Title
COURSE IN THE THEORY MID DESIGN OF PARTICLE ACCELERATORS - LECTURE 12

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Course in the Theory and Design of Particle Accelerators

LECTURE 12

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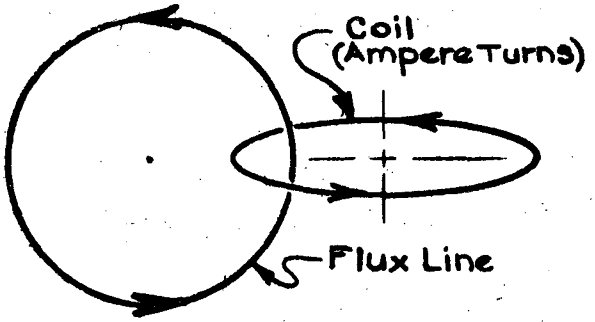
OUTLINE

I. D. C. MAGNETS

A. Fundamental Relationships

Magnetic Flux produced by an electric current:



$$\int H d\ell = \frac{4}{10} \pi NI$$

H = magnetic field intensity - Oersteds

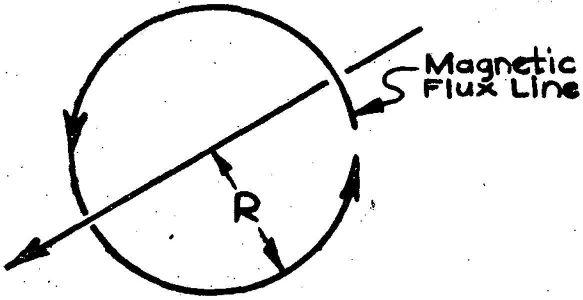
ℓ = flux path length - cm.

N = number of turns

I = electric current flowing - amperes

Figure 1

Consider the magnetic field around a wire carrying an electric current. From symmetry H is constant at constant R.



$$\int H d\ell = H(2\pi R) = \frac{4}{10} \pi NI$$

$$H = \frac{.4\pi NI}{2\pi R} = \frac{I}{5R}$$

Figure 2

In magnet design, one is usually interested in the magnetic flux density B, measured in Gauss. B and H are related by the magnetic permeability, μ, which varies with different materials.

$$\frac{B}{H} = \mu \quad \text{or} \quad B = \mu H$$

μ has dimensions: Gauss/Oersteds

μ is defined as 1 Gauss per Oersted for vacuum, air or non-magnetic material. For mild steel, μ is the order of 2000. Although B and H are entirely different fields and quantities, because of the selection of the size of the units, B and H have the same numerical value for air and non-magnetic materials.

The total magnetic flux, Φ, in a magnetic circuit is equal to $\int_A B dA$ and is expressed in Maxwells.

The following electro-magnetic analogies may be drawn:

Magnetic	Electric
H = Magnetic Field Intensity (Oersteds)	Voltage Gradient
B = Magnetic Flux Density (Gauss)	Current Density
Φ = Total Magnetic Flux (Maxwells)	Current
μ = Permeability (Gauss/Oersted)	Conductivity (amp./volt cm.)
$\mathcal{R}$ = Reluctance (Oersted/Gauss cm.)	Resistance (volts/amps.)

Magnetic circuits are much more difficult to treat theoretically than electric circuits because (1) the flux will not remain totally within the conductor provided (one cannot obtain good magnetic insulation) and (2) if the circuit contains iron its magnetic resistance varies non-linearly with the field intensity due to saturation effects (i.e. there is no simple "ohms law" for magnetic circuits containing ferromagnetic materials). The difficulties are comparable to attempting to predict the behavior of electric circuits immersed in salt water while at the same time the resistance of the conductors varies with the current.

If one plots the magnetic flux density (B) within an iron magnetic conductor vs. the intensity of the magnetic field (H), one finds at first that a change in H will produce a proportional change in B. As saturation is approached B rises much less rapidly. Finally, at saturation, the space being magnetized responds to increases in H as if no iron were there - it has an "incremental" permeability of 1 and $\Delta B = \mu_{air} \Delta H$. If one plots $(B - \mu_{air} H)$ vs. H the curve becomes flat. This last curve shows best the saturation effect.

