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Abstract

Recent studies suggest that five-year-old children can add and compare large numerical quantities through approximate representations of number. However, the nature of this understanding and its susceptibility to influence from canonical, learned mathematics remain unclear. The present study examined whether children's early competence depends on the canonical problem format (i.e., arithmetic operations presented on the left-hand side of space). Children (*M* age = 5 years, 3 months) viewed events that required them to add and compare large numbers. Events were shown in a canonical or non-canonical format. Children performed successfully on all tasks, regardless of format; however, they performed better when problems were presented in the canonical format. Thus, children's approximate number representations support arithmetic reasoning prior to formal schooling, but this reasoning may be shaped by learned mathematics.

Keywords: cognitive development; mathematical cognition; approximate arithmetic; competence/performance

Mathematics is important to our lives, whether we are engineers testing the strength of a beam or parents splitting four cookies between five children. Thus, it is no surprise that scientists have a long history of studying the foundations of mathematical thinking. After decades of research in this area, there is growing consensus that humans and nonhuman animals share a non-verbal system for representing approximate number (Dehaene, 1997; Brannon, 2006; Feigenson, Dehaene, & Spelke, 2004; Gordon, 2004; Gallistel & Gelman, 2000). This approximate number system operates according to Weber's Law and appears to be functional in infants as young as 6 months old (Brannon, Abbot, & Lutz, 2004; Lipton & Spelke, 2003; Xu & Spelke, 2000). This means that babies, children, and adults detect a difference between two numerical quantities based on the ratio between the quantities, not on their absolute difference. Some researchers have even suggested that this approximate number system supports arithmetic reasoning early in life without influence from learned mathematics (Barth, La Mont, Lipton, & Spelke, 2005; Gilmore, McCarthy, & Spelke, 2007; McCrink & Wynn, 2004). In the present study, we examine whether aspects of learned mathematics influence children's use of the approximate number system to reason arithmetically.

Barth and colleagues (2005) were among the first to provide evidence that young children use the approximate number system to reason about large numbers arithmetically. In Barth et al.'s approximate addition task, children see sets of animated dot events that represent addition problems. First, a set of blue dots appears on the left side of the screen and moves down behind an occluder. Then, another set of blue dots appears on the left and moves behind the occluder. Finally, a set of red dots appears on the right side of the screen and moves down beside the occluder. The blue and red dots differ by a specific ratio (e.g., 4:7, 3:5, 2:3, or 4:5 depending on the study). Children's goal is to judge whether there are more blue dots or more red dots. Five-year-old children are able to identify whether the sum of the blue dots is larger or smaller than the number of red dots at a rate significantly greater than chance. These results have been used to suggest that children can use their approximate number system to reason arithmetically before they learn arithmetic in school.

Until recently, early competence with approximate arithmetic was assumed to be limited to nonsymbolic stimuli, such as dots or tones, in which the mapping between the stimuli and the numerical magnitude is concrete (Barth et al., 2005; cf. Levine, Jordan, & Huttenlocher, 1992). However, a recent study by Gilmore et al. (2007) demonstrated early competence with symbolic number. The term "symbolic" here is used to refer to abstract representations of number (e.g., Arabic numerals and number words) that do not directly involve sets of concrete stimuli. In Gilmore et al.'s symbolic approximate addition task, the events unfold similarly to those in Barth et al., but the events are presented in the context of a boy and girl receiving candies. Instead of dots or actual candies, children see bags (of candy) with Arabic numerals written on them. Children's goal is to judge who gets more candies. Again, five-year-old children are able to identify whether the sum of the candies presented on the left side is larger or smaller than the number of candies presented on the right side at a rate significantly greater than chance. Thus, children seem to be able to draw on their approximate number system when performing symbolic arithmetic. These findings are consistent with the view that children map symbolic numbers onto their approximate number representations as early as age five (Lipton & Spelke, 2005; Temple & Posner, 1998).

