad J_{(d+1)d} = D \tag{1.3}
$$

For the Poincare group in quantum field theory, we classify the representations of the group via the little group $SO(d) \times SO(2)$ where $SO(d)$ gives the typical spin of the field. The $SO(2)$ charge is the scaling dimension of the field that transforms as $\phi(x) \to \phi'(x) = \lambda^{\Delta}\phi(\lambda x)$. The algebra of the group imply that the operator P_μ raises the dimension of the field, while the operator K_μ lowers it. Unitarity of the theory gives a lower bound on the dimension of fields (for scalar fields it is $\Delta \geq (d-2)/2$ [21]). Therefore each representation of the conformal group must have some operator of lowest conformal dimension, which must then be annihilated by K_μ at the origin. Such operators are called primary operators. Then all the local operators can be found via P_μ acting on the primaries.

The conformal symmetry group strongly constraints the n-point functions of local operators [22]. Via conformal transformations any three points can be mapped to any other, fixing the form of 2 and 3-point functions up to constant factors. Via scaling and Poincare invariance, the 2,3 and 4-point functions of primary scalar fields can be written

$$\langle \mathcal{O}_{\Delta_1}(x_1)\, \mathcal{O}_{\Delta_2}(x_2)\rangle = \delta_{1,2} \prod_{i<j}^{2} |x_{ij}|^{-\Delta} ,$$

$$\langle \mathcal{O}_{\Delta_1}(x_1)\, \mathcal{O}_{\Delta_2}(x_2)\, \mathcal{O}_{\Delta_3}(x_3)\rangle = c_{123} \prod_{i<j}^{3} |x_{ij}|^{\Delta-2\Delta_i-2\Delta_j} , \tag{1.4}$$

$$\langle \mathcal{O}_{\Delta_1}(x_1)\, \mathcal{O}_{\Delta_2}(x_2)\, \mathcal{O}_{\Delta_3}(x_3)\, \mathcal{O}_{\Delta_4}(x_4)\rangle = c_{1234}(u,v) \prod_{i<j}^{4} |x_{ij}|^{\frac{\Delta}{3}-\Delta_i-\Delta_j} .$$

Where $x_{ij} \equiv x_i - x_j$ and $\Delta \equiv \sum_i \Delta_i$ and the functions $x_{1234}(u,v)$ is a function of the two conformally invariant cross-ratios

$$u \equiv \frac{|x_{12}|\,|x_{34}|}{|x_{13}|\,|x_{24}|}, \quad v \equiv \frac{|x_{14}|\,|x_{23}|}{|x_{13}|\,|x_{24}|}. \tag{1.5}$$

The state-operator correspondence in a CFT follows from the scale invariance. States are defined in different time slices (represented by spheres centered at the origin). As one scales down the sphere to the origin, the effect of the state is equivalent to a local operator centered at the origin. This correspondence implies that the product of any two conformal primaries can be rewritten as a linear combination of conformal operators inserted at a nearby point (no other insertions in between the two primaries). This property of CFTs is called Operator Product Expansion (OPE), and for two scalar operators it can be written as

$$\mathcal{O}_{\Delta_i}(x)\mathcal{O}_{\Delta_j}(0) = \sum_k c_{ijk}|x|^{-\Delta_i-\Delta_j+\Delta_k} \left(\mathcal{O}_{\Delta_k}(0) + \text{ descendants }\right) \tag{1.6}$$

where the coefficients of each dependant are determined by conformal invariance. Any n-point function can be reduced to an infinite sum of (n-1)-point functions. Thus, the CFT is complete once the conformal dimension, the irreducible representation and the c_{ijk} of the 3-point function for each primary are determined. These coefficients are not random, since the 4-point function can be expressed in different channels leading to algebraic equations that constrain their possible values. Using this infinite set of crossing equations to identify possible conformal field theories is the basis of the conformal bootstrap program [23, 24].

1.1.2 Anti-de Sitter space

Anti-de Sitter spacetime [25, 26] is a solution to Einstein's field equations with negative cosmological constant that exhibits maximal isometries. It appears in string theory and in supergravity theories as vacua of gauged supergravity theories and Kaluza Klein reductions (KK reductions) of gravity theories [27].

AdS_{d+1} may be embedded into a d-dimensional hyperboloid hypersurface with spacetime $\left(X^0, X^1, \ldots, X^d, X^{d+1}\right) \in \mathbb{R}^{d,2}$, and metric $\bar{\eta} = \mathrm{diag}(-,+,+,\ldots,+,-)$,

$$\bar{\eta}_{MN} X^M X^N = -\left(X^0\right)^2 + \sum_{i=1}^{d} \left(X^i\right)^2 - \left(X^{d+1}\right)^2 = -L^2 \tag{1.7}$$

inside $\mathbb{R}^{d,2}$. L is the radius of curvature of the Anti-de Sitter space and the hypersurface is invariant under $SO(d,2)$ transformations acting on $\mathbb{R}^{d,2}$. $SO(d,2)$ has $(d+1)(d+2)/2$ generators, the same number of Killing generators in $(d+1)$ Minkowski spacetime. Therefore, as mentioned AdS space is also maximally symmetric.

There are different useful representations of the induced metric. Using the parametrization of the hypersurface

$$\begin{aligned} X^0 &= L \cosh\rho \cos\tau \\ X^{d+1} &= L \cosh\rho \sin\tau \\ X^i &= L\Omega_i \sinh\rho, \qquad \text{for } i = 1, \ldots, d \end{aligned} \tag{1.8}$$

The induced metric for the line element is

$$ds^2 = L^2 \left(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_{d-2}^2\right) \tag{1.9}$$

where Ω_i parametrize a sphere S^{d-1}. The remaining coordinates take the ranges $\rho \in \mathbb{R}_+$and $\tau \in [0, 2\pi[$. These coordinates are global coordinates of AdS_{d+1} since all points of the hypersurface are covered exactly once. τ being a periodic value brings closed time-like curves for fixed Ω_i and ρ. Unwrapping the time circle in which $-\infty < \tau < +\infty$ fixes those issues. The conformal boundary of these coordinates occurs as $\rho \to \infty$ where the induced metric is a cylinder $\mathbb{R} \times S^{d-1}$.

Another useful set of coordinates describing AdS_{d+1} is the Poincaré patch defined by

$$\begin{aligned} X^0 &= \frac{L}{z} x^0, & X^j &= \frac{L}{z} x^j \\ X^d &= \frac{z}{2}\left(\frac{L^2 - x^2}{z^2} - 1\right), & X^{d+1} &= \frac{z}{2}\left(\frac{L^2 + x^2}{z^2} + 1\right), \end{aligned} \tag{1.10}$$

where $x^2 = -\left(x^0\right)^2 + \sum_{j=1}^{d-1}\left(x^j\right)^2$ and the coordinate range of z is restricted to $0 < z < \infty$. The corresponding metric is conformal to half of flat Minkowski spacetime and takes the form

$$ds^2 = \frac{L^2}{z^2}\left[dz^2 - \left(dx^0\right)^2 + \left(dx^1\right)^2 + \ldots \left(dx^{d-1}\right)^2\right] \tag{1.11}$$

making the d-dimensional Poincaré symmetry of the coordinates manifest. The metric furthermore makes the scaling symmetry $z \to \lambda z$, $x^0 \to \lambda x^0$, $x^j \to \lambda x^j$ explicit.

Due to the state-operator correspondence and the invariance of the vacuum, the CFT vacuum corresponds to pure AdS. In order to get a correspondence with other CFT states, spacetimes whose metrics approach that of AdS near their boundaries must be analyzed. Solutions like black holes and p-branes can present an AdS asymptotical behavior. An example is Reissner–Nordström black hole in 4D

$$\mathrm{d}s^2 = -F(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2 \tag{1.12}$$

$$F(r) = 1 - \frac{2MG}{r} + \frac{q^2 G}{4\pi r^2}. \tag{1.13}$$

For the extremal case $q^2 = 4\pi M^2 G$, the boundary is located at $r = MG$. Introducing the change of coordinates $v = r - MG$ and taking the limit $v \to \infty$, we get the metric

$$\mathrm{d}s^2 \approx -\frac{v^2}{(MG)^2}\,\mathrm{d}t^2 + (MG)^2\frac{\mathrm{d}v^2}{v^2} + (MG)^2\,\mathrm{d}\Omega_2^2. \tag{1.14}$$

which has the AdS form in global coordinates where both the AdS scale and the radius of the sphere are equal to MG.

1.1.3 Holography

The AdS/CFT correspondence arises from matching the global symmetries on both sides. The AdS/CFT correspondence provides a mapping between fields in the bulk near the boundary and operators in the dual CFT. To illustrate this, consider a free massive scalar in AdS_{d+1} in Poincare patch coordinates ($ds^2 = \frac{1}{z^2}\left(dz^2 + dx_1^2 + \cdots + dx_d^2\right)$)

$$S_{\text{bulk}}\left[\phi\right] = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}m^2\phi^2\right), \tag{1.15}$$

Ignoring the backreaction and solving the KG equation, we find two linearly independent solutions. The boundary of AdS is at z $\to$ 0. Thus at leading order in z

$$\phi(z,x) \sim \phi_{(0)}(x)z^{\Delta_-} + \phi_{(+)}(x)z^{\Delta_+} + \cdots, \tag{1.16}$$

where $\Delta_\pm$ are the roots of $m^2L^2 = \Delta(\Delta - d)$ given by

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2L^2}. \tag{1.17}$$

For solutions to be real, the mass is bounded by $m^2 \geq -d^2/4$, which agrees with the unitarity bound [28, 29]. We can evaluate the on-shell action and realize that the source term (ϕ_0) is a non-normalizable term. We identify Δ_+ as the scaling dimension of the dual operator and notice that the term $\int \mathrm{d}^d x\phi_0(x)\mathcal{O}_\Delta(x)$ is scale invariant.

The boundary values of the fields are identified with sources that couple to the dual operator, and the on-shell bulk partition function with the generating functional of QFT correlation functions [14, 15]

$$Z_{\text{SUGRA}}\ \left[\phi_{(0)}\right]=\int_{\Phi\sim\phi_{(0)}} D\Phi\exp(-S[\Phi])=\left\langle\exp\left(-\int_{\partial AAdS}\phi_{(0)}O\right)\right\rangle_{QFT} \tag{1.18}$$

where $\phi_{(0)}$ is the boundary value of the field in the AdS side and a source to the field operator O. Correlation functions of the operator are now computed by functional differentiation with respect to the source

$$\langle O\left(x_1\right)\cdots O\left(x_n\right)\rangle=(-1)^{n+1}\left.\frac{\delta^n S_{\text{onshell}}}{\delta\phi_{(0)}\left(x_1\right)\cdots\delta\phi_{(0)}\left(x_n\right)}\right|_{\phi_{(0)}=0} \tag{1.19}$$

In general, the on-shell supergravity action diverges due to integration over an infinite AdS space (IR divergence). The long-distance (IR) is the same as near the boundary, and a good coordinate system to work with is the Fefferman-Graham coordinates (FG) with a cutoff at $z=\varepsilon$. In the field theory side, we have UV divergences from the FG cutoff. A tool to control the divergences on the gravity side is holographic renormalization [30,31]. The solutions will be valid near the boundary. Expanding around it

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{z^2}\left(\,\mathrm{d}z^2+g_{ij}(x,z)\mathrm{d}x^i\ \mathrm{d}x^j\right),\\
g_{ij}(x,z) &= g_{(0)ij}(x)+zg_{(1)ij}(x)+\cdots+z^d g_{(d)ij}(x)+z^d\log z\tilde{g}_{(d)ij}(x)+\cdots\\
\mathcal{F}(x,z) &= z^{\Delta_-}\left(f_{(0)}(x)+zf_{(1)}(x)+\cdots+z^{\Delta_+-\Delta_-}f_{\Delta_+}(x)+z^{\Delta_+-\Delta_-}\log z\tilde{f}_{\Delta_+}(x)+\cdots\right),
\end{aligned} \tag{1.20}$$

Using Einstein equations and the field equations, we can group terms order by order in z, solve iteratively and all terms except $g_{(d)ij}$ and f_{Δ_+} can be expressed as local functions of the leading terms $g_{(0)ij}$ and $f_{(0)}$. This is expected as the equation of motion for the scalar field requires two linearly independent solutions. These values will be associated with the expectation value of the corresponding CFT operator (stress tensor and scalar operator) in the presence of the sources.

To regularize the on-shell action, we restrict the range of the z integration $z \geq \varepsilon$ for some small constant $\varepsilon > 0$. Since the equation of motion is satisfied, the bulk contribution to the action vanishes and only the boundary terms at $z = \varepsilon$ survive. This gives a regularized action with a finite number of divergences of the form

$$S_{\text{reg}}\left[g_{(0)}, f_{(0)}, \varepsilon\right] = \int \mathrm{d}^d x \sqrt{g_{(0)}} \left[a_0 \varepsilon^{-\nu} + a_2 \varepsilon^{(-\nu+1)} + \ldots + a_\nu \ln \varepsilon + \mathcal{O}\left(\varepsilon^0\right)\right] \tag{1.21}$$

All coefficients a_ν are local functions of the sources $g_{(0)ij}$ and $f_{(0)}$. The divergences can be canceled out by adding local counterterms to the action. These are expressed in terms of the fields living on the regulating hypersurface for the subtraction to be covariant. Then, we have to invert the asymptotic expansion of the fields to find $f_{(0)} = f_{(0)}(\mathcal{F}(x,\varepsilon), g_{ij}(x,\varepsilon), \varepsilon)$ and $g_{(0)} = g_{(0)}(\mathcal{F}(x,\varepsilon), g_{ij}(x,\varepsilon), \varepsilon)$. The counterterm action is then defined as

$$S_{\text{ct}}[\mathcal{F}(x,\varepsilon), g_{ij}(x,\varepsilon); \varepsilon] = -\text{ divergent terms of } S_{\text{reg}}\ \left[f_{(0)}, g_{(0)}; \varepsilon\right] \tag{1.22}$$

We then define the normalized action as

$$S_{\text{ren}}\left[g_{(0)}, \phi_{(0)}\right] = \lim_{\varepsilon \to 0}\left(S_{\text{on-shell}} + S_{\text{ct}}\right). \tag{1.23}$$

which is finite and can be used to compute the correlation functions. For the one-point function

$$\begin{aligned} \langle T_{ij}(x)\rangle_s &\equiv \frac{2}{\sqrt{g_0}} \frac{\delta S_{\text{ren}}}{\delta g_{(j)}^{ij}(x)} \sim g_{(d)ij}(x) + C_{ij}\left(g_{(0)}, f_{(0)}\right), \\ \langle \mathcal{O}_\Delta(x)\rangle_s &\equiv \frac{1}{\sqrt{g_0}} \frac{\delta S_{\text{ren}}}{\delta \phi_{(0)}(x)} \sim f_{\Delta+}(x) + C\left(g_{(0)}, f_{(0)}\right). \end{aligned} \tag{1.24}$$

where the C's are local functions of the sources. Thus, an n-point function information is encoded in $f_{\Delta+}$ and $g_{(d)ij}$, which can be found from the bulk equation.

1.1.4 Maldacena limit

In type IIB string theory, the open string and closed string perspective applied to a stack of N D3-branes in flat spacetime in the low energy limit implies a correspondence between $\mathcal{N}=4$ super-Yang-Mills theory [32] in 4-dimensions and supergravity solutions on $AdS_5 \times S^5$.

For the open string perspective in a background of (9+1)D Minkowski and a stack of N coincident D3-branes, open strings are not stretched and can have arbitrary short lengths. Thus, in the low energy limit, the strings are massless. The massless effective action contains the closed string modes, open string modes and the interactions between them. The action of the last two can be expressed from the Dirac–Born–Infeld action and the Wess–Zumino term. Expanding around the metric fluctuations and to first order in α', we find

$$\begin{aligned}
\mathcal{S}_{\text{closed}} &= \frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{-g}e^{-2\phi}\left(R+4\partial_M\phi\partial^M\phi\right)+\cdots \\
&\sim -\frac{1}{2}\int \mathrm{d}^{10}x\partial_M h\partial^M h+\mathcal{O}(\kappa), \\
\mathcal{S}_{\text{open}} &= -\frac{1}{2\pi g_{\text{s}}}\int \mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i+\mathcal{O}\left(\alpha'\right)\right) \\
\mathcal{S}_{\text{int}} &= -\frac{1}{8\pi g_{\text{s}}}\int \mathrm{d}^4x\kappa\phi F_{\mu\nu}F^{\mu\nu}+\cdots
\end{aligned} \tag{1.25}$$

where h is the metric fluctuation $g=\eta+\kappa h$, g_s is the string coupling, $2\kappa^2=(2\pi)^7\alpha'^4 g_{\text{s}}^2$ and $(2\pi\alpha')^{-1}$ is the string tension.

In the Maldacena limit ($\alpha' \to 0$ while keeping N and g_s fixed), the interaction term can be neglected and the open and closed strings decouple. In this limit, $\mathcal{S}_{\text{open}}$ is the bosonic part of $\mathcal{N}=4$ Super Yang-Mills theory with the identification $2\pi g_{\text{s}}=g_{\text{YM}}^2$, while $\mathcal{S}_{\text{closed}}$ is approximated by Type IIB supergravity.

In the closed string perspective, we have the supergravity solution of the metric of N coincident D3 branes to be

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-\frac{1}{2}} \eta_{ij} dx^i dx^j + \left(1 + \frac{L^4}{r^4}\right)^{\frac{1}{2}} \left(dr^2 + r^2 d\Omega_5^2\right) \tag{1.26}$$

where L is found by calculating the charge via integration of the R-R flux of the D3-brane. Setting the charge to N, $L^4 = 4\pi g_s N \alpha'^2$ is found. For large L, the metric reduces to 10D supergravity in flat space. In the near horizon limit ($r \ll L$) and using the coordinate $u \equiv L^2/r$, the metric becomes

$$ds^2 = L^2 \left[\frac{1}{u^2}\eta_{ij} dx^i dx^j + \frac{du^2}{u^2} + d\Omega_5^2\right] \tag{1.27}$$

which has the $AdS_5 \times S^5$ form. $E_\infty = \sqrt{-g_{00}} E_r = H(r)^{-1/4} E_r$ implies that string excitations coming from the near horizon decouple from the flat supergravity for an observer at infinity.

Both perspectives should be physically equivalent, suggesting that four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory is equivalent to type IIB string theory on $\mathrm{AdS}_5 \times S^5$. Furthermore, the global symmetries of both theories match. $\mathcal{N} = 4$ SYM has a vanishing beta function and is hence conformal. The supergroup is $\mathrm{PSU}(2,2 \mid 4)$ where the bosonic subgroup is given by $SU(2,2) \sim SO(4,2)$ (which is the symmetry group of AdS_5) and the $\mathrm{SU}(4) \cong \mathrm{SO}(6)$ R-symmetry of $\mathcal{N} = 4$ SYM (symmetry group of S^5). This matches the isometries of $AdS_5 \times S^5$. The field-operator map between the theories can be found via KK reduction on the supergravity side and matching the representations on both sides [33, 34].

Classic supergravity appears as the limit of string theory when the string length is much smaller than the curvature radius of AdS L. $L^4 = 4\pi g_s N \alpha'^2$, which means $g_s N$ is large. Since $g_s N \sim g_{\mathrm{Y}M}^2 N$, this limit corresponds to large 't Hooft coupling in the field, and we obtain a strong/weak duality theory which allows us to study strong coupling dynamics of one theory from the weak coupling perturbative behaviour of the other.

1.2 Supergravity

If the symmetry algebra of a theory allows fermionic generators with anti-commutation relations, the Poincare symmetry extends to the supersymmetry algebra [35], where the new fermionic generators Q^I obey

$$\begin{aligned} \left\{Q^I, \bar{Q}^J\right\} &\sim \gamma^M P_M \delta^{IJ}, \quad \left\{Q^I, Q^J\right\} \sim Z^{IJ}, \\ \left[L_{MN}, Q^I\right] &\sim \Sigma_{MN} Q^I, \quad \left[P_M, Q^I\right] = 0, \qquad I, J = 1, 2..., \mathcal{N} \end{aligned} \tag{1.28}$$

where the generators Z^{IJ} are central charges which are antisymmetric and commute with all the other generators of the algebra, and γ^M and Σ_{MN} are the Dirac matrices and the generator of the spinor representation in $SO(1, D-1)$ obeying the algebra

$$\begin{aligned} \left\{\gamma^M, \gamma^N\right\} &= 2\eta^{MN}, \quad M, N = 0, \ldots, D-1. \\ \Sigma^{MN} &:= \frac{1}{4}\left[\gamma^M, \gamma^N\right] \end{aligned} \tag{1.29}$$

To get the representation of the algebra, we make a Lorentz boost to make the little group $SO(D-1)/SO(D-2)$ for the massless/massive case explicit. Writing the algebra, the generators Q^I can be separated into Q^+ and Q^- that obey the algebra of creation and annihilation operators for the massless case, and a BPS-bound algebra for the massive case. This implies that a supersymmetric multiplet can be constructed starting from a vacuum state annihilated by the Q^- and the multiplet is generated by the action of the supercharges Q^+ acting on the vacuum.

Supergravity is a supersymmetric model which is covariant under general coordinate transformations. This is equivalent to having local supersymmetry transformations. For gravity, the fundamental particle is the massless spin-2 graviton. Thus, to obtain local SUSY we need a massless spin-3/2 superpartner called the gravitino, which is the gauge field of the theory.

For $D > 4$ interacting massless fields of $S > 2$ have a trivial S-matrix [36]. Thus, for a physical interacting theory, we can only have 8 raising operators. The number of raising operators of a theory is determined by the number of supercharges and the dimension of the spinor representation. This gives us an upper bound for the amount of supercharges (32 maximum) and the dimension ($D \leq 11$) in which supergravity is possible [37]. Thus, we have pure supergravity where only the graviton and its superpartners are present, and matter-coupling supergravities that contain additional supermultiplets (chiral, vector, graviton, hyper and tensor multiples). For a detailed review of the topic, see for example [18, 38, 39]

1.2.1 Global Supersymmetry

Once a representation is obtained from the supersymmetry algebra, constructing a Lagrangian invariant under supersymmetry transformations is the next step, leading to the superfield formalism [40]. As an example for $\mathcal{N} = 1$, the idea is to enlarge Minkowski space to a superspace $\left(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right)$ where $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ are Grassman coordinates associated with the supersymmetry generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$. The most general superfield can be written as

$$\begin{aligned} S\left(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right) =& \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta M(x) + \bar{\theta}\bar{\theta}N(x) + \left(\theta\sigma^\mu\bar{\theta}\right) V_\mu(x) \\ &+ (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\rho(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x) \end{aligned} \tag{1.30}$$

Analogous to the transformation of a field under translations in Minkowski space, supersymmetry transformations of the superfield are realized linearly via the Qs operators.

$$\delta_\varepsilon S = (\varepsilon Q + \bar{\varepsilon}\bar{Q})S \tag{1.31}$$

where ε is a constant spinor and the differential operator Q can be written as

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{1.32}$$

The transformation of each field from the superfield is obtained via differentiation. It also follows that the integral

$$\int d^4x d^2\theta d^2\bar{\theta} S(x,\theta,\bar{\theta}) \tag{1.33}$$

is invariant under supersymmetric transformations if S is a superfield. Linear combinations of superfields are superfields, as well as $\partial_\mu S$ and $\mathcal{D}_\alpha S$ where $\mathcal{D}_\alpha$ is the covariant derivative $\mathcal{D}_\alpha := \partial_\alpha + i\left(\sigma^\mu\right)_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}}\partial_\mu$. To ensure supersymmetry, the Lagrangian must be a scalar under supersymmetry transformations. The common terms for the possible superfield are: the kinetic term $\mathcal{L}_{\text{kin}} = S^\dagger S$, interacting terms from the superpotential $\mathcal{L}_{\text{int}} = W(S) + W^\dagger\left(S^\dagger\right)$, and gauge terms for vector superfields $\mathcal{L}_{\text{gauge}} = \frac{1}{4g^2}\int d^2\theta W^\alpha W_\alpha +$ h.c. These last two terms are only integrated over half the superspace. Thus, we can write general Lagrangians through integration

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{A}(x;\theta,\bar{\theta}) = \int d^4x \mathcal{L}\left(\phi(x), \psi(x), A_\mu(x), F_{\mu\nu}\ldots\right) \tag{1.34}$$

where $\mathcal{A}$ is a combination of the kinetic term (integration over the full superspace) and the interaction terms and gauge terms (integrated over half the superspace). A general S is not in an irreducible representation, but imposing supersymmetric invariant constraints on S, a reducible representation can be found, and the explicit supersymmetric Lagrangian can be derived.

1.2.2 $D = 4$, $\mathcal{N} = 1$ pure supergravity

For supergravity to arise, supersymmetry has to be local. The study and cataloguing of lower dimension supergravities have been widely studied [41, 42]. In pure supergravity, we only have the massless graviton e^a_μ and its superpartner, a Majorana spinor gravitino ψ_μ. The e^a_μ is related to the metric via

$$g_{\mu\nu}(x) = e^m_\mu(x)\eta_{mn}e^n_\nu(x) \tag{1.35}$$

where η_{mn} is the Minkwoski metric. The Greek letters denote spacetime indices, while the latin ones denote flat space indices and couple to fermionic fields and the Dirac matrices.

A supersymmetric infinitesimal transformation is linear on the fields, and from the superspace formalism, the transformation changes bosonic into fermionic particles and vice versa. Thus, the general transformation of these fields

$$\delta B(x) = \bar{\varepsilon}(x) f_1(B(x)) F(x) + \mathcal{O}\left(F^3\right), \quad \delta F(x) = f_2(B(x))\varepsilon(x) + \mathcal{O}\left(F^2\right) \tag{1.36}$$

where the functions f_1 and f_2 include dirac matrices and covariant derivatives. For a classical solution, where the fermions vanish, the bosonic variation is trivially satisfied, and the spinor solutions of the fermionic transformations determine the amount of unbroken supersymmetry, giving us shorter multiplets of the theory (BPS multiplets).

