Title
SIMPLE CALCULATIONS OF RESONANCES IN BARYON-MESON TWO-PARTICLE CHANNELS

Author
Chu, Shu-Yuan

Publication Date
1964-08-19

UCRL-11622

University of California

Ernest O. Lawrence Radiation Laboratory

SIMPLE CALCULATIONS OF RESONANCES IN BARYON-MESON TWO-PARTICLE CHANNELS

Berkeley, California

Research and Development

UCRL-11622
UC-34 Physics
TID-4500 (30th Ed.)

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

SIMPLE CALCULATIONS OF RESONANCES IN BARYON-MESON TWO-PARTICLE CHANNELS

Shu-yuan Chu

August 19, 1964

UCRL 11622

SIMPLE CALCULATIONS OF RESONANCES IN BARYON-MESON TWO-PARTICLE CHANNELS*

Shu-yuan Chu

Lawrence Radiation Laboratory
University of California
Berkeley, California

August 19, 1964

ABSTRACT

The matrix formulation of the N/D method is applied in the static model to the case of n-coupled two-particle channels containing one baryon and one pseudoscalar meson. The N function is approximated by a single pole at the same position for all matrix elements, with residues given by exact SU_3 symmetry. The mass splitting of the $P_{3/2}$ baryon decuplet is studied, as well as the effect of the $K\Sigma$ channel on the position of the 3-3 resonance and the πN 3-3 phase shift.

II. FORMALISM

To be specific, we restrict ourselves to the $J = 3/2$, $T = 3/2$, $Y = 1$ case; the generalization to other cases is evident.

The partial-wave scattering amplitude A is a two-by-two matrix,

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

with 1, 2 the πN, $K\Sigma$ channels. The normalization convention is

$$A_{11} = \frac{e^{i\delta_{33}} \sin \delta_{33}}{q_1^3}$$

where δ_{33} is the πN 3-3 phase shift, and q is the magntiude of the momentum.

We assume the most important interaction comes from the exchange of a baryon in the crossed channel, so our only left-hand singularity is a short cut approximated by a pole, at the same position for all matrix elements and with the residues given by exact SU_3 symmetry.[3] In the present case we have

$$A \underset{w \to w_0}{\sim} \begin{pmatrix} 2g_{N\pi}^2 & g_{\Lambda K} g_{\Lambda\pi} - g_{\Sigma\pi} g_{\Sigma K} \\ g_{\Lambda\pi} g_{\Lambda K} - g_{\Sigma\pi} g_{\Sigma K} & 2h_{\Sigma K}^2 \end{pmatrix} \frac{1}{w - w_0} .$$

Here we follow the notation of reference 3, in which it has been shown that all the coupling constants can be characterized by two parameters g and f, with $(1 - f)g$ and fg as the coefficients of d- and F-type coupling.

We then write A as the product of two matrices;

$$A = ND^{-1} \quad ,$$

and the unitarity condition gives

$$D = 1 - \frac{1}{\pi}\int^{\infty} dw' \frac{\rho N}{w' - w} \quad ,$$

where ρ is a diagonal matrix with appropriate phase-space factors and θ-functions, i.e.,

$$\rho_{ij} = q_i^3 \theta(w - w_i)\delta_{ij} \quad , \quad \text{if} \quad w_i = M_i + m_i$$

is the threshold of the ith channel, and $q_i = \{(w - M_i)^2 - m_i^2\}^{1/2}$ is the magnitude of the momentum.

In actual calculations we put N equal to A near the interaction pole; thus we have to make a subtraction for D at w_0 and require $D(w_0) = 1$:

$$N = \begin{pmatrix} 2g_{N\pi}^2 & g_{\lambda\kappa}g_{\lambda\pi} - g_{\Sigma\kappa}g_{\Sigma\pi} \\ g_{\lambda\kappa}g_{\lambda\pi} - g_{\Sigma\kappa}g_{\Sigma\pi} & 2h_{\Sigma\kappa}^2 \end{pmatrix} \frac{1}{w - w_0}$$

$$D = 1 - \frac{w - w_0}{\pi}\int^{w_{max}} dw' \frac{\rho N}{(w' - w)(w' - w_0)} \quad .$$

The dispersion integral for D doesn't converge even after the subtraction, so we need the cutoff parameter w_{max}.

