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ABSTRACT

The problem of multiturn injection into the 340-Mev electron synchrotron utilizing the 2-Mev beam from the rf linear electron accelerator is discussed. The details associated with inflection are ignored; primary attention is given to estimating the acceptance requirements for the betatron and synchrotron oscillations. Two methods of injection are examined: injection at constant energy and injection with energy increasing with the magnetic field. Under somewhat idealized assumptions regarding the output properties of the linac beam, we estimate that a trapped-beam current could be obtained which is several factors larger than the present beam using betatron injection. Since the angular properties of the beam from the linac are appreciably poorer than the idealized assumption, it appears that the probability of increasing the present synchrotron beam intensity using these injection methods is small.

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An increase in beam intensity by a factor of 10 to 100 would open up a new range of experiments on the Berkeley synchrotron. The present beam is about 10^9 electrons per pulse (e. p. p.) with a maximum of 3×10^9 e. p. p. recorded. Considering the machine aperture and injection voltage of 100 kv, there is a theoretical limit from space-charge blow-up of 4.9×10^{10} particles. From the above numbers it can be seen that the machine is operating at about one tenth of the theoretical limit. To gain a factor of 10 to 100 would require raising the injection energy. The feasibility of using an electron linear accelerator as an injector is being considered. For the 2.5-Mev electrons from this linac, the space-charge limit is 2.2×10^{13} e. p. p.

The problem of the inflector and of bringing the electrons through the fringe field of the magnet will not be considered here. It will be assumed that the electrons can be placed tangent to an orbit at the desired radius and with the correct energy.

Two methods of injection will be considered. Injection at constant energy as the synchronous orbit contracts, and injection at increasing energy such that injection is always into a stable orbit with a minimum of betatron oscillation. In both cases injection is with the radiofrequency oscillator (rf) off as the orbits contract. The rf is then turned on at the appropriate time. If injection were attempted with the rf on, the synchronous orbit would be near the center of the radial aperture, causing betatron oscillations of maximum amplitude which would wipe the beam out on the inflector after several turns.

Consider first injection at constant energy. Injection starts as the synchronous orbit sweeps by the inflector and continues until the synchronous orbit is at the midpoint of the radial aperture. Any particles injected after the synchronous orbit passes the midpoint are lost to the inner wall on the first half-cycle of the betatron oscillation. When the synchronous orbit reaches the midpoint, the rf is turned on. The synchronous phase angle ϕ_s is about 54 deg. Therefore $(3 \times 54) / 360 = 0.45$ of the injected beam is within the necessary phase angle to be captured. Within a quarter of a period of the synchronous phase oscillation (synchrotron oscillation), i. e., within about twelve (12) turns, the maximum amplitude is reached. For $\Delta E = \pm 15.4$ kev, the acceptance-energy spread of the synchrotron oscillation, this amplitude appears as a ± 1.85 -cm radial oscillation. Superimposed on this is the radial betatron oscillation with amplitude equal to the distance from the synchronous orbit to the inflector at the time of injection. The radial aperture of the synchrotron is 8 centimeters. Therefore, there is a maximum distance of

4 centimeters for the radial amplitude of oscillation. The synchrotron oscillation takes 1.85 cm, which leaves 2.15 cm for betatron oscillation. The aperture for betatron oscillations is reduced by the following consideration: First, the beam has a finite diameter and divergence (1-cm diameter and 3.5×10^{-3} radian at the end of the inflector). Second, rf is constant and β changes from 0.98 at injection to 1 during the acceleration. This causes the orbit to expand by 2% during the acceleration. For a 100-cm orbit, this is a 2-cm expansion. However, during this orbit expansion, the amplitude of the synchrotron oscillation is damped inversely as the $3/4$ power of the energy, and the betatron oscillation is adiabatically damped inversely as the square root of the energy. These considerations reduce the useful radial aperture for betatron oscillations from 2.15 cm to 0.9 cm. This radial aperture allows for injection of 5.6 turns. The trapped beam from the succeeding six turns diminishes to zero (neglecting the effect of losing part of the beam by its striking the inflector on the second turn after injection). From Fig. 1 (beam-current-vs-energy curve) 20 ma at 2.5 Mev appears to be a reasonable number for the injected current. For 5.6 turns this is 112 ma. Of this, 0.45 is within the range of stable synchronous phase oscillation. Therefore 50 ma is caught. This is further reduced because, with a 1-cm-diameter beam and the orbit moving in at 0.175 cm per turn, only part of the beam will clear the inflector. Because of betatron oscillations, two turns are made before the beam strikes the inflector. The final beam current is thus reduced to 30 ma total. The present 10^9 e.p.p. is 8 ma of beam current.