The action should contain the free action for both the graviton(the Einstein-Hilbert action), the gravitino (Rarita-Schwinger action) and an interacting term to ensure supersymmetry in the theory. The action [43] is given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho + \mathcal{L}[e, \psi]\right) \tag{1.37}$$

where $D_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{4}\omega_\mu^{ab} \gamma_{ab} \psi_\nu$ and ω_μ^{ab} is the torsion-free spin connection defined by $de^a + \omega^a{}_b \wedge e^b = 0$. The supersymmetry transformations are

$$\begin{aligned} \delta e_\mu^a &= \frac{1}{2} \bar{\varepsilon} \gamma^a \psi_\mu \\ \delta \psi_\mu &= D_\mu \varepsilon(x) \equiv \partial_\mu \varepsilon + \frac{1}{4} \omega_{\mu ab} \gamma^{ab} \varepsilon \end{aligned} \tag{1.38}$$

The first equation follows from the bosonic transformation only involving fermionic fields. The second follows from the gravitino being the gauge field of the theory. To find the interacting term, we treat the spin connection as an independent variable, the equation of motion for the spin connection can be solved to obtain a connection with torsion. This result can then be substituted in the action to obtain the theory with torsion-free connection and explicit four-fermion interacting terms. This procedure gives

$$de^a + \omega^a{}_b \wedge e^b = 0 = -\frac{i}{2}\bar{\psi}_\mu \gamma^a \psi_\nu \tag{1.39}$$

with solution

$$\begin{aligned} \omega_{\mu ab} &= \omega_{\mu ab}(e) + K_{\mu ab}, \\ K_{\mu\nu\rho} &= -\frac{1}{4}\left(\bar{\psi}_\mu \gamma_\rho \psi_v - \bar{\psi}_\nu \gamma_\mu \psi_\rho + \bar{\psi}_\rho \gamma_\nu \psi_\mu\right) \end{aligned} \tag{1.40}$$

The supergravity action invariant under the supersymmetry transformations then is given by

$$\begin{aligned} S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \Big(R - \bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{1}{16}\left(\bar{\psi}^\rho \gamma^\mu \psi^\nu\right)\left(\bar{\psi}_\rho \gamma_\mu \psi_\nu + 2\bar{\psi}_\rho \gamma_\nu \psi_\mu\right) \\ + \frac{1}{4}\left(\bar{\psi}_\mu \gamma_\nu \psi^\nu\right)\left(\bar{\psi}^\mu \gamma_\rho \psi^\rho\right) \Big) \end{aligned} \tag{1.41}$$

which is invariant under the supersymmetry transformations

$$\begin{aligned} \delta e^a_\mu &= \frac{1}{2}\bar{\varepsilon}\gamma^a \psi_\mu \\ \delta \psi_\mu &= \partial_\mu \varepsilon + \frac{1}{4}\left(\omega^{ab}_\mu + K^{ab}_\mu\right)\gamma_{ab}\varepsilon \end{aligned} \tag{1.42}$$

1.2.3 Supergravity in higher dimensions

The importance of supergravity in $D > 4$ derives from supergravity being the low-energy effective action for fundamental theories such as string theories, which is the case for $D = 11$ supergravity and M-theory, as well as $D = 10$ and type IIA and IIB string theory. Besides, dimensional reduction of supergravity in higher dimensions can be used to construct extended supergravity in lower dimensions.

Kaluza-Klein reduction procedure can lower the dimension of supergravity in $\mathbb{R}^{(1,D-1)}$ to $\mathbb{R}^{(1,D-1-d)}$ that includes a d-dimensional compact manifold ($\mathbb{R}^{(1,D-1-d)} \times X^d$), where any isometry of the manifold becomes a gauge symmetry in $\mathbb{R}^{(1,D-1-d)}$ [44]. An example is a massless vector field $A_M\left(x^M\right)$ where $M = 0, 1, ..., D-1$ compactified in $\mathbb{R}^{1,D-2} \times S^1$ where the circle has radius R. The vector field becomes a scalar and a vector field in the Lorentz group $SO(1, D-2)$

$$A_\mu = \sum_{n=-\infty}^{\infty} A_\mu^n \exp\left(\frac{iny}{R}\right), \quad \rho = \sum_{n=-\infty}^{\infty} \rho_n \exp\left(\frac{iny}{R}\right) \tag{1.43}$$

If we write the field equation with a transverse gauge, we get a massless Klein-Gordon equation for each component indicating an infinite KK tower with masses $m_n^2 = n^2/R^2$. As $R \to 0$ all the modes, except for $n = 0$ become infinitely heavy and decouple. The action would get reduced into a massless gauge field and a massless scalar field

$$\mathcal{S}_D = \int d^{D-1}x dy \frac{1}{g_D^2} F_{MN} F^{MN} = \int d^{D-1}x \left(\frac{2\pi R}{g_D^2} F_{(0)}^{\mu\nu} F_{(0)\mu\nu} + \frac{2\pi R}{g_D^2} \partial_\mu \rho_0 \partial^\mu \rho_0 \right) = \mathcal{S}_{D-1} \tag{1.44}$$

$D = 11$ is the highest dimension allowed for a supergravity theory. The massless multiple (little group SO(9)) contains the metric(traceless symmetric with 44 d.o.f), and the vector-spinor gravitino (with 128 d.of). To match the fermionic and bosonic d.o.f, the theory needs an object with 84 d.o.f, which is the number of independent components of 3-form in 11D. Thus, 11D supergravity [45] in a circle is reduced to

$$\begin{aligned} g_{MN} &\to g_{\mu\nu}, \quad g_{\mu 10} \sim A_\mu, \quad g_{10,10} \sim \phi \\ A_{MNP} &\to A_{\mu\nu\rho}, \quad A_{\mu\nu 10} \sim B_{\mu\nu} \\ \Psi_M &\to (\Psi_{\mu,\alpha}, \Psi_{\mu\dot{\alpha}}), \quad (\Psi_{10\alpha} \sim \lambda_\alpha, \Psi_{10\dot{\alpha}} \sim \lambda_{\dot{\alpha}}) \end{aligned} \tag{1.45}$$

The local Lorentz invariance breaks from SO(1,10) to SO(1,9)x SO(1), allowing the triangular parametrization of the vielben

$$\hat{e}^{\hat{\alpha}}{}_{\hat{\mu}} = \begin{pmatrix} e^{\delta\phi} e^\alpha{}_\mu & 0 \\ e^\phi A_\mu & e^\phi \end{pmatrix} \quad \hat{\mu}, \hat{\alpha} = 0, \ldots, 10 \quad \mu, \alpha = 0, \ldots, 9 \tag{1.46}$$

With a similar procedure to the previous example, we can write the bosonic part of the action of the reduced supergravity to be [46]

$$
\begin{aligned}
S_{\mathrm{IIA}} =&\frac{1}{2}\int \mathrm{d}^{10}x\sqrt{-g}\left(e^{-\phi}\left(R+4\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}\left|H_3\right|^2\right)-\frac{1}{4}\left(\left|F_2\right|^2+\left|\hat{F}_4\right|^2\right)\right)\\
&-\frac{1}{4}\int \mathrm{d}^{10}x B_2\wedge F_4\wedge F_4+\ \text{(fermionic interactions)}\ ,
\end{aligned} \tag{1.47}
$$

where $F_2 = \mathrm{d}A_1.H_3 = \mathrm{d}B_2, F_4 = \mathrm{d}A_3$ and $\hat{F}_4 = F_4 - A_1 \wedge H_3$. Constructing consistent KK reductions can provide methods to construct explicit solutions in lower dimensions, which can be uplifted to the higher dimensional theory. Truncations and consistency of the KK towers have been studied in [47–49].

1.2.4 Ungauged and gauged supergravity

The KK procedure on $\mathbb{R}^{(1,D-1)} \times X^d$ induces gauge symmetries on the vector fields originated from the metric dimensional reduction [50,51]. This gauge is induced by the isometries of X^d. Starting from D=11 supergravity, maximal supergravities in lower dimensions are obtained by toroidal compactification, which preserves all the supersymmetries of the the parent theory. For toroidal KK reductions, the new matter fields are not charged under the abelian gauge group, and the matter fields of the KK tower are massless, obtaining an ungauged massless theory. More complex compactifications typically come with non-abelian gauge symmetries under which the matter fields are charged, and come with a scalar potential and masses for the matter fields.

Gauging an ungauged theory is possible via deformation, which consists of promoting a subgroup G_0 of the global symmetry group G of the ungauged theory, to a local gauge symmetry of the vector fields [52]. This procedure introduces mass deformations, scalar potential, and charged matter fields to maintain supersymmetry in the theory.

The bosonic field of supergravity is fixed by gauge covariance and diffeomorphism. Under the global symmetry, the scalar field and the gauge fields transform in non-linear and linear representations of G respectively. The scalar fields are parametrized by a coset G/K, where K is the maximal compact subgroup of G. A gauge fixing describing the scalar fields is given by the scalar matrix

$$\mathcal{V} = \exp\{\phi^a Y_a\} \tag{1.48}$$

where the generators Y_a span the lie algebra of the coset space. In this gauge, the global invariance of the scalar Lagrangian is manifest. We can write the Lagrangian sector via the current

$$J_\mu = \mathcal{V}^{-1}\partial_\mu \mathcal{V} = Q_\mu + P_\mu \tag{1.49}$$

where Q_μ is part of the Lie Algebra of K and behaves as a gauge field under it, taking the role of a composite connection for the fermion fields. P_μ is part of the orthogonal complement of the Lie Algebra of K, and can be used to build an invariant kinetic term under K

$$\mathcal{L}_{\text{scalar}} = -\frac{1}{2} e \operatorname{Tr}(P_\mu P^\mu) \tag{1.50}$$

Gauging of the theory can be done via the embedding tensor formalism. Choosing a subgroup G_0 of G by selecting a subset of the generators of G. This subgroup gets promoted to local symmetry by the standard covariant derivative definition. Labeling the generators of the subgroup by X_M, we can introduce the embedding tensor [53, 54]

$$X_M \equiv \Theta_M{}^\alpha t_\alpha \tag{1.51}$$

where t_α are the generators of the global symmetry G. Once a particular choice for $\Theta_M{}^\alpha$ is made, the gauge group G_0 is fixed [55]. The generators X_M must close under into a sub-lie algebra of G, which sets linear (from supersymmetry) and quadratic ($\Theta_M{}^\alpha$ invariant under G_0) constraints on the form of the embedding tensor [53, 56].

These constraints are not enough to make the standard non-abelian field strength covariant. A new term proportional to the coupling constant needs to be added to the field strength. Once the free Lagrangian for both the scalar field and the new gauge field is found, it has to be invariant under supersymmetry. The invariance requires an extra term involving fermionic mass terms and linear in the coupling constant. The supersymmetry transformations of the fermion fields add terms of order g^2, which can be canceled by a scalar potential. Examples of the deformations and Lagrangians can be found in [57, 58].

1.2.5 Domain walls

A CFT can be deformed by introducing relevant operators, which induce an RG flow connecting it to other CFTs. This introduces an energy scale, that breaks the conformal symmetry into Poincare symmetry. On the supergravity side, we can identify this energy scale with a radial direction that separates the regions, and gives different values to the matter fields on each region. A domain wall [59–61] is a supergravity solution separating two regions in space, which approach an AdS space. In these regions, we have two critical points of the potential, and the solutions of the EoM are two different AdS vacua which are dual to a CFT. The effective Lagrangian for a scalar on the boundary becomes

$$\mathcal{L}_{\rm QFT} = \mathcal{L}_{\rm CFT} + \phi_{(0)} \mathcal{O}_{\Delta} \tag{1.52}$$

where $\phi_{(0)}$ is the source for the dual operator. Relevant deformations of the CFT give rise to RG-flows that connect the two AdS vacua regions. The mapping between SUGRA and the CFT corresponds to: the critical point of SUGRA potential dual to fixed point in the beta function, warp-factor of the domain wall dual to energy scale, and the scalars dual to the coupling constant.

For a scalar-gravity toy model obeying Poincare symmetry, the most general ansatz for a domain-wall solution and the EoM are

$$\begin{gathered} \mathrm{d}s^2 = \mathrm{e}^{2A(r)} \delta_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j + \mathrm{d}r^2, \quad \phi = \phi(r) \\ A'^2 = \frac{1}{d(d-1)} \left[\phi'^2 - 2V(\phi) \right], \quad \phi'' + dA'\phi' = \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} \cdot \end{gathered} \tag{1.53}$$

From this ansatz, the critical points of the potential are solutions of the EoM for constant scalar ϕ_i and a warp factor of $A(r) = \pm(r + r_0)/L_i$ which matches the metric of an AdS space with boundary region at $r \to +\infty$ and deep interior at $r \to -\infty$. To get domain solutions that interpolate between the two critical points and are dual to the RG-flow, we work to the lowest order around the critical point where $\phi(r) = \phi_i + h(r)$, $A'(r) = 1/L_i + a'(r)$ and $V(\phi) \approx V(\phi_i) + \frac{1}{2}\frac{m_i^2}{L_i^2}h^2$. Using EoM, $a'(r)$ is of order h^2 and can be neglected. The scalar equation and its solutions are

$$h'' + \frac{d}{L_i}h' - m_i^2 h = 0$$

$$h(r) = B\mathrm{e}^{(\Delta_i - d)r/L_i} + C\mathrm{e}^{-\Delta_i \tau/L} \quad \text{with} \quad \Delta_i = \frac{1}{2}\left(d + \sqrt{d^2 + 4m_i^2 L_i^2}\right). \tag{1.54}$$

The fluctuation h has to vanish at the critical points. For $r \to \infty$, we need $d/2 < \Delta_1 < d$, which implies negative mass and a local maximum in the scalar potential, consistent with a relevant deformation of an ultraviolet CFT. For the second critical point $r \to -\infty$, the scale dimension obeys $\Delta_2 > d$ consistent with an infrared CFT, which approaches the UV critical point as $Be^{(\Delta_1 - d)r/L_1}$.

An equivalent solution for the domain wall ansatz is via an auxiliary quantity, the superpotential [62, 63] defined as

$$\frac{1}{2}\left(\frac{\mathrm{d}W}{\mathrm{d}\phi}\right)^2 - \frac{d}{2(d-1)}W^2 = V(\phi) \tag{1.55}$$

The energy of the toy model is minimized when

$$\phi'(r) = \frac{\mathrm{d}W}{\mathrm{d}\phi}, \quad A'(r) = -\frac{1}{d-1}W(\phi) \tag{1.56}$$

are obeyed. The solutions of these first-order equations are also solutions of the EoM. These emerge as BPS conditions for supersymmetric domain walls in supergravity theories. The superpotential can be extracted from the fermionic variation of the supergravity theory.

1.2.6 Janus ansatz

Non-supersymmetric backgrounds in string theory present challenges to computing well-behaved quantities in AdS/CFT. The Janus solution [64] provides an example where supersymmetry is broken. Even then, the theory is stable [65,66] and the scalar curvature and the string coupling can be kept small everywhere in spacetime. An explicit example is a dilaton deformation in Type IIB background $AdS_5 \times S_5$ that breaks the isometry $SO(4,2)$ of AdS_5 to $SO(3,2)$. In the dual gauge theory side, the Janus corresponds to having a different SYM coupling in each half-space. The Janus solution joins the two boundaries at a conformal invariant interface under $SO(3,2)$.

The Janus solution is based on a deformation of the AdS_{d+1} being sliced using AdS_d spaces. Starting from a Poincare patch metric $ds^2 = \frac{1}{z^2}\left(dz^2 + dx_\perp^2 - dt^2 + \sum_{i=2}^{d-1} dx_i^2\right)$. Using the mapping

$$x_\perp = y \sin\mu, \quad z = y\cos\mu, \tag{1.57}$$

gives the sliced metric

$$ds^2 = \frac{1}{\cos^2\mu}\left(d\mu^2 + \frac{dy^2 - dt^2 + \sum_{i=2}^{d-1} dx_i^2}{y^2}\right) = f(\mu)\left(d\mu^2 + ds^2_{AdS_d}\right) \tag{1.58}$$

where the two boundaries are located at $\mu = \pi/2$ and $\mu = -\pi/2$ and y = 0 corresponding to a $(d-1)$-dimensional interface where the two half-spaces are glued together.

The ansatz for a Janus solution of a dilatonic deformation of $AdS_5 \times S^5$ in type IIB string theory is

$$\begin{aligned} ds^2 &= f(\mu)\left(d\mu^2 + ds^2_{AdS_4}\right) + ds^2_{S^5}, \\ \phi &= \phi(\mu), \\ F_5 &= 2f(\mu)^{\frac{5}{2}} d\mu \wedge \omega_{AdS_4} + 2\omega_{S^5}, \end{aligned} \tag{1.59}$$

and the EoM are

$$
\begin{aligned}
&R_{\alpha\beta} - \frac{1}{2}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{4}F^2_{\alpha\beta} = 0 \\
&\partial_\alpha\left(\sqrt{g}g^{\alpha\beta}\partial_\beta\phi\right) = 0 \\
&* F_5 = F_5
\end{aligned}
\tag{1.60}
$$

The ansatz respects the $SO(2,3)\times SO(6)$ isometry. The solution for a constant dilaton is the AdS sliced metric, which implies a variation of the dilaton implies a deformation of the AdS space. Solving the scalar equation and the Einstein equations, we get

$$
\begin{aligned}
\phi'(\mu) &= \frac{c_0}{f^{\frac{3}{2}}(\mu)}. \\
f'f' &= 4f^3 - 4f^2 + \frac{c_0^2}{6}\frac{1}{f},
\end{aligned}
\tag{1.61}
$$

where the Einstein equation is equivalent to the motion of a particle with zero energy with a potential of minus the right-hand side. As $f \to \infty$, the dilaton is constant, and we recover the AdS sliced solution, which allows us to choose the branch of the potential that is physically relevant. These values are approached at the maximum of μ (the potential being monotonous in the branch). From the integration of both equations, we can get the jump on the dilaton between both boundaries to be

$$
2\Delta\phi_0 = \phi\left(\mu_0\right) - \phi\left(-\mu_0\right) = 2\int_{f_{\min}}^{\infty} \frac{c_0 df}{2f^{3/2}\sqrt{f^3 - f^2 + \frac{c_0^2}{24f}}} > 0 \tag{1.62}
$$

where μ_0 is the maximum value which grows with the constant c_0 until the solution becomes singular for $c_0 > 9/4\sqrt{2}$ (the potential becomes negative for any f). Since the value of the dilaton differs on both boundaries, the dual gauge coupling constant also jumps between boundaries, hence the dual gauge theory is considered an interface CFT.

Different deformations on the gauge theory side of the correspondence being dual to deformations on the string theory side have been studied [67,68], as well as the Janus solutions in different dimensions [69, 70]. Supersymmetric Janus solutions that realize superconformal interface theories can be found via the addition of interface operators whose support is confined to the interface [71–74].

1.3 Conformal interfaces

Conformal interfaces, a situation in which two non-trivial CFTs are glued together along a common interface, are pervasive both in condensed matter systems [75, 76] and in studies of holographic duality [77, 78]. The interface can be permeable, and the AdS/CFT correspondence provides an approach to calculate energy transmission through a large class of interfaces with holographic duals. The interface can be described as a boundary state in the tensor product theory using the folding trick along the interface so that both CFTs live on the same side. For detailed derivations and review of the topic, see [79, 80].

Here we review general properties of 2D ICFTs for a free boson. The gluing conditions for the free boson would be:

$$\begin{pmatrix} \partial_x \phi \\ \partial_t \phi \end{pmatrix}_{x=-0} = M \begin{pmatrix} \partial_x \phi \\ \partial_t \phi \end{pmatrix}_{x=+0} \tag{1.63}$$

where the interface is located at $x = 0$ and M a constant matrix 2×2. Energy conservation requires

$$T_{xt} = T_{++} - T_{--} = \partial_x \phi \partial_t \phi$$

to be continuous throughout the interface and we used light-cone coordinates $x^{\pm} = t \pm x$. This continuity implies that M is an element of $\mathrm{O}(1, 1)$, which has the components

$$M = \pm \begin{pmatrix} \tan\theta & 0 \\ 0 & \cot\theta \end{pmatrix} \quad \text{or} \quad M' = \pm \begin{pmatrix} 0 & \cot\theta \\ \tan\theta & 0 \end{pmatrix} \tag{1.64}$$

with $\theta \in [-\pi/2, \pi/2]$, being the singular cases perfectly reflecting interfaces, while $\theta =$

$\pm\pi/4$ perfectly transmitting cases. Another useful object is the scattering matrix S from which the reflection and transmission coefficients can be obtained.

Using the notation ϕ^1 for the field to the left of the interface and ϕ^2 for the field to the right, we can write the gluing conditions in terms of incoming ($\partial_-\phi^1$ and $\partial_+\phi^2$) and outgoing ($\partial_+\phi^1$ and $\partial_-\phi^2$) waves:

$$\begin{pmatrix}\partial_-\phi^1\\ \partial_+\phi^2\end{pmatrix} = S\begin{pmatrix}\partial_+\phi^1\\ \partial_-\phi^2\end{pmatrix} \tag{1.65}$$

Comparing both the M and S matrices and matching the gluing conditions, one obtains

$$S = \begin{pmatrix} -\cos 2\vartheta & \sin 2\vartheta \\ \sin 2\vartheta & \cos 2\vartheta \end{pmatrix} \quad \text{and} \quad S' = \begin{pmatrix} \cos 2\vartheta & -\sin 2\vartheta \\ \sin 2\vartheta & \cos 2\vartheta \end{pmatrix} \tag{1.66}$$

To describe the interface as a D-brane, we can use the folding trick [80] where we define a 'conjugate' field in the left-half plane by mirror reflecting the field on the right (ϕ^2)

$$\hat{\phi}^2(x,t) \equiv \phi^2(-x,t) \quad \text{for} \quad x \leq 0. \tag{1.67}$$

With this identification $x = 0$ becomes the boundary for $\mathrm{CFT}_1 \times \overline{\mathrm{CFT}}_2$, and S maps operators from the right-plane to a conjugate one on the left ($a^i_n \to S_{ij}\bar{a}^j_{-n}$). The gluing conditions for the M matrix are

$$\hat{\phi}^2(x,t) \equiv \phi^2(-x,t) \quad \text{for} \quad x \leq 0. \tag{1.68}$$

which are the boundary conditions for a D1-brane [81] in the $\left(\phi^1, \hat{\phi}^2\right)$ plane along θ. The conformal boundary state for a D1-brane

$$|b\rangle = \mathcal{N}\prod_{n=1}^{\infty} \exp\left(\frac{1}{n} a^i_{-n}\bar{a}^j_{-n} S_{ij}\right)|0\rangle \tag{1.69}$$

where $a_n^{1,2}$ are the left-moving modes for ϕ^1 and $\hat{\phi}^2$ such that $[a_m, a_n] = m\delta_{m+n,0}$. This state is annihilated by

$$\left(a^i_m - S_{ij}\bar{a}^j_{-m}\right)|b\rangle = 0, \tag{1.70}$$

A natural generalization for a general CFT is to use the Virasoro algebra instead of the normal modes of the free boson. A possible guess would be $\left(L^i_m - S_{ij}\bar{L}^j_{-m}\right)|b\rangle = 0$, but this is only consistent for total transmission or total reflection at the interface. In [79], they found that the matrix

$$R_{ij} = \frac{\langle 0|L^i_2\bar{L}^j_2|b\rangle}{\langle 0 \mid b\rangle} \tag{1.71}$$

describes the reflection and transmission coefficients for ICFTs

$$\begin{aligned} \mathcal{R} &= \frac{2}{c_1+c_2}\left(R_{11}+R_{22}\right) \\ \mathcal{T} &= \frac{2}{c_1+c_2}\left(R_{12}+R_{21}\right) \end{aligned} \tag{1.72}$$

Expanding the boundary state of the free boson

$$\begin{aligned} |b\rangle &= \mathcal{N}\prod_{n=1}^{\infty}\exp\left(\frac{1}{n}a^i_{-n}\bar{a}^j_{-n}S_{ij}\right)|0\rangle \\ &= \mathcal{N}\left(1 + a^i_{-1}\bar{a}^j_{-1}S_{ij} + \frac{1}{2}\left(a^i_{-1}\bar{a}^j_{-1}S_{ij}\right)^2 + a^i_{-2}\bar{a}^j_{-2}S_{ij} + \dots\right)|0\rangle \end{aligned} \tag{1.73}$$

Using $L^i_{-2}|0\rangle = \frac{1}{2}a^i_{-1}a^i_{-1}|0\rangle$ and the commutator of the normal modes, one notices that only the third term of the boundary state expansion survives. One obtains

$$R_{ij}\langle 0 \mid b\rangle = \langle 0|L^i_2\bar{L}^j_2|b\rangle = \frac{\mathcal{N}}{2}\left(S_{ij}\right)^2 \tag{1.74}$$

which gives us

$$\mathcal{R} = \cos^2(2\theta), \quad \mathcal{T} = \sin^2(2\theta). \tag{1.75}$$

.