The idea that the approximate number system plays a role in knowledge of symbolic mathematics is not new. Researchers have long argued that learned forms of mathematics, from Arabic numerals to basic addition facts, are automatically mapped onto preexisting approximate number representations (Dehaene, 1997; Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967, but see Ansari, 2008 for an alternative view). According to this account, understanding of symbolic mathematics is mediated by the approximate number system (Gallistel & Gelman, 2005; Halberda, Mazzocco, & Feigenson, 2008). This account generally implies that the influence is unidirectional (approximate numerical reasoning influences symbolic mathematical reasoning, not vice versa). Consistent with this view, some researchers have suggested that the approximate number system is a "core" system that lays the groundwork for higher-order mathematical knowledge (Dehaene, 1997; Feigenson et al., 2004).

Some findings, however, challenge the notion that the approximate number system is automatically deployed when reasoning about symbolic mathematics. For example, Booth and Siegler (2006) asked children to estimate solutions to large number symbolic addition problems, such as "Is 34 + 29 closest to 40, 50, or 60?" This task should be easy for five- and six-year-old children if they draw on their approximate number representations because they have the acuity to discriminate numbers differing by a 5:6 ratio (Halberda & Feigenson, 2008). However, kindergarteners only selected the more accurate estimate on 36% (SE = .03) of the problems (chance = 33%). First graders performed better, but they still only selected the more accurate estimate on 52% of the problems. These results suggest that children may not always utilize their approximate number system when reasoning about symbolic arithmetic problems. They also leave open the possibility that children's knowledge of symbolic, learned mathematics may affect when and how the approximate number system is used. Several researchers seem to be open to this view: Gallistel and Gelman (2005) discuss possible bidirectional effects between the approximate and symbolic number systems over the course of development; Ansari (2008) speculates that the acquisition of symbolic representations may alter pre-existing approximate number representations; and Dehaene (1997) suggests that poor instruction in exact, symbolic mathematics may suppress children's use of the approximate number system in mathematical reasoning.

In the present study, we examined whether an aspect of learned, symbolic mathematics affects children's performance on approximate addition tasks. Specifically, we tested whether children's success on approximate addition tasks depends on the canonical problem format. Research has shown that children's understanding of and performance solving exact arithmetic facts depends heavily on the canonical problem format, which has arithmetic operations on the left-hand side of space and the "answer" on the right-hand side (e.g., 3 + 4 = 7, Behr et al., 1980; Baroody & Ginsburg, 1983; Seo & Ginsburg, 2003). Children's behavior on a variety of math tasks suggests that children have internalized this canonical "operations on left side" format (McNeil & Alibali, 2004, 2005). For example, when children are asked to check the "correctness" of math sentences written by a child who "attends another school," most mark sentences such as 10 = 6 + 4 as "incorrect" and change them to 6 + 4 = 10, 4 + 6 = 10, or even 10 + 6 = 4 (Cobb, 1987; see also Behr et al., 1980, Baroody & Ginsburg, 1983; Rittle-Johnson & Alibali, 1999). Similarly, when children are asked to reconstruct a problem such as "3 + 5 = 6 + __" after viewing it briefly, many write "3 + 5 + 6 = __" (McNeil & Alibali, 2004). Children's misinterpretation of non-canonical problem formats such as "__ = 6 + 4" and "8 + 4 = __ + 5" appears as early as first grade (Carpenter, Franke, & Levi, 2003).

It is not known whether young children's understanding of approximate addition tasks also depends on the canonical format because previous studies have only presented problems in the canonical format. At first blush, it seems that the problem format should not affect performance because the approximate number system "[does] not emerge through learning or cultural transmission" (Feigenson et al., 2004, p. 307), and researchers have argued that success on approximate addition tasks does not depend on experience with learned, symbolic arithmetic (Barth et al., 2005; Gilmore et al., 2007). However, several studies have shown that experience with learned, exact symbolic mathematics has the potential to change how children represent approximate numerical quantities (Opfer & Siegler, 2007; Siegler & Opfer, 2003). This work leaves open the possibility that children's success on approximate addition tasks may also depend on the canonical problem format.

