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ABSTRACT

The orbital surface of a cyclotron magnetic field is defined and the similarity between particle orbit and a current-carrying flexible wire is noted. In a vertical cyclotron field, an equation for the vertical location of a wire loop is found. When the point of support is on the orbital surface, the force required to support the loop is shown to be independent of current. Application of the loop to location of the orbital surface is discussed. Factors involved in selection of a suitable wire material and wire size are considered.

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SHAPE OF THE LOOP IN RELATION TO ORBITAL SURFACE

The path of a particle in the magnetic field is a solution of

$$\frac{\vec{n}}{\rho} = -\vec{t} \times \vec{B}(e/p), \qquad (1)$$

where

$\frac{\vec{n}}{\rho}$ = curvature of path (directed away from center),
$\vec{t}$ = unit vector tangent to path and in direction of velocity,
$\vec{B}$ = magnetic flux density vector,
$\frac{e}{p}$ = charge-to-momentum ratio of particle.

A solution of Eq. (1) which is a closed smooth curve is an equilibrium orbit about which the nonrepeating orbits oscillate. The locus of all equilibrium orbits may be called the orbital surface of the cyclotron field. The orbital surface is, in general, different from the median surface as commonly defined.

A similarity between a current-carrying flexible wire and the particle path is evident from the differential equation for the wire:

$$\frac{\vec{n}}{\rho} = \vec{t} \times \vec{B}(i/T) + \overrightarrow{mg}/T, \qquad (2)$$

where

$\frac{\vec{n}}{\rho}$ = curvature of wire,
$\vec{t}$ = unit vector tangent to wire and in direction of current,
i = current,
T = tension,
$\overrightarrow{mg}$ = weight per unit length of wire.

If the weight of the wire could be neglected, then the differential equations are identical when $i/T = -e/p$, and the wire and the particle path would coincide for equal boundary conditions (end points).

It is desired to investigate the behavior of a complete loop of wire to determine the feasibility of locating the orbital surface from measurements on the wire. The case will be considered in which the field is approximately vertical and only the vertical positions of the equilibrium orbits are of interest. If the point where the ends of the wire meet is free to move radially (from loop center), the wire system has no radial forces not operating on a particle, and it may be assumed that the radial positions of the wire and of a corresponding particle are equal. Thus, only the vertical component of Eq. (2) is important and for convenience, the vertical position of the wire, z, is given in relation to that of the orbital surface, z_o, by introducing z_Δ, defined by

$$z = z_o + z_\Delta . \tag{3}$$

The vertical component of Eq. (2) is

$$-\frac{d^2z}{dx^2} = \frac{i}{T}\,(t_R B_\phi - t_\phi B_R) - mg/T, \tag{4}$$

where subscripts R and ϕ refer to radial and to azimuthal component, respectively, and x is the azimuthal distance along the wire. The terms of this equation may be expanded about their values at the equilibrium orbit (designated by subscript o):

$$z = z_o + z_\Delta,$$

$$t_\phi B_R \approx (t_\phi B_R)_o + \frac{\partial B_R}{\partial z}\, z_\Delta,$$

$$t_R B_\phi \approx (t_R B_\phi)_o,$$

$$mg/T \approx 0 + mg/T.$$

at the equilibrium orbit, we have the equation

$$\frac{d^2z_o}{dx^2} = i/T\left[(t_R B_\phi)_o - (t_\phi B_R)_o\right],$$

which may be subtracted from Eq. (4) to give the equation for the variations from the equilibrium orbit:

$$-\frac{d^2z_\Delta}{dx^2} = -\frac{i}{T}\,\frac{\partial B_R}{\partial z}\, z_\Delta - mg/T. \tag{5}$$

In a cyclotron field, the radial gradient of flux density does not vary greatly, and $\frac{\partial B_R}{\partial z}$ is conveniently written, at all azimuths, as

$$\frac{\partial B_R}{\partial z} = \frac{\partial B_z}{\partial R} = -\,\alpha B_o,$$

where B_o is the value of the vertical component of flux density. Also, the tension is given in terms of current by

$$T = i\frac{p}{e} = B_o R i, \tag{6}$$

Where R, as defined by Eq. (6), is a characteristic radius for the wire loop. When these quantities are introduced, Eq. (5) becomes

$$\frac{d^2z_\Delta}{dx^2} = \frac{mg}{B_o R i} - \frac{\alpha}{R}\, z_\Delta . \tag{5a}$$