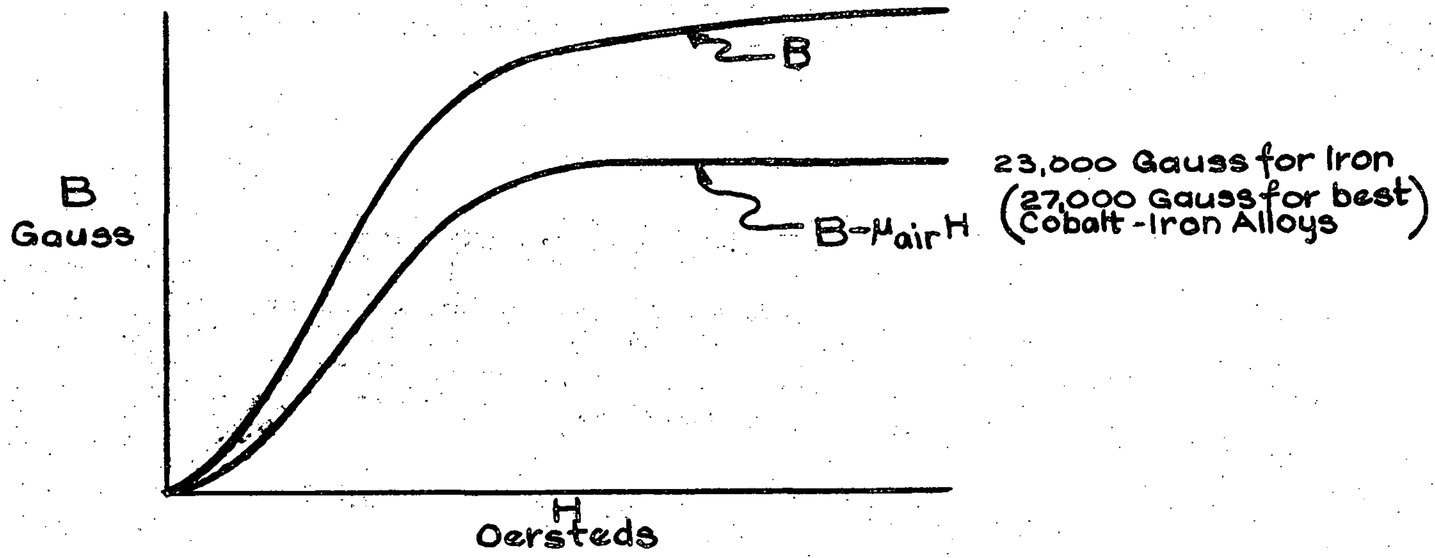
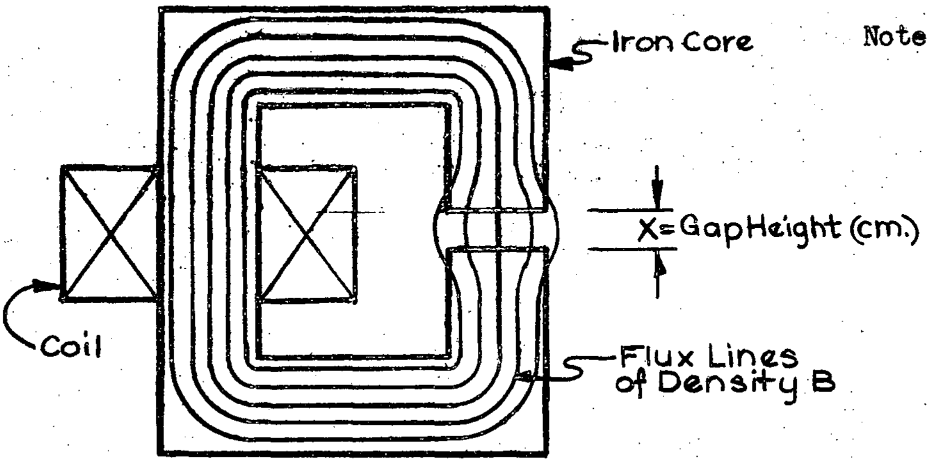


Figure 3

B. Magnetic Circuits

1. Assuming Infinite Permeability in Iron (H = 0 in iron)



$(Bx)_{gap} = 0.4\pi NI$

Note: 1. Each flux line is a closed loop.
2. Each flux loop links a conductor turn.
3. The $\int H d\ell$ is the same for any flux loop that links the entire coil.

