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SIMPLE CALCULATIONS OF RESONANCES IN BARYON-MESON TWO-PARTICLE CHANNELS

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**Berkeley, California**

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August 19, 1964

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## ABSTRACT

The matrix formulation of the N/D method is applied in the static model to the case of n-coupled two-particle channels containing one baryon and one pseudoscalar meson. The N function is approximated by a single pole at the same position for all matrix elements, with residues given by exact  $SU_3$  symmetry. The mass splitting of the  $P_{3/2}$  baryon decuplet is studied, as well as the effect of the  $\kappa$  channel on the position of the 3-3 resonance and the  $\pi N$  3-3 phase shift.

## II. FORMALISM

To be specific, we restrict ourselves to the  $J = 3/2$ ,  $T = 3/2$ ,  $Y = 1$  case; the generalization to other cases is evident.

The partial-wave scattering amplitude  $A$  is a two-by-two matrix,

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

with 1, 2 the  $\pi N$ ,  $\kappa \Sigma$  channels. The normalization convention is

$$A_{11} = \frac{e^{i\delta_{33}} \sin \delta_{33}}{q_1^3}$$

where  $\delta_{33}$  is the  $\pi N$  3-3 phase shift, and  $q$  is the magnitude of the momentum.

We assume the most important interaction comes from the exchange of a baryon in the crossed channel, so our only left-hand singularity is a short cut approximated by a pole, at the same position for all matrix elements and with the residues given by exact  $SU_3$  symmetry.<sup>3</sup>

In the present case we have

$$A_{w \rightarrow w_0} = \begin{pmatrix} 2g_{N\pi}^2 & \epsilon_{\lambda\kappa}\epsilon_{\lambda\pi} - \epsilon_{\Sigma\pi}\epsilon_{\Sigma\kappa} \\ \epsilon_{\lambda\pi}\epsilon_{\lambda\kappa} - \epsilon_{\Sigma\pi}\epsilon_{\Sigma\kappa} & 2h_{\Sigma\kappa}^2 \end{pmatrix} \frac{1}{w - w_0}$$

Here we follow the notation of reference 3, in which it has been shown that all the coupling constants can be characterized by two parameters  $g$  and  $f$ , with  $(1 - f)g$  and  $fg$  as the coefficients of d- and F-type coupling.

We then write A as the product of two matrices;

$$A = ND^{-1}$$

and the unitarity condition gives

$$D = 1 - \frac{1}{\pi} \int_{-\infty}^{\infty} dw' \frac{\rho N}{w' - w}$$

where  $\rho$  is a diagonal matrix with appropriate phase-space factors and  $\theta$ -functions, i.e.,

$$\rho_{ij} = q_i^3 \theta(w - w_i) \delta_{ij}, \quad \text{if } w_i = M_i + m_i$$

is the threshold of the  $i$ th channel, and  $q_i = \{(w - M_i)^2 - m_i^2\}^{1/2}$  is the magnitude of the momentum.

In actual calculations we put  $N$  equal to  $A$  near the interaction pole; thus we have to make a subtraction for  $D$  at  $w_0$  and require  $D(w_0) = 1$ :

$$N = \begin{pmatrix} 2g_{N\pi}^2 & g_{\lambda\kappa}g_{\lambda\pi} - g_{\Sigma\kappa}g_{\Sigma\pi} \\ g_{\lambda\kappa}g_{\lambda\pi} - g_{\Sigma\kappa}g_{\Sigma\pi} & 2h_{\Sigma\kappa}^2 \end{pmatrix} \frac{1}{w - w_0}$$

$$D = 1 - \frac{w - w_0}{\pi} \int_{-\infty}^{w_{\max}} dw' \frac{\rho N}{(w' - w)(w' - w_0)}$$

The dispersion integral for  $D$  doesn't converge even after the subtraction, so we need the cutoff parameter  $w_{\max}$ .

To find the position of the resonance, note that

$$D_{ij}^{-1} = \frac{D(ji)}{\det D}$$

where  $D(ji)$  is the minor of  $D$   $_{ji}$ , so

$$A_{ij} = \frac{N_{ik} D(jk)}{\det D}$$

and we identify the zero of the real part of the determinant of  $D$  as the position of the resonance.

If we define

$$a = \frac{d}{dw} \operatorname{Re} (\det D) \Big|_{w=w_{33}}$$

the residue of the 3-3 resonance pole in  $A_{11}$  is

$$\gamma_{33} = \frac{[N_{11} D_{22} - N_{12} D_{21}]_{w=w_{33}}}{a}$$

An additional point of interest is how the  $\pi N$  3-3 phase shift changes when both  $\pi N$  and  $\pi \Sigma$  channels are taken into account. We have:

$$\begin{aligned} A_{11} &= \frac{e^{i\delta_{33}} \sin \delta_{33}}{q_1^3} \\ &= (ND^{-1})_{11} \\ &= \frac{N_{11} D_{22} - N_{12} D_{21}}{\det D} \\ q_1^3 \cot \delta_{33} &= \frac{\operatorname{Re} (\det D)}{N_{11} D_{22} - N_{12} D_{21}} \end{aligned}$$

to be compared with  $\operatorname{Re} D_{11}/N$  in the single channel case.



