The Role of Vagueness in the Numerical Translation of Verbal Probabilities: A Fuzzy Approach

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Abstract

The paper describes a general two-step procedure for the numerical translation of linguistic terms using parametric fuzzy potential membership functions. In an empirical study 121 participants estimated numerical values that correspond to 13 verbal probability expressions. Among the estimates are the most typical numerical equivalent and the minimal and maximal values that just correspond to the given linguistic terms. These values serve as foundation for the proposed fuzzy approach. Positions and shapes of the resulting membership functions suggest that the verbal probability expressions are not distributed equidistantly along the probability scale and vary considerably in symmetry, vagueness and overlap. The role of vagueness for further investigations in reasoning and decision making is discussed and relations to knowledge representation and working memory are highlighted.

Keywords: verbal probability expressions; vagueness; fuzzy potential membership functions; knowledge representation; diagnostic reasoning; working memory

Introduction

Since the 1960s up to the present time researchers of different scientific areas have sustained an interest in studying the relationship between verbal and numerical probability expressions (Lichtenstein & Newman, 1967; Teigen & Brun, 2003; Smits & Hoorens, 2005). Among these are cognitive psychologists that inquire about the influence of uncertainty expressions on basic cognitive processes such as reasoning and decision making (Windschitl & Wells, 1996) as well as engineers, computer scientists and others that focus on the characterization (Zadeh, 1978, 2002) or on the treatment of uncertainty in applications such as medical decision support systems (Boegl, Adlassnig, Hayashi, Rothenfluh & Leitich, 2004). This broad interdisciplinary interest may be motivated by the essential role language plays in our daily life. Verbal probability terms, such as *probably* or *thinkable* are very widely used to express uncertainty about the occurrence of future events or about the degree of belief in hypotheses. For example, a typical statement that illustrates the use of linguistic terms in the conversation of stock market traders could be: "It is *very unlikely* that there will be a significant increase in the price of oil in the next month vice future.". Several studies consistently show that people prefer words over numbers to express uncertainty (e.g. Wallsten, Budescu, Zwick & Kemp, 1993). This preference may be explained by the possibility of saying something about two different kinds of subjective uncertainty by using only one word. First, the stochastic uncertainty about the occurrence of an event (e.g. the probability of an increase of the oil price) and second, the vagueness of the event (e.g. what is meant by "a significant increase").

The understanding of these two kinds of uncertainty, their relations to each other and the way in which they influence human reasoning and decision making is crucial for any application that aims to support decision makers for example in medicine, business, risk management, marketing or politics. In our view, in order to contribute to the understanding of uncertainty, it is essential to first uncover the underlying relationship between word meaning and mathematical concepts such as subjective probability or fuzzy membership. Therefore, we propose a general two-step procedure for the numerical translation of verbal probability expressions based on (1) empirical estimates modelled by (2) fuzzy membership functions (Zadeh, 1965, Bocklisch & Bitterlich, 1994).

The paper is structured as follows: first, we compare verbal and numerical probability expressions and discuss existing translation approaches. Second, we present our proposal that goes beyond other methodical issues and the results of an empirical investigation. Thereafter, the results are discussed and conclusions (e.g. for the construction of verbal probability scales for questionnaires) are highlighted. Further, potentialities of the fuzzy pattern classification method for reasoning and decision processes are pointed out.

Verbal and Numerical Probabilities

There is broad agreement concerning the different features of verbal and numerical expressions (see Teigen & Brun, 2003 for an overview). Numerical probabilities are commonly described as precise, unambiguous and especially useful for calculations. Additionally, the quality of numerical expressions can be evaluated and compared to predictions of normative models such as Bayes nets. Currently many researchers in the area of cognitive

