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L. Jackson Laslett

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Trajectory for Minimum Transit Time Through the Earth

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January 31, 1966

The tantalizing theoretical possibility of very rapid transit between points on the earth's surface by gravitational fall through frictionless tunnels¹⁻³ has recently been discussed by Cooper.⁴ The differential equation for the plane curve leading to a minimum transit time between two given points on the surface is given in Cooper's note, and the results of a computer solution are presented graphically. The differential equation may be written conveniently in a form that expresses the dependence of θ as a function of r , and in these terms it becomes

$$(d/dr) \left\{ r^2 [R^2 - r^2]^{-1/2} [1 + r^2 (d\theta/dr)^2]^{-1/2} (d\theta/dr) \right\} = 0. \quad (1)$$

A solution to Eq. (1), symmetric about $\theta = 0$, may then be obtained as

$$\theta = \pm \frac{r_0}{R} \int_{r_0}^r \frac{1}{r} \left(\frac{R^2 - r^2}{r^2 - r_0^2} \right)^{1/2} dr \quad (2a)$$

$$= \pm \left\{ \sin^{-1} \left[\frac{R}{r} \left(\frac{r^2 - r_0^2}{R^2 - r_0^2} \right)^{1/2} \right] - \frac{r_0}{R} \sin^{-1} \left(\frac{r^2 - r_0^2}{R^2 - r_0^2} \right)^{1/2} \right\}, \quad (2b)$$

in which the constant of integration r_0 may be identified with the radius of closest approach to the earth's center.

For class presentation, Eq. (1) is most directly obtained from the variational statement

$$\delta \int_A^B ds/v = \delta \int_A^B [(g/R)(R^2 - r^2)]^{-1/2} [(dr)^2 + (r d\theta)^2]^{1/2} = 0 \quad (3)$$

[in which we neglect the rotation of the earth and employ the speed acquired from the potential-energy change $\Delta V = -\frac{1}{2} mg (R^2 - r^2)/R$] by writing the Euler-Lagrange equation that results from regarding r as the independent variable. The first integral

$$r^2 [R^2 - r^2]^{-1/2} [r^2 + (dr/d\theta)^2]^{-1/2} = r_0 [R^2 - r_0^2]^{-1/2} \quad (4)$$

then follows immediately, and the resulting explicit expression for $d\theta/dr$ may be integrated as shown by Eqs. (2a,b). The solution given by Eq. (2b) corresponds to a diametrical trajectory ($\theta = \pm \pi/2$) in the limiting case for which r_0 vanishes.

For journeys between points separated by more than a few kilometers, very high maximum speeds will be attained on a path of the form given by Eq. (2b). An interesting problem for the student is an evaluation of the "number of g's" experienced by a passenger (or the force with which he presses on the seat of the train) at the lowest point of the trajectory. In addition, an elementary evaluation of the integral in Eq. (3) provides the transit time as a function of r_0 .

Footnotes

* Work assisted by the United States Atomic Energy Commission.

1. L. K. Edwards, Sci. American 213, 30 (August 1965).
2. M. Gardner, Sci. American 213, 10 (September 1965).
3. Cf. L. Lessing, Fortune 71, 124 (April 1965).
4. P. W. Cooper, Am. J. Phys. 34, 68 (1966).

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