We can also compute the transmission coefficient for two AdS_3 slices separated by a string of tension σ. The calculation [82] is done by scattering surface-gravity waves in a semiclassical geometry dual to the ground state of the ICFT and gluing the matching conditions

$$\begin{aligned} \gamma_{L,\alpha\beta} &= \gamma_{R,\alpha\beta} \\ [K_{\alpha\beta}] - [\operatorname{tr} K]\gamma_{\alpha\beta} &= 8\pi G\sigma\gamma_{\alpha\beta} \end{aligned} \tag{1.76}$$

where $\gamma_{L,R}$ and $K_{L,R}$ are the induced metric and extrinsic curvature of the metric $ds_L^2 = \frac{\ell_L^2}{y_L^2}\left[dy_L^2 + du_L^2 - dt_L^2\right]$ for $u_L \leq y_L \tan\theta_L$, and analogous for the right side. These equations give

$$\ell_W = \frac{\ell_L}{\cos\theta_L} = \frac{\ell_R}{\cos\theta_R} = \frac{\tan\theta_L + \tan\theta_R}{8\pi G\sigma} \tag{1.77}$$

which gives a relation for the string tension of the interface in terms of the AdS_3 slices geometric properties and the metric. Matching these gluing conditions to the incoming scattering surface-gravity waves and using the no outgoing wave condition [83], one obtains

$$\mathcal{T}_L = \frac{2\cos\theta_R}{\cos\theta_R\,(1+\sin\theta_L) + \cos\theta_L\,(1+\sin\theta_R)}. \tag{1.78}$$

combining it with (1.77), we get

$$\mathcal{T}_{L,R} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G\sigma\right]^{-1} \tag{1.79}$$

We can also compute transmission coefficients for general 2D ICFTs that admit a smooth holographic dual of 3D gravity coupled to any matter. Instead of one thin brane, we can model it as a discrete set of thin branes, using the argument that transmission past a pair of thin branes interfaces with tensions σ_1, σ_2 is the same as for a single thin brane with tension $\sigma_1 + \sigma_2$ [84]. In this case (1.79) is still valid with the replacement

$$\sigma \to \int_{-\infty}^{\infty} \frac{d\sigma}{dy} dy \text{ and } \begin{cases} \ell_L \to 1/\sqrt{-\Lambda(-\infty)} \\ \ell_R \to 1/\sqrt{-\Lambda(\infty)} \end{cases} \tag{1.80}$$

In [85], the authors derive a relation between the string tension and a scalar field in AdS_n with a thin-brane array separating left and right, by replacing the array with a "pizza" of AdS_n slices separated by thin tensile branes obtaining

$$d\sigma = (\phi')^2 \, dy \tag{1.81}$$

where y is the radial coordinate of AdS.

CHAPTER 2

Janus and RG-flow interfaces in three-dimensional gauged supergravity

Janus solutions provide holographic constructions of interface conformal field theories (CFTs). In many known examples, the solutions are constructed by considering an AdS_d slicing of a $(d+1)$-dimensional space where the scalar fields depend non-trivially on the slicing coordinate. One of the most well-known examples is the Janus solution of [64], which is a deformation of the $\mathrm{AdS}_5 \times S^5$ vacuum of type IIB and is given by an AdS_4 slicing where the dilation depends non-trivially on the slicing coordinate. The solution is dual to an interface of $N = 4$ super Yang-Mills (SYM) theory where the coupling g_{YM} jumps across a co-dimension one interface [86]. This solution breaks all the supersymmetries, but a general solution given by an $\mathrm{AdS}_4 \times S^2 \times S^2$ space warped over a Riemann surface can preserve half the supersymmetries of the $\mathrm{AdS}_5 \times S^5$ vacuum [87] and is dual to supersymmetric interface theories in $N = 4$ SYM [88–90]. For other examples of Janus solutions in type II and M-theory, see e.g. [69, 71, 91].

Instead of constructing solutions in ten or eleven dimensions, it is often useful to use lower-dimensional gauged supergravities since the ansatz and resulting equations are simpler. Often the resulting solutions can be uplifted to higher dimensions, but even if the uplift is not known the gauged supergravity solutions are useful for studying universal and qualitative features of interface solutions. For an incomplete list of such solutions in various dimensions, see e.g. [70, 72, 92–97].

A related construction is given by holographic RG-flows, which consider a Poincaré slicing instead of an AdS slicing. If the solutions asymptotically approach two AdS vacua with different cosmological constants, we can interpret this solution as an RG-flow from a CFT in the UV to a CFT in the IR which is triggered by turning on a relevant deformation in the UV [61, 98, 99]. On the other hand, for most examples of AdS-sliced holographic interface solutions the CFTs on both sides of the interface are the same and differ only by a marginal deformation (such as different values of $g_{\rm YM}$ in the example discussed above) or a position-dependent profile of the expectation value of a relevant operator [69].

In [100], Gaiotto proposed the idea of a RG-interface in two-dimensional CFTs, where the two sides of the interface are CFTs related by a RG-flow. The goal of the current paper is to construct holographic solutions which realize this idea.[1] We consider three-dimensional $\mathcal{N} = 8$ gauged supergravity with $n = 4$ vector multiplets, first discussed in [104]. This theory has an AdS_3 vacuum with maximal $\mathcal{N} = (4, 4)$ supersymmetry as well as two families of AdS_3 vacua with $\mathcal{N} = (1, 1)$ supersymmetry [105]. The theory gauges a $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry of the $\mathrm{SO}(8, 4)/\,\mathrm{SO}(8)\times\mathrm{SO}(4)$ coset. The gauging depends on a continuous parameter α where the superconformal algebra of the $\mathcal{N} = (4, 4)$ vacuum is given by the "large" superconformal algebra $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$, and the three-dimensional supergravity is believed to be a truncation of M-theory on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ [106–109]. In this paper, we primarily focus on the case $\alpha = 1$ for which the expressions for the flow equations are the simplest.

In [105], the Poincaré-sliced holographic RG-flow solutions were constructed for the $\mathcal{N} = 8$ gauged supergravity, which interpolate between the $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ vacua. The goal of the present paper is to construct Janus solutions which realize the interface between the same CFT at different points in the moduli space as well as RG-flow interfaces between CFTs preserving different numbers of supersymmetries.

The structure of this paper is as follows. In section 2.1, we review the $\mathcal{N} = 8$ gauged supergravity theory we will be using in the paper and discuss the different supersymmetric

[1] See [101–103] for earlier attempts in this direction.

AdS_3 vacua. In section 2.2, we derive the BPS equations for a Janus ansatz following from the vanishing of the gravitino and spin-$\frac{1}{2}$ supersymmetry variations. In section 2.3, we solve the BPS equations for various truncations which make the analysis of the flow equations manageable. For a Janus interface between the $\mathcal{N} = (4, 4)$ vacuum we find an analytic solution, whereas for the RG-flow interfaces the flow equations can only be solved numerically. We present the solutions and provide evidence for our interpretation of these solutions as RG-flow interfaces. In section 2.4, we close with a discussion and open questions.

2.1 Three-dimensional $\mathcal{N} = 8$ gauged supergravity

In this section, we review the $\mathcal{N} = 8$ gauged supergravity first constructed in [104]. The theory is characterized by the number n of vector multiplets. The bosonic field content consists of a graviton $g_{\mu\nu}$, Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$, and scalars fields living in a $G/H = \mathrm{SO}(8, n)/\,\mathrm{SO}(8)\times\mathrm{SO}(n)$ coset, which has $8n$ degrees of freedom before gauging. The scalar fields can be parametrized by a G-valued matrix $L(x)$ in the vector representation, which transforms under H and the gauge group $G_0 \subseteq G$ by

$$L(x) \to g_0(x)L(x)h^{-1}(x) \tag{2.1}$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations.

For future reference, we use the following index conventions:

- $I, J, \ldots = 1, 2, \ldots, 8$ for SO(8).
- $r, s, \ldots = 9, 10, \ldots, n + 8$ for SO(n).
- $\bar{I}, \bar{J}, \ldots = 1, 2, \ldots, n + 8$ for SO($8, n$).
- $\mathcal{M}, \mathcal{N}, \ldots$ for generators of SO($8, n$).

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the noncompact generators. Explicitly, the generators of the vector representation are given by

$$(t^{\bar{I}\bar{J}})^{\bar{K}}{}_{\bar{L}} = \eta^{\bar{I}\bar{K}}\delta^{\bar{J}}_{\bar{L}} - \eta^{\bar{J}\bar{K}}\delta^{\bar{I}}_{\bar{L}} \tag{2.2}$$

where $\eta^{\bar{I}\bar{J}} = \text{diag}(++++++++-\cdots)$ is the SO$(8,n)$-invariant tensor. These generators satisfy the typical SO$(8,n)$ commutation relations,

$$[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}] = 2\Big(\eta^{\bar{I}[\bar{K}}t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}}t^{\bar{L}]\bar{I}}\Big) \tag{2.3}$$

The gauging of the supergravity is characterized by an embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ (which has to satisfy various identities [110]) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry transformations and action depending on the gauge coupling g. We will look at the particular case in [105] where $n \geq 4$ and the gauged subgroup is the $G_0 = \text{SO}(4) \times \text{SO}(4)$ subset of the SO$(8) \subset$ SO$(8,n)$. The embedding tensor has the entries,[2]

$$\Theta_{\bar{I}\bar{J},\bar{K}\bar{L}} = \begin{cases} \alpha\varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I},\bar{J},\bar{K},\bar{L} \in \{1,2,3,4\} \\ \varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I},\bar{J},\bar{K},\bar{L} \in \{5,6,7,8\} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

Note that the gauging depends on a real parameter α. As discussed in [105], the maximally supersymmetric AdS$_3$ vacuum has an isometry group,

$$D^1(2,1;\alpha) \times D^1(2,1;\alpha) \tag{2.5}$$

which corresponds to the family of "large" superconformal algebras of the dual SCFT. In the following, we will consider the special case $\alpha = 1$ for which $D^1(2,1;1) = \text{OSp}(4|2)$ and the form of many quantities defined below are most compact. We expect that for generic values of α the qualitative behavior of the solutions will be similar.

[2]We use the conventions $\varepsilon_{1234} = \varepsilon_{5678} = 1$.

From the embedding tensor, the G_0-covariant currents can be obtained,

$$L^{-1}(\partial_\mu + g\Theta_{\mathcal{MN}} B_\mu^{\mathcal{M}} t^{\mathcal{N}})L = \frac{1}{2}\mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir} \tag{2.6}$$

It is convenient to define the $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ tensors,

$$L^{-1} t^{\mathcal{M}} L = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} t^{\mathcal{A}} = \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ} X^{IJ} + \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{rs} X^{rs} + \mathcal{V}^{\mathcal{M}}{}_{Ir} Y^{Ir} \tag{2.7}$$

and the T-tensor,

$$T_{\mathcal{A}|\mathcal{B}} = \Theta_{\mathcal{MN}} \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \tag{2.8}$$

The T-tensor is used to construct the tensors $A_{1,2,3}$ which will appear in the scalar potential and the supersymmetry transformations,

$$\begin{aligned}
A_1^{AB} &= -\frac{1}{48}\Gamma^{IJKL}_{AB} T_{IJ|KL} \\
A_2^{A\dot{A}r} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}} T_{IJ|Kr} \\
A_3^{\dot{A}r\dot{B}s} &= \frac{1}{48}\delta^{rs}\Gamma^{IJKL}_{\dot{A}\dot{B}} T_{IJ|KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}} T_{IJ|rs}
\end{aligned} \tag{2.9}$$

where A, B and $\dot{A}, \dot{B}$ are SO(8)-spinor indices. Our conventions for the SO(8) Gamma matrices are presented in the appendix.

We take the spacetime signature $\eta^{ab} = \mathrm{diag}(+--)$ to be mostly negative. The bosonic Lagrangian and scalar potential are

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{bos}} &= -\frac{1}{4}R + \frac{1}{4}\mathcal{P}_\mu^{Ir}\mathcal{P}^{Ir\,\mu} + W - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho} g\Theta_{\mathcal{MN}} B_\mu^{\mathcal{M}}\left(\partial_\nu B_\rho^{\mathcal{N}} + \frac{1}{3}g\Theta_{\mathcal{KL}} f^{\mathcal{NK}}{}_{\mathcal{P}} B_\nu^{\mathcal{L}} B_\rho^{\mathcal{P}}\right) \\
W &= \frac{1}{4}g^2\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\right)
\end{aligned} \tag{2.10}$$

The supersymmetry variations are

$$\begin{aligned}
\delta\chi^{\dot{A}r} &= \frac{1}{2}i\Gamma^I_{A\dot{A}}\gamma^\mu\varepsilon^A\mathcal{P}_\mu^{Ir} + gA_2^{A\dot{A}r}\varepsilon^A \\
\delta\psi_\mu^A &= \left(\partial_\mu\varepsilon^A + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon^A + \frac{1}{4}\mathcal{Q}_\mu^{IJ}\Gamma^{IJ}_{AB}\varepsilon^B\right) + igA_1^{AB}\gamma_\mu\varepsilon^B
\end{aligned} \tag{2.11}$$

The Einstein equation of motion is

$$R_{\mu\nu} - \mathcal{P}^{Ir}_{\mu}\mathcal{P}^{Ir}_{\nu} - 4W g_{\mu\nu} = 0 \tag{2.12}$$

and the gauge field equation of motion is

$$e\mathcal{P}^{Ir\,\lambda}\Theta_{\mathcal{QM}}\mathcal{V}^{\mathcal{M}}{}_{Ir} = \varepsilon^{\lambda\mu\nu}\left(\Theta_{\mathcal{QM}}\partial_\mu B^{\mathcal{M}}_{\nu} + \frac{1}{6}g B^{\mathcal{M}}_{\mu}B^{\mathcal{K}}_{\nu}\left(\Theta_{\mathcal{MN}}\Theta_{\mathcal{KL}}f^{\mathcal{NL}}{}_{\mathcal{Q}} + 2\Theta_{\mathcal{MN}}f^{\mathcal{LN}}{}_{\mathcal{K}}\Theta_{\mathcal{LQ}}\right)\right) \tag{2.13}$$

2.1.1 The $n = 4$ case

Let us focus on the case of four vector multiplets, i.e. $n = 4$. The symmetries consist of a local $G_0 = \mathrm{SO}(4) \times \mathrm{SO}(4)$ and a global $\mathrm{SO}(n = 4)$. Thus, the scalar potential only depends on $8 \cdot 4 - 3 \cdot 6 = 14$ parameters out of the original 32. Moreover, we will only consider a further consistent truncation outlined in [105] where the coset representative depends on eight of the fourteen scalars.

$$L = \begin{pmatrix} \cos A & \sin A \cosh B & \sin A \sinh B \\ -\sin A & \cos A \cosh B & \cos A \sinh B \\ 0 & \sinh B & \cosh B \end{pmatrix}$$
$$A = \mathrm{diag}(p_1, p_2, p_3, p_4)\ , \qquad B = \mathrm{diag}(q_1, q_2, q_3, q_4) \tag{2.14}$$

With this truncation, the scalar potential is[3]

$$g^{-2}W = 1 + \prod_{i=1}^{4}\cosh q_i + \frac{1}{4}\sum_{i=1}^{4}\sinh^2 q_i - \frac{1}{4}\sum_{i<j<k}(x_i^2x_j^2x_k^2 + y_i^2y_j^2y_k^2) - \frac{1}{2}\left(\prod_{i=1}^{4}x_i + \prod_{i=1}^{4}y_i\right)^2$$
$$x_i = \cos p_i \sinh q_i\ , \qquad y_i = \sin p_i \sinh q_i \tag{2.15}$$

[3] We correct a small typo in the potential given in [105].

The $\mathcal{Q}_\mu$ and $\mathcal{P}_\mu$ currents, excluding the $g\Theta_{\mathcal{M}\mathcal{N}}B_\mu^{\mathcal{M}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{A}}$ term, are

$$\mathcal{Q}_\mu^{IJ} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cosh q_1\partial_\mu p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cosh q_2\partial_\mu p_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_3\partial_\mu p_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_4\partial_\mu p_4 \\ -\cosh q_1\partial_\mu p_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\cosh q_2\partial_\mu p_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cosh q_3\partial_\mu p_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cosh q_4\partial_\mu p_4 & 0 & 0 & 0 & 0 \end{pmatrix}_{IJ}$$

$$\mathcal{Q}_\mu^{rs} = 0$$

$$\mathcal{P}_\mu^{Ir} = \begin{pmatrix} \sinh q_1\partial_\mu p_1 & 0 & 0 & 0 \\ 0 & \sinh q_2\partial_\mu p_2 & 0 & 0 \\ 0 & 0 & \sinh q_3\partial_\mu p_3 & 0 \\ 0 & 0 & 0 & \sinh q_4\partial_\mu p_4 \\ \partial_\mu q_1 & 0 & 0 & 0 \\ 0 & \partial_\mu q_2 & 0 & 0 \\ 0 & 0 & \partial_\mu q_3 & 0 \\ 0 & 0 & 0 & \partial_\mu q_4 \end{pmatrix}_{Ir} \tag{2.16}$$

Using these matrices, we can check that the combination $\mathcal{P}_\mu^{Ir}\mathcal{V}^{JK}{}_{Ir}$ vanishes whenever the indices $J, K \in \{1, 2, 3, 4\}$ or $J, K \in \{5, 6, 7, 8\}$. This implies that there is no source for $B_\mu^{\mathcal{M}}$ in the gauge field equation of motion (2.13), so it is consistent to set $B_\mu^{\mathcal{M}} = 0$. We will make this choice from now on.

2.1.2 Truncations and supersymmetric AdS$_3$ vacua

In order to make our analysis more tractable, we make further truncations to reduce the number of independent scalar fields. Below we consider three truncations, which together explore the AdS$_3$ vacua with $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ supersymmetry.

2.1.2.1 Truncation 1

The first truncation is given by calling $q_1 = q$, $p_1 = p$ and setting all remaining $q_i = p_i = 0$ for $i = 2, 3, 4$. The scalar potential is

$$W = \frac{g^2}{2}\cosh^2\frac{q}{2}\,(3 + \cosh q) \tag{2.17}$$

The $\mathcal{N} = (4, 4)$ vacuum is given by setting $q = 0$ and the vacuum potential is $W_0 = 2g^2$.

2.1.2.2 Truncation 2

The second truncation is given by setting all the qs and ps equal, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3, 4$. The scalar potential is

$$W = \frac{g^2}{8192}\Big(8103 + 6856\cosh 2q + 1452\cosh 4q - 8\cosh 6q - 19\cosh 8q \\ - 768(3+\cosh 2q)\cos 4p \sinh^6 q - 128\cos 8p\sinh^8 q\Big) \tag{2.18}$$

The $\mathcal{N} = (4,4)$ vacuum is given by $q = 0$ as before, and $\mathcal{N} = (1,1)$ vacua are given by $q = \pm\sinh^{-1}\sqrt{2}$ and $p = \pi(\mathbb{Z}/2 + 1/4)$ which have a vacuum potential of $W_0 = 8g^2$.

2.1.2.3 Truncation 3

The third truncation is given by setting the first three qs and ps equal, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3$, and setting the remaining $q_4 = p_4 = 0$. The scalar potential is

$$W = \frac{g^2}{1024}\Big(690 + 768\cosh q + 309\cosh 2q + 256\cosh 3q \\ + 30\cosh 4q - 5\cosh 6q - 96\cos 4p\sinh^6 q\Big) \tag{2.19}$$

The $\mathcal{N} = (4,4)$ vacuum is given by $q = 0$ as before, and $\mathcal{N} = (1,1)$ vacua are given by $q = \pm\sinh^{-1}\sqrt{2+2\sqrt{2}}$ and $p = \pi(\mathbb{Z}/2 + 1/4)$ which have a vacuum potential of $W_0 = 2(1+\sqrt{2})^2 g^2$.

2.2 Janus flow equations

In this section, we present the equations of motion and supersymmetry variations for a Janus ansatz where the three-dimensional metric is written as an AdS_2 slicing and the scalar fields only depend on the slicing coordinate. We will also set the Chern-Simons gauge $B_\mu^{\mathcal{M}}$ fields to zero, which is consistent as argued in section 2.2.1. Hence, the Janus ansatz is given by

$$\begin{aligned} \mathrm{d}s^2 &= e^{2B(u)}\left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2 \ , \qquad B_\mu^{\mathcal{M}} = 0 \\ q_i &= q_i(u) \ , \qquad p_i = p_i(u) \end{aligned} \tag{2.20}$$

The Ricci tensor has the non-zero components,

$$\begin{aligned} R_{tt} = -R_{zz} &= z^{-2}\big(1 + e^{2B}(2B'^2 + B'')\big) \\ R_{uu} &= -2(B'^2 + B'') \end{aligned} \tag{2.21}$$

The prime $'$ denotes a derivative with respect to the slicing coordinate u. The gravitino supersymmetry variation $\delta\psi_\mu^A = 0$ is

$$\begin{aligned} 0 &= \partial_t\varepsilon + \frac{1}{2z}\gamma_0\big(\gamma_1 - B'e^B\gamma_2 + 2ige^BA_1\big)\varepsilon \\ 0 &= \partial_z\varepsilon + \frac{1}{2z}\gamma_1\big(-B'e^B\gamma_2 + 2ige^BA_1\big)\varepsilon \\ 0 &= \partial_u\varepsilon + \frac{1}{4}\mathcal{Q}_u^{IJ}\Gamma^{IJ}\varepsilon + ig\gamma_2A_1\varepsilon \end{aligned} \tag{2.22}$$

where we have suppressed the SO(8)-spinor indices of ε^A and A_1^{AB}. There are two integrability conditions which can be derived from the gravitino variations (2.22) in the t, z and z, u directions respectively

$$\begin{aligned} 0 &= \big(1 - (2ge^BA_1)^2 + (B'e^B)^2\big)\varepsilon \\ 0 &= 2ige^B\left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}_u^{IJ}\Gamma^{IJ}]\right)\varepsilon + \left(-\frac{\mathrm{d}}{\mathrm{d}u}\big(B'e^B\big) + (2ge^BA_1)^2e^{-B}\right)\gamma_2\varepsilon \end{aligned} \tag{2.23}$$

We can use the first integrability condition to express the second one as

$$2ig\left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}_u^{IJ}\Gamma^{IJ}]\right)\varepsilon + \big(-B'' + e^{-2B}\big)\gamma_2\varepsilon = 0 \tag{2.24}$$

The spin-$\frac{1}{2}$ variation $\delta\chi^{\dot{A}} = 0$ is

$$\left(-\frac{i}{2}\Gamma^I\mathcal{P}_u^{Ir}\gamma_2 + gA_2^r\right)_{A\dot{A}}\varepsilon^A = 0 \ , \qquad r = 9, 10, \ldots, 8+n \tag{2.25}$$

2.2.1 Eigenvectors of A_1

It follows from (2.9) that A_1 is a 8×8 matrix which has eigenvectors

$$A_1^{AB} n_\pm^{(i)B} = \pm w_i n_\pm^{(i)A} \,, \qquad i = 1, 2, 3, 4 \tag{2.26}$$

The eigenvalues w_i determine whether a supersymmetric AdS_3 vacuum can exist. In the following we denote the positive supersymmetric eigenvalue $w(u)$, which can be determined as follows: for the AdS_2-sliced metric given in (2.20), the AdS_3 vacuum solution with potential W_0 is given by

$$B_{\mathrm{vac}}(u) = \ln \frac{\cosh\left(\sqrt{2W_0} u\right)}{\sqrt{2W_0}} \tag{2.27}$$

which satisfies

$$B_{\mathrm{vac}}'^2 + e^{-2B_{\mathrm{vac}}} - 2W_0 = 0 \tag{2.28}$$

When we expand the spinors ε^A in terms of the eigenvectors of A_1, the first equation in (2.23) implies for the spinor component associated with the eigenvalue w that

$$B'^2 + e^{-2B} - 4g^2 w^2 = 0 \tag{2.29}$$

For the AdS_3 vacuum solution (2.27), this condition relates the eigenvalue evaluated at the vacuum w_{vac} to the potential W_0 via

$$w_{\mathrm{vac}}^2 = \frac{W_0}{2g^2} \tag{2.30}$$

As discussed in section 2.3 for truncation 1, A_1 has eight eigenvectors $n_\pm^{(i)}$ for $i = 1, 2, 3, 4$ all with the same with eigenvalue $\pm w$ that satisfy the supersymmetry condition (2.30) for the AdS_3 vacuum with $q = 0$. Hence, this vacuum preserves $\mathcal{N} = (4, 4)$ supersymmetry. On the other hand for truncations 2 and 3, there are only two eigenvectors $n_\pm$ with an eigenvalue $\pm w$ that satisfy (2.30) for the AdS_3 vacua with non-trivial values for the scalars. Consequently, these vacua only preserve $\mathcal{N} = (1, 1)$ supersymmetry.

For the RG-flow solutions which interpolate between the different vacua, we expand the spinors in the basis of the eigenspinors that correspond to the supersymmetric vacuum when the scalars take their vacuum values. This implies that (2.29) can be solved for B',

$$\begin{aligned} B' &= \pm\sqrt{4g^2w^2 - e^{-2B}} \\ &= \pm 2gw\gamma \end{aligned} \tag{2.31}$$

where we defined the convenient combinations,

$$\gamma(u) = \sqrt{1 - \frac{e^{-2B}}{4g^2w^2}}\ , \qquad \sqrt{1-\gamma^2(u)} = \frac{e^{-B}}{2gw} \tag{2.32}$$

which will be useful later on. The two signs in (2.31) are two branches of solutions which for Janus solutions will be patched together—the numerical evolution usually breaks down at $B' = 0$ and this is the location where the two branches will be glued together.

2.2.2 AdS_2 Killing spinors

The Killing spinors for a unit radius AdS_2 with metric,

$$\mathrm{d}s^2_{\mathrm{AdS}_2} = \frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2} \tag{2.33}$$

satisfy the following equation,

$$D_\mu \zeta_\eta = i\frac{\eta}{2}\gamma_\mu \zeta_\eta\ , \qquad \mu = t, z \tag{2.34}$$

with $\eta = \pm 1$. The covariant derivatives on AdS_2 take the form,

$$D_t = \partial_t \varepsilon + \frac{1}{2z}\gamma_0\gamma_1\ , \qquad D_z = \partial_z \tag{2.35}$$

Since the general spinor in AdS_2 is a two-component spinor, the $\zeta_\pm$ form a basis of two-component spinors. Since spinors in three dimensions are also two-component spinors, the $\zeta_\pm$ are also a basis of the spinors in three dimensions. Note that $\gamma_2 = i\gamma_\#$ where $\gamma_\#^2 = 1$ and

$$i\gamma_2\zeta_\eta = \zeta_{-\eta}\ , \qquad \eta = \pm 1 \tag{2.36}$$

The general ansatz for ε^A is given by

$$\varepsilon^A = \sum_i \big(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A}\big)\zeta_+ + \big(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A}\big)\zeta_- \tag{2.37}$$

For truncation 1 we have $i = 1, 2, 3, 4$, which label four eigenvectors of A_1, whereas in truncations 2 and 3 the index i is dropped.