psychology utilize subjective probabilities for the modelling of human reasoning (e.g. Bayes nets in inductive learning and reasoning (Tenenbaum, Griffiths & Kemp, 2006)). This approach is very fruitful and the obtained results contribute highly to the understanding of psychological processes but, at the same time, it focuses only on the probability dimension of uncertainty. Generally, vagueness is another facet of people's subjective uncertainty and should not be neglected. The effects of vagueness, such as exemplarily described by Kuhn and Budescu (1996) for hazard risk decisions, have received much less research attention in psychology. Although it is investigated more in engineering and other domains, where the practical significance is clearly observable from its prevalence in real-world decisions, vagueness is also crucial for psychological approaches. Zadeh (1965) proposed the fuzzy framework for the handling of vagueness and pointed out that probability theory and fuzzy approaches are complementary rather than competitive (Zadeh, 1995). Hence, it is possible to combine probability and fuzzy accounts and the advantages of bridging the gaps have been discussed recently (Singpurwalla & Booker, 2004).

In contrast to numerical probabilities, probability words are vague, with ambiguous meaning. They cannot be easily used for calculations and their meaning is often only clarified by means of a context (such as domain, speakers' prior knowledge and experience, reference point or prior probabilities and base rates of events). Nevertheless, most people in most everyday situations use words rather than numbers when describing their own uncertainty. Words are perceived as more natural, easier to understand and communicate and they are useful in situations when uncertainty can not at all be verbalized exactly. Numerical and verbal expressions are closely associated and refer to the underlying concept of probability and there is evidence that people can use numbers and words interchangeably (Jaffe-Katz, Budescu & Wallsten, 1989). But, at the same time, words and numbers do not mean exactly the same thing.

Furthermore, it can be assumed from various experiments that the use of numbers versus words affects human reasoning processes under certain circumstances. Windschitl and Wells (1996) show that numeric measures of uncertainty tend to sway people toward rule-based, deliberate thinking, whereas verbal expressions tend to elicit more associative and intuitive reasoning. These findings are of particular importance for reasoning situations that create conflicts between logical reasoning and intuitive beliefs (e.g. the belief-bias effect (Evans, 2003)).

In belief updating processes, such as customers product evaluation, there is evidence for the influence of information format (verbal vs. numerical) on order effects. Shen and Hue (2007) report that numerical information lead to order effects whereas verbal expressions do not. It can be assumed that the utilization of numerical vs. verbal expression formats result in different cognitive processes that in turn have different consequences for decisions.

Translating Words Into Numbers

In order to investigate the impact of verbal versus numerical probability expressions on order effects, decision making and the communication of uncertainty methods have to be developed for the "translation" of verbal into numerical expressions. There are already a number of translation studies that utilized different estimation and translation procedures. Among these are empirical approaches using direct estimation techniques for instance on a scale from 0 to 100 (Beyth-Marom, 1982) or pair comparison methods (Wallsten, Budescu, Rapoport, Zwick & Forsyth, 1986) as well as expert consultations for example to create knowledge bases for decision support systems (Boegl et al., 2004). A summary and discussion of different estimation approaches, that map verbal probabilities onto a numerical probability scale, is provided by Teigen and Brun (2003).

Recurrent findings in the studies using empirical estimations are that the mean estimates of the verbal probability expressions are reasonably similar supporting the idea that words are translatable. At the same time, there is a large variability between individuals indicating inconsistency in word understanding which may lead to communication problems. Although there are different views on whether verbal probability expressions are quantifiable or not (Teigen & Brun, 2003), we agree with Budescu et al. (2003). They propose to treat probability words as fuzzy sets and use fuzzy membership functions (MFs) over the probability scale to represent their vague meanings. They elicited judgments of membership by using a multiple stimuli estimation method in which probability values (0, 0.1, …, 0.9, 1) are presented simultaneously with a verbal probability expression. Their results show, that the peak value and skew of the MF describing a probability expression depends on the words meaning. Therefore, they conclude that properties of the MF can predict for example the directionality (positive vs. negative verbal expressions, such as probable vs. improbable) of probability words.