Figure 4
Hypothetical Case

Around any flux line:

$$\int_{iron} H d\ell + \int_{gap} H d\ell = 0 + (Bx)_{gap} = 0.4\pi NI \qquad (1)$$

$$\text{For gap, } \mu = \frac{B}{H} = 1 \therefore \int_{gap} \mu H d\ell = (Bx)_{gap}$$

B_{gap}, rather than H_{gap}, is usually plotted vs. $(NI)_{gap}$ for a specific magnet in which x is a constant. This plot is a straight line shown in Figure 6. The NI given by Equation (1) is required to drive the flux across the gap only and is called the NI in the gap.

2. Considering Actual Permeability of Iron

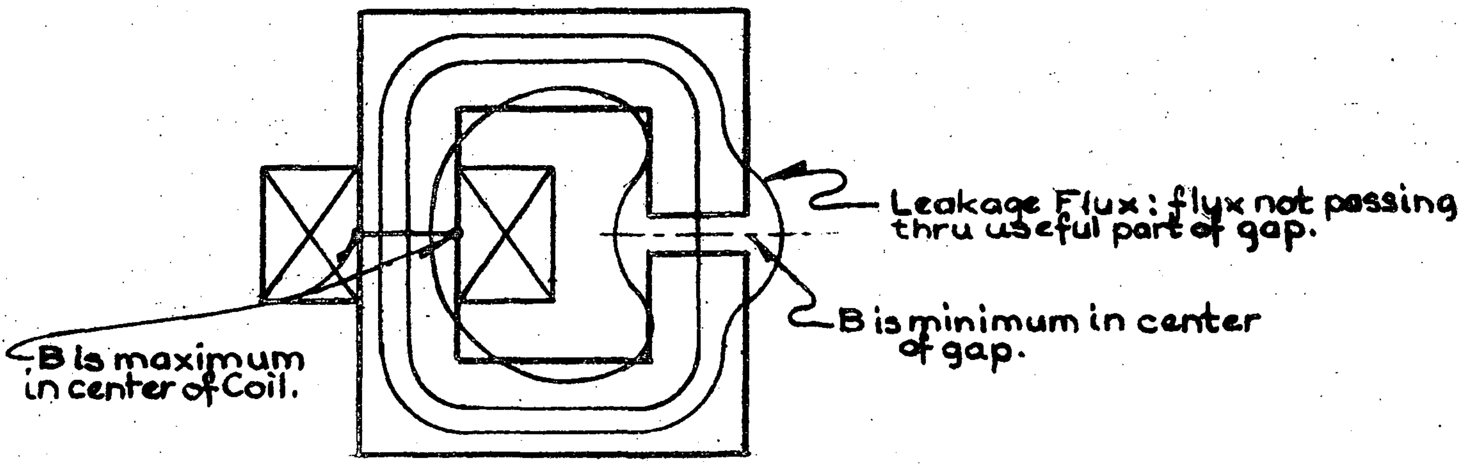


Figure 5. More Realistic Case

B is not constant. Iron saturates so μ_{iron} must be considered.

$$\int_{iron} H\,d\ell + \int_{gap} H\,d\ell = \int_{iron} \frac{B}{\mu}\,d\ell + \left(\frac{Bx}{\mu}\right)_{gap} = 0.4\pi(NI)_{total} \qquad (2)$$

An estimate of the leakage flux is required in order to solve Equation (2). For this purpose a term called the "leakage coefficient" is useful.

$$\text{Leakage coefficient } K = \frac{\text{Flux thru Core}}{\text{Flux thru Gap}}$$

The value of K depends upon where the flux is measured. Unless otherwise specified:

$$K = \frac{\text{max. } \Phi \text{ in iron}}{(\text{B in gap center}) \times (\text{area of pole})}$$

Max. Φ usually is in center of length of coil.

K may be determined by several methods:

a. flux plots
b. model tests
c. experience with similar shapes

The method used depends on the time available and economics.

After K is determined, B can be determined from the area of the iron and H from the μ of iron at the corresponding B.

Another term, other than K, is used to take into account the reluctance (analogous to electrical resistance) of the magnetic circuit. This term is the "magnet efficiency".

$$\text{Magnet Efficiency} = \frac{(Hx) \text{ in gap}}{\int H\,d\ell \text{ total}} = \frac{\text{NI in gap}}{\text{NI total}}$$

The magnet efficiency varies from 80 to 30%, being approximately 33% for the 184 inch Cyclotron and 55% for the Bevatron. Magnet efficiency is not a ratio of power; it is more like a volumetric efficiency. A high magnetic efficiency does not necessarily indicate an optimum design. It may mean that too much capital cost has been paid for iron in order to decrease operating costs. However, high efficiency permits future increase in field without building a new core.