2.2.3 First projector

With this ansatz for the spinors ε^A, the first two components of the gravitino variation,

$$\begin{aligned} 0 &= \partial_t \varepsilon + \frac{1}{2z}\gamma_0\big(\gamma_1 - B' e^B \gamma_2 + 2ige^B A_1\big)\varepsilon \\ 0 &= \partial_z \varepsilon + \frac{1}{2z}\gamma_1\big(-B' e^B \gamma_2 + 2ige^B A_1\big)\varepsilon \end{aligned} \tag{2.38}$$

can be expressed as follows by using the properties of the AdS_2 Killing spinors,

$$0 = i\Big\{\big(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A}\big)\zeta_+ - \big(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A}\big)\zeta_-\Big\} \tag{2.39}$$

$$+ iB' e^{-B} i\gamma_2 \Big\{\big(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A}\big)\zeta_+ + \big(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A}\big)\zeta_-\Big\}$$

$$+ 2igwe^B \Big\{\big(f_+^{(i)} n_+^{(i)A} - f_-^{(i)} n_-^{(i)A}\big)\zeta_+ + \big(g_+^{(i)} n_+^{(i)A} - g_-^{(i)} n_-^{(i)A}\big)\zeta_-\Big\} \tag{2.40}$$

Using $i\gamma_2\zeta_\eta = \zeta_{-\eta}$ and the linear independence of the $n_\pm^{(i)}$ and $\zeta_\pm$, one obtains a set of equations,

$$\begin{aligned} f_+ + B' e^B g_+ + 2gwe^B f_+ &= 0 \\ -g_+ + B' e^B f_+ + 2gwe^B g_+ &= 0 \\ f_- + B' e^B g_- - 2gwe^B f_- &= 0 \\ -g_- + B' e^B f_- - 2gwe^B g_- &= 0 \end{aligned} \tag{2.41}$$

which are consistent if the integrability condition (2.29) holds. In terms of the $\gamma(u)$ defined in (2.32) we have

$$f_+ = \frac{\sqrt{1-\gamma^2} - 1}{\gamma} g_+ \ , \qquad f_- = \frac{\sqrt{1-\gamma^2} + 1}{\gamma} g_- \tag{2.42}$$

2.2.4 Second projector

The spin-$\frac{1}{2}$ variation (2.25) can be rewritten in the following form

$$\left[-\frac{1}{2g}\Big((A_2^r)^T\Big)^{-1}\Big(\Gamma^I\mathcal{P}_u^{Ir}\Big)^T i\gamma_2+1\right]^{AB}\varepsilon^B=0\ ,\qquad r=9,10,11,12 \tag{2.43}$$

Since $\mathcal{P}_u^{Ir}$ contains the first derivatives of the scalar fields, the flow equations for the scalars can be derived from the condition of vanishing of this supersymmetry variation. The projectors for $r=9,10,11,12$ take the form

$$\Big(M^{AB}i\gamma_2+\delta^{AB}\Big)\varepsilon^B=0 \tag{2.44}$$

For consistency, the matrix must satisify $M^{AB}M^{BC}=\delta^{AC}$. Plugging in the ansatz (2.37) for the spinors ε^A, we get

$$\begin{aligned}0=&\ \big(f_+^{(i)}n_+^{(i)A}+f_-^{(i)}n_-^{(i)A}\big)\zeta_++\big(g_+^{(i)}n_+^{(i)A}+g_-^{(i)}n_-^{(i)A}\big)\zeta_-\\&+M^{AB}i\gamma_2\Big\{\big(f_+^{(i)}n_+^{(i)B}+f_-^{(i)}n_-^{(i)B}\big)\zeta_++\big(g_+^{(i)}n_+^{(i)B}+g_-^{(i)}n_-^{(i)B}\big)\zeta_-\Big\}\end{aligned} \tag{2.45}$$

Using the fact that the eigenvectors can be orthonormalized,

$$n_+^{(i)A}n_+^{(j)A}=\delta^{ij}\ ,\qquad n_-^{(i)A}n_-^{(j)A}=\delta^{ij}\ ,\qquad n_+^{(i)A}n_-^{(j)A}=0 \tag{2.46}$$

and projecting onto $n_\pm^{(i)}$ gives

$$\begin{aligned}f_++M_{++}g_++M_{+-}g_-&=0\\g_++M_{++}f_++M_{+-}f_-&=0\\f_-+M_{+-}g_++M_{--}g_-&=0\\g_++M_{+-}f_++M_{--}f_-&=0\end{aligned} \tag{2.47}$$

where we define

$$M_{++}=n_+^AM^{AB}n_+^B\ ,\qquad M_{--}=n_-^AM^{AB}n_-^B\ ,\qquad M_{+-}=M_{-+}=n_+^AM^{AB}n_-^B \tag{2.48}$$

If there is more than one $n_\pm$ (as in truncation 1) the $M_{\pm\pm}, M_{\pm\mp}$ have to take the same form for all $n_\pm^{(i)}$, which is a consistency condition. Using (3.41) it can be shown that equations (2.47) can only[4] be satisfied if we have

$$M_{++} = \gamma \ , \qquad M_{--} = -\gamma \ , \qquad M_{+-} = M_{-+} = \sqrt{1-\gamma^2} \tag{2.49}$$

The relations (2.49) for the matrix M^{AB} given in (2.43) provide first-order flow equations for the scalar fields. Note that the second integrability condition for the gravitino variation in (2.23) is also the form of (2.44). For all the solutions which we find, this condition is automatically satisfied and does not constrain the flow further.

2.3 Janus and RG-flow solutions

In this section we obtain the flow equations and solve them. Only for truncation 1 are we able to solve the system analytically. For truncations 2 and 3 we solve the resulting flow equations numerically.

2.3.1 Truncation 1

For the truncation to a single scalar described in section 2.1.2.1, the matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 & 0 & \sin p \cosh^2 \frac{q}{2} & 0 \\ 0 & 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & \sin p \cosh^2 \frac{q}{2} \\ 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 \\ \cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 \\ 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & -\cos p \cosh^2 \frac{q}{2} \\ 0 & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 \\ \sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 & 0 \\ 0 & \sin p \cosh^2 \frac{q}{2} & 0 & 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 \end{pmatrix} \tag{2.50}$$

[4]We can also have $M_{+-} = M_{-+} = -\sqrt{1-\gamma^2}$, which gives a similar solution. For example, in section 2.3.1 for truncation 1, this sends $p(u) \to p(-u)$. This resolves an ambiguity in the definition of our eigenvectors, as we can freely send $n_+ \to -n_+$ or $n_- \to -n_-$.

We have four pairs of eigenvectors $n_\pm^{(i)}$ for $i = 1, 2, 3, 4$ with the same eigenvalues $\pm w$, where

$$w = \cosh^2 \frac{q}{2} \tag{2.51}$$

so the supersymmetry condition (2.30) is satisfied for the vacuum where $q = 0$. The pairs of eigenvectors are

$$\begin{aligned}
n_+^{(1)} &= \left\{\tfrac{1}{\sqrt{2}}, 0, 0, \tfrac{\cos p}{\sqrt{2}}, 0, 0, \tfrac{\sin p}{\sqrt{2}}, 0\right\}, & n_-^{(1)} &= \left\{0, 0, 0, -\tfrac{\sin p}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}, \tfrac{\cos p}{\sqrt{2}}, 0\right\} \\
n_+^{(2)} &= \left\{0, \tfrac{1}{\sqrt{2}}, -\tfrac{\cos p}{\sqrt{2}}, 0, 0, 0, 0, \tfrac{\sin p}{\sqrt{2}}\right\}, & n_-^{(2)} &= \left\{0, 0, \tfrac{\sin p}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}, 0, 0, \tfrac{\cos p}{\sqrt{2}}\right\} \\
n_+^{(3)} &= \left\{0, 0, -\tfrac{\sin p}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}, 0, 0, -\tfrac{\cos p}{\sqrt{2}}\right\}, & n_-^{(3)} &= \left\{0, -\tfrac{1}{\sqrt{2}}, -\tfrac{\cos p}{\sqrt{2}}, 0, 0, 0, 0, \tfrac{\sin p}{\sqrt{2}}\right\} \\
n_+^{(4)} &= \left\{0, 0, 0, -\tfrac{\sin p}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}, \tfrac{\cos p}{\sqrt{2}}, 0\right\}, & n_-^{(4)} &= \left\{\tfrac{1}{\sqrt{2}}, 0, 0, -\tfrac{\cos p}{\sqrt{2}}, 0, 0, -\tfrac{\sin p}{\sqrt{2}}, 0\right\}
\end{aligned} \tag{2.52}$$

Given the eigenvectors, we can compute the M_{++} and M_{+-} matrix elements for the matrix in (2.43) for any pair of $n_\pm^{(i)}$. Note that for this truncation, only the flow equation for index $r = 9$ is nontrivial while the others are identically vanishing. Then (2.49) gives us flow equations for the scalars q and p. The remaining flow equation for the metric factor B comes from (2.31). The flow equations are

$$q' = -g\gamma \sinh q\ , \qquad p' = g\sqrt{1-\gamma^2}\ , \qquad B' = \pm 2g\gamma \cosh^2 \frac{q}{2} \tag{2.53}$$

which solve the equations of motion. We can solve for a flow where $p(0) = p_0$, $q(0) = q_0$ are arbitrary and $B'(0) = 0$, which is equivalent to $\gamma(0) = 0$. This first-order system can be rewritten using the function γ in lieu of B, in which case the third equation above is replaced with

$$\gamma' = 2g(1-\gamma^2) \tag{2.54}$$

The solution is

$$\begin{aligned}
\gamma(u) &= \tanh(2gu) \\
\tanh \frac{q(u)}{2} &= \sqrt{\operatorname{sech}(2gu)} \tanh \frac{q_0}{2} \\
\tan[p(u) - p_0] &= \tanh(gu)
\end{aligned} \tag{2.55}$$

which gives the metric factor

$$e^{B(u)} = \frac{\cosh(2gu)}{2g} \operatorname{sech}^2 \frac{q(u)}{2} \tag{2.56}$$

This is a Janus solution which approaches the $\mathcal{N} = (4, 4)$ vacuum at the two endpoints $u \to \pm\infty$. We note that the flow equation is the same for each pair of eigenvectors $n_\pm^{(i)}$ and hence the solution preserves eight of the sixteen supersymmetries of the $\mathcal{N} = (4, 4)$ vacuum. We present plots for three choices of the parameters $q_0 = 0.5, 1.0, 1.5$ in figure 2.1 and set $p_0 = 0$ for all three. The qualitative behavior is very similar for all three choices and corresponds to a Janus interface which interpolates between different values of $p(u)$ as $u \to \pm\infty$.

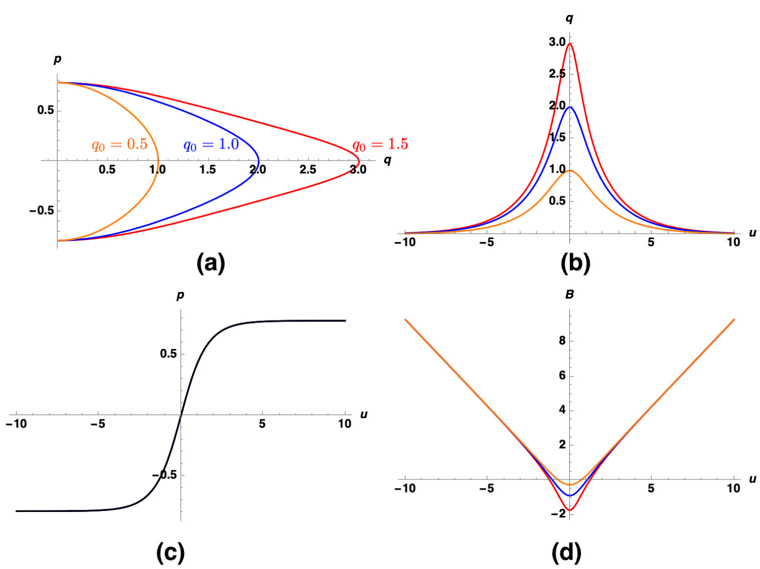


Figure 2.1: (a) pq parametric plot, (b) plot of q , (c) plot of p, (d) plot of the metric function B as functions of the AdS$_2$ slicing coordinate u for truncation 1. The colors denote three different values for q_0. $p_0 = 0$ for all three plots. The behavior of p is the same for all three examples.

2.3.2 Truncation 2

Recall that the truncation presented in section 2.1.2.2 sets all the q_i equal and p_i equal. The matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & a_1+a_3 & 0 & 0 & 0 & 0 \\ 0 & -2a_3 & -a_1+a_3 & 0 & a_2 & 0 & 0 & a_2 \\ 0 & -a_1+a_3 & -2a_3 & 0 & -a_2 & 0 & 0 & -a_2 \\ a_1+a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & -a_2 & 0 & 2a_3 & 0 & 0 & -a_1+a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_1+a_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1+a_3 & 0 & 0 \\ 0 & a_2 & -a_2 & 0 & -a_1+a_3 & 0 & 0 & 2a_3 \end{pmatrix} \tag{2.57}$$

where we define, for this truncation,

$$\begin{aligned} a_1 &= \frac{1}{8}(3+\cos 4p)(1+\cosh^4 q) \\ a_2 &= \frac{1}{8}(3+\cosh 2q)\cosh q \sin 4p \\ a_3 &= 2\cos^2 p \sin^2 p \cosh^2 q \end{aligned} \tag{2.58}$$

The eigenvectors and eigenvalues can be obtained by considering the matrix $(A_1)^2$ first. There are two eigenvalues, the first is $(a_1+a_3)^2$ which is six-fold degenerate but does not satisfy the condition (2.30) for the $\mathcal{N}=(1,1)$ vacuum. The second eigenvalue is $4a_2^2+(a_1-3a_3)^2$ which is two-fold degenerate and does satisfy (2.30) for the $\mathcal{N}=(1,1)$ vacua. The corresponding eigenvectors of $(A_1)^2$ take the form

$$v_1 = \left\{0, \tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}, 0,0,0,0,0\right\} \ , \qquad v_2 = \left\{0,0,0,0,\tfrac{1}{\sqrt{2}},0,0,\tfrac{1}{\sqrt{2}}\right\} \tag{2.59}$$

Let

$$w = \sqrt{4a_2^2+(a_1-3a_3)^2} \tag{2.60}$$

be the positive eigenvalue of A_1. One can reduce the A_1 matrix on the subspace spanned by v_1, v_2 and find that the (not yet normalized) eigenvectors $v_\pm$ of A_1 with eigenvalue $\pm w$ are given by

$$v_\pm = 2a_2 v_1 + (-a_1+3a_3\pm w)v_2 \tag{2.61}$$

The flow equations take the form of a first-order system of ordinary differential equations for the functions $p(u)$, $q(u)$, and $B(u)$. These equations do not take a simple form and are too unwieldy to be presented here. Using Mathemtica, we have checked that the flow equations imply that the equations of motion are satisfied as well as the second integrability condition of the gravitino variation (2.24).

The flow equations can be numerically integrated.[5] In figure 2.2 we present some examples for the numerical solutions of the flow equations. By fine-tuning initial conditions we can produce flows that (i) look like the Janus solutions in truncation 1 plotted, in red in figure 2.2, (ii) connect $\mathcal{N} = (4,4)$ and $\mathcal{N} = (1,1)$ vacua, plotted in blue in figure 2.2, and (iii) connect two $\mathcal{N} = (1,1)$ vacua which are related by flipping signs of p, plotted in orange in figure 2.2. In the pq parametric plot in figure 2.2(a), the locations of the $\mathcal{N} = (1,1)$ vacua $p = \pm\frac{\pi}{4}$, $q = \sinh^{-1}\sqrt{2}$ are denoted by black dots.

In the numerical integration, the $\mathcal{N} = (1,1)$ points are repulsive and require fine-tuning in order to obtain flows that approach these points. This can be explained as follows. The choice $g = 1/2$ sets the $\mathcal{N} = (4,4)$ vacuum potential to $W_0 = 1/2$ and the AdS_3 length scale to unity. For the $\mathcal{N} = (1,1)$ point, the vacuum potential becomes $W_0 = 2$ and the AdS_3 length scale is $L = 1/2$. Taking the linear expansion around the $\mathcal{N} = (1,1)$ point,

$$\begin{aligned} p(u) &= \frac{\pi}{4} + \delta p(u) + \mathcal{O}\big(\delta p^2\big) \\ q(u) &= \sinh^{-1}\sqrt{2} + \delta q(u) + \mathcal{O}\big(\delta q^2\big) \end{aligned} \tag{2.62}$$

the mass-squares for the δp and δq fluctuations are

$$m_p^2 L^2 = \frac{5}{4}\,, \qquad m_q^2 L^2 = \frac{21}{4} \tag{2.63}$$

Using $\Delta(\Delta - 2) = m^2 L^2$, the scaling dimensions of the corresponding dual operators are

$$\Delta_p = \frac{5}{2}\,, \qquad \Delta_q = \frac{7}{2} \tag{2.64}$$

[5] We use the method described in [94]: we choose the location $p(0), q(0)$ of a turning point where $B' = 0$ and use the flow equations to determine $p'(0), q'(0)$. These values provide the initial conditions for the second order equations of motion to give $p(u), q(u)$ and $B(u)$.

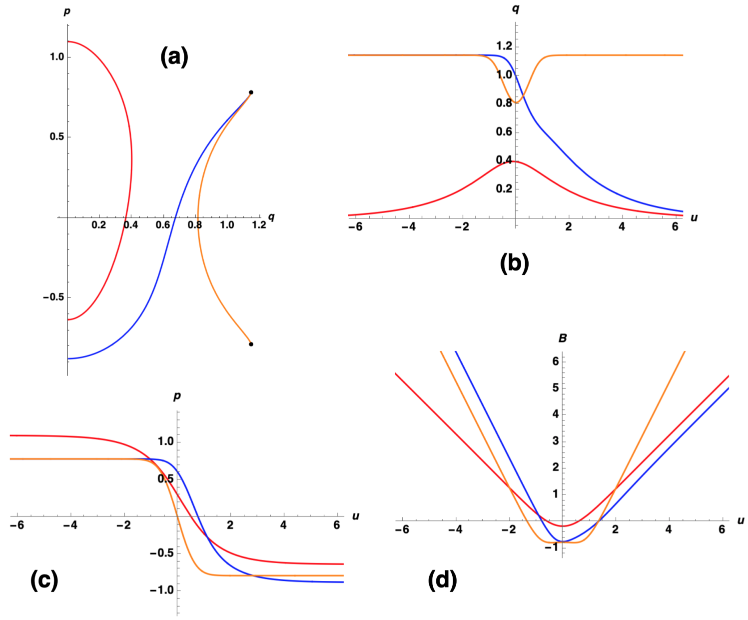


Figure 2.2: (a) pq parametric plot, (b) plot of q , (c) plot of p, (d) plot of the metric function B as functions of the AdS$_2$ slicing coordinate u.

In our AdS-sliced coordinates, the boundary is given by the two AdS$_2$ components at $u = \pm\infty$, which are joined together at the $z = 0$ interface. The coordinates (z, u) can be mapped to Fefferman-Graham coordinates (ρ, x) where the boundary is located at $\rho = 0$.[6] Let us consider the boundary at $u \to +\infty$. In the (ρ, x) coordinates, the metric factor B has the expansion $e^B = \rho^{-1} + \mathcal{O}(\rho^0)$ near the boundary. But in the (z, u) coordinates, from (2.27) the expansion near the boundary is $B = u/L + \cdots$. Therefore, the asymptotic form of the

[6]Recall that the AdS$_3$ metric in Poincaré coordinates,

$$\mathrm{d}s^2 = \frac{-\,\mathrm{d}\rho^2 + \mathrm{d}t^2 - \mathrm{d}x^2}{\rho^2}$$

is related to an AdS$_2$-sliced metric by the coordinate change,

$$z = \sqrt{x^2 + \rho^2} \qquad\qquad \sinh u = x/\rho$$

coordinate change $(\rho, x) \mapsto (z, u)$ takes the form,

$$e^u = \rho^{-L} + \cdots \tag{2.65}$$

The linearized flow equations around the $\mathcal{N} = (1,1)$ point are

$$\begin{aligned} \delta p' &= -5\delta p + \cdots \\ \delta q' &= 3\delta q + \cdots \end{aligned} \tag{2.66}$$

which are solved by $\delta p \sim C_p e^{-5u}$ and $\delta q \sim C_q e^{3u}$, or in terms of ρ,

$$\delta p \sim C_p \rho^{5/2} \qquad\qquad \delta q \sim C_q \rho^{-3/2} \tag{2.67}$$

These asymptotic forms are consistent with the scaling dimensions in (2.64), as we either have solutions that scale as ρ^{Δ} or $\rho^{2-\Delta}$. We see that the q scalar diverges as we approach the $\mathcal{N} = (1,1)$ point as $\rho \to 0$ unless we fine-tune the coefficient C_q to zero. This is identified with turning off the source for an operator with scaling dimension larger than 2 on the boundary CFT.

A similar counting as before shows that the flow preserves two of the four supersymmetries of the $\mathcal{N} = (1,1)$ vacuum. Therefore, we have RG-flow interfaces between a CFT with central charge $c^{(4,4)}$ and a CFT with central charge $c^{(1,1)}$, where [9, 111]

$$\frac{c^{(1,1)}}{c^{(4,4)}} = \sqrt{\frac{W_0^{(4,4)}}{W_0^{(1,1)}}} = \frac{1}{2} \tag{2.68}$$

2.3.3 Truncation 3

The analysis of the flow equations and their solutions for truncation 3 proceeds very similarly to the one for truncation 2, presented in the previous section. The matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & (a_1 - a_2 + b_2)\cos p & 0 & (-a_1 + a_2 + b_1)\sin p & 0 & 0 \\ 0 & -2a_1 \cos p & (a_1 + a_2 - b_2)\cos p & 0 & (a_1 + a_2 - b_1)\sin p & 0 & 0 & 2a_2 \sin p \\ 0 & (a_1 + a_2 - b_2)\cos p & -2a_1 \cos p & 0 & -2a_2 \sin p & 0 & 0 & (-a_1 - a_2 + b_1)\sin p \\ (a_1 - a_2 + b_2)\cos p & 0 & 0 & 0 & 0 & 0 & (a_1 - a_2 - b_1)\sin p & 0 \\ 0 & (a_1 + a_2 - b_1)\sin p & -2a_2 \sin p & 0 & 2a_1 \cos p & 0 & 0 & (a_1 + a_2 - b_2)\cos p \\ (-a_1 + a_2 + b_1)\sin p & 0 & 0 & 0 & 0 & 0 & (a_1 - a_2 + b_2)\cos p & 0 \\ 0 & 0 & 0 & (a1 - a2 - b1)\sin p & 0 & (a_1 - a_2 + b_2)\cos p & 0 & 0 \\ 0 & 2a_2 \sin p & (-a_1 - a_2 + b_1)\sin p & 0 & (a_1 + a_2 - b_2)\cos p & 0 & 0 & 2a_1 \cos p \end{pmatrix} \tag{2.69}$$

where we define, for this truncation,

$$a_1 = \frac{1}{2}(1+\cosh q)\cosh q \sin^2 p \ , \qquad a_2 = \frac{1}{2}(1+\cosh q)\cosh q \cos^2 p$$
$$b_1 = \frac{1}{2}(1+\cosh q)(1+\cosh^2 q)\sin^2 p \ , \quad b_2 = \frac{1}{2}(1+\cosh q)(1+\cosh^2 q)\cos^2 p \tag{2.70}$$

As with trunctation 2, there are two eigenvalues of $(A_1)^2$: one with six-fold degeneracy that does not satisfy (3.34), and one with two-fold degeneracy that does. This eigenvalue is

$$w^2 = \frac{1}{64}\cosh^4\frac{q}{2}\big(175 - 224\cosh q + 140\cosh 2q - 32\cosh 3q + 5\cosh 4q + 24\cos 4p \sinh^4 q\big) \tag{2.71}$$

The corresponding eigenvectors of A_1 with eigenvalue $\pm w$ are

$$v_\pm = \big(-(3a_1 + a_2 - b_2)\cos p \pm w\big)v_1 + (a_1 + 3a_2 - b_1)\sin p \ v_2 \tag{2.72}$$

where v_1, v_2 are defined as before in (2.59). The flow equations once again do not take a simple form and must be solved numerically. In figure 2.3 we present some examples for the numerical solutions of the flow equations for truncation 3, which exhibit very similar features to the solutions of the flow equations for truncation 2. By fine-tuning initial conditions we can produce flows that (i) look like the Janus solutions in truncation 1, plotted in red in figure 2.3, (ii) connect $\mathcal{N} = (4,4)$ and $\mathcal{N} = (1,1)$ vacua, plotted in blue in figure 2.3, and (iii) connect two $\mathcal{N} = (1,1)$ vacua which are related by flipping signs of p, plotted in orange in figure 2.3. In the pq parametric plot given in figure 2.3(a), the location of the $\mathcal{N} = (1,1)$ vacua $p = \pm\frac{\pi}{4}$, $q = \sinh^{-1}\sqrt{2+2\sqrt{2}}$ are denoted by black dots.

The $\mathcal{N} = (1,1)$ points are again repulsive. With $g = 1/2$, the $\mathcal{N} = (1,1)$ vacuum potential is $W_0 = (1+\sqrt{2})^2/2$ and the AdS_3 length scale is $L = \sqrt{2} - 1$. The linear expansion around the vacuum yields the following mass-squares for δp and δq fluctuations,

$$m_p^2 L^2 = 1 \ , \qquad m_q^2 L^2 = 2 + 2\sqrt{2} \tag{2.73}$$

which correspond to the scaling dimensions,

$$\Delta_p = 1 + \sqrt{2} \ , \qquad \Delta_q = 2 + \sqrt{2} \tag{2.74}$$

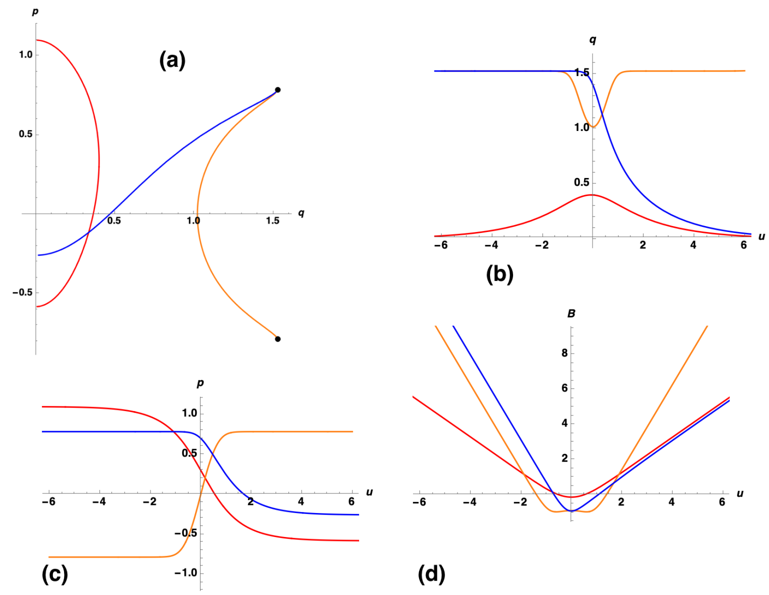


Figure 2.3: (a) pq parametric plot, (b) plot of q, (c) plot of p, (d) plot of the metric function B as functions of the AdS$_2$ slicing coordinate u for truncation 3.