Objective of the Paper

This paper has the goal to present a general two-step procedure for the numerical translation of linguistic terms. It is composed of (1) a direct empirical estimation method that yields numerical data participants assigned to presented words and (2) a fuzzy approach for the analysis of the data resulting in parametric membership functions (MFs) of the potential type (Bocklisch & Bitterlich, 1994). We outline this method for verbal probability expressions (e.g. *possible*) but the proposed procedure can also be applied for other linguistic terms such as expressions of frequency (e.g. *occasionally*), strength (e.g. *strong*) or others and is therefore of potential interest for many research areas and applications. Furthermore, our method goes beyond existing approaches for two reasons: first, the presented direct estimation method is frugal, efficient and easy to use to yield data from human decision makers. Therefore, it is suitable for research purposes and especially for applications where expert knowledge is crucial but also rare

or expensive. Second, the proposed parametric MFs of the potential type bring along advantages compared to other MFs (Zadeh, 1965; Budescu et al., 2003). For instance, they are able to account for asymmetric probability terms and are defined continuously over the numerical probability scale. Hence, linguistic terms can be modelled very realistically. In addition, the MFs can be implemented directly in applications (e.g. in fuzzy decision support systems) and the fuzzy pattern classification approach has potentials for psychological research (see Future Prospects at the end of this paper).

In contrast to Boegl et al. (2004) we do not expect that the MFs of the probability words are distributed equidistantly along the numerical probability scale and just like Budescu et al. (2003) we predict the functions to be asymmetric in shape.

Two-Step Translation Procedure

In this section we present the details of the two-step translation procedure for the numerical translation of verbal probability expressions. At first, the estimation technique and the method we used in the empirical study is outlined. Thereafter, the fuzzy analysis and the MFs are specified.

Empirical Investigation

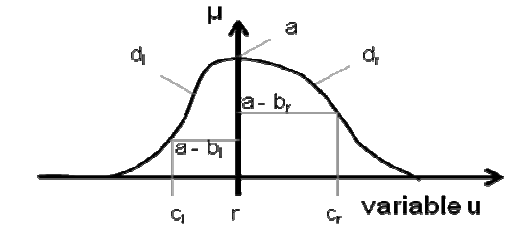
Participants. 121 participants (19 males) took part in the study mainly for exchange of credits. The majority were undergraduate students of the Universities of Chemnitz, Göttingen and Zurich with an average age of 23 years (*SD*=4.6).

Materials and Procedure. Participants read a short contextual story from the area of medical decision making and were requested to take over the perspective of a physician. Then they assigned three numerical values to each of 13 exemplars of probability words (see translated words in Table 1, the original material was presented in German language) that were chosen from previous studies (e.g. Budescu et al., 2003). Among the three numerical values that had to be estimated were: (1) the one that represents the given probability word best and the (2) minimal and (3) maximal values that just correspond. The estimations can be interpreted according to the semantic meaning of the words: the first value characterizes the most typical numerical equivalent for the word, whereas the other values indicate the lower and upper border of the verbal probability expression. Participants were instructed to give their estimates in the frequency format (e.g. "In how many of 100 cases a certain diagnosis is correct if it is for instance *improbable*?"). This frequency format of estimation was proved to be better than for instance the estimation of percentages (Gigerenzer & Hoffrage, 1998). Participants used a PDF online questionnaire to provide their estimates.

Fuzzy Analysis

Fuzzy Membership Functions. Membership functions are truth value functions. The membership value (μ) represents the value of truth that an object belongs to a specific class (e.g. that the numerical probability value 0.25 belongs to the word *doubtful*). For the analysis of the empirical data provided by the 121 participants a parametric membership function of the potential type (Bocklisch & Bitterlich, 1994; Hempel & Bocklisch, 2009) was used.