Figure 6 shows the relationship between $(NI)_{gap}$, $(NI)_{iron}$, and $(NI)_{total}$.

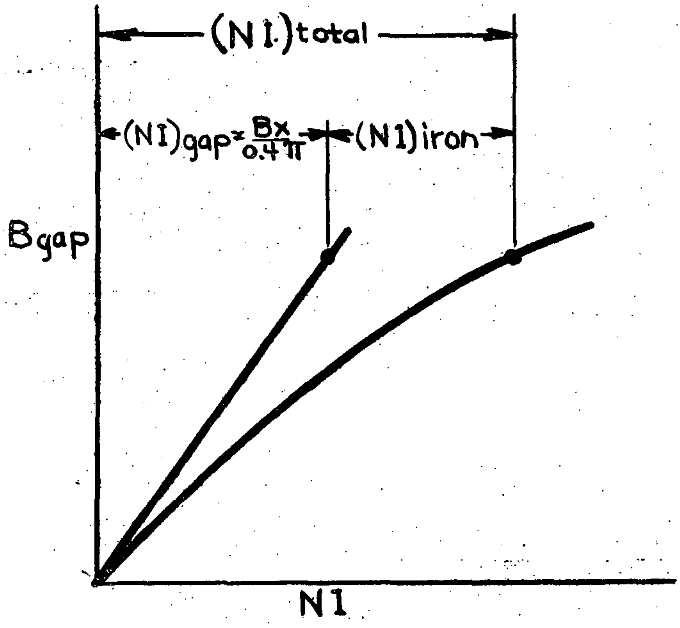


Figure 6. B vs NI

C. D.C. Coil Design

1. Design Requirements

a. Cooling
b. Resist magnetic forces
c. Avoid electrical breakdown
d. Suitable supply voltage
e. Compactness
f. "Optimum" quantity of copper
(The "Optimum" quantity depends on costs, space and other factors)

2. Space Factor

$$\text{Space Factor} = \frac{\text{area of copper}}{\text{cross section of coil}}$$

There are two kinds of space factor:

a. $$\text{Net Space Factor} = \frac{\text{area of copper}}{\text{area Cu + insulation + cooling space}}$$

b. Gross Space Factor $= \frac{\text{area Cu}}{\text{total coil area}}$

Total coil area includes area of Cu, insulation, cooling space, coil box, coil spring pads, clamps, etc., i.e. everything related to the coil that takes up space for an appreciable angular sector of the coil. Thus a pipe flange which takes up a fraction of a degree of arc and could be moved to allow space for other equipment would not be included as part of the total coil area.

TABLE I. Space Factors for some UCRL Coils

Coil	Cooling	Net Space Factor	Gross Space Factor	Conductor
90 inch Cyclotron	oil	55%	35%	strap
184 inch Cyclotron Main Coils	oil	69	48	strap
Bevatron	air	28	15	stranded cable
40 inch Meson Magnet	water	63	37	hollow conductor
184 inch Cyclotron Auxiliary Coils	water	70	42	hollow conductor
18 inch I.D. Ion Pump	water*	80	55	strap

* "Sandwich" cooling, i.e., copper sheet, with cooling tube around periphery, placed between conductor layers.

3. Conductor Insulation

TABLE II. AIEE Electrical Insulating Materials

Class.	Description	Example	Temperature of "Hottest Spot" (degrees C.)
O	Organic material neither impregnated nor immersed under liquid dielectric	paper, bakelite, cotton, silk	90
A	Inorganic material when either impregnated or immersed under liquid dielectric	paper under oil	105
B	Inorganic material with organic binder	mica with shellac	130
C	Inorganic material	mica, asbestos fiber glass, ceramic	no std.
H	Inorganic material with silicone binder		no std.

4. Current Density

The cooling medium and insulation determine the current density permissible. For air and oil I/A is typically 1000 amps./sq. in. For water, I/A is not limited - 2000 to 3000 amps./sq. in. is common; over 100,000 amps./sq.in. has been used with short water paths.