To find the position of the resonance, note that

$$D_{ij}^{-1} = \frac{D(ji)}{\det D} \quad ,$$

where D(ji) is the minor of D_{ji}, so

$$A_{ij} = \frac{N_{ik} D(jk)}{\det D} ,$$

and we identify the zero of the real part of the determinant of D as the position of the resonance.

If we define

$$a = \frac{d}{dw} \mathrm{Re}\, (\det D) \Big|_{w=w_{33}} ,$$

the residue of the 3-3 resonance pole in A_{11} is

$$\gamma_{33} = \frac{[N_{11} D_{22} - N_{12} D_{21}]_{w=w_{33}}}{a} .$$

An additional point of interest is how the πN 3-3 phase shift changes when both πN and κΣ channels are taken into account. We have:

$$A_{11} = \frac{e^{i\delta_{33}} \sin \delta_{33}}{q_1^3}$$

$$= (ND^{-1})_{11}$$

$$= \frac{N_{11} D_{22} - N_{12} D_{21}}{\det D}$$

$$q_1^3 \cot \delta_{33} = \frac{\mathrm{Re}\, (\det D)}{N_{11} D_{22} - N_{12} D_{21}} ,$$

to be compared with $\mathrm{Re}\, D_{11}/N$ in the single channel case.

III. NUMERICAL CALCULATIONS AND RESULTS

The adjustable parameters are g,f, w_{max}, and w_0. We set $2g^2 = (4/9)\gamma_{11} \sim 0.1$,[1] and the value of f is chosen to be 0.35,[4] following the considerations in reference 3.

Two different choices of w_0 and w_{max} are made on the following basis: (a) The decuplet mass splitting case: we fix w_0 = 1060 Mev and w_{max} = 2866 Mev by requiring that the masses of Δ and Ω be given correctly by single-channel calculations. We then calculate the positions of Σ* and Ξ* as well as the two-channel Δ mass. The results are shown in Fig. 1, where it may be seen that the effect of the κΣ channel on the Δ mass is small. (b) To study more accurately the change in γ_{33} and the energy dependence of πN 3-3 phase shift due to the inclusion of the κΣ channel, we choose w_0 to be the nucleon mass (939 Mev) and w_{max} is fixed by requiring that the position of the 3-3 resonance be given correctly by considering the πN channel only. We then find w_{max} = 2340 Mev. The results are shown in Table I and Figs. 2 and 3.

ACKNOWLEDGMENT

It is a pleasure to acknowledge my indedtness to Professor Geoffrey F. Chew for suggestion of the problem and his constant guidance throughout this work.

Table I. Value of γ_{33}, where w_0 = 939 Mev and w_{max} = 2340 Mev.

	1-channel	2-channel
a(MeV^{-1})	3.7×10^{-3}	3.4×10^{-3}
γ_{33}	0.097	0.099

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U.S Atomic Energy Commission.

1. G. F. Chew, Phys. Rev. Letters 9, 233 (1962).

2. J. D. Bjorken, Phys. Rev. Letters 4, 473 (1960).

3. A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963); see also A. W. Martin and K. C. Wali, Meson-Baryon Resonances in the Octet Model (Argonne National Laboratory preprint).

4. K. C. Wali and R. L. Warnock, Breaking of SU(3) Symmetry in the $3/2^+$ Meson-Baryon Decuplet (Argonne National Laboratory preprint).