This function (see Figure 1) is based on a set of eight parameters: r marks the position of the mean value, a is representing the maximum value of the membership function. Regarding a class structure, a expresses the "weight" of the class in the given structure (we use a fixed a=1 in this investigation). The parameters b_l and b_r assign left and right-sided membership values at the borders of the function. Hence, they represent the border memberships whereas c_l and c_r characterize the left and right-sided expansions of the class and therefore mark the range of the class (in a crisp sense). The parameters d_l and d_r specify the continuous decline of the membership function starting from the class centre, being denoted as representative of a class. They determine the shape of the function and hence the fuzziness of the class.



$$\mu(u) = \begin{cases} \dfrac{a}{1 + (\frac{1}{bl} - 1)(\frac{r-u}{cl})^{dl}} & u \leq r \\ \dfrac{a}{1 + (\frac{1}{br} - 1)(\frac{u-r}{cr})^{dr}} & u \geq r \end{cases}$$

Figure 1: Parameters of the membership function (for r=0)

A continuous range of membership functions, varying from a high degree of fuzziness to crisp, is available. This function type allows considering asymmetry in fuzzy classes by individual parameters for the left and right hand branches of the function. As we expect the MFs for the probability expressions to be asymmetric, this feature is especially important for the present study.

Results

In this paragraph we present the results of the statistical and fuzzy analysis of the present study. The descriptive statistics were calculated with the help of SPSS software. For the fuzzy analysis and the modelling of the MFs a software package (Fuzzy Toolbox, 2008) was used.

Descriptive Statistics

Table 1 shows the descriptive statistics for the empirical estimates of the most typical values that correspond to the

presented words. The minimal and maximal estimates, that indicate the borders of the semantic meaning of the linguistic terms, were necessary for modelling the MFs.

Results show that the probability words are distributed all over the numerical probability scale with varying distances. The standard deviation and kurtosis show a systematic pattern: probability words near to the borders of the numerical probability scale (e.g. *impossible* and *certain*) have small standard deviations but high values of kurtosis. And probability words in the middle (e.g. *thinkable* and *possible*) offer a larger spread but smaller kurtosis values. Also systematic differences exist for the skew indicating that probability expressions with means smaller than P=0.5 are skewed to the right whereas words with means higher than P=0.5 are asymmetric to the left. These findings are consistent with the results reported by Budescu et al. (2003).

Table 1. *Descriptive statistics for the estimates (most typical values)*

probability words	Mean	SD	Skew	Kurtosis
Impossible	1.44	3.01	3.25	13.39
very improbable	5.53	5.48	1.71	2.72
quite improbable	9.99	7.94	1.42	2.2
Improbable	11.68	9.03	1.43	1.82
hardly probable	17.01	11.05	1.15	1.02
sparsely probable	18.57	12.19	1.12	.89
Doubtful	21.34	13.61	.72	.32
Thinkable	49.33	20.24	.35	.1
Possible	51.49	21.6	.54	.53
Probable	67.68	12.49	-.01	-.85
quite probable	75.07	12.89	-1.01	1.02
very probable	83.95	9.08	-1.02	1.2
Certain	96.28	6.45	-2.87	9.99

Fuzzy Analysis

Figure 2 shows the MFs for the 13 verbal probability expressions. The representative values (r) indicating the highest memberships are identical to the reported means in Table 1.

Obviously, the functions differ considerably in shape, symmetry, overlap and vagueness. Functions at the borders (e.g. *impossible*) are narrower than those in the middle (e.g. *thinkable*) which is consistent with the observed standard deviations and kurtosis values. Most functions are asymmetric and are not distributed equidistantly along the probability scale. From the functions' positions, three clusters arise, that may be described by (1) *low* (MFs 1-7), (2) *medium* (MFs 8 and 9) and (3) *high* (MFs 10 - 13) probability ranges. The 13 MFs overlap in large parts and especially when they belong to the same cluster.

To test whether the probability expressions are distinct or not, participants' estimates were reclassified. Table 2 shows the results of the reclassification.