The second method of injection is to increase the injection energy so as always to inject at a synchronous orbit. This is considered as follows. There are small betatron oscillations in each turn with a maximum amplitude slightly greater than 1 cm. On the second turn after injection, 75% of the beam is lost to the inflector. However, because the betatron oscillations are very small, the accepted beam can now be spread over the full radial aperture as determined by the energy acceptance of the rf. This is $2 \times 1.85 = 3.70$ cm, which gives 21 turns. The maximum current in this case is then $20 \times 0.45 \times 0.25 \times 21 = 47$ ma total synchrotron current.

Phase oscillations

For a particle moving in a closed path, Φ is given by

$$\Phi = 2\pi = 2\pi \frac{L}{vt},$$

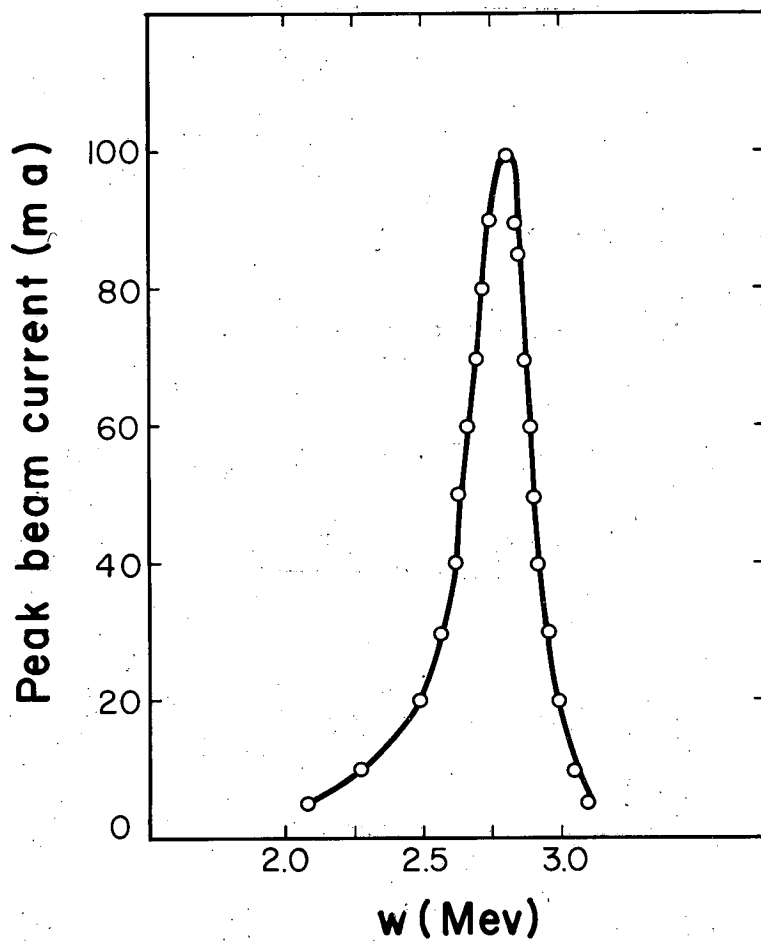
where

L = path length,

v = velocity,

t = time to move distance L, and

Φ = phase angle.



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Fig. 1. Typical current-vs-energy curve for electron linear accelerator. Actual curve for the linac used was not available at the reduced energy level. Some broadening of the energy is to be expected but has not been used here.

For circular orbits, we have $L = 2\pi r$. Therefore, we obtain

$$\bar{\Phi} = 2\pi \frac{2\pi r}{\beta c \tau} = \frac{4\pi^2 r \nu}{\beta c},$$

where

τ = period,
 ν = frequency,
 c = velocity of light, and
 $\beta = v/c$.

For a particle in a synchronous orbit, the synchronous phase angle $\bar{\Phi}_s$ is given by

$$\begin{aligned} \bar{\Phi}_s &= \frac{4\pi^2 r_s \nu_s}{c \beta_s} \\ \bar{\Phi} &= \frac{4\pi^2 r \nu_s}{c \beta} = \frac{4\pi^2}{c} \frac{r_s + \Delta r}{\beta_s + \Delta \beta} \\ \bar{\Phi} &= \frac{4\pi^2}{c} \frac{r_s}{\beta_s} \nu_s \left(1 + \frac{\Delta r}{r_s}\right) \left(1 - \frac{\Delta \beta}{\beta_s}\right) \end{aligned}$$