Placing the points of support at $x = \pm \pi R$ and at a height $z_o + z_A = z_s$ gives us the solution of Eq. (5a),

$$z_\Delta = \frac{m g}{B_o \alpha i}\left[1 - \frac{\cos\left(\pi\sqrt{\alpha R}\,\frac{x}{\pi R}\right)}{\cos\left(\pi\sqrt{\alpha R}\right)}\right] + (z_s - z_o)\frac{\cos\left(\pi\sqrt{\alpha R}\,\frac{x}{\pi R}\right)}{\cos\left(\pi\sqrt{\alpha R}\right)} . \qquad (7)$$

From this solution, the total sag of the loop (at $x = 0$) and the force required to support it at $z = z_s$ are readily found to be

$$\text{Sag} = (z_s - z_o) - z_\Delta\Big|_{x=0}$$

$$= \left[\frac{m g (\pi R)^2}{2 B_o R i} - (z_s - z_o)\, 1/2\pi^2 \alpha R\right] \frac{\sec\left(\pi\sqrt{\alpha R}\right) - 1}{1/2\pi^2 \alpha R}, \qquad (8)$$

$$\text{Force} = 2\,T\,\frac{\partial z_\Delta}{\partial x}\Big|_{x = \pi R}$$

$$= \left[2\pi R\, m g + (z_s - z_o)\, 2\pi\alpha\, B_o R i\right] \frac{\tan\left(\pi\sqrt{\alpha R}\right)}{\pi\sqrt{\alpha R}} . \qquad (9)$$

Commonly the quantity αR is small in a cyclotron, so that approximate formulae for this case are useful. For $\alpha R < 1/10$,

$$\text{Sag} = \left[\frac{m g (\pi R)^2}{2 B_o R i} - (z_s - z_o) 1/2\pi^2 \alpha R\right]\left(1 + \frac{5}{12}\pi^2 \alpha R + \frac{61}{360}(\pi^2 \alpha R)^2 + \ldots\right), \qquad (8a)$$

$$\text{Force} = \left[2\pi R m g + (z_s - z_o) 2\pi\alpha B_o R i\right]\left(1 + \frac{1}{3}\pi^2 \alpha R + \frac{2}{15}(\pi^2 \alpha R)^2 + \ldots\right). \qquad (9a)$$

Note that the loop becomes unstable at values of the field index $\alpha R \geqslant 0.25$.

Equation (9) reveals the interesting property of the loop that the force does not depend upon current if the point of support is on the orbital surface. This behavior makes the loop a useful device for locating the orbital surface. Certainly, for a loop which sagged excessively, or which differed markedly from circular, one would not expect Eqs. (8) and (9) to be exact. An allowable sag might be that which does not carry the loop beyond the region where the radial component of B is essentially a linear function of excursion from the orbital surface. For a usable cyclotron field in which α is definable, a tight loop of wire should display the calculated behavior. The assumptions necessary to interpretation of the behavior of the wire loop limit its usefulness as a testing device to fields in which the region near the orbital surface is without large local aberrations.

THE LOOP ON A SPRING SUPPORT

In order to use the property shown by Eq. (9), the loop must be mounted on a force-sensitive device. Typically, this is done by supporting the loop on a single "hydrometer"; a torsion balance or any of several other mechanisms would seem adaptable to this purpose. In principle, then, detection of the orbital surface involves detection of the location of the point of support at which a change in current produces no motion of the supporting device. The position of the point of support will obey the relation

$$z_s - z_o = \frac{(z_H - \frac{FW}{K}) - z_o}{1 - \frac{2\Delta \alpha B_o RiF}{K}},$$

where z_s = height of point of support,

z_o = height of point on orbital surface (at same azimuth),

z_H = height of support when loop is detached (unloaded),

K = force constant of supporting device (force/displacement),

W = weight of loop of wire,

R = radius of loop,

B_o = flux density,

i = current in wire,

$\alpha = -\frac{1}{B_o} \frac{\partial B}{\partial R}$,

$F = \frac{\tan (\pi \sqrt{\alpha R})}{\pi \sqrt{\alpha R}}$.

When the loop is not supported exactly on the orbital surface, its displacement from the orbital surface depends upon current; as the current is increased, the point of support moves farther from the orbital surface. This behavior is suitable for determining the location of the orbital surface by successive approaches. A critical current exists for which the denominator of Eq. (10) becomes zero; at this and greater currents, no stable equilibrium for the loop exists.