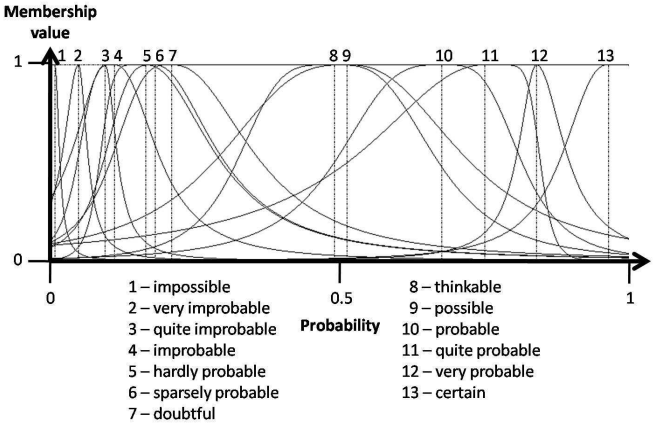


Figure 2: Membership functions of the 13 verbal probability expressions

The second column of the table presents percentages of the corresponding estimation data that was reclassified correctly. According to these results, some of the probability words are unambiguous and the reclassification was very successful (e.g. *certain*; 93.5% reclassified correctly). Others are inconclusive and almost no estimation data point that was used to describe the MF was reclassified correctly (e.g. *improbable*; 2.5 % classified correctly). Instead, the data was classified as belonging to the neighboring functions.

Table 2. *Percentages correct reclassification*

probability words	Scale (13)	Scale (5)
impossible	80.0	95.0
very improbable	33.1	
quite improbable	24.8	
improbable	2.5	
hardly probable	15.1	
sparsely probable	2.5	
doubtful	42.4	77.1
thinkable	41.2	61.3
possible	6.6	
probable	44.2	72.5
quite probable	33.9	
very probable	18.4	
certain	93.5	93.5

For a verbal probability scale that could be employed in psychological research or application, a scale with 13 probability words would not be useful because the words are too indifferent according to their meanings. But if a few words with small overlaps are selected, it is possible to create a scale that differentiates very well (see reclassification rate computed by the Fuzzy Toolbox Software in column three of Table 2). Figure 3 shows an example scale with five probability words described by their MFs.

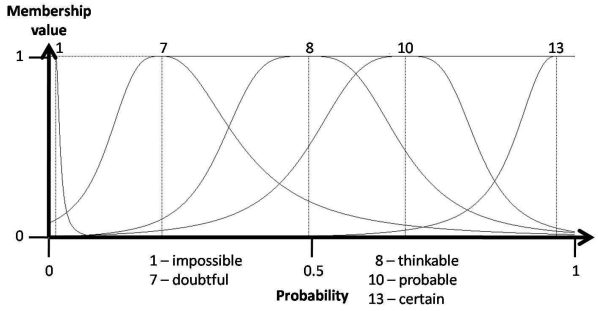


Figure 3: Membership functions of 5 selected verbal probability expressions

Discussion

This paper aims to present a two-step procedure for the numerical translation of linguistic terms that goes beyond existing approaches. First of all, the estimation of three numerical values for each linguistic term (the most typical, minimal and maximal corresponding values) is very frugal and data can be gained very efficiently, whereas most alternative procedures are more costly (Budescu et al., 2003). The resulting estimation data can be analyzed using the proposed parametric MFs of the potential type. Results show, that the functions are able to model the data in a very efficient way, creating averaged membership functions that describe the linguistic terms continuously over the numerical probability scale. Because of the eight parameters, the functions take into account asymmetry, which was indeed found in the empirical data. Parametric MFs with fewer parameters would model the data without considering asymmetry and would therefore be less accurate and suitable for the reported data. Another advantage of the proposed function type is that the parameters can be interpreted in terms of content on a semantic meta level and illustrate the vague meaning of probability words very realistically.

Large overlaps of the functions (see Figure 2) indicate that the words are very similar in their meanings. Despite the imprecision of natural language, the MFs allow identifying words that are more distinct in their meaning than others. Just as Dhami and Wallsten (2005) we also found five probability expressions (see Figure 3) that are sufficiently distinct. This is especially useful for the creation of verbal probability scales for purposes of research and application that should include unambiguous words when possible.