The change $\bar{\Phi}$ per turn k is given by

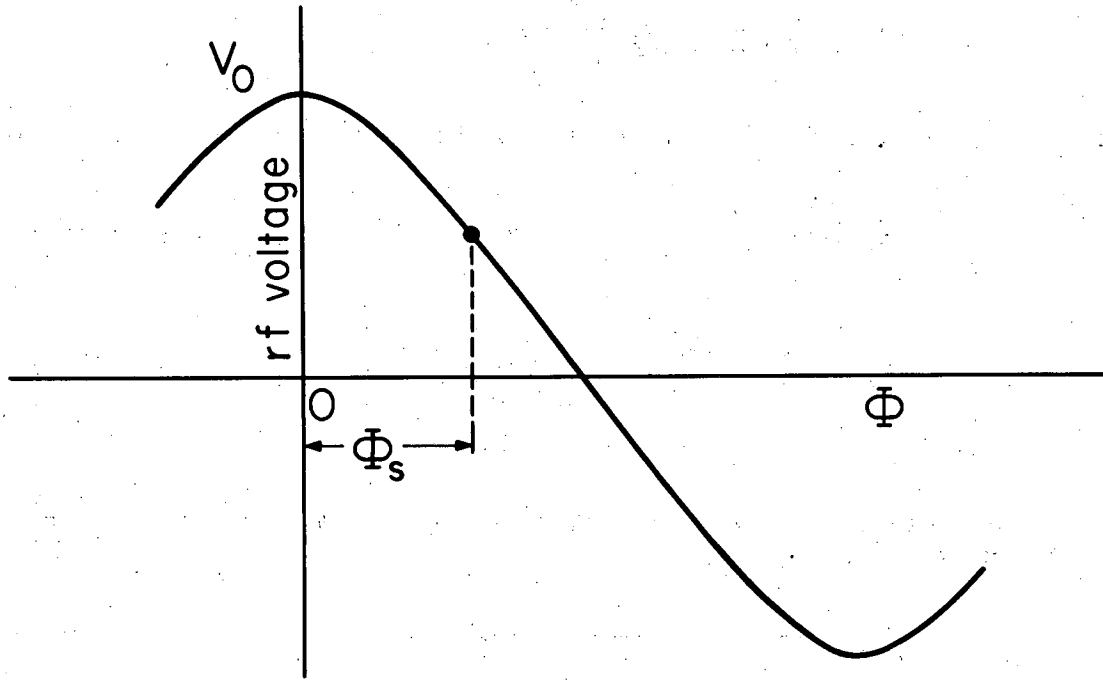
$$\frac{d\bar{\Phi}}{dk} = \bar{\Phi} - \bar{\Phi}_s = \frac{4\pi^2 r_s}{\lambda_s \beta_s} \left(\frac{\Delta r}{r_s} - \frac{\Delta \beta}{\beta_s} \right), \quad (1)$$

where

$$\lambda_s = c/\nu_s$$

The energy gain per turn from the rf oscillator is given by (see Fig. 2):

$$\begin{aligned} \frac{dw_s}{dk} &= e V_0 \cos \bar{\Phi}_s, \\ \frac{dw}{dk} &= e V_0 \cos (\bar{\Phi}_s + \bar{\Phi}), \\ \frac{d(w-w_s)}{dk} &= e V_0 \left[\cos (\bar{\Phi}_s + \bar{\Phi}) - \cos \bar{\Phi}_s \right]. \end{aligned} \quad (2)$$



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Fig. 2 Graph showing measurement of Φ_s .

In the above equations the quantities are as follows:

$$\begin{aligned}
 w &= \text{total energy of the particle} = mc^2 = m_0 \gamma c^2, \\
 w_s &= \text{synchronous phase energy,} \\
 m_s &= \text{electron mass,} \\
 m_0 &= \text{rest mass of electron,} \\
 e &= \text{electron charge,} \\
 V_0 &= \text{peak of gap voltage, and} \\
 \gamma &= (1 - \beta^2)^{-1/2}.
 \end{aligned}$$

Substituting w into Eq. (2) and expanding the right-hand member yields

$$\begin{aligned}
 \frac{d}{dk} (\gamma - \gamma_s) m_0 c^2 &= eV_0 (\cos \bar{\Phi}_s \cos \bar{\Phi} - \sin \bar{\Phi}_s \sin \bar{\Phi} - \cos \bar{\Phi}_s) \\
 \frac{d\Delta\gamma}{dk} &= - \frac{eV_0}{m_0 c^2} \sin \bar{\Phi}_s \bar{\Phi}, \quad (3)
 \end{aligned}$$

for ϕ small.