It is important to test whether or not children's performance on approximate addition tasks depends on the canonical format not only because it can shed light on potential bidirectional effects between learned, exact symbolic mathematics and the approximate number system, but also because it can provide much needed data about the nature of children's early understanding of arithmetic. When children misinterpret non-canonical problem formats, it reveals their tendency to interpret addition events ($x + y = z$) operationally, rather than relationally. The term "operational" here should not be confused with Piaget's use of the term. It is used in mathematics education to refer to children's tendency to see $x + y = z$ as a directional process with $x + y$ producing the result z, rather than as an equivalence relation between $x + y$ and z. This type of operational thinking is problematic because makes it difficult for children to learn algebra down the road (Knuth, Stephens, McNeil, & Alibali, 2006). We already know that elementary school children interpret exact arithmetic facts this way (Baroody & Ginsburg, 1983; Carpenter et al., 2003; McNeil & Alibali, 2005); however, we do not know whether children exhibit this bias prior to formal schooling when relying on their approximate number representations.

Based on children's success on approximate arithmetic tasks in previous studies, it is tempting to conclude that young children understand the general principles of arithmetic, including the equivalence relation inherent in the structure, regardless of problem format. However, the problems in all previous studies were presented in the canonical "operations on left side" format, so children could have succeeded with only an operational, directional view. The next logical step is to test if young children can also succeed on approximate arithmetic problems presented in a non-canonical format. We tested the effect of the canonical problem format on children's performance on approximate arithmetic tasks in an experiment designed to replicate and extend the experiments of Barth et al. (2005) and Gilmore et al. (2007). Children solved approximate addition problems presented either in the canonical format with the operation on the left-hand side of space, or in a non-canonical format with the operation on the right-hand side.

Method

Participants

Participants were 60 children (M age = 5 years, 3 months; 27 boys and 33 girls). The study was conducted at childcare centers located on two college campuses in the Midwest. The centers are open to the community, but they use a weighted lottery system, with equal first precedence given to children of staff, students, and faculty at the universities. Tuition is based on a sliding scale, and 30% of children receive some form of reduced tuition.

Design

Children participated in one of four conditions in a 2 (number representation: non-symbolic or symbolic) x 2 (problem format: canonical or non-canonical) factorial design. In the non-symbolic condition, number was represented by arrays of dots. In the symbolic condition, number was not represented by sets of concrete stimuli, such as dots, but by Arabic numerals (e.g., 10) and the corresponding words (e.g., "ten"). In the canonical problem format condition, the two values to be added were presented first on the left side of the screen, and then the comparison value was presented on the right side of the screen. In the non-canonical problem format condition, the comparison value was presented first on the left, and then the two values to be added were presented on the right (see Figures 1 and 2). All problems, regardless of format, were read from left to right by the experimenter.

Procedure

Testing took place in a quiet room in the daycare. All problems were presented as animated events on a Mac Laptop with screen resolution 1440 x 900. The problems were the same as those used at a 2:3 ratio by Barth et al. (2005) and Gilmore et al. (2007). This ratio is appropriate for children ages 4-5 (Halberda & Feigenson, 2008). All participants received four practice problems at a 1:5 ratio,