### III. NUMERICAL CALCULATIONS AND RESULTS

The adjustable parameters are  $g, f, w_{\max}$ , and  $w_0$ . We set  $2g^2 = (4/9)\gamma_{11} \sim 0.1$ ,<sup>1</sup> and the value of  $f$  is chosen to be 0.35,<sup>4</sup> following the considerations in reference 3.

Two different choices of  $w_0$  and  $w_{\max}$  are made on the following basis: (a) The decuplet mass splitting case: we fix  $w_0 = 1060$  Mev and  $w_{\max} = 2866$  Mev by requiring that the masses of  $\Delta$  and  $\Omega$  be given correctly by single-channel calculations. We then calculate the positions of  $\Sigma^*$  and  $\Xi^*$  as well as the two-channel  $\Delta$  mass. The results are shown in Fig. 1, where it may be seen that the effect of the  $\kappa\Sigma$  channel on the  $\Delta$  mass is small. (b) To study more accurately the change in  $\gamma_{33}$  and the energy dependence of  $\pi N$  3-3 phase shift due to the inclusion of the  $\kappa\Sigma$  channel, we choose  $w_0$  to be the nucleon mass (939 Mev) and  $w_{\max}$  is fixed by requiring that the position of the 3-3 resonance be given correctly by considering the  $\pi N$  channel only. We then find  $w_{\max} = 2340$  Mev. The results are shown in Table I and Figs. 2 and 3.

#### ACKNOWLEDGMENT

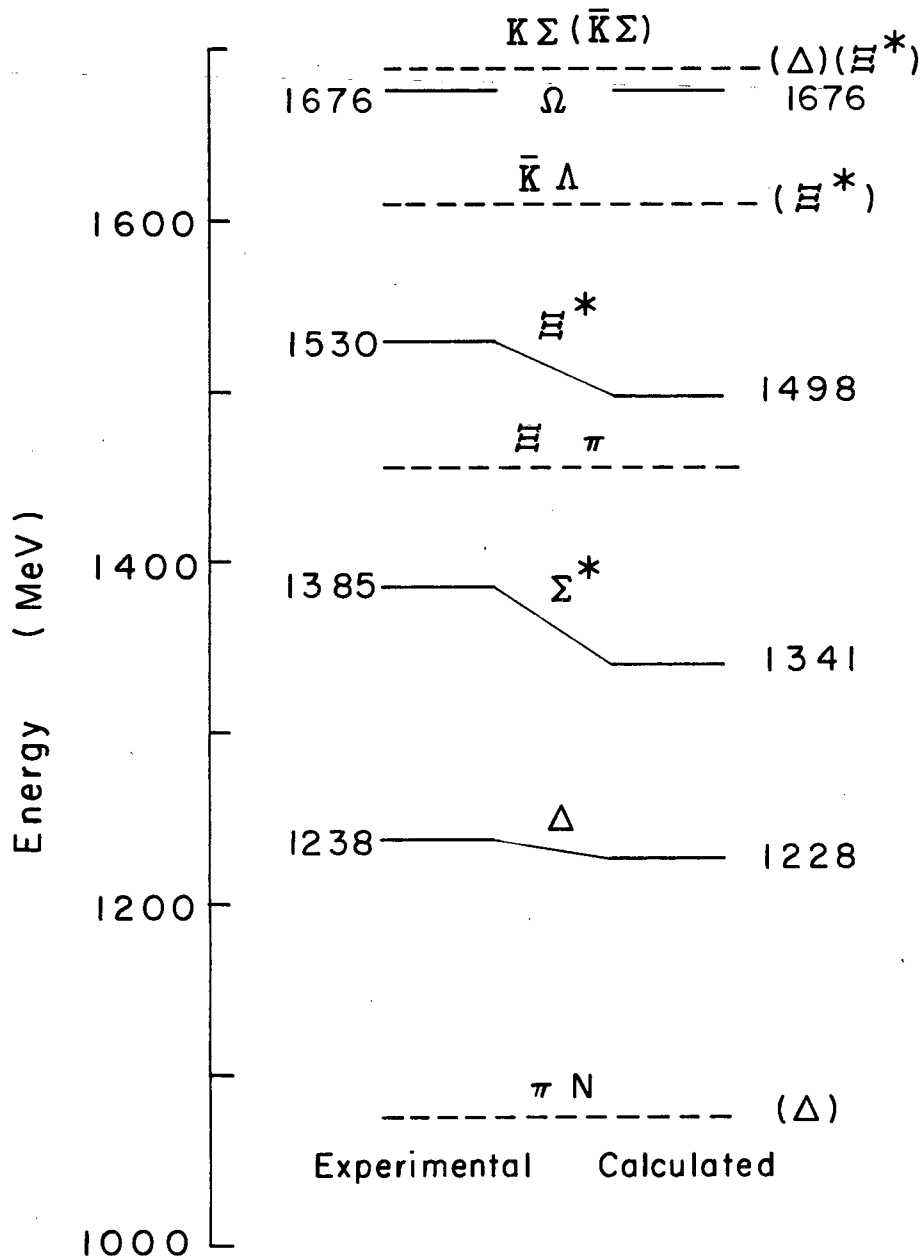
It is a pleasure to acknowledge my indebtedness to Professor Geoffrey F. Chew for suggestion of the problem and his constant guidance throughout this work.