For a particle moving in a circular path in a magnetic field of strength B , we have

$$\begin{aligned}
 r &= \frac{mv}{eB} = \frac{m_0 c}{e} \frac{\gamma \beta}{B}, \\
 \Delta r &= \frac{m_0 c}{e} \left(\Delta \gamma \frac{\beta}{B} + \Delta \beta \frac{\gamma}{B} - \frac{\gamma \beta}{B^2} \Delta B \right) \\
 \frac{\Delta r}{r} &= \frac{\Delta \gamma}{\gamma} + \frac{\Delta \beta}{\beta} - \frac{\Delta B}{B}. \quad (4)
 \end{aligned}$$

For the synchrotron, we have

$$B = B_0 \frac{r_0^n}{r^n},$$

where n is the magnetic-field index and B_0 is the field strength at r_0 . For this case, B_0 is a constant. Differentiating, we obtain

$$\Delta B = Bn \frac{\Delta r}{r}.$$

Dividing by B , we get

$$\frac{\Delta B}{B} = -n \frac{\Delta r}{r}.$$

Substituting the above into Eq. (4), we now obtain

$$\frac{\Delta r}{r} (1-n) = \frac{\Delta \gamma}{\gamma} + \frac{\Delta \beta}{\beta} \quad (5)$$

Next, we differentiate $\gamma = (1-\beta^2)^{-1/2}$ and divide by γ :

$$\frac{\Delta \gamma}{\gamma} = \frac{\beta^2}{1-\beta^2} \frac{\Delta \beta}{\beta}$$

For $\beta = 0.98$ we have

$$\begin{aligned} \frac{\Delta \gamma}{\gamma} &= \frac{0.96}{0.04} \frac{\Delta \beta}{\beta} \\ &= 24 \frac{\Delta \beta}{\beta} \end{aligned}$$

Because $\Delta \gamma/\gamma \gg \Delta \beta/\beta$, Eq. (5) becomes

$$\frac{\Delta \gamma}{\gamma} = (1-n) \frac{\Delta r}{r} \quad (6)$$

Also, Eq. (1) becomes

$$\frac{d\bar{\phi}}{dk} \approx \frac{4\pi^2 r_s}{\beta_s \lambda_s} \frac{\Delta r}{r_s} \quad (7)$$

Differentiating with respect to k gives

$$\begin{aligned} \frac{d}{dk} \frac{d\bar{\phi}}{dk} &= \frac{d^2 \bar{\phi}}{dk^2} \\ &= \frac{4\pi^2 r_s}{\beta_s \lambda_s} \frac{d}{dk} \left(\frac{\Delta r}{r_s} \right) \\ &\approx \frac{4\pi^2 r_s}{\beta_s \lambda_s} \frac{d}{dk} \left[\frac{\Delta \gamma}{\gamma_s} \frac{1}{(1-n)} \right] \\ \frac{d^2 \bar{\phi}}{dk^2} &= \frac{4\pi^2 r_s}{\beta_s \lambda_s \gamma_s (1-n)} \frac{d}{dk} \Delta \gamma \end{aligned}$$

Substituting from Eq. (3), we obtain

$$\frac{d^2 \bar{\Phi}}{dk^2} = \frac{4\pi^2 r_s}{\beta_s \lambda_s \gamma_s (1-n)} \left(-\frac{eV_0}{m_0 c^2} \sin \bar{\Phi}_s \bar{\Phi} \right),$$

$$= \left[\frac{4\pi^2 r_s eV_0 \sin \bar{\Phi}_s}{\beta_s \lambda_s \gamma_s (1-n) m_0 c^2} \right] \bar{\Phi};$$

$$\frac{d^2 \bar{\Phi}}{dk^2} + \frac{2\pi}{\gamma_s} \frac{eV_0 \sin \bar{\Phi}_s}{(1-n) m_0 c^2} \bar{\Phi} = 0.$$