SELECTION OF WIRE FOR THE LOOP

In any application of the wire loop, the question of wire material and wire diameter arises. Generally, it is desired to have (a) a large vertical force constant, and (b) a small sag in the loop. From Eq. (9), the vertical force constant is

$$\frac{\partial F}{\partial z_s} = 2\pi \alpha B_o RiF, \qquad (11)$$

and from Eq. (8) the sag in the loop when $z_s = z_o$ is

$$Sag_o = \frac{\pi D^2 dg}{4} \frac{(\pi R)^2}{2B_o Ri} \quad G, \qquad (12)$$

where D = diameter of wire cross section,

d = density of wire material,

$G = \frac{\sec (\pi \sqrt{\alpha R}) - 1}{1/2\pi^2 \alpha R}$.

Clearly, the desire for a large force constant implies that the current be made large. The current is limited by the maximum tension the wire can support; this dependence is expressed by

$$T = B_o Ri \leq \frac{\pi D^2}{4} S, \qquad (13)$$

where S is the maximum working stress the wire material will allow. Combining Eqs. (11) and (13), we may write the sag

$$Sag_o \geqslant \frac{(\pi R)^2 dg}{2S} . \qquad (14)$$

At first this result would seem to indicate that the minimum sag, being a function of the ratio of density to stress, is independent of wire size. This is not the case, however, for the current causes heating of the wire and this affects the working tensile stress. Air cooling becomes more favorable as wire diameter decreases; seeking minimum sag thus leads one away from large wire diameters.

From the foregoing considerations, the desirable characteristics of a wire material appear: the conductivity should be high, the ratio of usable tensile stress to density should be high, and the wire should be flexible and nonmagnetic. Usable tensile stress will be temperature-sensitive, so that it is not sufficient to compare materials at room temperature only. It seems possible that no one material is best for all cases. At low values of B_oR, heating of the loop places emphasis upon properties at high temperatures; at high values of B_oR, the loop is limited by breaking at comparatively low currents. Among the common materials, molybdenum and tungsten offer high tensile strength and resistance to temperature; however, their low conductivity would degrade over-all performance somewhat. The high-conductivity materials such as aluminum and copper offer a suitable combination of properties. Aluminum suffers from marked loss of strength with increasing temperature; perhaps an aluminum alloy combining improved resistance to temperature with acceptable conductivity could be found. Except at low temperatures, copper is a better material than pure aluminum; not an inconsiderable advantage of copper is its ready availability in many sizes, including multistranded flexible leads.

Measurements of heat transfer from horizontal cylindrical wires in air have been made by Madden and Piret,[1] and they present an empirical formula applicable to wires in the neighborhood of 0.002 inch to 0.010 inch in diameter. Inserting constants for air in this equation give the numerical form

$$hD = \frac{4.766 \times 10^{-4}}{\ln\left(\frac{1.316\,(P_o/P)^{2/3}}{D\,(\Delta t)^{1/3}}\right)} \text{ watt/ cm } ^{\circ}C,$$

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[1] A. J. Madden Jr. and E. L. Piret, "Proceedings of the General Discussion on Heat Transfer," London, Institution of Mech. Eng., and N. Y. A. S. M. E., 1951, pp. 328-333, 382, 388.

where

h = surface coefficient of heat transfer (watt/cm^2 $^\circ$C),

D = diameter of wire (cm),

Δt = temperature difference (wire above ambient) ($^\circ$C),

P/P_o = air pressure (atmospheres).

Such a formula is very useful in estimating operating conditions with a wire loop.

Selection of a wire diameter is a special problem for each application; the values of B_oR, R, and α will be fundamental parameters. Some judgment of acceptable sag must be made; perhaps this may be related to the aperture in which beam oscillations are to be allowed. Examination, as a function of diameter, of the performance of the loop in a given field will be a basis for selection. In the particular case of a proposed study of the field in the UCRL 184-inch cyclotron, the range of B_oR is from 2.5×10^5 gauss inches to 1.7×10^6 gauss inches, with an expected α of 0.0005 per inch. Tests must be made, operating under approximately 1 mm Hg air pressure as well as at atmospheric pressure. Each loop will be made of fourteen strands of No. 44 gauge bare copper wire. Temperature rise of the various loops is expected to range from 5°C to about 150°C.

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