Current Density (I/A) is an important variable in coil design since it determines, for example, the ratio of capital to operating costs and the compactness of the coil. See page 10 for more detailed information.

5. Relationship between I/A and P/W

$$P = I^2R = I^2\rho\frac{N\ell}{A}$$

$$W = \delta N \ell A$$

A is in sq. inches,
ℓ is in inches

$$\frac{P}{W} = \frac{I^2\rho N\ell}{A\delta N\ell A} = \left(\frac{I}{A}\right)^2 \frac{\rho}{\delta}$$

$$\left(\frac{KW}{ton}\right) = 4.52 \left(\frac{\text{kiloamps}}{\text{sq. in.}}\right)^2$$ for copper at 40 C.

6. Rate of Temperature Rise without Cooling

$$\frac{P}{cW} = \frac{1}{c}\left(\frac{I}{A}\right)^2 \frac{\rho}{\delta}$$

$$\left(\frac{°C}{sec}\right) = .013 \left(\frac{\text{kiloamps}}{\text{sq. in.}}\right)^2$$

7. Sample Calculation

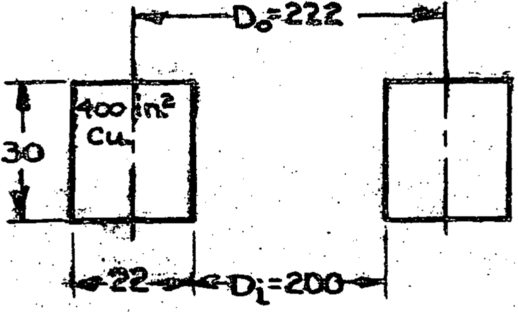
Given: NI = 1,200,000 amp. turns (same order of magnitude as 184")

D_i = 200 inches

Generator voltage = 600, Coil Terminal voltage = 575

Assume: I/A = 3000 amps./sq. in.

Gross Space Factor = 60%



Then: Area of Cu = $\frac{1,200,000}{3,000}$ = 400 sq. in.

Area of Coil Space = $\frac{400}{.6}$ = 670 sq. in.

Assume: Coil Cross Section = 30 in. x 22 in.

Then: Mean Dia. D_o = 200 + 22 = 222 in.

Wt. of Cu = 222π (400)(.321) = 90,000 lbs.

Power = $(3)^2$ x 4.52 x 45 = 1860 KW (at 40 C. mean temp. of Cu)

I = $\frac{1,860,000}{575}$ = 3,240 amps.

Conductor area = $\frac{3240}{3000}$ = 1.08 sq. in.

$$N = \frac{1,200,000}{3240} = 370$$

In the above we guessed at the space factor. We now can calculate a more realistic space factor and repeat the calculations until the assumptions agree with the answers.

Calculation should be repeated for several current densities to determine effect on costs, etc.

8. Factors to Consider in Coil Design:

a. Rigidity: The coil must not move as the magnetic field is applied. In addition to a changed field at the magnet gap, coil motion chafes insulation and will eventually result in short circuits. Since coils will loosen slightly during operation, good design also provides means for tightening the coil assembly periodically.

b. Coils Must be Kept Dry: Water will cause short circuits, corrosion and insulation deterioration. Coils should be fully enclosed where the possibility of water entrance exists.

c. Methods of Coil Cooling:

(1) Air Cooling: Air cooled coils provide good insulation for high voltage, are easy to wind and repair. However, they usually have poor space factor, require large equipment and extensive ducting for forced air circulation, and are also noisy due to the forced circulation.

(2) Oil Cooling: Oil cooled coils are usually of copper bar immersed in oil. This provides good electrical insulation and the coils are easy to wind. Space factor is low but better than that of air cooled coils. Heat exchangers and filter systems are required. Oil systems are a fire hazard and CO_2 fire extinguisher systems are usually provided. Always, they stink.

(3) Water Cooling: Good space factors and high permissible current densities are obtained with water cooled coils (extreme example - an early 184" model magnet was run with current density of 160,000 amperes per square inch). Water cooled coils are expensive to wind and there is always the possibility of water leaks with resultant shorting and corrosion damage. De-ionized water is usually used, and an elaborate system of flow indicators and interlocks is required.

d. **Economics:**

One wants the total cost to be a minimum.