Letting

$$\omega_{\bar{\Phi}}^2 = \frac{2\pi}{\gamma_s} \frac{eV_0 \sin \bar{\Phi}_s}{(1-n) m_0 c^2}$$

gives

$$\frac{d^2 \bar{\Phi}}{dk^2} + \omega_{\bar{\Phi}}^2 \bar{\Phi} = 0 \tag{8}$$

as the equation of motion for phase oscillations. To determine $\bar{\Phi}_s$ we must first determine the energy gain per turn. We recall the cyclotron equation:

$$\omega = \frac{eB}{m}$$

Solving for B, we obtain

$$B = \frac{\omega m}{e} = \frac{\omega m_0 \gamma}{e}$$

For this case ω is a constant. Therefore, B is equal to a constant times γ . Differentiating gives

$$\Delta B = (\text{constant}) \Delta \gamma.$$

From this we obtain

$$\frac{\Delta B}{B} = \frac{\Delta \gamma}{\gamma}$$

From the relation $w = m_0 c^2 \gamma$ we can obtain

$$\frac{\Delta w}{w} = \frac{\Delta \gamma}{\gamma}$$

Therefore, we have

$$\frac{\Delta B}{B} = \frac{\Delta w}{w}$$

Multiplying by $\Delta t / \Delta t$ gives

$$\frac{\Delta B}{B} \frac{\Delta t}{\Delta t} = \frac{\Delta B}{\Delta t} \frac{\Delta t}{B} = \frac{\dot{B}}{B} \tau$$

Therefore, we obtain

$$\frac{\Delta w_k}{w} = \frac{\dot{B}}{B} \tau_k$$

where Δw is the energy gain per turn and τ_k is the period of rotation.

For the synchrotron, \dot{B} is 2.3 gauss per microsecond, B is 82 gauss for 2.5-Mev electrons, and τ_k is 2.1×10^{-2} μsec ; therefore, Δw_k equals 1470 ev per turn. The rf peak voltage is 2500 volts; therefore, Φ_s is 54 deg.

Equation (8) has a solution:

$$\Phi = \Phi_{\max} \sin(\omega_{\Phi} k)$$

for ω_{Φ} constant. Differentiating gives

$$\frac{d\Phi}{dk} = \Phi_{\max} \omega_{\Phi} \cos(\omega_{\Phi} k)$$

For $\cos \omega_{\Phi} k = 1$, we obtain

$$\left(\frac{d\bar{\Phi}}{dk}\right)_{\max} = \bar{\Phi}_{\max} \omega_{\Phi}$$

as the condition for maximum kinetic energy of oscillation. From the values for the linac and the synchrotron given, we have $\omega_{\Phi} = 1.55 \times 10^{-2}$ and $\tau_s = 50.2$ turns per phase oscillation. Using Eqs. (6) and (7), we find

$$\Delta w_{\max} = w(1-n) \frac{\beta_s \tau_s}{4\pi^2 r_s} \bar{\Phi}_{\max} \omega_{\Phi}$$

$$\Delta w_{\max} = \pm 15.4 \text{ kev,}$$

for the values given in Table I. From $\Delta w/w = (1-n) \Delta r/r$, we find that $\Delta r = \pm 1.85 \text{ cm}$ is the aperture needed for the synchrotron oscillations.

Orbit contraction

The rate at which the orbit contracts can be determined from the following:

$$r = \frac{\gamma m_0 c \beta}{e B},$$

$$\Delta r = - \frac{\gamma m_0 c \beta}{e} \frac{\Delta B}{B^2},$$

for γ and β constant. Determine ΔB from $B = B(r, t)$; substituting and dividing by r gives

$$\frac{\Delta r}{r} = \frac{1}{B} \left(\frac{\delta B}{\delta r} \Delta r + \frac{\delta B}{\delta t} \Delta t \right).$$

For the synchrotron field we have

$$n = - \frac{\delta B}{\delta r} \frac{r}{B}$$

Table I

Linac specifications	
Energy	2-4 Mev
ΔE for half intensity	0.2 Mev
Divergence at output of inflector	3.5×10^{-3} radian
Beam size at output of inflector	1 cm
Beam current (total)	100 ma
Synchrotron specifications	
Injection energy (present)	100 kv
Beam current	8 ma
Radius ($\beta = 1$)	100 cm
Radiofrequency	48 Mc
Rf voltage, peak	2500 kv
H - magnetic field at 2.5 Mev	82 gauss
H - rate of change of H at 82 gauss	2.3 gauss/ μ sec
Radius of inflector	104 cm
Radius of target	96 cm

Substituting for $\delta B/\delta r$ and collecting terms gives

$$\Delta t = - \Delta r (1 - n) \frac{B}{rB} \quad (9)$$

(Δr is negative for B positive).