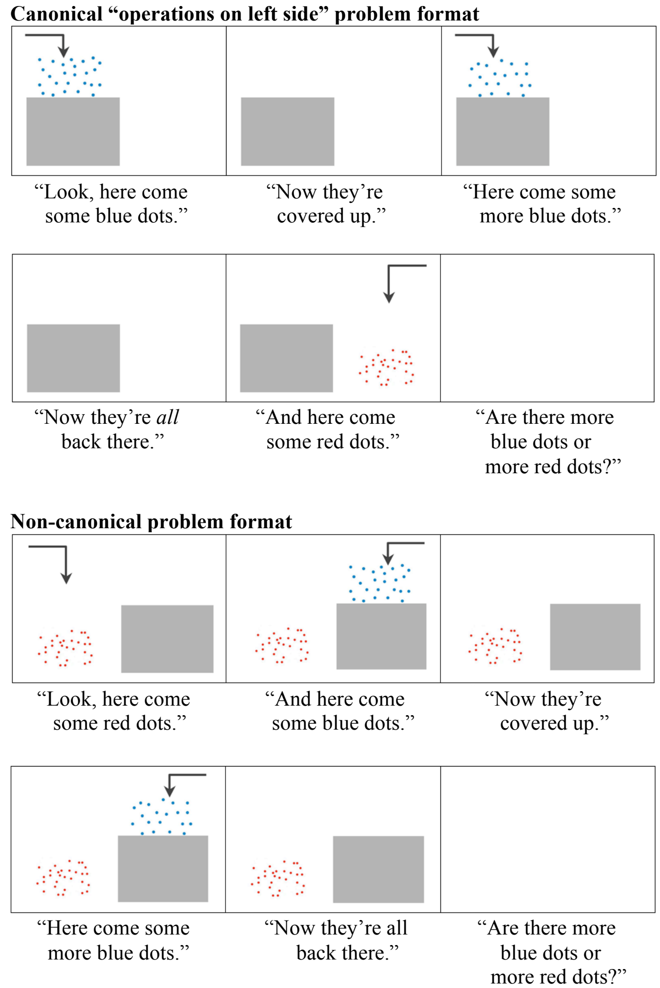

Figure 1: Example of a nonsymbolic, approximate addition problem presented in the canonical format (top panel) and the non-canonical format (bottom panel)

followed by 8 test problems presented at a 2:3 ratio. The procedure used in each condition is described next.

Non-symbolic condition. The procedure was a replication of Barth et al. (2005). Children were told that they would be playing a game in which they would guess whether there were more blue dots or more red dots. They were then presented with animated events accompanied by a narrative (see Figure 1). The animated events consisted of arrays of 5-51 dots (either 2 mm or 3 mm) in virtual enclosure of one of two sizes (7 x 5 cm or 9 x 6 cm). As in Barth et al. (2005), dot size, total contour length, summed dot area, and density were negatively correlated with number on half of the trials; on the remaining trials, the correlations were reversed. Blue dots were the same size within trials. In the *canonical condition*, an occluder appeared on the bottom left of the screen (1300 ms). A set of blue dots then moved in from the upper left side of the screen (1300 ms) and down behind the behind the occluder (1450 ms). After a pause