Total Cost = (Capital Costs) + (Operating Costs) x (Operating Life)

= [Capital costs of iron core, coil, coil power supply]

+ [Operating Cost] x [Operating Life]

The operating life is difficult to estimate, and a low figure, say five years, should be used for an experimental magnet with an uncertain future.

There may be an impression that a high current density is desirable. This may be so where space is limited but for magnets in steady operation for long periods, as Cyclotrons, over approximately 1500 amps./in.^{2} is probably uneconomical. Figures 7 and 8 show the relationship between costs and I/A and W.

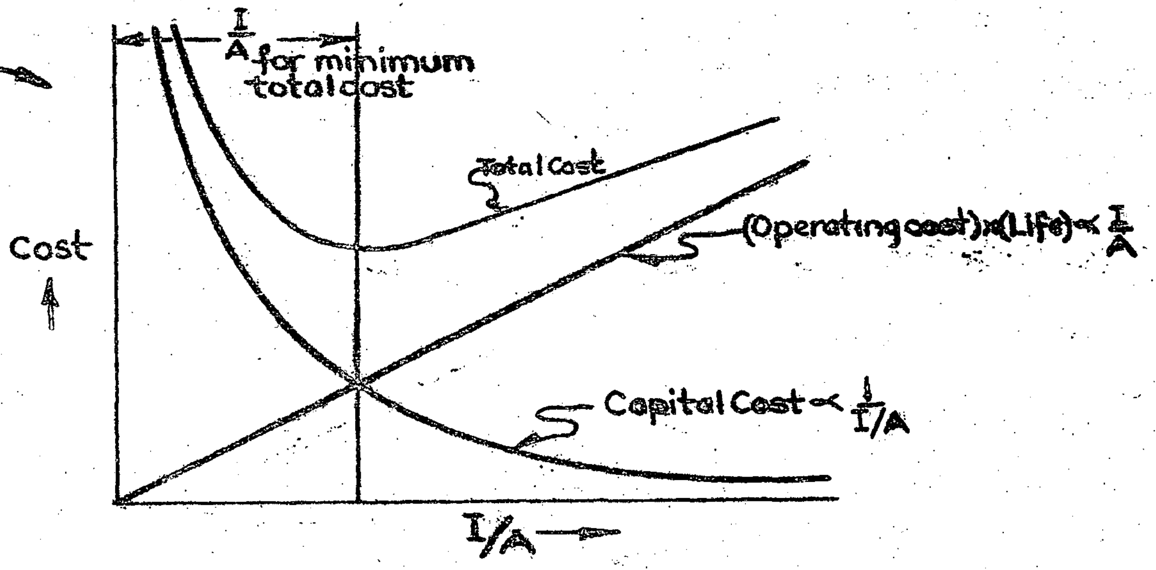


Figure 7. Coil and Power Supply Costs for constant NI and constant mean turn length

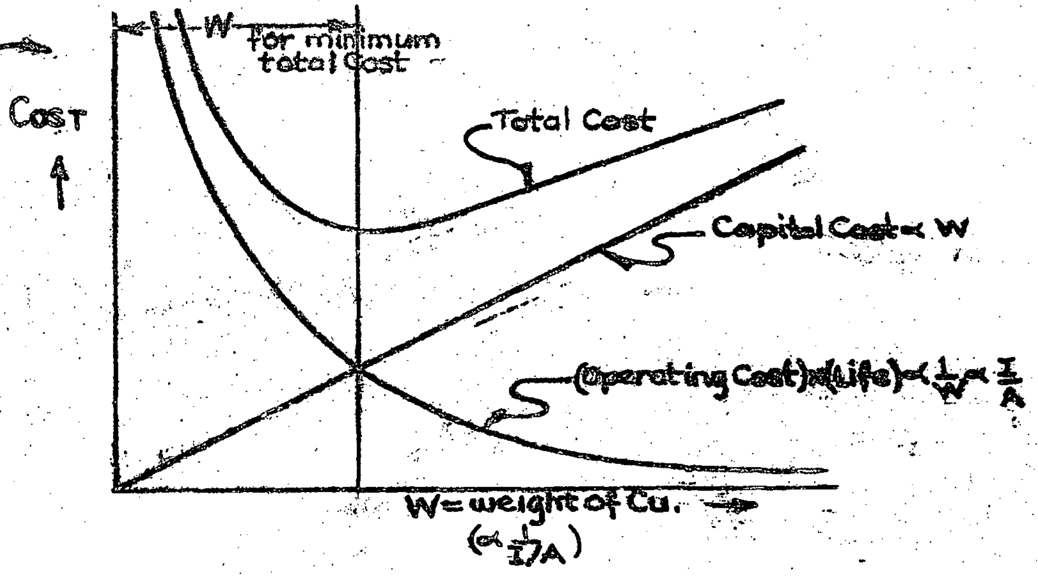


Figure 8. Coil and Power Supply Costs for constant NI and constant mean turn length