Betatron oscillations

The equation of motion for a particle moving in a betatron-type field, $B = B_0 r_0^n / r^n$, is

$$m \frac{d^2 r}{dt^2} = m \dot{\Phi}^2 r - evB,$$

neglecting space-charge blow-up, which can be shown to be negligible compared to the focusing forces in the betatron for the conditions being considered here. For the condition $mv^2/r_0 = evB_0$ and letting $r = r_0 + \Delta r$, we obtain for the equation of motion:

$$\frac{d^2 \Delta r}{dt^2} = \frac{v^2}{r_0^2} \Delta r (1 - n). \quad (10)$$

We change variables as follows:

$$\frac{d\Delta r}{dt} = \frac{d\Delta r}{d\Phi} \frac{d\Phi}{dt}$$

and use the condition $\dot{\Phi} = \omega = \text{constant}$. The equation for betatron oscillations as a function of angular position is then

$$\frac{d^2 \Delta r}{d\Phi^2} + (1 - n) \Delta r = 0. \quad (11)$$

The solution to this equation is

$$\Delta r = a \sin [(1 - n)^{1/2} \Phi + \delta]$$

at injection, where

$$\Phi_i = 0,$$

$$\frac{d\Delta r_i}{d\Phi} = a v_0 \quad (\text{see Fig. 3}),$$

a = divergence of the beam, and

Δr_i = the distance of the beam from synchronous orbit at injection. (In the above equation δ is a constant of integration.)

Using the initial conditions and solving for the maximum amplitude (a), we obtain

$$a^2 = 0.369 + (\Delta r_i)^2, \quad (12)$$

where Δr_i and a are expressed in centimeters.

Damping

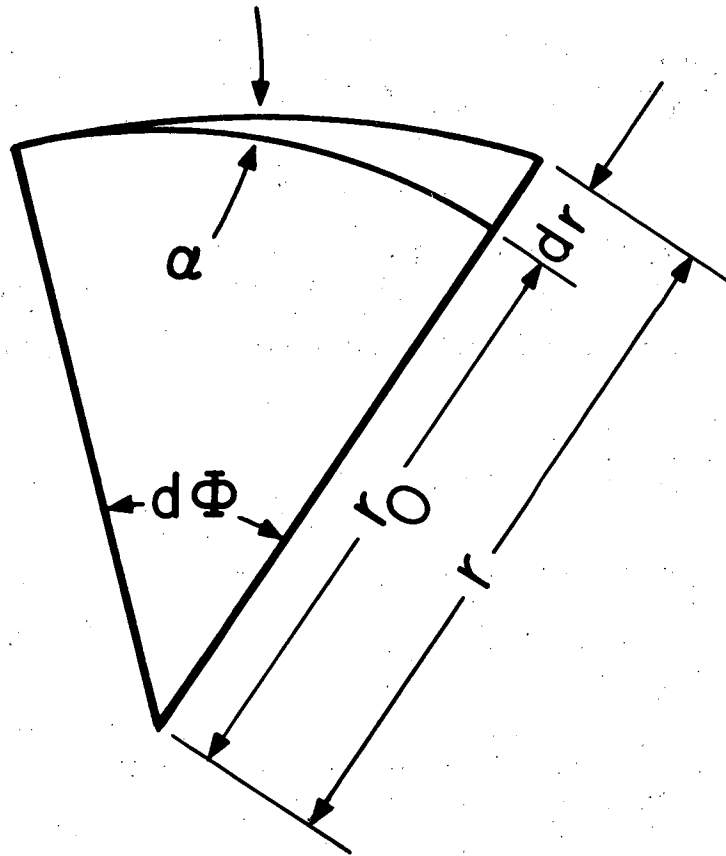
When the rf is turned on and as the electrons gain energy, β increases from 0.98 to 1. The orbit radius r is given by $r = (m_0 c \beta / e) (\gamma / B)$. The ratio γ / B is a constant for fixed-frequency acceleration. Therefore, as β increases, r increases. This expansion is about 2 cm for this machine. The available aperture for betatron oscillations after allowing 1.85 cm for synchrotron oscillations is only 2.15 cm. Without sufficient damping of the oscillations to overcome the orbit expansion, there is essentially no radial aperture left unless the rf is modulated to hold the orbit radius constant.

The change in radius for the synchrotron oscillations can be determined as follows. From Eq. (6) we have

$$\frac{\Delta \gamma}{\gamma} = \frac{\Delta E}{E} = (1 - n) \frac{\Delta r}{r}$$

$$\frac{\Delta E_1 / E_1}{\Delta E_2 / E_2} = \frac{(1 - n)}{(1 - n)} \frac{\Delta r_1 / r_1}{\Delta r_2 / r_2}$$

$$\frac{\Delta E_1}{\Delta E_2} \frac{E_2}{E_1} = \frac{\Delta r_1}{\Delta r_2} \frac{r_2}{r_1},$$



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Fig. 3. Diagram showing relation of α to other variables.
Differentiating $r = r_0 + \Delta r$ gives $dr = \alpha r_0 d\Phi$.

where ΔE is proportional to $E^{3/4}$. * Therefore, we get

$$\frac{\Delta r_2}{\Delta r_1} = \frac{E_1}{E_2} \frac{E_2^{1/4}}{E_1^{1/4}} \frac{r_2}{r_1},$$

$$\frac{\Delta r_2}{\Delta r_1} \approx \left(\frac{E_1}{E_2} \right)^{3/4}$$

for $r_1 \approx r_2$.