The linearized flow equations around the $\mathcal{N} = (1,1)$ point are

$$\begin{aligned} \delta p' &= -(3 + 2\sqrt{2})\delta p + \cdots \\ \delta q' &= (2 + \sqrt{2})\delta q + \cdots \end{aligned} \tag{2.75}$$

which are solved by $\delta p \sim C_p e^{-(3+2\sqrt{2})u}$ and $\delta q \sim C_q e^{(2+\sqrt{2})u}$, or in terms of ρ by substituting $e^u \sim \rho^{-L}$,

$$\delta p \sim C_p \rho^{1+\sqrt{2}} \qquad\qquad \delta q \sim C_q \rho^{-\sqrt{2}} \tag{2.76}$$

These asymptotic forms are consistent with the scaling dimensions in (2.74). Again, we see that the q scalar diverges as $\rho \to 0$ unless we fine-tune the coefficient C_q to zero, which turns off the source for an operator with scaling dimension larger than 2 on the boundary CFT.

A similar counting as before shows that the flow preserves two of the four supersymmetries of the $\mathcal{N} = (1,1)$ vacuum. Therefore, we have RG-flow interfaces between a CFT with central charge $c^{(4,4)}$ and a CFT with central charge $c^{(1,1)}$, where

$$\frac{c^{(1,1)}}{c^{(4,4)}} = \sqrt{\frac{W_0^{(4,4)}}{W_0^{(1,1)}}} = \sqrt{2} - 1 \tag{2.77}$$

2.4 Discussion

In the paper, we constructed new solutions of three-dimensional gauged supergravity. The solutions produced describe interface CFTs holographically. We considered three different truncations of the scalar fields which are associated with three different supersymmetric AdS_3 vacua with $\mathcal{N} = (4,4)$ and $\mathcal{N} = (1,1)$ supersymmetry. The solution in the first truncation is a Janus solution that is very similar to the one found in [112] for a simpler three-dimensional gauged supergravity. The CFTs on both sides of the interface are deformations of the $\mathcal{N} = (4,4)$ vacuum with a source for a marginal operator turned on as well as a position-dependent expectation value for a relevant operator. The interface preserves half of the sixteen supersymmetries of the $\mathcal{N} = (4,4)$ vacuum.

The solutions for the other two truncations represent RG-flow interfaces in the sense that the solutions we find have different CFTs on each side of the interface. For example, we find solutions where the interface connects the $\mathcal{N} = (4,4)$ CFT (with a marginal operator and relevant expectation value) and the $\mathcal{N} = (1,1)$ CFT (with an irrelevant source). Note that there is no clear distinction between the UV and the IR in the RG-flow Janus solutions, since both sides put together form the boundary of the asymptotically AdS space. This is to be contrasted with a Poincaré-sliced RG-flow solution, where the AdS boundary with the larger curvature radius (or central charge) is viewed as describing the UV CFT. A irrelevant source is turned on near the $\mathcal{N} = (1,1)$ asymptotic AdS, which means that, from the perspective of the flow, this constitutes a repulsive direction. To find a flow that comes very close to the $\mathcal{N} = (1,1)$ vacuum, we have to fine-tune our initial conditions, which corresponds to fine-tuning the source of the irrelevant operator.

We have worked in truncations where the dynamics of the eight scalar fields q_i, p_i for $i = 1, 2, 3, 4$ are reduced to the dynamics of two scalars q, p, where in all three cases $q = 0$ corresponds to the $\mathcal{N} = (4, 4)$ vacuum. Hence, we could find interface CFTs of the $\mathcal{N} = (4, 4)$ CFT with one of the $\mathcal{N} = (1, 1)$ CFTs. In this truncation, we cannot find an interface solution connecting the two distinct $\mathcal{N} = (1, 1)$ vacua. For such a solution, we would have to consider the flow equations with at least four independent scalars. The fine-tuning of the initial conditions to produce the interface solution would also be more challenging.

The SO(4) × SO(4) gauging depends on a real parameter α and in this paper we have only considered the case $\alpha = 1$ which simplifies the expression of the A_i matrices and the scalar potential. We expect that the solutions for other choices of α behave qualitatively the same, since the supersymmetric vacua exist for other values of α. It would also be interesting to consider holographic observables such as the entanglement entropy around the interface or correlation functions.

CHAPTER 3

Janus and RG interfaces in three-dimensional gauged supergravity II: General α

Janus solutions provide a holographic description of interface conformal field theories. Generally, the solutions are constructed by considering an AdS_d slicing of a higher dimensional space where the other fields depend non-trivially on the slicing coordinate(s). For example the original Janus solution [64], deforms the $\mathrm{AdS}_5 \times S^5$ vacuum of type IIB and is given by an AdS_4 slicing where the dilation depends non-trivially on a single the slicing coordinate and approaches two different values on the two boundary components. The solution is dual to an interface of $N = 4$ super Yang-Mills theory where the coupling g_{YM} jumps across a codimension one interface [86]. More general Janus solutions preserving supersymmetry were constructed as $\mathrm{AdS}_4 \times S^2 \times S^2$ space warped over a Riemann surface [87]. These solutions are dual to supersymmetric interface theories in $N = 4$ SYM [88–90]. For other Janus solutions in ten and eleven dimensions, see e.g. [69, 71, 91, 113]. In general, constructing such solutions is quite difficult due to the fact that the supersymmetry variations, as well the equations of motion, depend on more than one warping coordinate and the resulting equations are nonlinear partial differential equations. A useful approach is to construct Janus solutions in lower dimensional gauged supergravities (see for example [70, 72, 92–97, 114–116]). Such solutions are often easier to obtain, can be uplifted to ten or eleven dimensions or can be used to explore qualitative features of Janus solution in a bottom-up approach.

In lower dimensional gauged supergravities it is often the case that in addition to a maximally supersymmetric AdS vacuum there are extrema with a reduced amount of supersymmetry. One of the aims of the present paper is to construct holographic Janus solutions which correspond to RG interfaces [100], between different AdS vacua[1]. This paper is a continuation of the work presented [1], which considered three-dimensional $\mathcal{N} = 8$ gauged supergravity with $n = 4$ vector multiplets, first discussed in [104]. This theory has an AdS_3 vacuum with maximal $\mathcal{N} = (4, 4)$ supersymmetry as well as two families of AdS_3 vacua with $\mathcal{N} = (1, 1)$ supersymmetry [105]. The gauged supergravity has a parameter α on which the embedding tensor for the gauged supergravity depends. For this theory the dual superconformal algebra of the $\mathcal{N} = (4, 4)$ vacuum is given by the "large" superconformal algebra $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$, and the three-dimensional supergravity is believed to be a truncation of M-theory on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ [106–109]. In the previous paper we considered the special case of $\alpha = 1$ for which the explicit expressions become simpler. Here we will analyze the case for general α, using both analytical and numerical methods.

The structure of this paper is as follows: In section 4.1 we review the three dimensional gauged supergravity with $n = 4$ vector multiplets used in the paper. We consider three truncations where the gauge fields as well as some scalars can consistently be set to zero and fix the $N = (1, 1)$ vacua for general α. In section 4.2 we derive the BPS flow equations for an AdS_2 sliced Janus ansatz, this generalizes and streamlines the discussion of [1]. In section 4.3 we derive the flow equations and integrate them numerically for the three truncations. For the second and third truncations where $N = (1, 1)$ AdS vacua exists we present examples of RG-flow interfaces. In section 4.4 we use the solutions to calculate some holographic observables. In particular we determine the masses of the fluctuating scalars around the $N = (1, 1)$ vacua, the mass squared is positive and quite large, which means that the scalar fluctuations around fixed point are repulsive in the UV. This implies that the initial conditions have to be fine tuned in order to reach the fixed point. We discuss our results and possible directions for future research in section 4.5.

[1]See [101–103] for other examples of holographic RG-flow interfaces.

3.1 Three-dimensional $\mathcal{N} = 8$ gauged supergravity

In this section, we review the $\mathcal{N} = 8$ gauged supergravity first constructed in [104] mainly following the conventions of [1]. The bosonic field content consists of a graviton $g_{\mu\nu}$, Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$, and scalars fields living in a $G/H = \mathrm{SO}(8,n)/\,\mathrm{SO}(8)\times\mathrm{SO}(n)$ coset, which has $8n$ degrees of freedom before gauging. The scalar fields are parametrized by a G-valued matrix $L(x)$ in the vector representation, which transforms under H and the gauge group $G_0 \subseteq G$ by right and left multiplication of group elements respectively.

$$L(x) \to g_0(x)L(x)h^{-1}(x) \tag{3.1}$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations. In this paper we use the following index conventions:

- $I, J, \ldots = 1, 2, \ldots, 8$ for SO(8).
- $r, s, \ldots = 9, 10, \ldots, n + 8$ for SO(n).
- $\bar{I}, \bar{J}, \ldots = 1, 2, \ldots, n + 8$ for SO($8, n$).
- $\mathcal{M}, \mathcal{N}, \ldots$ for generators of SO($8, n$).

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the noncompact generators. Explicitly, the generators of the vector representation are given by

$$(t^{\bar{I}\bar{J}})^{\bar{K}}{}_{\bar{L}} = \eta^{\bar{I}\bar{K}}\delta^{\bar{J}}_{\bar{L}} - \eta^{\bar{J}\bar{K}}\delta^{\bar{I}}_{\bar{L}} \tag{3.2}$$

where $\eta^{\bar{I}\bar{J}} = \mathrm{diag}(++++++++-\cdots)$ is an SO($8, n$)-invariant tensor. These generators satisfy the typical SO($8, n$) commutation relations,

$$[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}] = 2\Big(\eta^{\bar{I}[\bar{K}}t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}}t^{\bar{L}]\bar{I}}\Big) \tag{3.3}$$

The gauging of the supergravity is characterized by an embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ (which has to satisfy various identities [110] in order to define a consistent theory) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry transformations and action depending on the gauge coupling g. We will look at the particular case in [105] where $n \geq 4$ and the gauged subgroup is the $G_0 =$ SO(4) × SO(4) subset of the SO(8) ⊂ SO(8, n). The embedding tensor has the non vanishing entries,[2]

$$\Theta_{\bar{I}\bar{J},\bar{K}\bar{L}} = \begin{cases} \alpha \varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{1,2,3,4\} \\ \varepsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{5,6,7,8\} \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

Note that the gauging depends on a real parameter α. As discussed in [105], the maximally supersymmetric AdS_3 vacuum is obtained where the potential has a local maximum at $L(x) = I_{8n}$, and it has an isometry group,

$$D^1(2,1;\alpha) \times D^1(2,1;\alpha) \tag{3.5}$$

which corresponds to the family of "large" superconformal algebras of the dual SCFT. The bosonic subalgebra of $D^1(2,1,\alpha)$ is $SL(2,R) \times SU(2) \times SU(2)$. Representations are labeled by the $SU(2)$ quantum numbers $l^{\pm}$and the conformal weight h (which can be obtained from $\Delta = h + \bar{h}$ and the relation between Δ and $m^2 L_0^2$), and can be denoted by (l^+, l^-, h). Unitarity implies the bound $h \geq \gamma l^- + (1-\gamma) l^+$. In this paper we generalize the analysis of [1] where the case $\alpha = 1$ was considered to the case of general α. Note that in the spacial case $\alpha = 1$ the super algebra becomes more familiar $D^1(2,1;1) = \mathrm{OSp}(4|2)$.

From the embedding tensor, the G_0-covariant currents can be obtained,

$$L^{-1}(\partial_\mu + g\Theta_{\mathcal{M}\mathcal{N}} B_\mu^{\mathcal{M}} t^{\mathcal{N}})L = \frac{1}{2}\mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir} \tag{3.6}$$

It is convenient to define the ${\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}}$ tensors,

$$L^{-1} t^{\mathcal{M}} L = {\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}} t^{\mathcal{A}} = \frac{1}{2}{\mathcal{V}^{\mathcal{M}}}_{IJ} X^{IJ} + \frac{1}{2}{\mathcal{V}^{\mathcal{M}}}_{rs} X^{rs} + {\mathcal{V}^{\mathcal{M}}}_{Ir} Y^{Ir} \tag{3.7}$$

[2]We use the conventions $\varepsilon_{1234} = \varepsilon_{5678} = 1$.

and the T-tensor,

$$T_{\mathcal{A}|\mathcal{B}} = \Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \tag{3.8}$$

The T-tensor is used to construct the tensors $A_{1,2,3}$ which will appear in the scalar potential and the supersymmetry transformations,

$$\begin{aligned}
A_1^{AB} &= -\frac{1}{48}\Gamma^{IJKL}_{AB}T_{IJ|KL} \\
A_2^{A\dot{A}r} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}}T_{IJ|Kr} \\
A_3^{\dot{A}r\dot{B}s} &= \frac{1}{48}\delta^{rs}\Gamma^{IJKL}_{\dot{A}\dot{B}}T_{IJ|KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}}T_{IJ|rs}
\end{aligned} \tag{3.9}$$

where A, B and $\dot{A}, \dot{B}$ are SO(8)-spinor indices. Our conventions for the SO(8) Gamma matrices are presented in the appendix.

Here we choose the spacetime signature $\eta^{ab} = \text{diag}(+--)$ as mostly negative. The bosonic Lagrangian and scalar potential are given by

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{bos}} &= -\frac{1}{4}R + \frac{1}{4}\mathcal{P}^{Ir}_{\mu}\mathcal{P}^{Ir\,\mu} + V - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho}g\Theta_{\mathcal{M}\mathcal{N}}B^{\mathcal{M}}_{\mu}\left(\partial_\nu B^{\mathcal{N}}_{\rho} + \frac{1}{3}g\Theta_{\mathcal{K}\mathcal{L}}f^{\mathcal{N}\mathcal{K}}{}_{\mathcal{P}}B^{\mathcal{L}}_{\nu}B^{\mathcal{P}}_{\rho}\right) \\
V &= \frac{1}{4}g^2\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\right)
\end{aligned} \tag{3.10}$$

The supersymmetry variations take the following form

$$\begin{aligned}
\delta\chi^{\dot{A}r} &= \frac{1}{2}i\Gamma^{I}_{A\dot{A}}\gamma^{\mu}\varepsilon^{A}\mathcal{P}^{Ir}_{\mu} + gA_2^{A\dot{A}r}\varepsilon^{A} \\
\delta\psi^{A}_{\mu} &= \left(\partial_\mu\varepsilon^{A} + \frac{1}{4}\omega^{ab}_{\mu}\gamma_{ab}\varepsilon^{A} + \frac{1}{4}\mathcal{Q}^{IJ}_{\mu}\Gamma^{IJ}_{AB}\varepsilon^{B}\right) + igA_1^{AB}\gamma_\mu\varepsilon^{B}
\end{aligned} \tag{3.11}$$

The Einstein equations of motion are

$$R_{\mu\nu} - \mathcal{P}^{Ir}_{\mu}\mathcal{P}^{Ir}_{\nu} - 4Vg_{\mu\nu} = 0 \tag{3.12}$$

and the gauge field equations of motion are

$$e\mathcal{P}^{Ir\,\lambda}\Theta_{\mathcal{Q}\mathcal{M}}\mathcal{V}^{\mathcal{M}}{}_{Ir} = \varepsilon^{\lambda\mu\nu}\left(\Theta_{\mathcal{Q}\mathcal{M}}\partial_\mu B^{\mathcal{M}}_{\nu} + \frac{1}{6}gB^{\mathcal{M}}_{\mu}B^{\mathcal{K}}_{\nu}\left(\Theta_{\mathcal{M}\mathcal{N}}\Theta_{\mathcal{K}\mathcal{L}}f^{\mathcal{N}\mathcal{L}}{}_{\mathcal{Q}} + 2\Theta_{\mathcal{M}\mathcal{N}}f^{\mathcal{L}\mathcal{N}}{}_{\mathcal{K}}\Theta_{\mathcal{L}\mathcal{Q}}\right)\right) \tag{3.13}$$

3.1.1 The $n = 4$ case

The smallest number of matter multiplets where multiple supersymmetric vacua exist is $n = 4$. The symmetries of the theory are a local $G_0 = \mathrm{SO}(4) \times \mathrm{SO}(4)$ and a global $\mathrm{SO}(n)$ with $n = 4$. Consequently, the scalar potential only depends on $8 \cdot 4 - 3 \cdot 6 = 14$ fields out of the original 32. Moreover, a further consistent truncation outlined in [105] is performed where the coset representative depends only on eight of the fourteen scalars.

$$L = \begin{pmatrix} \cos A & \sin A \cosh B & \sin A \sinh B \\ -\sin A & \cos A \cosh B & \cos A \sinh B \\ 0 & \sinh B & \cosh B \end{pmatrix}$$

$$A = \mathrm{diag}(p_1, p_2, p_3, p_4)\ , \qquad B = \mathrm{diag}(q_1, q_2, q_3, q_4) \tag{3.14}$$

We will not display the general form of the tensors A_1 and A_2 defined in (3.9) here. The scalar potential has terms up to order α^2.

$$\begin{aligned} g^{-2}V = &\frac{1}{2} + \frac{1}{4}\sum_i x_i^2 - \frac{1}{4}\sum_{i<j<k} x_i^2 x_j^2 x_k^2 - \frac{1}{2}\prod_i x_i^2 + \alpha\left(-\prod_i x_i y_i + \prod_i \sqrt{1 + x_i^2 + y_i^2}\right) \\ &+ \alpha^2\left(\frac{1}{2} + \frac{1}{4}\sum_i y_i^2 - \frac{1}{4}\sum_{i<j<k} y_i^2 y_j^2 y_k^2 - \frac{1}{2}\prod_i y_i^2\right) \end{aligned} \tag{3.15}$$

where all indices run form 1 to 4 unless otherwise indicated and we used the following defined scalar fields

$$x_i = \cos p_i \sinh q_i\ , \qquad y_i = \sin p_i \sinh q_i \tag{3.16}$$

The $\mathcal{Q}_\mu$ and $\mathcal{P}_\mu$ currents do not depend on α, excluding the $g\Theta_{\mathcal{M}\mathcal{N}}B_\mu^{\mathcal{M}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{A}}$ term, they are given by

$$\mathcal{Q}_\mu^{IJ} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cosh q_1\partial_\mu p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cosh q_2\partial_\mu p_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_3\partial_\mu p_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_4\partial_\mu p_4 \\ -\cosh q_1\partial_\mu p_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\cosh q_2\partial_\mu p_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cosh q_3\partial_\mu p_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cosh q_4\partial_\mu p_4 & 0 & 0 & 0 & 0 \end{pmatrix}_{IJ}$$

$$\mathcal{Q}_\mu^{rs} = 0$$

$$\mathcal{P}_\mu^{Ir} = \begin{pmatrix} \sinh q_1\partial_\mu p_1 & 0 & 0 & 0 \\ 0 & \sinh q_2\partial_\mu p_2 & 0 & 0 \\ 0 & 0 & \sinh q_3\partial_\mu p_3 & 0 \\ 0 & 0 & 0 & \sinh q_4\partial_\mu p_4 \\ \partial_\mu q_1 & 0 & 0 & 0 \\ 0 & \partial_\mu q_2 & 0 & 0 \\ 0 & 0 & \partial_\mu q_3 & 0 \\ 0 & 0 & 0 & \partial_\mu q_4 \end{pmatrix}_{Ir} \tag{3.17}$$

Using these matrices, we can check that the combination $\mathcal{P}_\mu^{Ir}\mathcal{V}^{JK}{}_{Ir}$ vanishes whenever the indices $J, K \in \{1,2,3,4\}$ or $J, K \in \{5,6,7,8\}$. This implies that there is no source for $B_\mu^{\mathcal{M}}$ in the gauge field equation of motion (3.13), so it is consistent to set $B_\mu^{\mathcal{M}} = 0$. We will make this choice from now on.

The kinetic term for the scalars in the action (3.10) can be expressed in terms of the x_i and y_i using the relations (3.16) and take the form

$$\frac{1}{4}\mathcal{P}_\mu^{Ir}\mathcal{P}^{Ir\,\mu} = -\frac{1}{4}\sum_{i=1}^{4}\frac{1}{1+x_i^2+y_i^2}\Big((1+y_i^2)(\partial_\mu\partial^\mu x_i - 2x_iy_i\partial_\mu x_i\partial^\mu y_i' + (1+x_i^2)\partial y_i\partial^\mu y_i\Big) \tag{3.18}$$

This expression will be needed for determining masses of the fluctuations of the scalar fields around the supersymmetric vacua.

3.1.2 Truncations and supersymmetric AdS$_3$ vacua

In order to make our analysis more tractable, we make further truncations to reduce the number of independent scalar fields. Below we consider three consistent truncations, which together explore the AdS$_3$ vacua with $\mathcal{N} = (4,4)$ and $\mathcal{N} = (1,1)$ supersymmetry. All of the results are generalizations of the $\alpha = 1$ case discussed in [1].

3.1.2.1 Truncation 1

The first truncation is given by denoting $q_1 = q$, $p_1 = p$ and setting all remaining $q_i = p_i = 0$ for $i = 2, 3, 4$. The scalar potential is

$$V = \frac{g^2}{4}\left(2(1+\alpha^2) + 4\alpha\cosh q + (\cos^2 p + \alpha^2 \sin^2 p)\sinh^2 q\right) \tag{3.19}$$

The $\mathcal{N} = (4,4)$ vacuum is given by setting $q = 0$ and the vacuum potential is $V_0 = \frac{1}{2}g^2(1+\alpha)^2$. In the x, y coordinates the $\mathcal{N} = (4,4)$ vacuum is given by $x_i = y_i = 0$. This is the only supersymmetric vacuum for this truncation. We note that for the choice $\alpha = 1$ the potential is independent of the scalar field p. We note we will chose $g = 1/(1+\alpha)$ in order to set the potential at the $N = (4,4)$ vacuum to be $V_0 = \frac{1}{2}$, which corresponds to a unit radius AdS_3.

3.1.2.2 Truncation 2

The second truncation is given by setting all the q and p equal respectively, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3, 4$. The scalar potential becomes

$$\begin{aligned} V = \frac{g^2}{2}\Big\{&(1-\cos^2 p\sinh^2 q)(1+\cos^2 p\sinh^2 q)^3 + \alpha^2(1-\sin^2 p\sinh^2 q)(1+\sin^2 p\sinh^2 q)^3 \\ &+ \alpha(2 + 4\sinh^2 q + 2\sinh^4 q - 2\sin^4 p\cos^4 p\sinh^8 q)\Big\} \end{aligned} \tag{3.20}$$

or in terms of the x, y fields, the potential will take the following form

$$V = \frac{g^2}{2}\Big\{(x^2-1)(x^2+1)^3 + \alpha^2(y^2-1)(y^2+1)^3 + 2\alpha\big(2x^2(1+y^2) + (1+y^2)^2 - x^4(y^4-1)\big)\Big\} \tag{3.21}$$

As before the $\mathcal{N} = (4,4)$ vacuum is given by $q = 0$ or $x = y = 0$. There are $\mathcal{N} = (1,1)$ vacua which are located at

$$\begin{aligned} x &= \pm\frac{1}{6\sqrt{3}\alpha}\left(-12\alpha^2 - \frac{2^{\frac{2}{3}}Y^{\frac{1}{3}}(Y + 2\alpha^2(-3+8\alpha))}{(3+2\alpha)} - \frac{2^{\frac{1}{3}}y^{\frac{2}{3}}(Y + 2\alpha(-18+\alpha(-15+8\alpha)))}{(3+2\alpha)^2}\right)^{\frac{1}{2}} \\ y &= \pm\frac{1}{3}\Big(-1 + \frac{2^{\frac{2}{3}}Y^{\frac{1}{3}}}{\alpha} + \frac{4\,2^{\frac{1}{3}}(3+2\alpha)}{Y^{\frac{1}{3}}}\Big)^{\frac{1}{2}} \end{aligned} \tag{3.22}$$

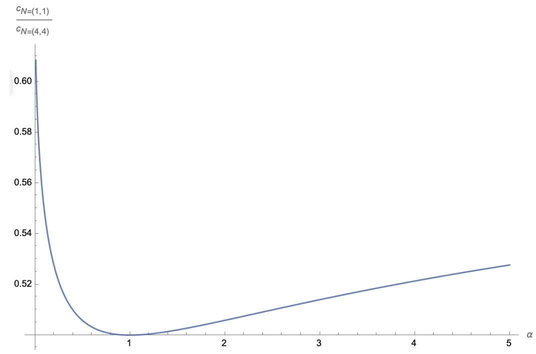


Figure 3.1: Ration of central charges for the $N = (1, 1)$ and $N = (4, 4)$ vacua.

where Y is given by

$$Y = 3i\alpha^{\frac{3}{2}}\sqrt{96 + 3\alpha(61 + 32\alpha)} - \alpha^2(9 + 16\alpha) \tag{3.23}$$

The central charge of the dual CFT is related to the AdS radius and the value of the potential V_0 at its minimum

$$c = \frac{R_{AdS}}{4G_N} = \frac{1}{\sqrt{2V_0}}\frac{1}{4G_N} \tag{3.24}$$

Choosing $g = 1/(1 + \alpha)$ sets the AdS radius of the $N = (4, 4)$ vacuum to one and the ratio of the central charge of the $N = (4, 4)$ to the $N = (1, 1)$ vacuum as a function of α becomes

$$\frac{c_{N=(1,1)}}{c_{N=(4,4)}} = \frac{1}{\sqrt{2V_0^{N=(1,1)}(\alpha)}} \tag{3.25}$$

The expressions derived (3.22) are not very illuminating and we present a plot of the ratio of the central charges for the two vacua in the figure 3.1. It is interesting to note that the ratio of central charges is minimized for the special value $\alpha = 1$.