Table I. Value of  $\gamma_{33}$ , where  $w_0 = 939$  Mev and  $w_{max} = 2340$  Mev.

	1-channel	2-channel
$a(\text{MeV}^{-1})$	$3.7 \times 10^{-3}$	$3.4 \times 10^{-3}$
$\gamma_{33}$	0.097	0.099

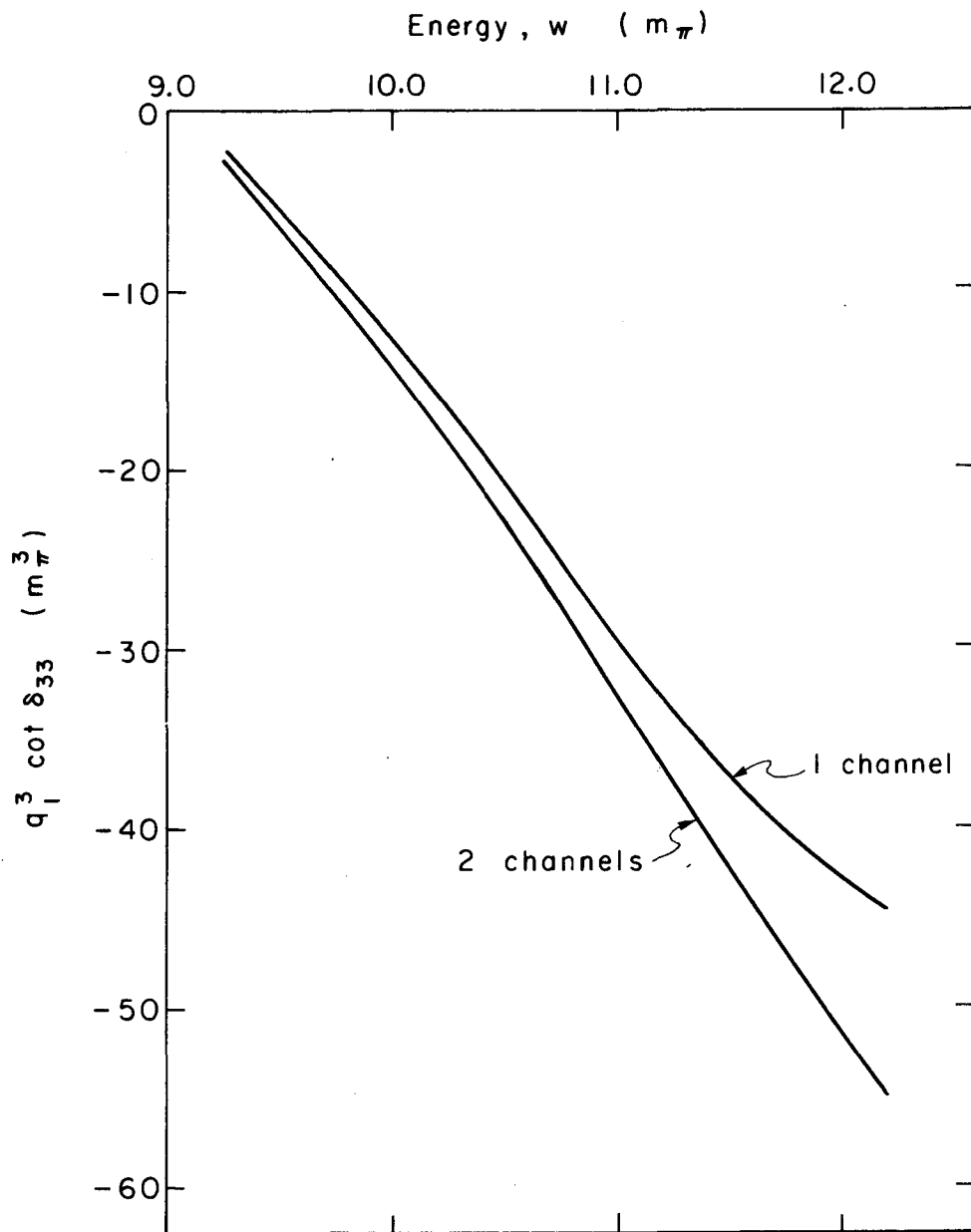
FOOTNOTES AND REFERENCES

- \* This work was done under the auspices of the U.S Atomic Energy Commission.
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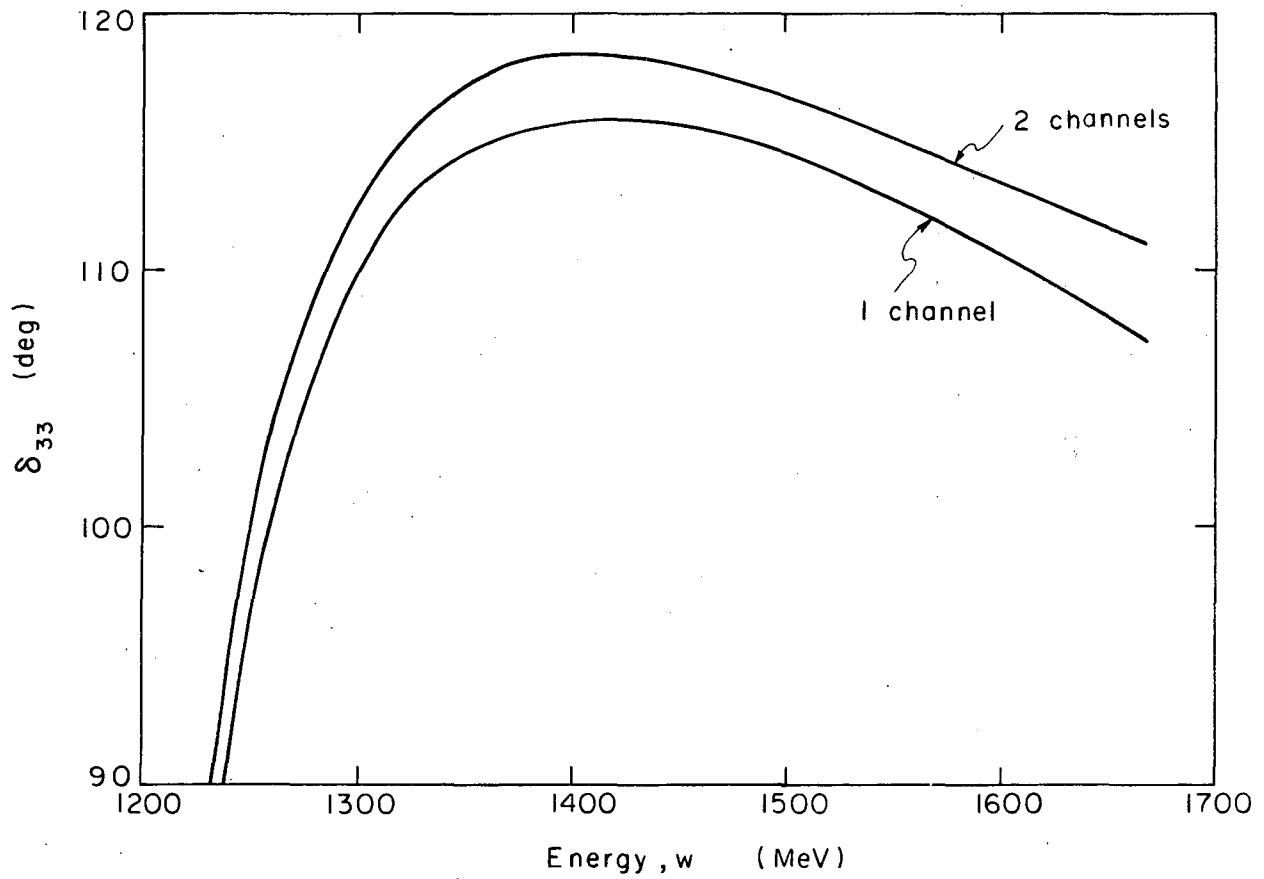
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Fig. 1. Comparison of calculated resonance positions with experiment. Dotted lines are the related thresholds.  $w_n = 1059.5$  MeV,  $w_{max} = 2866$  MeV.



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Fig. 2.  $q_1^3 \cot \delta_{33}$  vs energy for the 1- and 2-channel cases.  
 $w_0 = 939 \text{ MeV}$ ,  $w_{\text{max}} = 2340 \text{ MeV}$ .



MUB-3946

Fig. 3.  $\delta_{33}$  vs energy for the 1- and 2-channel cases.

$w_0 = 939$  MeV,  $w_{\max} = 2340$  MeV.

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