The change in radius for the betatron oscillation is determined from Eq. (10). Substituting the equilibrium condition $v/r_0 = eB_0/m$ into Eq. (10), we obtain

$$\frac{d^2 \Delta r}{dt^2} = \frac{e^2 B_0^2}{m^2} (1-n) \Delta r.$$

The force F is $(e^2 B_0^2) (1-n) \Delta r/m$. The energy E of a harmonic oscillator is given by

$$\begin{aligned} E &= \int_0^a F d\Delta r = \int_0^a \left[e^2 B_0^2 (1-n) \Delta r/m \right] d\Delta r, \\ &= \frac{e^2 B_0^2 (1-n)}{m} \frac{a^2}{2}, \end{aligned}$$

where a is the maximum amplitude of radial oscillation. From the above equilibrium condition it can be shown that B_0 is proportional to w , the

* The calculation for synchrotron phase angle used in this paper assumes that the synchronous energy of the particle does not change very much during our time of observation. It is valid at any particular time but cannot be used to determine the change in parameters in going from one energy condition to another. We therefore refer to the complete derivation as given by Edwin McMillan. [E.M. McMillan, Particle Accelerators, in Experimental Nuclear Physics, E. Segre et al. Eds., Vol. III (Wiley, New York, 1959), Part XII.]

total particle energy. Also, m is proportional to w . Therefore, E is proportional to wa^2 . As the process is adiabatic, E is a constant. Therefore,

$$w_1 a_1^2 = w_2 a_2^2,$$

$$\frac{a_2}{a_1} = \left(\frac{w_1}{w_2} \right)^{1/2}.$$

Letting A_β be the amplitude of the betatron oscillation and A_ϕ be the amplitude resulting from the synchrotron oscillation, we can tabulate the maximum radial displacement $r + A_\phi + A_\beta$. (See Table II.) From this it can be seen that an initial maximum radial displacement of 2 cm is available for the betatron oscillation, neglecting the finite width of the beam.

Injection at constant energy

Constant-energy injection starts when the synchronous orbit is at the inflector and continues to the midpoint of the radial aperture. The rf is then turned on. Not all of the beam between the inflector and the midpoint is trapped by the rf, but only that beam in the region between the synchronous orbit (r_s) and a distance out from r_s equal to the maximum allowed amplitude for betatron oscillations. This is determined from Eq. (12):

$$a^2 = 0.369 + (\Delta r_i)^2.$$

For $a = 2$ cm, we have $\Delta r_i = 1.9$ cm. This is reduced by the injected-beam diameter of 1 cm, giving a useful amplitude of oscillation Δr_i of 0.9 cm. From Eq. (9) and the values in Table I we obtain $\Delta t = 1.05 \times 10^{-7}$ sec. The period of revolution of the electron is 0.0208 μ sec per turn. Therefore, the total number of turns T accepted in time Δt is

$$T = \frac{0.105}{0.208} \approx 6.$$

Some of the beam in each turn is lost by striking the inflector. The betatron oscillation causes the beam to miss the inflector on the first turn, but some of it is wiped off by the inflector on the second turn. From the solution to Eq. (11), we have

$$\Delta r = a \sin(0.577 \phi + \delta),$$

Table II

Radial position as a function of energy w
of the injected electrons

w	γ	β	r	A_{β}	A_{Φ}	$r + A_{\beta} + A_{\Phi}$
2.51	4.9	0.9794	100	2.15	1.85	104.1
3.0	5.9	0.9856	100.6	1.98	1.61	104.19
4.0	7.8	0.9918	101.2	1.70	1.31	104.21
5.0	9.8	0.9948	101.5	1.52	1.10	104.10
6.0	10.7	0.9956	101.6	1.46	1.04	103.95
2.51	4.9	0.9794	100	2.00	1.85	103.85
3.0	5.9	0.9856	100.6	1.73	1.61	103.94
4.0	7.8	0.9918	101.2	1.51	1.31	104.01
5.0	9.8	0.9948	101.5	1.34	1.10	103.94