D. Factors in Core Design (Cyclotron)

1. Homogeneous Steel: In the design of accelerator magnets one of the principal requirements is that the magnetic field vary in a predetermined manner over the useful pole face. Unexpected local perturbations are very undesirable. Therefore, steel of uniform magnetic properties is highly desirable and some quality control should be established over this factor during magnet fabrication.

   The choice of steel is generally limited to the lowest carbon content without extra cost, i.e. ~0.15% C. Lower carbon is not worth the extra cost. Alloys do not improve μ at high B except for cobalt alloys, which are prohibitively expensive.

2. Symmetry About the Median Plane: The magnetic structure should be symmetrical about the desired position of the magnetic median plane. The location of the magnetic median plane is extremely sensitive to assymetries in the magnetic field. In the usual case, one desired to have the magnetic median plane coincide with the geometric center of the magnet gap, so that equal clearances and focussing forces exist for particle axial oscillations about the magnetic median plane.

   Example: An oil leak in the 184" magnet was corrected by installing a mild steel barrier ring (182" I.D. x 1/4" thick x 13" high) between the vacuum barrier disc and the pole tip disc. This ring was installed on the upper pole only and upset the median plane position to such an extent that it was necessary to disconnect one layer of one coil to correct the displacement.

3. The magnet structure must resist the magnetic forces. Often this is more a problem of stiffness (permissible deflections) than of ultimate strength.

4. The magnet must resist earthquake loads.

5. Working Space around Gap: Effort should be made to provide maximum unobstructed space around the magnet gap for installation of experimental equipment. This space is purchased by tolerating larger leakage flux and more iron in yokes, and hence, a less efficient (magnetically) machine. Some years ago Swedish scientists built a very efficient magnet in terms of minimum leakage flux but, as the sketch indicates, access to the air gap was severely restricted.

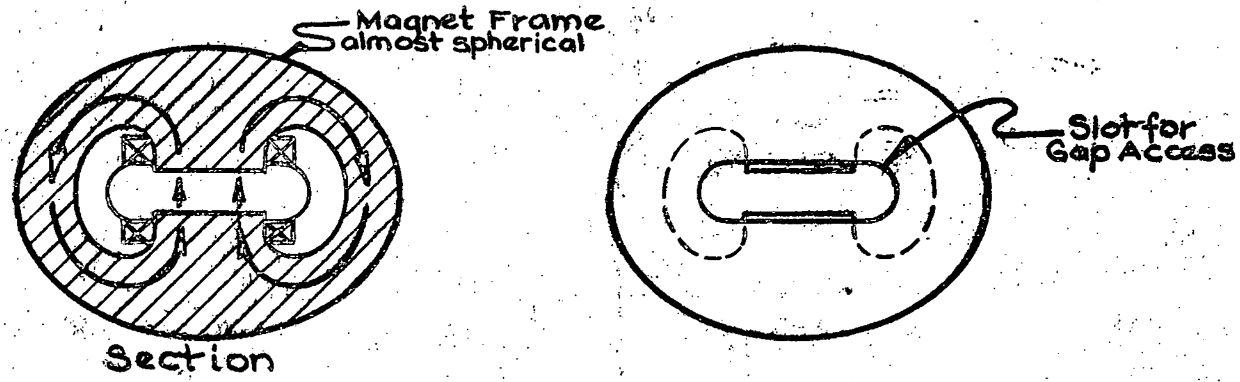


Figure 9

6. **Keep Coils Close to Pole Tips:** The closer the coils are to the pole tips the smaller the leakage flux.