3.1.2.3 Truncation 3

The third truncation is given by setting the first three q and p equal, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3$, and setting the remaining $q_4 = p_4 = 0$. The scalar potential is

$$V = \frac{g^2}{4}\Big(2 + 3\cos^2 p \sinh^2 q - \cos^6 p \sinh^6 q + 4\alpha \cosh^3 q + \alpha^2(2 + 3\sin^2 p \sinh^2 q - \sin^6 p \sinh^6 q)\Big) \tag{3.26}$$

or in the x, y variables

$$V = \frac{g^2}{4}\Big((2 + 3x^2 - x^6) + 4\alpha(1 + x^2 + y^2)^{\frac{3}{2}} + \alpha^2(2 + 3y^2 - y^6)\Big) \tag{3.27}$$

The $\mathcal{N} = (4, 4)$ vacuum is given by $q = 0$ or $x = y = 0$ as before, and $\mathcal{N} = (1, 1)$ vacua can be determined by finding the extrema for the potential (3.27) away from the origin.

$$y = \pm\sqrt{\frac{1}{2}}\left(1 + (\alpha)\sqrt{1 - \frac{4}{X^{\frac{1}{3}}} + \frac{2X^{\frac{1}{3}}}{3\alpha^2}} + \sqrt{2 + \frac{4}{X^{\frac{1}{3}}} - \frac{2X^{\frac{1}{3}}}{3\alpha^2} + \frac{2(\alpha^2 - 4)}{\alpha^2\sqrt{1 - \frac{4}{X^{\frac{1}{3}}} + \frac{2X^{\frac{1}{3}}}{3\alpha^2}}}}\right)^{\frac{1}{2}}$$

$$x = \pm\left(\frac{\alpha^2}{4}(y^4 - 1)^4 - 1 - y^2\right)^{\frac{1}{2}} \tag{3.28}$$

where we used the abriviation

$$X = 3\alpha^2\big(9 - 9\alpha^2 + \sqrt{81 - 138\alpha^2 + 81\alpha^4}\big) \tag{3.29}$$

The $\varepsilon(\alpha)$ is a sign which selects a branch of the solutions which gives real x, y depending on α and we have $\varepsilon(\alpha) = +1$ for $\alpha < 2$ and $\varepsilon(\alpha) = -1$ for $\alpha > 2$. We can plot the ratio of the central charges which is given by (3.25), determined from the potential (3.27). We note that the qualitative behavior of the ratio for truncation 2 and 3 is very similar, in particular the central charge for the $N = (1, 1)$ vacuum is minimized at $\alpha = 1$.

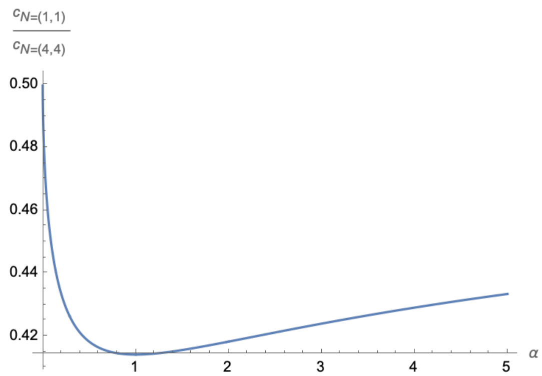


Figure 3.2: Ration of central charges for the $N = (1,1)$ and $N = (4,4)$ vacua.

3.2 Janus flow equations

In this section we will derive the BPS flow equations, expanding on the construction in our previous paper [1]. The Janus ansatz for the bosonic fields is give by

$$\mathrm{d}s^2 = e^{2B(u)}\left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2 \;, \qquad q_i = q_i(u) \;, \qquad p_i = p_i(u) \tag{3.30}$$

The Chern-Simons gauge fields is set to zero $B_\mu^{\mathcal{M}} = 0$.We will check that the source term on the right hand side of the gauge field equation of motion (3.13) is zero for the solutions considered in this paper.

The gravitino supersymmetry variation $\delta\psi_\mu^A = 0$ is

$$\begin{aligned}
0 &= \partial_t\varepsilon + \frac{1}{2z}\gamma_0\big(\gamma_1 - B'e^B\gamma_2 + 2ige^B A_1\big)\varepsilon \\
0 &= \partial_z\varepsilon + \frac{1}{2z}\gamma_1\big(-B'e^B\gamma_2 + 2ige^B A_1\big)\varepsilon \\
0 &= \partial_u\varepsilon + \frac{1}{4}\mathcal{Q}_u^{IJ}\Gamma^{IJ}\varepsilon + ig\gamma_2 A_1\varepsilon
\end{aligned} \tag{3.31}$$

where we have suppressed the SO(8)-spinor indices of ε^A and A_1^{AB}. The spin-$\frac{1}{2}$ variation $\delta\chi^{\dot{A}r} = 0$ is

$$\left(-\frac{i}{2}\Gamma^I\mathcal{P}_u^{Ir}\gamma_2 + gA_2^r\right)_{A\dot{A}}\varepsilon^A = 0 \;, \qquad r = 9, 10, \ldots, 8+n \tag{3.32}$$

The matrix A_1 defined in (3.9) has eigenvectors

$$A_1^{AB} n_\pm^{(i)B} = \pm w_i n_\pm^{(i)A} \ , \qquad i = 1, 2, 3, 4 \tag{3.33}$$

For a supersymmetric AdS_3 vacuum the eigenvalue w_i is related to the value of the potential evaluated at the vacuum via

$$w_{\rm vac}^2 = \frac{V_{\rm vac}}{2g^2} \tag{3.34}$$

and the associated eigenvectors $n_\pm^{(i)}$ determine the supersymmetries of the vacuum. For the $\mathcal{N} = (4, 4)$ vacuum the $w_i, i = 1, \cdots 4$ all satisfy (3.34) and hence the vacuum preserves eight supersymmetries. For the $\mathcal{N} = (1, 1)$ vacuum only one of the for $n_\pm^{(i)}$ and w_i satisfies (3.34). In the following we drop the index $^{(i)}$ to denote the supersymmetric eigenvalue w and the eigenvector $n_\pm^A$.

The general ansatz for unbroken supersymmetry ε^A for the Janus solution is given by

$$\varepsilon^A = \big(f_+ n_+ + f_- n_-^A\big)\zeta_+ + \big(g_+ n_+^A + g_- n_-^A\big)\zeta_- \tag{3.35}$$

where $\zeta_\pm$ are Killing spinors for a unit radius AdS_2

$$D_\mu \zeta_\eta = i\frac{\eta}{2}\gamma_\mu \zeta_\eta \ , \qquad \mu = t, z, \quad \eta = \pm 1 \tag{3.36}$$

3.2.1 Gravitino variation

The t, z components of the gravitino variation can be expressed as follows by using the properties of the AdS$_2$ Killing spinors,

$$\begin{aligned} 0 = {} & i\big\{\big(f_+ n_+^A + f_- n_-^A\big)\zeta_+ - \big(g_+ n_+^A + g_- n_-^A\big)\zeta_-\big\} & (3.37) \\ & + iB' e^B i\gamma_2 \Big\{\big(f_+ n_+^A + f_- n_-^A\big)\zeta_+ + \big(g_+ n_+^A + g_- n_-^A\big)\zeta_-\Big\} & \\ & + 2igwe^B \Big\{\big(f_+ n_+^A - f_- n_-^A\big)\zeta_+ + \big(g_+ n_+^A - g_- n_-^A\big)\zeta_-\Big\} & (3.38) \end{aligned}$$

Using $i\gamma_2\zeta_\eta = \zeta_{-\eta}$ and the linear independence of the $n_\pm$ and $\zeta_\pm$, one obtains a set of equations,

$$\begin{aligned} f_+ + B'e^B g_+ + 2gwe^B f_+ &= 0 \\ -g_+ + B'e^B f_+ + 2gwe^B g_+ &= 0 \\ f_- + B'e^B g_- - 2gwe^B f_- &= 0 \\ -g_- + B'e^B f_- - 2gwe^B g_- &= 0 \end{aligned} \tag{3.39}$$

It is convenient to define the following expressions

$$\gamma(u) = \sqrt{1 - \frac{e^{-2B}}{4g^2w^2}}\ , \qquad \sqrt{1-\gamma^2(u)} = \frac{e^{-B}}{2gw} \tag{3.40}$$

The equations (3.39) can then be solved by

$$f_+ = \frac{\sqrt{1-\gamma^2}-1}{\gamma} g_+\ , \qquad f_- = \frac{\sqrt{1-\gamma^2}+1}{\gamma} g_- \tag{3.41}$$

if the integrability condition

$$\begin{aligned} B' &= \pm\sqrt{4g^2w^2 - e^{-2B}} \\ &= \pm 2gw\gamma \end{aligned} \tag{3.42}$$

is satisfied. This equations provides us with a differential equation for the metric factor B.

3.2.2 Spin $\frac{1}{2}$ variation

The spin-$\frac{1}{2}$ variation (3.32) takes the following of a projector

$$\Big(M^{AB} i\gamma_2 + \delta^{AB}\Big)\varepsilon^B = 0 \tag{3.43}$$

where

$$M^{(r)}_{AB} = -\frac{1}{2g}\Big(\Gamma^I \mathcal{P}^{Ir}_u (A^r_2)^{-1}\Big)^T_{AB} \tag{3.44}$$

Note that there is a projector for each r, which all have to be satisfied and the resulting flow equations are mutually consistent for a supersymmetric Janus solution to exist. This analysis will be performed for the particular truncations presented in section 3.1.2.

Inserting ε^A is given by (3.35) and into the spin $\frac{1}{2}$ projector gives

$$\begin{aligned} 0 = &\big(f_+ n_+^A + f_- n_-^A\big)\zeta_+ + \big(g_+ n_+^A + g_- n_-^A\big)\zeta_- \\ &+ M^{AB} i\gamma_2 \Big\{ \big(f_+ n_+^B + f_- n_-^B\big)\zeta_+ + \big(g_+ n_+^B + g_- n_-^B\big)\zeta_- \Big\} \end{aligned} \tag{3.45}$$

We have dropped the index r for notational convenience. Using the fact that the two dimensional Killing spinors are orthogonal we can project (3.45) onto the $n_\pm$ and $\zeta_\pm$ components . This produces four equations

$$\begin{aligned} f_+ n_+^2 + M_{++} g_+ + M_{+-} g_- &= 0 \\ g_+ n_+^2 + M_{++} f_+ + M_{+-} f_- &= 0 \\ f_- n_-^2 + M_{+-} g_+ + M_{--} g_- &= 0 \\ g_+ n_-^2 + M_{+-} f_+ + M_{--} f_- &= 0 \end{aligned} \tag{3.46}$$

where we denoted $n_\pm^2 = n_\pm^A n_\pm^A$ and we define

$$M_{++} = n_+^A M^{AB} n_+^B \ , \qquad M_{--} = n_-^A M^{AB} n_-^B \ , \qquad M_{+-} = M_{-+} = n_+^A M^{AB} n_-^B \tag{3.47}$$

If there is more than one $n_\pm$ (as in truncation 1) one has to choose linear combinations for which $M_{\pm\pm}, M_{\pm\mp}$ take the same form for all $n_\pm^{(i)}$, which is a consistency condition. Using (3.41) it can be shown that equations (3.46) can only be satisfied if we have

$$M_{++} = \gamma n_+^2 \ , \qquad M_{--} = -\gamma n_-^2 \ , \qquad M_{+-} = M_{-+} = \sqrt{1-\gamma^2}\sqrt{n_+^2 n_-^2} \tag{3.48}$$

In all cases we consider, the M_{--} equation is automatically satisfied if the M_{++} is satisfied. Hence (3.48) provides two independent equations. It follows from (3.44) that these equations are linear in the first derivatives of the scalar fields and provide the BPS flow equations for the scalars. The complete set of flow equations is given by these equations and the flow equation for the metric factor (3.42), coming from the gravitino variation.

3.3 Janus and RG-flow solutions

In this section we obtain the flow equations and solve them numerically for the three truncations considered in this paper. Since the first truncation does not have $N = (1, 1)$ vacua the BPS flows will correspond to Janus solutions interpolating between $N = (4, 4)$ vacua. For the two other truncations we find Janus as well as RG-flow interface solutions.

3.3.1 Truncation 1

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & -a & 0 & 0 & 0 & 0 & b \\ 0 & -a & 0 & 0 & -b & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 & -b & 0 & 0 \\ 0 & 0 & -b & 0 & 0 & 0 & 0 & -a \\ 0 & 0 & 0 & -b & 0 & 0 & a & 0 \\ b & 0 & 0 & 0 & 0 & a & 0 & 0 \\ 0 & b & 0 & 0 & -a & 0 & 0 & 0 \end{pmatrix} \tag{3.49}$$

where

$$a = \frac{1}{2}\cos p(\alpha + \cosh q), \quad b = \frac{1}{2}\sin p(1 + \alpha \cosh q) \tag{3.50}$$

The eigenvalue of A_1 are $\pm w_0$ which is given by

$$w_0 = \sqrt{a^2 + b^2} = \frac{1}{2}\sqrt{\cos^2 p(\alpha + \cosh q)^2 + \sin^2 p(1 + \alpha \cosh q)^2} \tag{3.51}$$

The eigenvectors are given by

$$n_\pm^{(1)} = \begin{pmatrix} 0 \\ \frac{a\pm w_0}{b} \\ \frac{-a\mp w_0}{b} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad n_\pm^{(2)} = \begin{pmatrix} 0 \\ \frac{-a\pm w_0}{b} \\ \frac{-a\pm w_0}{b} \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad n_\pm^{(3)} = \begin{pmatrix} \frac{-a\pm w_0}{b} \\ 0 \\ 0 \\ \frac{a\mp w_0}{b} \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad , n_\pm^{(4)} = \begin{pmatrix} \frac{a\pm w_0}{b} \\ 0 \\ 0 \\ \frac{a\pm w_0}{b} \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \tag{3.52}$$

The matrix M^{AB} defined in (3.44) takes the following form for the truncation 1

$$M = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 & -m_2 & 0 \\ 0 & 0 & -m_1 & 0 & 0 & 0 & 0 & -m_2 \\ 0 & -m_1 & 0 & 0 & m_2 & 0 & 0 & 0 \\ m_1 & 0 & 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & m_2 & 0 & 0 & 0 & 0 & -m_1 \\ 0 & 0 & 0 & m_2 & 0 & 0 & m_1 & 0 \\ -m_2 & 0 & 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & -m_2 & 0 & 0 & -m_1 & 0 & 0 & 0 \end{pmatrix} \tag{3.53}$$

with

$$m_1 = \frac{\alpha \sin p\, p' + \cos p \operatorname{csch} q\, q'}{g(\cos^2 p + \alpha^2 \sin^2 p)}, \qquad m_2 = \frac{\cos p\, p' - \alpha \sin p \operatorname{csch} q\, q'}{g(\cos^2 p + \alpha^2 \sin^2 p)}, \tag{3.54}$$

Using the definitions (3.50) and (3.51) the flow equations (3.48) for the scalars p, q and

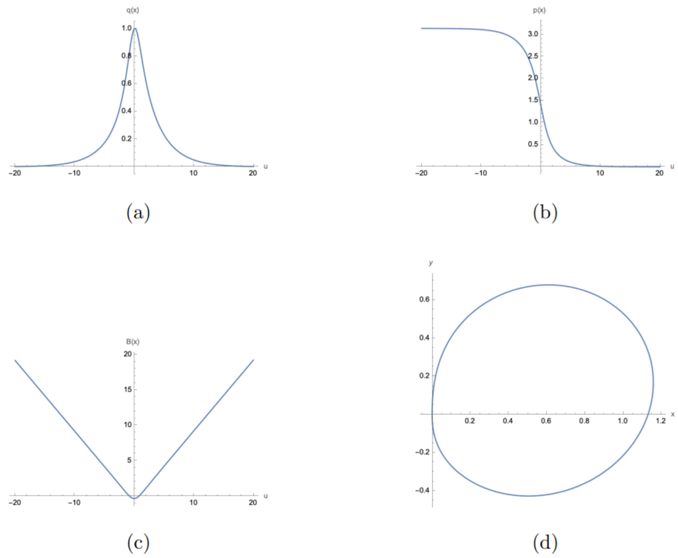


Figure 3.3: (a)-(c) plots of p, q, B respectively, (d) parametric plot of the Janus flow in the x, y variables. The initial conditions are $q(0) = 1.0$ and $p(0) = 1.5$ and $\alpha = 2.3$.

the metric function B (3.42) can be written relatively compactly

$$\begin{aligned} p' &= \frac{g}{w_0}\Big(\alpha(a\gamma - b\sqrt{1-\gamma^2})\sin p - (b\gamma + a\sqrt{1-\gamma^2})\cos p\Big) \\ q' &= \frac{g\sinh q}{w_0}\Big(\alpha(b\gamma + a\sqrt{1-\gamma^2})\sin p + (a\gamma - b\sqrt{1-\gamma^2})\cos p\Big) \\ B' &= \pm\sqrt{4g^2w_0^2 - e^{-2B}} \end{aligned} \tag{3.55}$$

This system of ordinary differential equations can only be integrated numerically. We will choose the coordinate u such that the turning point of the metric function where $B'(u) = 0$ is located at $u = 0$. We then use the BPS equations (3.55) to determine $p'(0)$, $q'(0)$ and $B(0)$ for a given $q(0)$ and $p(0)$. We then integrate the equations of motion following from the variation of the Lagrangian (3.10). This means that all our solutions depend on two initial conditions $q(0)$ and $p(0)$. We have given an illustrative example of the flows we can obtain in figure 3.3.

3.3.2 Truncation 2

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{a+c}{4} & 0 & 0 & 0 & 0 \\ 0 & -\frac{c}{2} & \frac{-a+c}{4} & 0 & b & 0 & 0 & b \\ 0 & \frac{-a+c}{4} & -\frac{c}{2} & 0 & -b & 0 & 0 & -b \\ \frac{a+c}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b & -b & 0 & \frac{c}{2} & 0 & 0 & \frac{-a+c}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{a+c}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a+c}{4} & 0 & 0 \\ 0 & b & -b & 0 & \frac{-a+c}{4} & 0 & 0 & \frac{c}{2} \end{pmatrix} \tag{3.56}$$

where

$$\begin{aligned} a &= 2\cos^4 p(\alpha \cosh^4 q) + 2\sin^4 p(1 + \alpha \cosh^4 q) \\ b &= \sin p \cos p \cosh q\big(\cos^2 p(\alpha + \cosh^2 q) - \sin^2 p(1 + \alpha \cosh^2 q)\big) \\ c &= (1+\alpha)\cosh^2 q \sin^2 2p \end{aligned} \tag{3.57}$$

The eigenvectors $n_{\pm}^{(1)}$ of A_1 with eigenvalues $\pm w_0$ corresponding to the unbroken $N = (1,1)$ supersymmetries are given by

$$n_{\pm}^{(1)} = \begin{pmatrix} 0 \\ \frac{a-3c\pm 4w_0}{8b} \\ -\frac{a-3c\pm 4w_0}{8b} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad w_0 = \frac{1}{4}\sqrt{64b^2 + (3c-a)^2} \tag{3.58}$$

We have checked that the extremum (3.22) does satisfy the supersymmetry condition (3.34) for the w_0 defined above and hence corresponds to an AdS vacuum with $N = (1,1)$ supersymmetry. The rest of the eigenvectors of A_1 do not have eigenvalues which satisfy the supersymmetry condition (3.34) for the $N = (1,1)$ vacuum. We chose the initial conditions the same way as in section 3.3.1.

In figure 3.4 we display examples of solutions to the flow equations representing Janus flows between $N = (4,4)$ vacua, $N = (1,1)$ vacua and RG-flow Janus solutions between $N = (4,4)$ and $N = (1,1)$ vacua. We note that the flows involving the $N = (1,1)$ vacua are a new feature of the truncation. As discussed in section 3.4.1 the $N = (1,1)$ is a repulsive fixed point of the flow and to obtain the numerical solutions one has to fine-tune the initial conditions at the turning point to approach the $N = (1,1)$ vacuum. This implies that choosing an initial $p(0)$ the initial $q(0)$ for which an RG-flow solution exists is fixed (if such a numerical solution exists). A third kind of low solution corresponds to a Janus solution interpolating between $N = (1,1)$ vacua, since both vacua are repulsive such solutions only exist for a discrete set of initial conditions.

3.3.3 Truncation 3

The matrix A_1 for this truncation is given by

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \frac{b+c}{2} & 0 & \frac{a+d}{2} & 0 & 0 \\ 0 & -c & \frac{-b+c}{2} & 0 & \frac{-a+d}{2} & 0 & 0 & d \\ 0 & \frac{-b+c}{4} & -c & 0 & -d & 0 & 0 & \frac{a-d}{2} \\ \frac{b+c}{2} & 0 & 0 & 0 & 0 & 0 & \frac{-a-d}{2} & 0 \\ 0 & \frac{-a+d}{2} & -d & 0 & c & 0 & 0 & \frac{-b+c}{2} \\ \frac{a+d}{2} & 0 & 0 & 0 & 0 & 0 & \frac{b+c}{2} & 0 \\ 0 & 0 & 0 & \frac{-a-d}{2} & 0 & \frac{b+c}{2} & 0 & 0 \\ 0 & d & \frac{a-d}{2} & 0 & \frac{-b+c}{2} & 0 & 0 & c \end{pmatrix} \tag{3.59}$$

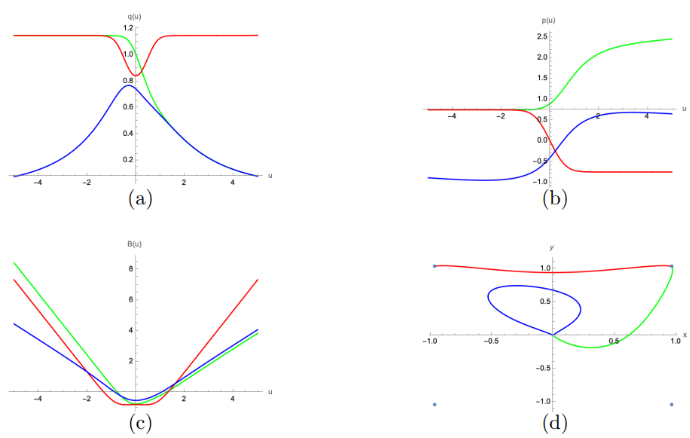


Figure 3.4: Truncation 2: (a)-(c) plots of p, q, B respectively, (d) parametric plot of the Janus flow in the x, y variables, the $N = (4, 4)$ vacuum is at the origin and the dots denote the locations of the $N = (1, 1)$ vacua. Blue: Janus between $N = (4, 4)$ vacua, red: Janus between $N = (1, 1)$ vacua, green: RG-Janus between $N = (4, 4)$ and $N = (1, 1)$. We have set $\alpha = 1.2$ for these examples.

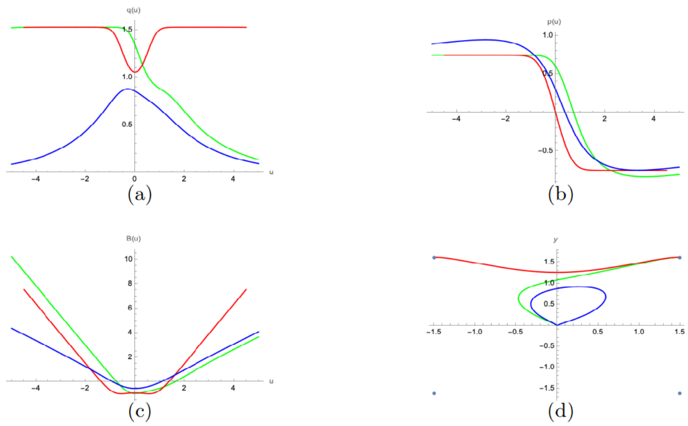


Figure 3.5: Truncation 3: (a)-(c) plots of p, q, B respectively, (d) parametric plot of the Janus flow in the x, y variables, the $N = (4,4)$ vacuum is at the origin and the dots denote the locations of the $N = (1,1)$ vacua. Blue: Janus between $N = (4,4)$ vacua, red: Janus between $N = (1,1)$ vacua, green: RG-Janus between $N = (4,4)$ and $N = (1,1)$. We have set $\alpha = 1.2$ for these examples.

where

$$\begin{aligned} a &= \sin^3 p(1 + \alpha \cosh^3 q) \\ b &= \cos^3 p(\alpha + \cosh^3 q) \\ c &= \sin^2 p \cos p \cosh q(1 + \alpha \cosh q) \\ d &= \cos^2 p \sin p \cosh q(\alpha + \cosh q) \end{aligned} \tag{3.60}$$

The eigenvectors $n_{\pm}^{(1)}$ of A_1 with eigenvalues $\pm w_0$ corresponding to the unbroken $N =$

$(1,1)$ supersymmetries are given by

$$n_{\pm}^{(1)} = \begin{pmatrix} 0 \\ -\frac{b-3c\pm 2w_0}{a-3d} \\ \frac{b-3c\pm 2w_0}{a-3d} \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad w_0 = \frac{1}{2}\sqrt{(a-3d)^2+(b-3c)^2} \tag{3.61}$$

The rest of the eigenvectors of A_1 do not have eigenvalues which satisfy the supersymmetry condition (3.34) for the $N=(1,1)$ vacuum. Note that all of them reduce to the ones of truncation 1 for the $N=(4,4)$ vacuum.

In figure 3.5 we display a sample of solutions to the flow equations representing Janus flows between $N=(4,4)$ vacua, $N=(1,1)$ vacua and RG-flow Janus solutions between an $N=(4,4)$ and $N=(1,1)$ vacuum. We note that the solutions behave qualitatively similar to the ones displayed for truncation 2.