Finally, the presented translation procedure serves as foundation for future investigations concerning the influence of contexts on word understanding. This influence can then be quantified by changes in the parameters defining the MFs. As these parameters can be semantically interpreted the influence of context on the interpretation of the expressions can be investigated in detailed way. As Wallsten and Budescu (1990) claimed, it is a promising instrument to uncover the various communication roles that probability phrases serve. For instance, it is likely that some of the ambiguous probability words are clarified by the context in which they are used and therefore will become less vague which can be observed in the MFs.

Future Prospects

Finally, we will present a short outlook that highlights the potentials of the fuzzy approach for further psychological research in the area of diagnostic reasoning and decision making.

An advantage of the proposed MFs and the underlying fuzzy pattern classification method (Bocklisch & Bitterlich, 1994) is that the functions serve for the representation and combination of various kinds of vague knowledge (e.g. fuzzy degrees of symptom intensity such as "*high* fever" or "*low* blood pressure") in a multidimensional way. For example, a physician considering the likelihood that a patient has a certain disease presumably takes into account the intensity of two (or more) present symptoms in combination prior to stating the diagnosis. Figure 4 exemplifies the content of a possible mental model in a simplified manner: three fuzzy classes (diseases A, B and C) resulting out of the multivariate combination of two features (intensities of the symptoms 1 and 2) that are described by fuzzy potential membership functions.

Furthermore, it is possible to integrate both vague and crisp information (such as precise predictions of probabilistic models) in this framework.

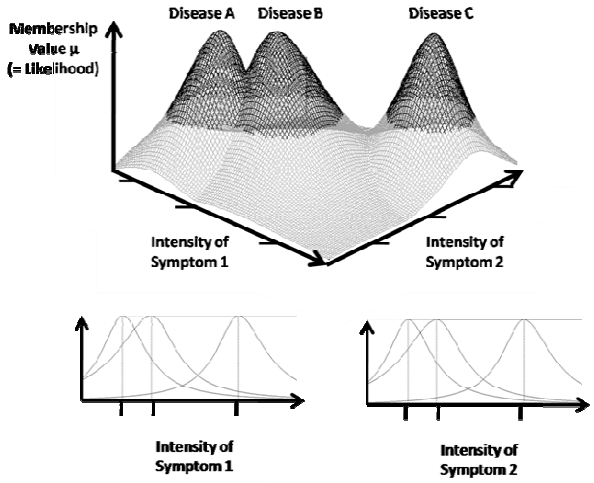


Figure 4: Representation of medical knowledge using fuzzy pattern classification method

The distance of the classes as well as their overlap can be interpreted in terms of similarity (disease classes A and B are near to each other and therefore cause similar symptom intensities, whereas disease C is apart and less similar to the other diseases). Furthermore, shapes and positions of the classes provide information about the discriminability of items in working memory which in turn affects reasoning performance. According to Oberauer, Süß, Wilhelm and Wittman (2003), the coordination function of working memory (WM) allows the integration of information (such as symptoms in a diagnostic reasoning process). Therefore,

WM provides simultaneous access to independently varying elements (such as symptoms and diseases) by placing them in a common coordinate system. The coordinate system has limited capacity to hold information and keep them separated from each other. Hence, it is likely that the precision or vagueness of the information elements (as it is described by the MFs) is an important variable influencing diagnostic reasoning processes and decision making performance. Moreover, it seems possible to predict to which extent relevant and irrelevant diagnostic hypotheses will interfere during the reasoning process (Dougherty & Sprenger, 2006) from the fuzzy knowledge representation. For example, it is plausible to assume that irrelevant diagnostic hypotheses that show a strong overlap with the relevant ones interfere more than irrelevant hypotheses that show less overlap. And the overlap can be quantified with this fuzzy approach. This is currently the object of further investigation.

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