and from Eq. (12) we have

$$a^2 = 0.369 + (\Delta r_0)^2 .$$

We can now calculate a and δ . The term Δr can then be calculated for two turns ($\Phi = 4\pi$). By calculating the Δr for the inner and outer radius, one can determine the percentage of the beam that clears the inflector for each of the twelve turns allowed during the injection cycle. In this calculation of the number of turns, the outer radius was used as the maximum amplitude in determining the loss due to betatron oscillations as the synchronous orbit expanded. Owing to the clipping at injection, only the inner part of the beam is accepted at any time. The total number of turns accepted will be between the six turns calculated and twelve turns when the inner radius reaches the 1.9-cm limit in amplitude. This is shown in Table III. The total current from ten turns is 33 ma compared to the present peak beam current of 8 ma.

Injection on synchronous orbit

The second method of injection is by increasing the energy of injected electrons to match the increasing magnetic field such that injection is always into a synchronous orbit. This minimizes the betatron oscillations. In the section on synchrotron phase oscillations the acceptance energy was calculated as ± 15.4 kev. This corresponded to a radial aperture of $\Delta r = \pm 1.85$ cm or a total Δr equal to 3.7 cm. From Eq. (9) we find that $\Delta t = 0.44$ μ sec is the time for the orbit to sweep 3.7 cm. This is equivalent to 21 turns. From the results of the previous section, 25% of the beam clears the inflector on each turn. The total current is then $I_T = 20 \times 0.45 \times 0.25 \times N_T = 9 \times 0.25 \times 21$. $I_T = 47$ ma. The initial amplitude for betatron oscillation is 1.77 cm. After clipping, it is 0.42 cm maximum. From previous calculation, Table II, there is room for 2-cm betatron oscillation without any loss when orbit expands. There will be no loss due to betatron oscillations for this method of injection after the initial clipping by the inflector.

Vertical oscillations

The equation of motion for particles in the vertical direction is

$$m \frac{d^2 z}{dt^2} = - \frac{mv_0^2}{R_s^2} \quad nz \quad \left[1 - (n+1) \frac{r}{R_s} \right] ,$$

Table III

Percent beam remaining after second turn for constant-energy injections

<u>Turn</u>	<u>Beam remaining (%)</u>	<u>Current (ma)</u>
1st	22	2.2
2	0	0
3	0.5	0
4	31	3.1
5	27	2.7
6	54	5.4
7	51	5.1
8	56	5.6
9	59	5.9
10	30	3.0
		<u>33.0</u> ma

Total

where z is the vertical distance from the equilibrium plane. For a particle on the synchronous orbit, r is 0; therefore, we have

$$\frac{d^2 z}{dt^2} = - \frac{v_0^2}{R_s^2} n z,$$

which has a solution $z = A \sin(\omega_z t + b)$. For the initial conditions, $t = 0$ and $z_0 = 0.5$ cm, we have $z_0 = A \sin b$ and $\dot{z}_0 = a \omega_z \cos b$. Squaring and adding, we get

$$z_0^2 + \frac{\dot{z}_0^2}{\omega_z^2} = A^2.$$

For vertical oscillations we have $\omega_z = v_0 n^{1/2} / R_s$. The vertical divergence of the beam is given by $\dot{z}_0 / v_0^2 = a$. Substituting these two quantities into the above equation for A gives

$$A^2 = z_0^2 + \frac{a^2 R_s^2}{n}$$

$$A^2 = 0.5^2 + \frac{(3.5 \times 10^{-3})^2 (10^2)^2}{2/3} = 0.435,$$

$$A = 0.6 \text{ cm.}$$

The half-aperture is about 2 cm; therefore, there is no problem from vertical oscillations.

Conclusion

The values for peak trapped-beam current are from 3 to 6 times the present operating current and about 2 times the maximum current ever recorded on the synchrotron. However, these values have been calculated assuming optimum conditions for injection. The characteristics of the linac beam and the difficulty of bringing this beam through the fringe field of the synchrotron and bending it into an orbit will probably reduce the beam current below that calculated. In addition, any perturbations in fields could increase the amplitude of the radial betatron oscillations, wiping out even more beam on the inflector.

The injection orbit sweeps radially inward of the rate of 0.9 cm in 10^{-7} sec. In about 4×10^{-7} sec the beam will be completely swept into the inner wall if the rf voltage is not turned on. The Q of the resonator section requires a turn-on time of 10^{-6} sec. Therefore, we may not be able to turn on the rf voltage fast enough to trap the beam.

In conclusion, we believe we will be fortunate to get as high a beam current as we have at present. The chance of increasing it by injecting with a linear accelerator would appear small.

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