7. **Tapered or Square Pole Tips:**

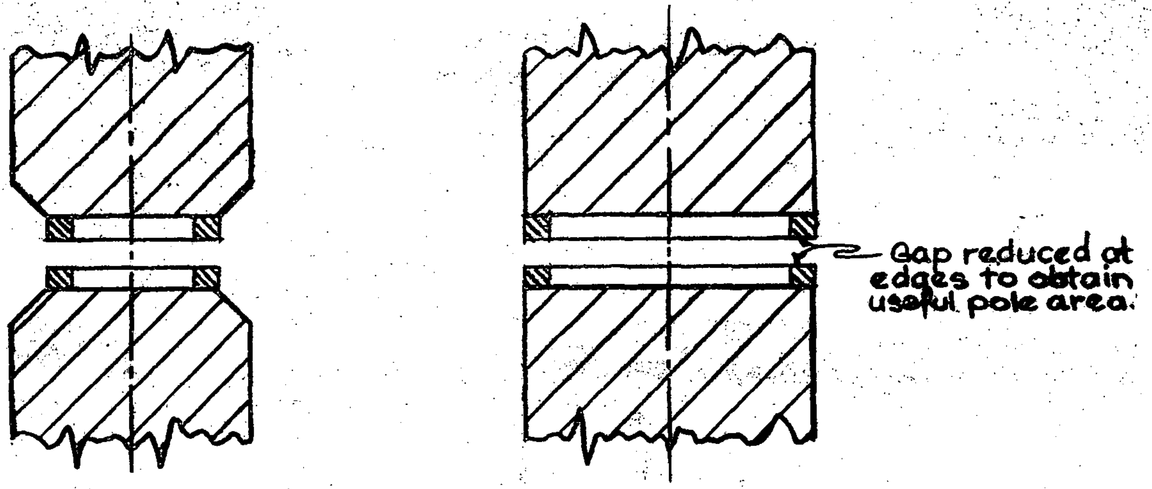


Figure 10

Tapered pole tips trade radius for flux density and allow a somewhat smaller vacuum tank installation. The resultant magnetic force on the tapered tip may be such as to pull the two tips together - a point which must be carefully investigated for each tapered pole design.

Historically, vacuum tanks used to have the pole face built into the tank. A shimming gap was left between the tank and the magnet pole. However, the vacuum tank pole greatly attenuated the effect of the shims, when so placed. Large machines need bolts attached to tank covers for support, which interfere with shimming. In general, shims are now placed on the pole tip inside the vacuum tank.

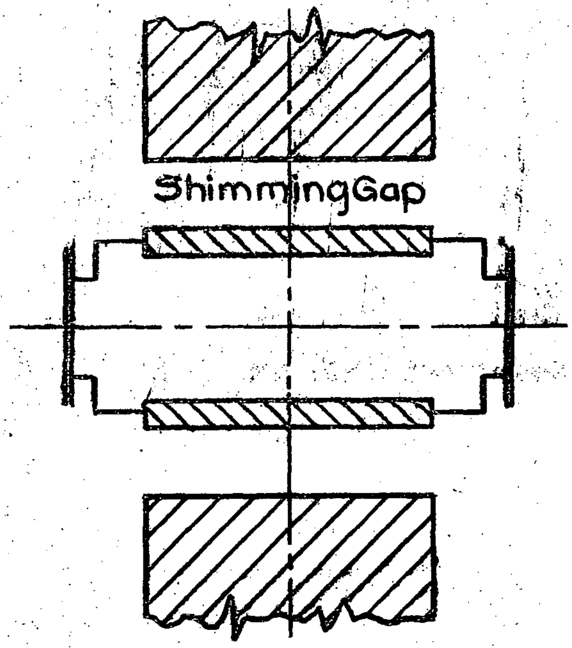


Figure 11

II. TABLE OF SYMBOLS AND UNITS

(Unless otherwise noted in text)

Symbols	Description	Dimensions
Φ	Total Flux	Maxwells, Lines
H	Magnetic Field Intensity	Oersteds
B	Magnetic Flux Density	Gauss
μ	Permeability	Gauss/Oersteds
$\mathcal{R}$	Reluctance	Oersted/Gauss cm.
NI	Magnetomotive Force (MMF)	Ampere Turns
ℓ	Length	cm.
x	Air Gap in Magnet	cm.
K	Flux Leakage Coefficient	Ratio
P	Power	Watts
R	Resistance	Ohms
W	Weight	Lbs.
δ	Unit Weight	Lbs./Cu. Inches
A	Area	Square cm.
ρ	Resistivity	Ohm Inches
c	Specific Heat	Watts/°C. Lb.