3.4 Holographic calculations

In this section we will perform some holographic calculations for the solutions obtained in the section 4.3. In particular we will calculate the masses for the fluctuations of the scalar fields around the $N=(4,4)$ and $N=(1,1)$ vacua. This will allow us to identify the dimensions of the dual operators which are turned on in the flows. One of the results is that for truncation 2 and 3 the mass squared of the fluctuations are positive, corresponding to operators with scaling dimensions $\Delta > 2$. Since the behavior near the AdS vacuum is given by

$$\lim_{\varepsilon\to 0}\phi \sim \bar{\phi} + c_1\varepsilon^{\Delta} + c_2\varepsilon^{2-\Delta} + \cdots \tag{3.62}$$

where $\varepsilon \to 0$ corresponds to approaching the AdS boundary, the initial conditions have to be fine tuned in order to make the repulsive term $c_2\varepsilon^{2-\Delta}$ very small. In addition we consider the entanglement entropy of a symmetric region around the defect [78, 117–120] and give a prescription to obtain the defect entropy (or g-factor) [121].

3.4.1 Operator spectrum

The $N = (4,4)$ vacuum has $q_i = 0, i = 1,2,3,4$. Since the kinetic terms for p_i are vanishing the x_i, y_i defined in (3.16) are better suited to analyze the fluctuations. Expanding around the $x_i = y_i = 0$ vacuum one finds for the quadratic term of the fluctuations.

$$\frac{1}{e}\mathcal{L}_{(2)} = \frac{1}{4}\sum_i \Big(\partial_\mu \delta x_i \partial^\mu \delta x_i + \partial_\mu \delta y_i \partial^\mu \delta y_i\Big) + \frac{g_c^2}{4}\sum_i \Big((1+2\alpha)\delta x_i^2 + \alpha(\alpha+2)\delta y_i^2\Big) \tag{3.63}$$

From which we can read off the masses of the scalar fluctuations. The masses the determine the conformal scaling dimensions

$$\Delta_\pm = 1 \pm \sqrt{1 + m^2 R^2} \tag{3.64}$$

where R is the AdS radius of the vacuum. Setting $g_c = 1/(1+\alpha)$ to obtain a unit radius AdS_3 for the $N = (4,4)$ vacuum and the standard AdS/CFT relation the conformal dimensions of the dual operators are displayed in table 3.1. Note that Δ_+ gives the scaling dimension of

$N = (4,4)$	m^2	Δ_+	Δ_-
δx_i	$-\frac{1+2\alpha}{(1+\alpha)^2}$	$\frac{1+2\alpha}{1+\alpha}$	$\frac{1}{1+\alpha}$
δy_i	$-\frac{\alpha(2+\alpha)}{(1+\alpha)^2}$	$\frac{2+\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$

Table 3.1: Mass and conformal dimensions of scalar fluctuations for the $N = (4,4)$ vacuum

the dual operator in the standard quantization which takes values between $1 < \Delta_+ < 2$ for $\alpha > 0$, whereas Δ_- corresponds to the alternative quantization and $0 < \Delta_- < 1$ for $\alpha > 0$. Supersymmetric flows are related to the standard quantization which we will adapt in the

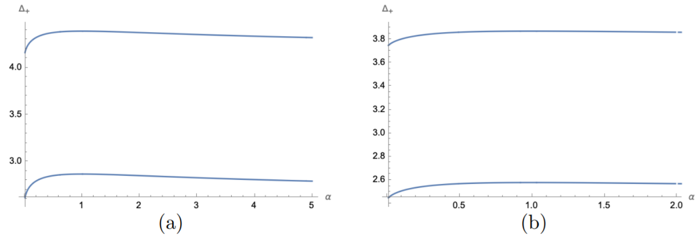


Figure 3.6: (a) Conformal dimension of operator dual to scalar fluctuations around the $N = (1, 1)$ vacuum for truncation 2, (b) same for truncation 3.

following [99]. We note that the $N = (4, 4)$ vacuum is attractive since both x and y are dual to operators with $\Delta < 2$ and the initial conditions do not have to be fine tuned for (3.62) to approach the vacuum value.

For truncations 2 and 3 we can determine the scaling dimensions of the operators at the $N = (1, 1)$ vacuum by expanding the scalar action around the vacuum to second order and diagonalizing the resulting scalar Lagrangian. The resulting expressions are quite unwieldy and we present the plots of the scaling dimensions of the two modes as a function of α in figure 3.6. We note that the scaling dimensions are larger than 2 and hence the $N = (1, 1)$ corresponds to a repulsive fixed point and the initial conditions have to be fine-tuned.

3.4.2 Holographic entanglement entropy

The Ruy-Takayanagi prescription [122] relates holographic entanglement entropy to the area of a minimal surface in the bulk which when approaching the AdS boundary ends at the border of the entangling surface. For a three dimensional static bulk spacetime this corresponds to a geodesic in the bulk which terminates at the ends of the entangling interval on the boundary. For the AdS_2 sliced metric (3.30) and an entangling surface which is symmetric about the defect and of length $2L$, such a geodesic is simply parameterized by u and constant $z = L$. The entanglement entropy is then given by

$$S_{EE}(L) = \frac{1}{4G_N}\int_{u_{-\infty}}^{u_{+\infty}} du = \frac{1}{4G_N}\big(u_{-\infty} - u_{+\infty}\big) \tag{3.65}$$

Where $u_{\pm\infty}$ will be related to an UV Fefferman-Graham cutoff in the following, we will generalize the derivation of [117, 118] to the case of an RG-flow interface where the AdS radius and hence the central charge take different values on both sides of the interface. The asymptotic behavior of the metric is determined by the metric function $B(u)$ as $u \to \pm\infty$

$$\lim_{u\to\pm\infty} B(u) = \pm\frac{u}{R_\pm} + \ln\lambda_\pm - \ln 2 + o(\frac{1}{u}) \tag{3.66}$$

In the two asymptotic regions we can define a Fefferman-Graham coordinate system by defining a new coordinates $\hat{u}_\pm$

$$u \to \pm\infty: \quad u = R_\pm \hat{u}_\pm \mp R_\pm \ln\lambda_\pm + o(\frac{1}{u}) \tag{3.67}$$

and then the coordinates $\zeta_\pm, \eta$

$$\begin{aligned} u \to +\infty: \qquad & e^{-2\hat{u}_+} = \frac{1}{4}\frac{\zeta_+^2}{\eta^2} + o(\zeta_+^4), \qquad z = \eta\Big(1 + \frac{1}{2}\frac{\zeta_+^2}{\eta^2}\Big) + o(\zeta_+^4) \\ u \to -\infty: \qquad & e^{2\hat{u}_-} = \frac{1}{4}\frac{\zeta_-^2}{\eta^2} + o(\zeta_-^4), \qquad z = -\eta\Big(1 + \frac{1}{2}\frac{\zeta_-^2}{\eta^2}\Big) + o(\zeta_-^4) \end{aligned} \tag{3.68}$$

This expansion is valid for $\eta >> \zeta_\pm$, i.e. if we consider an entanglement interval which is far away from the interface. In this limit the metric becomes

$$\zeta_\pm \to 0 \qquad ds^2 = R_\pm^2\Big(\frac{-d\zeta_\pm^2 - d\eta^2 + dt^2}{\zeta_\pm^2}\Big) + o(1) \tag{3.69}$$

It follows that $R_\pm$ defined in (3.66), corresponds to the asymptotic AdS radius and the left and right side of the interface respectively and a Fefferman-Graham cutoff is given by setting $\zeta_\pm = \varepsilon$. For the entanglement region located at $z = L$ in follows from (3.69) that the FG cutoff is related to the $u_\pm$ cutoff as follows

$$u_\pm = \mp R_{\pm\infty} \ln\left(\frac{1}{2}\frac{\varepsilon}{L}\right) \mp R_\pm \ln\lambda_\pm \tag{3.70}$$

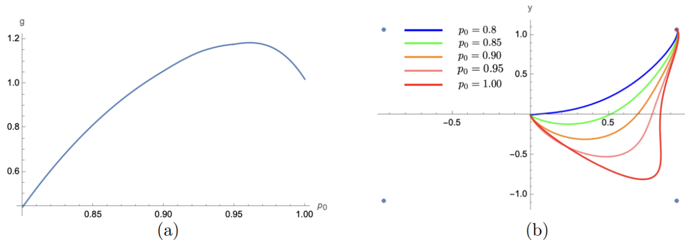


Figure 3.7: (a) Plot of boundary entropy for RG-flow interface for truncation 2, as a function of initial condition p_0 at the turning point for $\alpha = 1.4$. (b) Illustration of RG-flows for some initial values of p_0.

Plugging this into (3.65) gives the entanglement entropy

$$\begin{aligned} S_{EE} &= \frac{1}{4G_N}\Big((R_+ + R_-)\ln\frac{L}{2\varepsilon} - R_+\ln\lambda_+ - R_-\ln\lambda_-\Big) \\ &= \frac{c_+ + c_-}{6}\ln\frac{L}{2\varepsilon} - \frac{c_+}{6}\ln\lambda_+ - \frac{c_-}{6}\ln\lambda_- + o(\varepsilon) \end{aligned} \tag{3.71}$$

the constant term gives the boundary entropy

$$g = -\frac{c_+}{6}\ln\lambda_+ - \frac{c_-}{6}\ln\lambda_- \tag{3.72}$$

Where $c_\pm$ is the central charge for the two CFTs on either side of the RG interface. The g-factor is given by the second and third term in (3.71). For a Janus interface we have $c_+ = c_- = c$, whereas the central charges differ on both sides of the interface for a RG-flow interface. It is straightforward to determine the $R_\pm$ and $\ln\lambda_\pm$ by numerically fitting the metric functions (see plot (c) in figures 3.4 and 3.5) to determine the slope and the intercept (3.66) in the limit of large $|u|$. We will give an example of numerical results by presenting the g-factors as for the RG-interface between the $N = (4, 4)$ vacuum and a $N = (1, 1)$ vacuum in truncation 2. As discussed in section 3.3.2 there exists a unique RG-flow interface for a choice on initial condition p_0. In figure 3.7 we present the g-factor as a function of the initial condition for a particular value of $\alpha = 1.4$.

3.5 Discussion

In this paper we found holographic interface solution in three dimensional gauged supergravity theories. An important feature of these theories is that they have AdS vacua which preserve $N = (1, 1)$ supersymmetry in addition to the $N = (4, 4)$ AdS vacuum. This feature allows us to find solutions which correspond to interfaces between two $N = (4, 4)$ vacua on both side, $N = (1, 1)$ on both sides, as well as RG-flow interfaces which have a $N = (4, 4)$ on one side and $N = (1, 1)$ vacuum on the other. We derived BPS flow equations which are three first order nonlinear differential equations for the two scalars p, q which are non zero in the truncations as well as the warp factor B of the AdS_2 slicing. By using the freedom to shift the warping coordinate u by a constant we can choose the initial conditions for the flow as the value of p and q at the turning point of the warp factor, where $B' = 0$. In fact we use the BPS equations to determine the initial conditions for the second order equation motion following from the variation of the action. The numerical accuracy of the solution is tested by checking the BPS equations away from the point where the initial conditions were fixed.

The $N = (1, 1)$ extrema are repulsive fixed points of the flow and hence the initial condition have to be fine tuned using a shooting method. This is possible by fixing one scalar initial condition and varying the other in order to come closer and closer to the $N = (1, 1)$ vacuum in the flow. Our results indicate that the qualitative behavior of the solutions for general α is quite similar to the behavior of the $\alpha = 1$ solutions obtained in [1]. In addition we have considered entanglement entropy for the Janus and RG-flow solutions. Since for the RG-flow solutions the central charges and hence AdS radii are different on both sides of the interface and one has to carefully consider the UV cut-off. It is possible to determine the g-function or interface entropy from the numerical solution by a linear fit of the warp factor B.

We have considered truncations of the scalars to two nonzero scalars q and p (or x and

y), it would be interesting to generalize this since it would then be possible to consider more complicated flows between different $N = (1,1)$ vacua. It would also be interesting to investigate the solutions we have found can be lifted and tell and have a representation in $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ holography. It would also be interesting to see whether the prescription for the interface entropy can be applied to other examples of RG-flow interfaces.

CHAPTER 4

Janus and RG-interfaces in minimal 3d gauged supergravity

Janus solutions are solutions of supergravity theories which describe interface CFTs in the AdS/CFT correspondence. The first such solution [64] was constructed in type IIB supergravity describing $N = 4$ Super Yang-Mills theory with the gauge coupling jumping across a planar interface. There are two different approaches to constructing such solutions, one is the top-down approach where ten or eleven dimensional solutions type II or M-theory are constructed as products involving AdS and spherical factors warped over a Riemann surface with boundary (see e.g. [69, 73, 87, 88, 91]). A guiding principle is to look for solutions preserving half the number of supersymmetries of the AdS vacua which allows to construct the explicit solutions from harmonic functions on the Riemann surface with certain boundary conditions.

A second approach is to construct supersymmetric Janus solutions in lower dimensional gauged supergravities (see e.g. [70, 72, 74, 92–94, 96, 116, 123–127]). Such solutions are often easier to obtain since all fields only depend on a single AdS-slicing coordinate and the Killing spinor equations are simpler. In many cases, the gauged supergravity theories are consistent truncations of ten and eleven dimensional supergravities and lower dimensional solutions can be uplifted. In addition, the simpler form of the solution allows to calculate holographic observables and handle holographic renormalization more easily.

Another reason to consider lower dimensional gauged supergravity is that these theories often have more than one AdS vacuum, coming from multiple extrema of the scalar potential.

Apart from the maximally supersymmetric vacuum, the other vacua can have a lower number or no supersymmetry and in general will correspond to an AdS space with different values of the cosmological constant, which translates into CFTs with a different central charge. For an ansatz with a Poincare sliced metric, it is possible to construct solutions which correspond to holographic RG-flows relating the two CFTs (see e.g. [61, 128–130]). Using the Janus AdS-slicing it is possible to construct RG-flow interfaces which describe an interface between two different CFTs which are related by an RG-flow. On the field theory side RG-flow interfaces were discussed in [100, 131–135] and examples of holographic RG-flow interface solutions are [1, 2, 101, 102]. The goal of this paper is to find Janus and RG-flow solutions in one of the minimal theories in three dimensions, namely $N = 2, d = 3$ gauged supergravity, in order to have a set of simple (numerical) solutions for which holographic observables can be calculated. The ones we focus on in this paper are the interface entropy $\ln(g_A)$ for an entangling surface which is symmetric about the interface, the effective central charge c_{eff} associated with the entanglement entropy where the entangling surface ends at the interface and the reflection coefficient c_{LR} for the scattering of stress tensor modes off the interface. We use the solutions to test bounds and relations between the latter two quantities which have been investigated recently [136–138]. The structure of this note is as follows: In section 4.1 we review the $N = 2, d = 3$ gauged supergravity for which we will construct Janus and RG-flow solutions. In section 4.2, we set up the equations of motion for an AdS_2 slicing ansatz and generate families of numerical solutions both for Janus solutions which have the same CFT on both sides of the interface and RG-flow interfaces between two different CFTs. In section 4.3 we briefly review the holographic observables we calculate and plot the results for some example solutions. While the results for the minimal $N = 2, d = 3$ gauged supergravity are numerical, there exists a solution of $N = 8, d = 3$ gauged supergravity found previously by one of the authors in [112] which is exact and preserves half the supersymmetries. In section 4.4 we calculate the holographic observables and observe that the relation between c_{eff} and c_{LR}, which was pointed out to hold for the ten dimensional supersymmetric Janus solutions in [138] also hold for the solutions constructed in this paper. We close the note with a discussion of the results and some future research directions in section 4.5.

4.1 $N = 2, d = 3$ gauged supergravity

In this note we will use a minimal form of $N = 2, d = 3$ gauged supergravity where the bosonic sector is given by three dimensional gravity, a complex scalar and a Chern-Simons $U(1)$ gauge field. We will set the fermionic degrees of freedom to vanish and use the fermionic supersymmetry variations to test whether supersymmetries are preserved by the solutions. The action was constructed in [139] and we will follow the conventions of [140, 141]. The Lagrangian is given by

$$S = \frac{1}{4}\int d^3x\sqrt{g}\Big(R - \frac{4|D_\mu\Phi|^2}{a^2(1-|\Phi|^2)^2} - V(\Phi)\Big) + \frac{1}{4ma^4}\int A\wedge dA \tag{4.1}$$

The covariant derivative coupling the complex scalar and the $U(1)$ gauge field is given by

$$D_\mu\Phi = \partial_\mu\Phi + iA_\mu\Phi \tag{4.2}$$

The scalar potential can be most conveniently expressed using the following parameterization

$$C = \frac{1+|\Phi|^2}{1-|\Phi|^2}, \quad S = \frac{2\Phi}{1-|\Phi|^2} \tag{4.3}$$

and is given by

$$V = 8m^2C^2(2a^2|S|^2 - C^2) \tag{4.4}$$

The Chern-Simons gauge field couples to the phase of the complex scalar field Φ

$$\Phi = |\Phi|e^{i\theta} \tag{4.5}$$

It is convenient to introduce one more change of variable for the absolute value of the scalar field

$$|\Phi| = \tanh\left(\frac{a\phi}{2\sqrt{2}}\right) \tag{4.6}$$

which implies

$$C = \cosh\left(\frac{a\phi}{\sqrt{2}}\right), \quad |S| = \sinh\left(\frac{a\phi}{\sqrt{2}}\right) \tag{4.7}$$

The action can then be written in terms of the fields ϕ, θ

$$\begin{aligned} S = &\frac{1}{4}\int d^3x\sqrt{g}\Big(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big) \\ &+ \frac{1}{4}\int d^3x\sqrt{g}\Big(- \frac{\sinh^2\phi}{a^2}(\partial_\mu\theta + A_\mu)(\partial^\mu\theta + A^\mu) + \frac{1}{4ma^4}\int A\wedge dA \end{aligned} \tag{4.8}$$

In order to construct Janus and RG-flow interface solutions it is possible to set $A_\mu = \theta = 0$ consistently. The action is then given by the first line in (4.8), i.e. three dimensional gravity minimally coupled to a real scalar field ϕ with a potential V

$$V(\phi) = -8m^2\cosh^2\left(\frac{a\phi}{\sqrt{2}}\right)\left[\cosh^2\left(\frac{a\phi}{\sqrt{2}}\right) - 2a^2\sinh^2\left(\frac{a\phi}{\sqrt{2}}\right)\right] \tag{4.9}$$

The $d = 3, N = 2$ supersymmetry transformation of the gravitino and dilatino for the truncated Lagrangian takes the following form

$$\begin{aligned} \delta\psi_\mu &= (\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab})\varepsilon + \frac{1}{2}W\gamma_\mu\varepsilon \\ \delta\lambda &= \frac{1}{2}(-\gamma^\mu\partial_\mu\phi - \frac{2}{a}\frac{\partial W}{\partial\phi})\varepsilon \end{aligned} \tag{4.10}$$

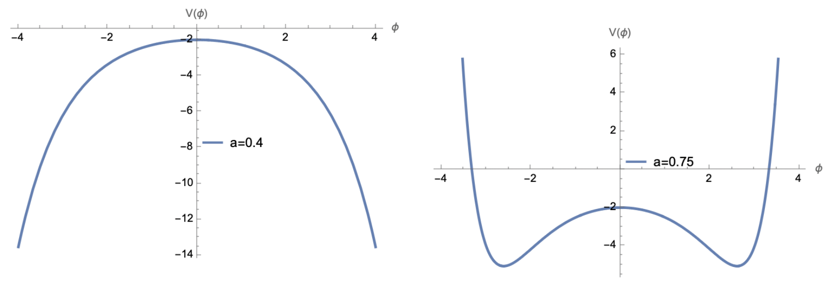
where the superpotential is given by

$$W = 2m\cosh^2\left(\frac{a\phi}{\sqrt{2}}\right) \tag{4.11}$$

and the potential is related to the superpotential by the following relation

$$V = 2\left(\frac{\partial W}{\partial\phi}\right)^2 - 2W^2 \tag{4.12}$$

Note that m only appears as an overall multiplicative factor in the potential, we will set $m = \frac{1}{2}$ which leads to a unit radius AdS_3 vacuum for $\phi = 0$. The shape of the potential and the number and nature of extrema depend on the parameter a, representative plots for the three different cases are shown in figure 4.1.



(a) $V(\phi)$ for $a = 0.4$ (b) $V(\phi)$ for $a = 0.75$ (c) $V(\phi)$ for $a = 1.3$

Figure 4.1: Example of potential $V(\phi)$ for three cases (a) $a < \frac{1}{\sqrt{2}}$, (b) $\frac{1}{\sqrt{2}} < a < 1$, (c) $a > 1$

For any value of a there is an extremum at $\phi = \phi^{(1)} = 0$. Expanding around it allows to read off the mass of the small fluctuation around $\phi = \delta\phi$

$$V \sim -2 - 2a^2(1-a^2)\delta\phi^2 + o(\delta\phi^4) \tag{4.13}$$

As mentioned before we have $l_{AdS}^{(1)} = 1$ Using the standard relation of the mass and conformal dimension of the dual operator one obtains

$$\Delta_{\pm}^{(1)} = 1 \pm |1 - 2a^2| \tag{4.14}$$

which is valid for all $a \in R$. This implies that the dual operator is relevant for $0 < a < 1$ and irrelevant for $a > 1$. For $\frac{1}{\sqrt{2}} < a < 1$, there are two additional extrema of the potential located at

$$\phi^{(2),(3)} = \pm \frac{1}{\sqrt{2}a} \ln \left(\frac{1 + 2a\sqrt{1-a^2}}{2a^2 - 1} \right) \tag{4.15}$$

Expanding $\phi = \phi^{(2,3)} + \delta\phi$, gives

$$V = -\frac{2a^4}{2a^2 - 1} - \frac{4a^4(a^2-1)}{2a^2-1}\delta\phi^2 + o(\delta\phi^3) \tag{4.16}$$

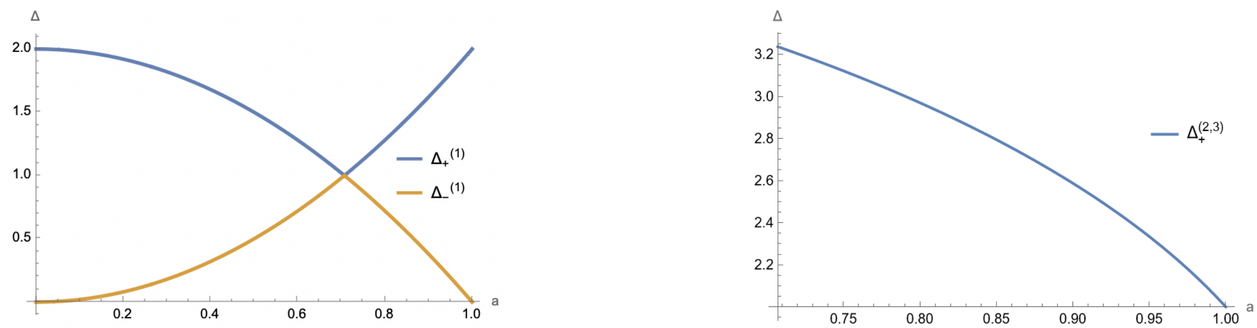
The AdS_3 vacuum has a curvature radius

$$l_{AdS}^{(2,3)} = \frac{\sqrt{2a^2-1}}{a^2} \tag{4.17}$$

and from (4.16) we can read off the mass and determine the conformal dimension of the operator dual to the scalar fluctuation around the extremum.

$$\Delta_{+}^{(2,3)} = 1 + \sqrt{1 + 8(1 - a^2)} \tag{4.18}$$

Consequently, the dual operator will always be irrelevant for the values of a where the additional extrema and AdS vacua exist (see figure 4.2).



(a) $\Delta_{\pm}^{(1)}$ for $a < 1$ (b) $\Delta_{+}^{(2,3)}$ for $\frac{1}{\sqrt{2}} < a < 1$

Figure 4.2: Conformal dimension of operator dual to fluctuation around extrema.

The simplicity of the minimal gauged supergravity makes the construction of analytic, as well as numerical solutions, relatively easy. For example, Poincare sliced domain wall solution representing RG-flows have been constructed in [140–143] and string and vortex solutions have been constructed in [139, 144–147]. In this note, we utilize an AdS_2 slicing ansatz to find Janus and RG-flow interface solutions in this theory.

4.2 Janus and RG-interfaces

The equations of motion following from the $\theta = A_\mu = 0$ truncation of the action (4.8) are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \frac{1}{2} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\sigma \phi \partial^\sigma \phi \right) - \frac{1}{2} g_{\mu\nu} V(\phi) \\ 0 &= \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) - V'(\phi) \end{aligned} \tag{4.19}$$

The ansatz for Janus and RG-flow interfaces is given by taking an AdS_2 slicing of the three dimensional metric and demanding that the scalar field ϕ only depends on the slicing coordinate u.

$$ds^2 = du^2 + e^{2B(u)}\frac{dx^2 - dt^2}{x^2}, \qquad \phi = \phi(u) \tag{4.20}$$

The equations of motion (4.19) then become a system of second order ordinary differential equations, for B and ϕ

$$B'' + 2(B')^2 + V + e^{-2B} = 0 \tag{4.21}$$

$$\phi'' + 2B'\phi' - \frac{\partial V}{\partial \phi} = 0 \tag{4.22}$$

Subject to a constraint

$$(B')^2 - \frac{1}{4}(\phi')^2 + e^{-2B} + \frac{1}{2}V = 0 \tag{4.23}$$

In order to determine whether Janus or RG-flow interface solutions exist which preserve some supersymmetry, it is sufficient to, first, consider the vanishing of gravitino variation in the AdS_2 direction

$$\begin{aligned}\delta\psi_t &= \partial_t\varepsilon + \frac{1}{2z}\gamma_0\Big(-\gamma_1 + B'e^B\gamma_2 + e^B W\Big)\varepsilon = 0\\ \delta\psi_z &= \partial_z\varepsilon + \frac{1}{2z}\gamma_1\Big(B'e^B\gamma_2 + e^B W\Big)\varepsilon = 0\end{aligned} \tag{4.24}$$

where the integrability $(\partial_t\partial_z - \partial_z\partial_t)\varepsilon = 0$ condition produces the following equation

$$1 - e^{2B}W^2 + e^{2B}(B')^2 = 0 \tag{4.25}$$

Secondly, the dilatino variation

$$\delta\lambda = -\frac{1}{2}\Big(\gamma_2\phi' + \frac{2}{a}\frac{\partial W}{\partial \phi}\Big)\varepsilon \tag{4.26}$$

corresponds to a projector on the susy parameter ε if

$$(\phi')^2 = \frac{4}{a^2}\left(\frac{\partial W}{\partial \phi}\right)^2 \tag{4.27}$$

It is straightforward to verify that the conditions (4.25) and (4.27) are inconsistent with the equations of motion (4.22) unless $\phi = \phi^{(1)} = 0$ which is the supersymmetric AdS_3 vacuum. Consequently, the additional AdS_3 $\phi = \phi^{(2,3)}$ which exist for $\frac{1}{\sqrt{2}} < a < 1$ as well as any AdS_2 sliced flow solution for which ϕ' is not vanishing, will break all the supersymmetries.