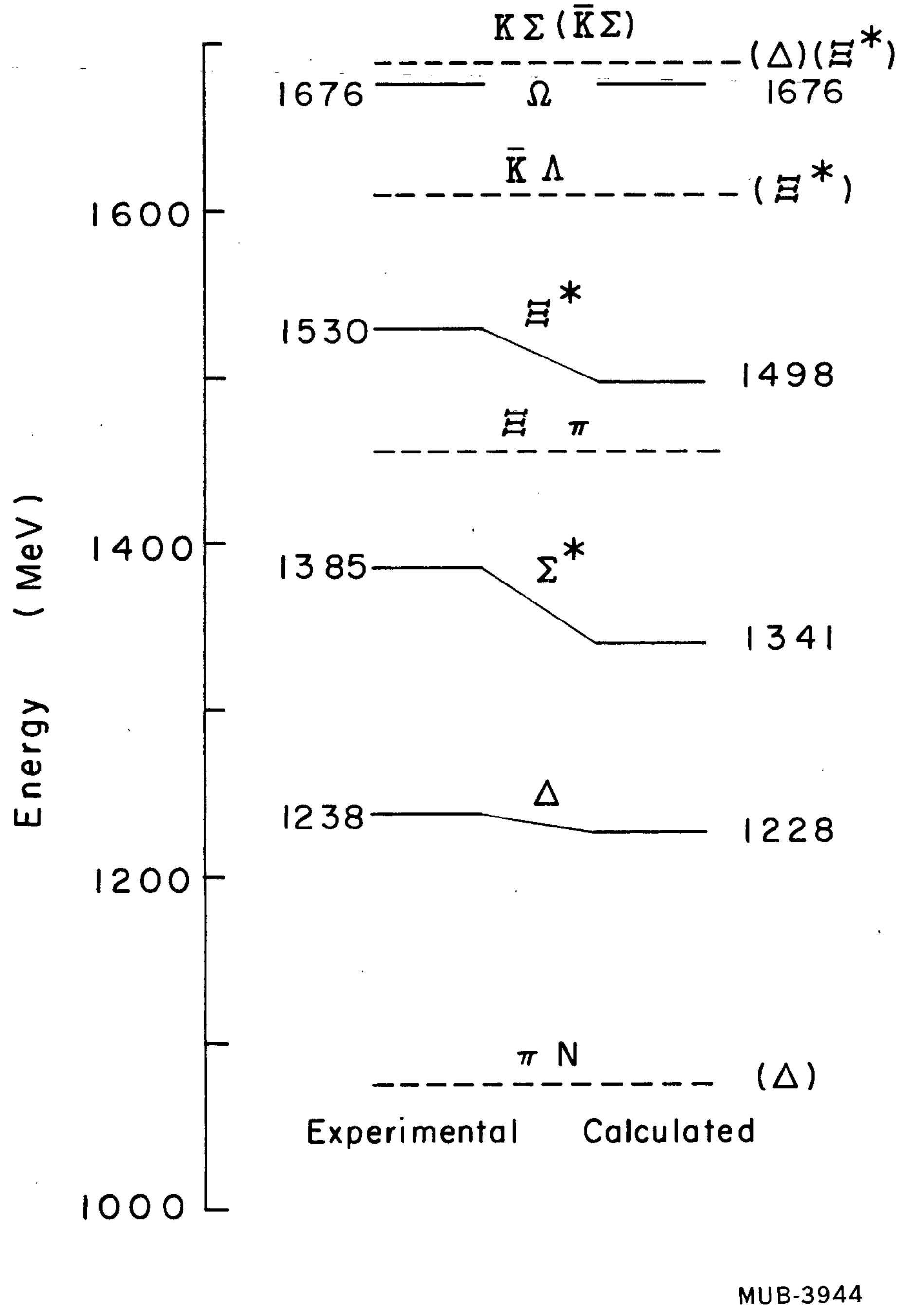
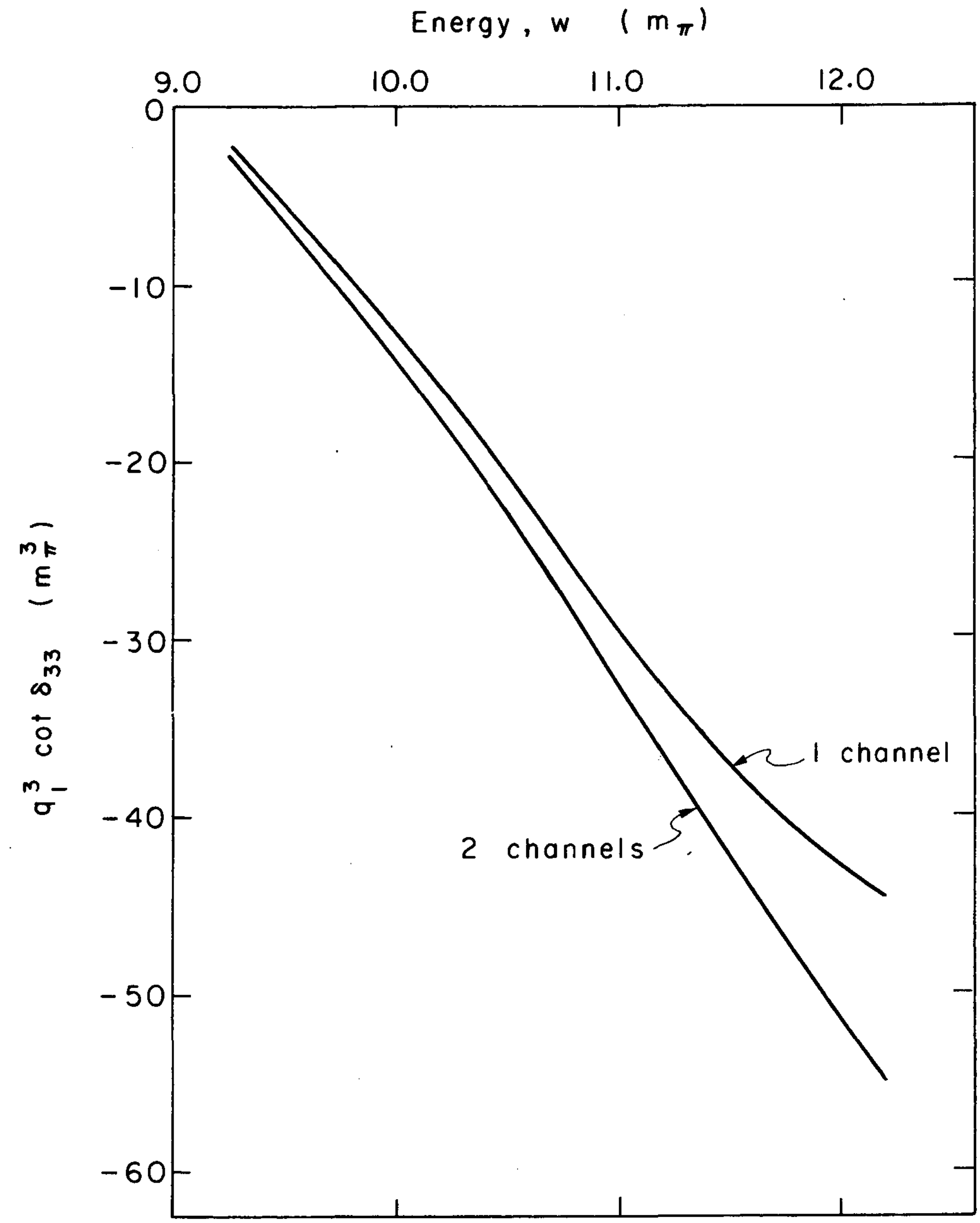
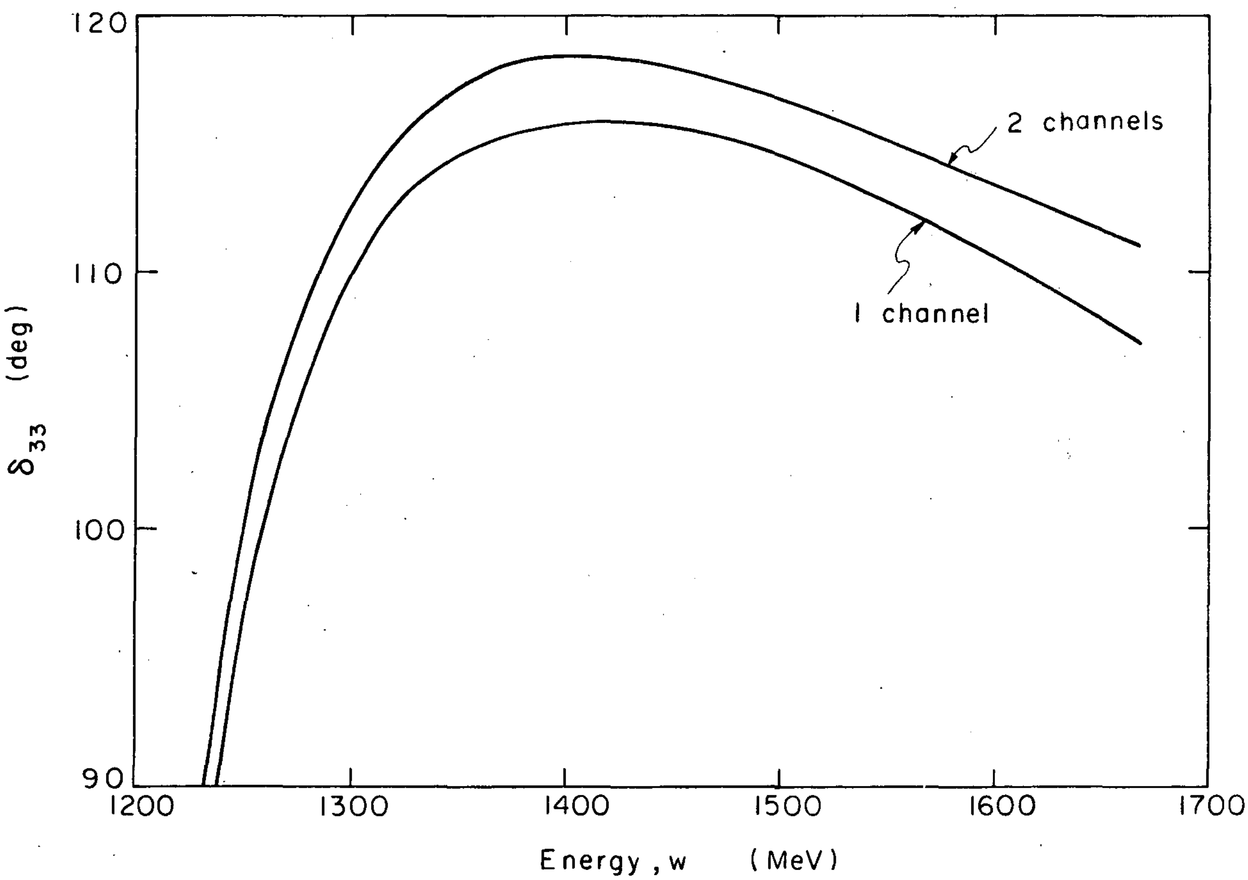


Fig. 1. Comparison of calculated resonance positions with experiment. Dotted lines are the related thresholds. w_0 = 1059.5 MeV, w_{max} = 2866 MeV.



MUB-3945

Fig. 2. q_1^3 cot δ_{33} vs energy for the 1- and 2-channel cases. w_0 = 939 MeV, w_{max} = 2340 MeV.



MUB-3946

Fig. 3. δ_{33} vs energy for the 1- and 2-channel cases. w_0 = 939 MeV, w_{max} = 2340 MeV.