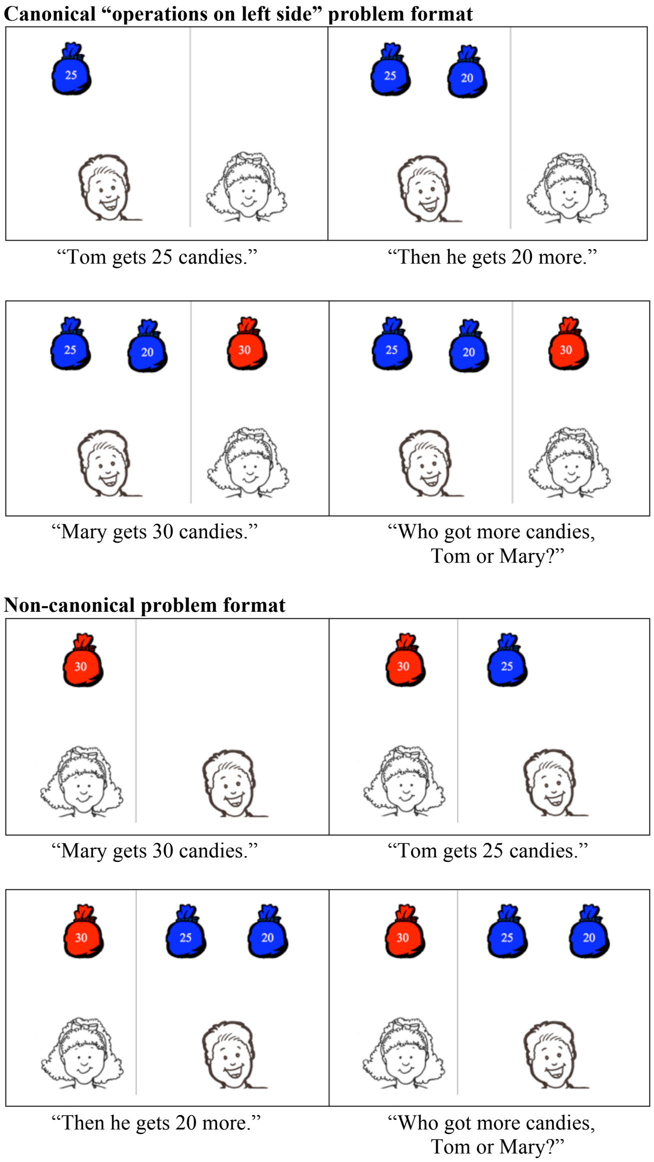

Figure 2: Example of a symbolic, approximate addition problem presented in the canonical format (top panel) and the non-canonical format (bottom panel)

(1300 ms), another array of blue dots moved in from the upper left side and behind the occluder with the same timing. After both sets of blue dots were behind the occluder, a set of red dots moved in from the upper right side of the screen (1300 ms) and came to rest beside the occluder (1450 ms), after which the question was posed: "are there more blue dots or more red dots?" Half of the children saw the problems presented in this way, and the other half saw the same sequence with two occluders, one for the blue dots and one for the red dots. Preliminary analyses revealed no significant effect of the number of occluders, so the data were collapsed in the main analysis. The events in the *non-canonical condition* were the same as those in the canonical condition, except the red dots were presented first on the left side before the two sets of blue dots were presented on the right side. On half of the trials, the number of blue dots exceeded the number of red dots, and on the other half, the number of red dots exceeded the number of blue dots.

Symbolic condition. The procedure was a replication of Gilmore et al. (2007). Children were introduced to two characters, Tom and Mary, and were told they would have to figure out who gets more candy. They were then presented with animated events accompanied by a narrative (see Figure 2). In the *canonical condition*, a boy and a girl were presented on each side of the screen. One blue bag of candies appeared above the head of the character on the left (1500 ms). Then, a second blue bag appeared (1500 ms). Finally, one red bag appeared above the character on the right. An Arabic numeral was written on each bag. In the *non-canonical condition*, the procedure was the same, except the red bag appeared above the character on the left side and the two blue bags above the character on the right side. On half of the trials, there were more candies in the two blue bags, and on half of the trials there were more candies in the red bag. Also, Tom had more candy on half of the trials, and Mary had more candy on half the trials.

Results

Children performed significantly above chance on the problems, M = 5.60, $t(59)$ = 10.24, p < .001. This result held for the youngest half of participants (M age = 4 years, 10 months), M = 5.47, $t(31)$ = 6.42, p < .001, and for the oldest half of participants (M age = 5 years, 8 months), M = 5.75, $t(27)$ = 8.34, p < .001. Children performed significantly above chance in every condition: non-symbolic canonical format, M = 5.64, $t(13)$ = 5.34, p < .001; non-symbolic non-canonical format, M = 5.00, $t(13)$ = 2.46, p = .03; symbolic canonical format, M = 6.19, $t(15)$ = 8.92, p < .001; and symbolic non-canonical format, M = 5.50, $t(15)$ = 6.21, p < .001. Across all conditions, only two (of 60) children answered fewer than four problems (out of 8) correctly. Thus, consistent with Barth et al. (2005) and Gilmore et al. (2007), young children were able to add two values together and compare that sum to a third value, regardless of whether number was represented non-symbolically or symbolically. Moreover, they were able to do so even when the problems were not presented in the canonical format.

We performed a 2 (number representation: non-symbolic or symbolic) x 2 (problem format: canonical or non-canonical) ANOVA with number correct (out of 8) as the dependent measure. Results revealed a main effect of problem format, $F(1, 56) = 4.89$, $p = .03$, $\eta_p^2 = .08$. Children performed better when problems were presented in the canonical problem format (M = 5.92, SE = 0.21) than when they were presented in the non-canonical problem format (M = 5.25, SE = 0.21). Neither the main effect of number representation, nor the interaction between number representation and problem format was significant, F(1, 56)

= 3.01, p = .09 and $F(1, 56) = 0.01$, p = .94 respectively. Results were unchanged when controlling for age: main effect of problem format, $F(1, 55) = 4.82$, p = .03; main effect of number representation, $F(1, 55) = 1.92$, p = .17; interaction, $F(1, 55) < 0.01$, p = .96.