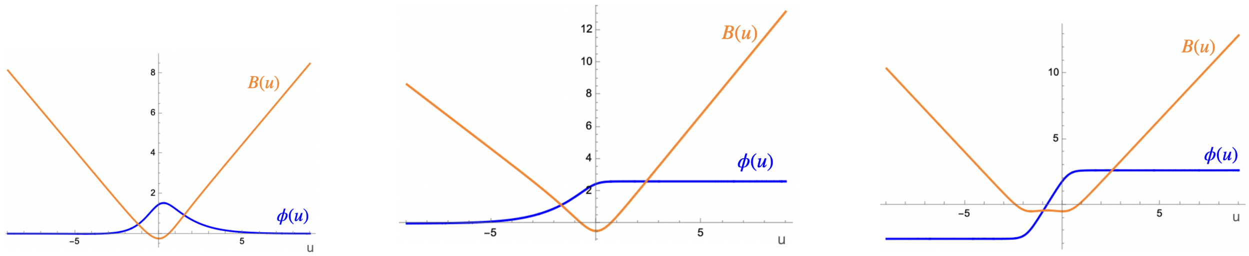
It is possible to rewrite the equations of motion (4.21)- (4.22) as a system of first order equations, however as pointed out already in [98] this is not very useful in obtaining closed form or even numerical solutions. Here we will employ the following strategy to obtain numerical solutions of the equations of motion: The uu component of Einstein equations (4.22) is a constraint for reparametrizations of the coordinate u. If it is imposed at a fixed u it will be satisfied for all u for solutions of the second order equations of motion. In addition, we look for Janus or RG-interface solutions. These all have the feature that the warping factor e^{2B} has a minimum. Other solutions are possible but they will generally develop a naked singularity or become non-physical (for example B will diverge or the signature of the metric changes).

Consequently, we impose the initial conditions at the turning point where $B' = 0$ which we set by a translation of the coordinate u to be localized at $u = 0$. The constraint (4.23) then becomes

$$(\phi')^2 - 2V - 4e^{-2B}\big|_{u=0} = 0 \tag{4.28}$$

and one can determine $B(0)$ from specifying the initial conditions $\phi'(0)$ and $\phi(0)$. The numerical solutions can then be obtained by integrating the second order equations (4.21) and (4.22) using a shooting method in Mathematica.

We will illustrate this for the example $a = \frac{3}{4}$ for which the potential has three extrema. There are three types of interface solutions as illustrated for some representative initial conditions in figure 4.3.



(a) Janus interface with $\phi^{(1)}$ vacuum on both sides

(b) RG-flow interface interpolating between $\phi^{(1)}$ and $\phi^{(2)}$ vacua

(c) RG-flow interface interpolating between $\phi^{(3)}$ and $\phi^{(2)}$ vacua

Figure 4.3: Examples of interface solutions for representative initial conditions

The plot (a) depicts a Janus interface between the supersymmetric vacuum $\phi^{(1)}$ as $u \to \pm\infty$. To obtain such a solution the initial conditions do not have to be fine-tuned, in figure 4.4 the initial conditions leading to Janus solutions are in the yellow area. The plot (b) depicts an RG-flow interface interpolating between the supersymmetric vacuum $\phi^{(1)}$ as $u \to -\infty$ and the vacuum $\phi^{(2)}$ as $u \to \infty$. The initial conditions have to be fine-tuned in figure 4.4 where they correspond to the blue line. The red line in figure 4.4 corresponds to initial conditions which lead to RG-flow interface interpolating between the supersymmetric vacuum $\phi^{(3)}$ as $u \to -\infty$ and the vacuum $\phi^{(1)}$ as $u \to \infty$. The plot (c) corresponds to a solution that interpolates between the vacuum $\phi^{(3)}$ as $u \to -\infty$ and the vacuum $\phi^{(2)}$ as $u \to \infty$. Here both initial conditions have to be fine-tuned and in figure 4.4 this solution corresponds to the green dot. Initial conditions outside the colored region lead to solutions which develop a naked singularity at a finite value of u.

For values of $a < \frac{1}{\sqrt{2}}$ only the supersymmetric vacuum $\phi^{(1)}$ exists and the solutions are all Janus solutions which look qualitatively similar to (a) in figure 4.3. For value $a > 1$ all interface solutions develop naked singularities.

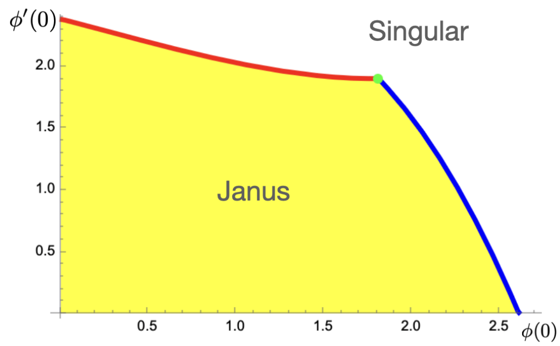
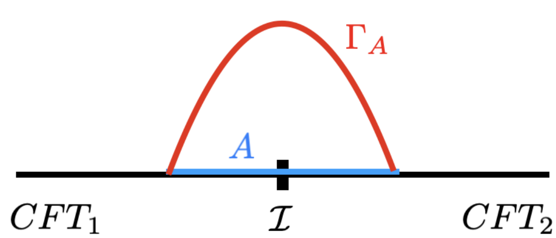


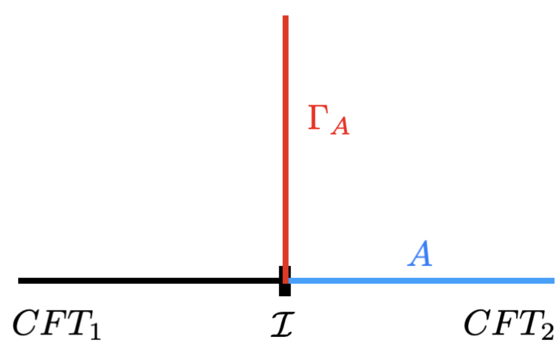
Figure 4.4: Phase diagram for interface solutions for the $a = 0.75$. The diagram is extended to the other quadrants using $\phi(0) \to -\phi(0)$ and $\phi'(0) \to -\phi'(0)$ maps.

4.3 Holographic observables

The numerical solutions obtained in the previous sections can be used to calculate holographic observables. Here we focus on a few, namely the entanglement entropy of an interval both symmetrically about the interface [117, 118] and at the interface [78, 148], as well as the transmission coefficient [82, 85, 138, 149].



(a) Entangling surface A symmetric about interface I



(b) Entangling surface A at interface I

Figure 4.5

Correlations functions in the background of Janus solutions have been discussed in [66, 150–152]. However these correlators are more difficult to obtain if the RG-flow solutions of the background are known only numerically. There are other holographic observables such as the on-shell action and volume measures of complexity (see e.g. [153–155]) which will not be discussed in this note.

4.3.1 Symmetric entanglement entropy

The Ryu-Takayanagi prescription [122, 156] allows to calculate the entanglement entropy holographically, for an interval A the entanglement entropy is given by

$$S_{EE}(A) = \frac{\text{Length}[\Gamma_A]}{4G_N} \tag{4.29}$$

where Γ_A is the geodesic in the bulk of spacetime which ends at the interval A on the boundary. For AdS_2 sliced solution describing an interface, there are two simple geometries one can consider. Firstly, we can choose the interval to be symmetric about the interface [117, 118, 148, 157] and the entanglement entropy takes the following form

$$S_A = \frac{c_L + c_R}{6} \ln \frac{l}{\varepsilon} + \ln g_A \tag{4.30}$$

Here $c_{L/R}$ are the central charges of the CFTs on either side of the interface. For a Janus interface they are equal, whereas for an RG-flow interface, they will be different. Furthermore, $2l$ is the length of the interval A, which is symmetric about the interface and ε is a UV cutoff and $\ln(g_A)$ is the g-factor (or interface entropy) which is a physical quantity associated with the number of degrees of freedom localized on the interface.

To apply the Ryu-Takayanagi formula (4.29) for the AdS_2 sliced metric (4.20), it was shown in [118] that the geodesic is parameterized by choosing a fixed $z = l$ and $u \in [-\infty, \infty]$. This geodesic corresponds to an entanglement interval symmetric about the interface at the origin, i.e. $A = [-l, l]$. In the following, we apply the holographic calculation [118, 148, 157], where details can be found. The length of the geodesic is divergent

$$\text{Length}[\Gamma_A] = \int_{u_{-\infty}}^{u_\infty} du = u_\infty - u_{-\infty} \tag{4.31}$$

and must be regulated by mapping the AdS_2 sliced metric (4.20) in the asymptotic regions $u \to \pm\infty$ to a Fefferman-Graham coordinate and then introducing a uniform UV cutoff ε.

For the RG-flow and Janus interface solutions obtained in section 4.2 the warp factor takes the following form for large $|u|$

$$\lim_{u\to+\infty} B(u) = \frac{u}{L_R} + \ln\gamma_R + o(\frac{1}{u}), \qquad \lim_{u\to-\infty} B(u) = -\frac{u}{L_L} - \ln\gamma_L + o(\frac{1}{u}) \tag{4.32}$$

For large $u \to \pm\infty$ the AdS_2 sliced metric can be mapped asymptotically to a Poincare sliced AdS_3 with radius $L_{R/L}$ respectively by the following coordinate change

$$\begin{aligned} u \to +\infty: \quad & u = L_R\Big(\ln\frac{x}{z} - \ln\gamma_R + \ln L_R\Big) + o(z), \\ u \to -\infty: \quad & u = -L_L\Big(\ln\frac{x}{z} - \ln\gamma_L + \ln L_L\Big) + o(z) \end{aligned} \tag{4.33}$$

Here z is the radial coordinate in the asymptotically AdS_3 in Poincare coordinates and the Fefferman-Graham UV-cutoff is $z = \varepsilon$. The boundary of the entangling surface is located at $x = l$. Using The Brown-Henneaux formula for the central charge on the left and right sides of the interface

$$\frac{c_{L/R}}{6} = \frac{L_{L/R}}{4G_N} \tag{4.34}$$

it follows that the holographic entanglement entropy (4.29) takes the form (4.30) with the g-factor given by

$$\ln g_A = -\frac{1}{6}\Big(c_R \ln\frac{\gamma_R}{L_R} + c_L \ln\frac{\gamma_L}{L_L}\Big) \tag{4.35}$$

For the numerical Janus or RG-flow interfaces obtained in section 4.2 this can be calculated by fitting $B(u)$ for large $|u|$ to (4.33) to obtain $L_{L/R}$ and $\ln\big(\gamma_{L/R}\big)$. We illustrate this here by presenting a plot of $\ln g_A$ for the RG-flow interfaces interpolating between two distinct vacua for initial conditions given by the red and blue curves in figure 4.4.

4.3.2 Entanglement entropy at the interface

Secondly, one can consider an entanglement interval A which ends on the defect. For simple CFTs the entanglement entropy can be calculated using the replica trick [158, 159] and it

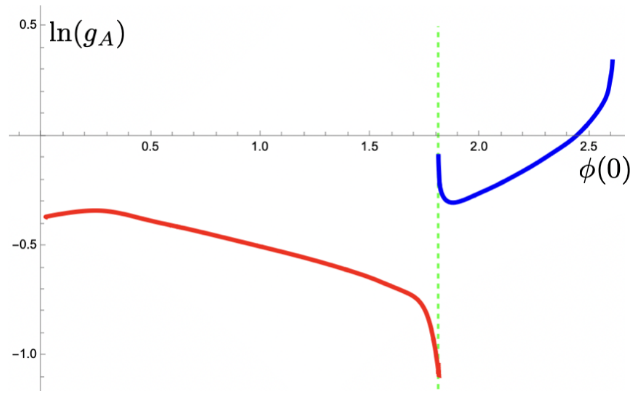


Figure 4.6: Interface entropy $\ln(g_A)$ for the RG-flow interfaces depending on the initial condition $\phi(0)$.

takes the following form

$$S_A = \frac{c_{eff}}{6} \ln \frac{l}{\varepsilon} \tag{4.36}$$

Where c_{eff} is an effective central charge, which depends on the details of the interface and measures the amount of entanglement across the interface.

The entanglement entropy at the interface has been calculated holographically in [78] where it has been shown for the AdS sliced metric (4.20) that the Ryu-Takanayagi geodesic is along the z coordinate and u is fixed at the minimum of $B(u)$, which was chosen to be at $u = 0$ for the numerical solutions constructed in section 4.2. One obtains for the numerical solutions of section 4.2

$$S_A = \frac{l}{4G_N} e^{B(0)} \int \frac{dz}{z} = \frac{1}{4G_N} e^{B(0)} \log \frac{l}{\varepsilon} \tag{4.37}$$

Here ε is a UV cutoff and l is the length of the interval which we take to to be very large in order to eliminate the contribution from the other end of the interval. Hence the effective central charge (4.36) is given by

$$c_{eff} = \frac{3}{2G_N} e^{B(0)} \tag{4.38}$$

The effective central charge c_{eff} has interesting properties such as a universal bound [137] and a relation to the transmission coefficient discussed below for certain supersymmetric Janus solutions [138].

4.3.3 Transmission and reflection coefficients

Another quantity is the energy reflection and transmission amplitude which describe the flow though and reflection of energy from the CFT interface. In the CFT the transmission amplitude can be expressed as a normalized two-point function of the stress tensors $T_{1,2}$ of the CFTs on the two sides of the interface [79, 80, 160]

$$\mathcal{T} = \frac{c_{LR}}{c_L + c_R} = \frac{\langle T_1 T_2 + \bar{T}_1 \bar{T}_2 \rangle}{\langle (T_1 + \bar{T}_1)(T_2 + \bar{T}_2) \rangle} \tag{4.39}$$

where c_L and c_R are the central charges of the two CFTs on either side of the interface. The reflection amplitude $\mathcal{R}$ is determined by unitarity $\mathcal{R} + \mathcal{T} = 1$. A holographic expression for c_{LR} has been obtained in [85], by taking a continuum limit for the reflection and transmission of energy in an array of probe branes.

$$c_{LR} = \frac{3}{G_N} \left(\frac{1}{l_R} + \frac{1}{l_L} + 8\pi G_N \sigma \right)^{-1} \tag{4.40}$$

Where $l_{L,R}$ are the AdS radius of the asymptotic AdS_3 half-regions close to the boundary on either side of the interface. The quantity σ depends on the scalar field kinetic energy, for the action (4.8) is given by

$$\sigma = \int_{-\infty}^{\infty} (\phi')^2 du \tag{4.41}$$

and can be calculated using the numerical solution obtained in the previous section, for both the Janus solution as well as the RG-flow interface solution.

In [137] a set of inequalities relating c_{LR} (and hence the transmission coefficient $\mathcal{T}$) to c_{eff} and the central charges on either side of the interface was proposed

$$0 \leq c_{LR} \leq c_{eff} \leq \min(c_L, c_R) \tag{4.42}$$

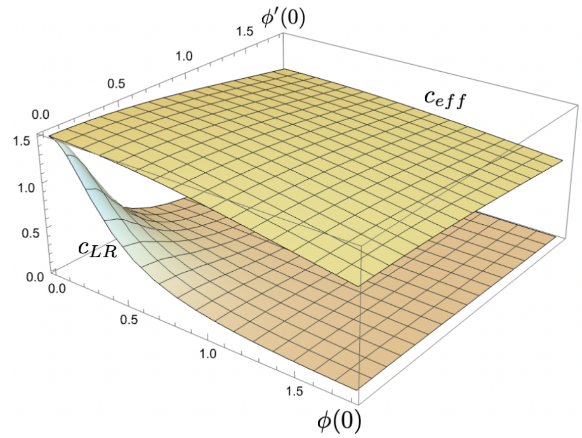


Figure 4.7: Plot of c_{LR} and c_{eff} for $a = \frac{3}{4}$ of a function of initial conditions $\phi(0), \phi'(0)$.

In [137] some holographic and CFT examples were checked and it was argued that the inequality between c_{LR} and c_{eff} is only becoming an equality for a completely reflective or transmissive interface. We can use the numerical solutions in our simple supergravity model to check and we found that the strict inequality holds for all Janus and RG-flow solutions. We can illustrate the validity of this inequality with the plot in figure 4.7 of c_{LR} and c_{eff} for $a = \frac{3}{4}$. Note that the point in the plot where $c_{LR} = c_{eff}$ corresponds to $\phi(0) = \phi'(0)$ which is the supersymmetric AdS_3 vacuum and hence corresponds to a trivial topological interface, i.e. no interface at all.

4.4 Transmission coefficient for $N = 8, d = 3$ gauged supergravity

In the previous section entanglement entropy and reflection coefficients were calculated for the non-supersymmetric Janus and RG-flow interfaces in minimal $N = 2$ gauged supergravity. Recently, it has been observed [138] that there is a relation of the transmission coefficient and the entanglement entropy at the interface for a class of supersymmetric Janus solutions constructed as $AdS_2 \times S_2 \times T_4 \times \Sigma_2$ solutions of type IIB supergravity in [138]. In this section we use supersymmetric Janus solutions of $d = 3, N = 8$ gauged supergravity which were obtained some time ago [112] to show that the relation of these two quantities holds for these solutions as well.

In the following, we will follow the construction [104]. The scalar fields of $d=3$, $N=8$ gauged supergravity take values in a $G/H = SO(8,n)/\big(SO(8)\times SO(n)\big)$ coset. There are $8n$ independent scalar degrees of freedom. The three dimensional theory can be constructed as a truncation of six-dimensional $N=(2,0)$ supergravity on $AdS_3\times S^3$ coupled to $n_T\geq 1$ tensor multiplets, where the number of tensor multiplets is fixed by $n_T=n-3$. The special cases $n_T=5$ and 21 are related to compactifications of ten-dimensional type IIB on T^3 and $K3$, respectively and hence are related to low energy limits of consistent string theories. Smaller values of n can be obtained by consistent truncations, see [161] for a discussion of consistent truncations of six-dimensional $N=(1,1)$ and $N=(2,0)$ using exceptional field theory. The action, gauging and supersymmerty transformations were constructed in [104] using the embedding tensor formalism. The details of the action and the construction of the half-BPS Janus solution can be found in [112]

The Janus solution considers the simplest case with $n=1$ for which there are eight coset scalars $\phi_i, i=1,2,\cdots 8$. It was shown in [112] that one can further consistently truncate the theory where only two denoted as ϕ_4,ϕ_5 have a nontrivial profile and all others are set to zero. The truncated bosonic action takes the following form

$$S=\frac{1}{2}\int d^3x\sqrt{-g}\Big\{R-P^I_\mu P^{\mu\, I}-V\Big\} \tag{4.43}$$

Where the notation $\Phi=\sqrt{\phi_4^2+\phi_5^2}$ is used for compactness. The kinetic energy term and the potential for non-vanishing scalars are given by

$$\begin{aligned}
P^I_\mu P^{\mu\, I} &= \frac{\phi_4^2+(\sinh^2\Phi+\phi_4^2)\phi_5^2}{\Phi^4}\partial_\mu\phi_4\partial^\mu\phi_4-\frac{\phi_5^4+(\sinh^2\Phi+\phi_5^2)\phi_4^2}{\Phi^4}\partial_\mu\phi_5\partial^\mu\phi_5\\
&\quad -\frac{2(\Phi^2-\sinh^2\Phi)\phi_4\phi_5}{\Phi^4}\partial_\mu\phi_4\partial^\mu\phi_5\\
V &= -\Big(\frac{\sinh^2(\Phi)\phi_4^2}{\Phi^2}+2\Big)
\end{aligned} \tag{4.44}$$

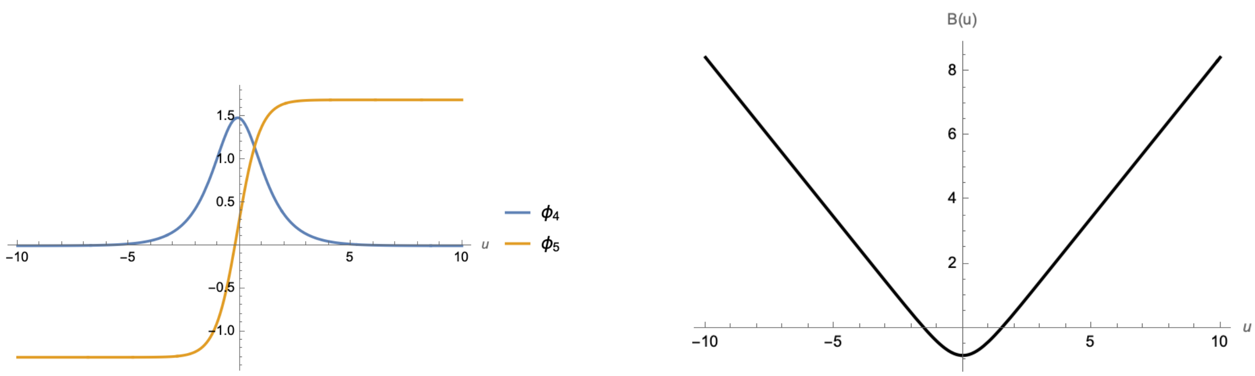
It was shown in [112] that a solution of the equations of motion which preserves half the supersymmetries of the $N=8$ gauged supergravity is given by the scalar profiles $\phi_4(u),\phi_5(u)$

which are implicitly defined and depend on two real parameters p, q

$$\frac{|\phi_4| \sinh \Phi}{\Phi} = |\sinh q| \,\mathrm{sech} u$$
$$\frac{\phi_5 \sinh \Phi}{\Phi} = \sinh p \cosh q + \cosh p \sinh q \tanh u \tag{4.45}$$

The AdS_2 sliced metric is given by

$$ds^2 = du^2 + \mathrm{sech}^2 \mathrm{q} \cosh^2 u \frac{d\xi^2 - dt^2}{\xi^2} \tag{4.46}$$



(a) Scalar profile for ϕ_4, ϕ_5 for $q = \frac{3}{2}, q = \frac{1}{5}$

(b) Warp factor B for $q = \frac{3}{2}, q = \frac{1}{5}$

Figure 4.8: Scalar Profiles and warp factors for the half-BPS solution.

A plot of the scalars and warp factor as a function of the slicing coordinate u is presented in figure 4.8. Note that the solution with $q = 0$ corresponds to the unit radius AdS_3 vacuum where the massless scalar is constant and given by $\phi_5 = \sinh p$. The g factor for a symmetric entanglement entropy for this solution was calculated in [112] and can easily be reproduced using the expression given in section 4.3.1 and one obtains

$$\ln(g_A) = \frac{c}{3} \ln(\cosh q) \tag{4.47}$$

For the entanglement interval at the interface, we can use (4.37) and the metric (4.46) to obtain the holographic result for the effective central charge

$$\frac{c_{eff}}{c} = e^{B(0)} = \frac{1}{\cosh q} \tag{4.48}$$

For the transmission coefficient, the relevant quantity is the integral of $d\sigma$ which for the action (4.43) is given by (choosing units such that $8\pi G_N = 1$).

$$\sigma = \int_{-\infty}^{\infty} du \; P_u^I P_u^I = 2\sinh^2 q \tag{4.49}$$

Plugging this result in the expression (4.40) and noting that $l_R = l_L = 1$ one obtains

$$c_{LR} = 2c\frac{1}{2(1+\sinh^2 q)} = \frac{c}{\cosh^2 q} \tag{4.50}$$

Hence we observe that the transmission coefficient and the effective central charge obey the following relation

$$\frac{c_{LR}}{c} = \left(\frac{c_{eff}}{c}\right)^2 \tag{4.51}$$

The same relation was found in [138] for the ten dimensional half BPS Janus Janus solution of [114,118]. Note that the exact relation has a different form than the inequalities discussed at the end of section 4.3.3, it is however easy to verify that the supersymmetric solutions also satisfy these inequalities.

4.5 Discussion

In this paper, we used minimal $d = 3, N = 2$ gauge supergravity with a single scalar field to construct solutions that represent Janus and RG-flow interface solutions. The model depends on a single parameter a, for $a < \frac{1}{\sqrt{2}}$ there is a single supersymmetric AdS_3 vacuum and AdS_2 sliced solution are Janus interface solution. For $\frac{1}{\sqrt{2}} < a < 1$ there are two additional non-supersymmetric vacua and depending on initial conditions there are Janus solutions as well as fine-tuned RG-flow interface solutions that interpolate between the different vacua. We showed that the single scalar model does not allow for interface solutions that preserve any supersymmetry and solutions are obtained by numerical integration. We calculated holographic observables such as symmetric and interface entanglement entropy and transmission coefficients using the numerical solutions and confirmed that the inequalities involving c_{LR}

and c_{eff} proposed in [137] are satisfied for the solutions obtained in this paper. The simplicity of the model and solutions makes it a good model to calculate other holographic observables such as correlation functions, complexity measures or other entanglement entropy and check whether other inequalities involving these quantities can be discovered.

For supersymmetric Janus solutions previously obtained in [112] we showed that an exact relation between the entanglement entropy and the reflection coefficient first obtained in [138] is satisfied. It would be interesting to find a proof of this relation for all supersymmetric AdS_3 Janus solutions, but we have not been able to find one so far.

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