Discussion

Children successfully added and compared large numerical quantities. Surprisingly, they did so not only when problems were presented in the canonical "operations on left side" format, but also when problems were presented in a non-canonical "operations on right side" format. These findings add to the evidence suggesting that young children can draw on their approximate number representations to understand the logic of large number addition, even before they receive formal arithmetic instruction in school (Barth et al., 2005; Gilmore et al., 2007). The findings also provide some initial support for the notion that children understand the equivalence relation inherent in the structure of arithmetic before they enter school, and only develop the operational bias after receiving formal arithmetic instruction (McNeil, 2007). Teachers may be able to build on this early understanding of equivalence to improve children's comprehension of non-canonical problem formats when working with exact symbolic arithmetic problems in school.

Although children successfully solved approximate addition problems regardless of format, they did perform significantly better when problems were presented in the canonical "operations on left side" format. This finding supports the view that children's use of the approximate number system may be susceptible to early influence from the environment. Although there are other possible explanations for this finding (e.g., disproportionate attention to the left-hand side of space that is either innate or learned through early experiences with non-mathematical stimuli), it is consistent with the suggestion that there may be bidirectional effects between the approximate and symbolic number systems (e.g., Gallistel and Gelman, 2005). Specifically, experiences with exact, symbolic arithmetic problems may shape children's ability to reason arithmetically based on their approximate number system.

When young children see arithmetic problems in books, on toys, or during TV shows, the problems are typically presented in the canonical "operations on left side" format (e.g., 1 + 1 = 2). The internalization of this convention may facilitate children's performance on both exact and approximate mathematics problems that are presented in the canonical format. However, it may hinder performance on problems presented in non-canonical formats, and it may ultimately lead to the conceptual difficulties with the equal sign that children develop in elementary school (e.g., Baroody & Ginsburg, 1983; McNeil, 2007).

According to this account, we should expect to see the difference in performance between the canonical and non-canonical versions of approximate addition tasks to increase over the course of the elementary school years. This is because as children progress from kindergarten to third grade, they continue to gain experience and practice with the canonical problem format (McNeil, 2007). If experience with and internalization of this format negatively affects children's ability to deploy their approximate number system in the face of non-canonical problems, then the discrepancy in performance on canonical versus non-canonical approximate addition problems should be even greater for older elementary school children than it was for the five-year-old children in the present study.

Of course, we do not yet have empirical evidence to support this prediction, and it is possible that the opposite pattern could hold. Indeed, other environmental factors may play an important role in children's ability to deploy their approximate number system, and at least some of these factors could, theoretically, work to hinder children's performance on canonical approximate addition problems. For example, as children begin formal instruction in arithmetic, they not only gain experience with the canonical problem format, but also learn that their goal is to determine the "right answer." The "right answer" is almost always an exact value, as opposed to an approximate one (e.g., children will not be given credit for answering "9" to the problem 4 + 4 = __). If children learn to devote all of their resources to getting the right answer when they are presented with a canonical arithmetic problem, then their ability to deploy their approximate number system may suffer on these problems (cf. Dehaene, 1997). According to this account, the difference between performance in the canonical and non-canonical versions of approximate addition tasks may decrease—and possibly even reverse direction—over the course of the elementary school years. Future research should examine theoretically relevant aspects of the environment and how they affect children's ability to deploy their approximate number system.

More generally, the results of the present study highlight the classic tension between competence and performance. Just as we cannot dismiss the fact that children may have richer conceptual knowledge of arithmetic than they are able to show when asked to solve exact symbolic arithmetic problems (e.g., Gilmore & Spelke, 2008), we also cannot dismiss the possibility that children who perform well on approximate addition tasks may nonetheless have important limitations in conceptual knowledge that might be revealed in certain testing situations (e.g., Booth & Siegler, 2006). As Sophian (1997) argued, to truly understand the nature of developing knowledge, "we need to pay attention not just to the conditions in which performance is best, but also to those in which children have more difficulty" (pp. 290-291).

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