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POLARIZATION-INVARIANT DIRECTIONAL CLOAKING BY TRANSFORMATION OPTICS

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Abstract—We propose a three-dimensional directional cloak for arbitrarily polarized incoming electromagnetic waves, motivated by the fact that there will be negligible scattering when the direction of impinging wave coincides with the axial direction of a very thin and elongated perfectly electric conducting (PEC) scatterer. The performance of the cloak under different polarizations of incoming waves are numerically investigated. The case in which the direction of incoming wave is perturbed off the ideal direction is also quantitatively studied. Numerical simulations show that the directional cloaking device is able to tolerate a large range of tilted angles of incoming waves.

1. INTRODUCTION

Research on electromagnetic cloaks has become very popular in the past few years due to their exciting property of completely hiding objects from electromagnetic detection. There exist various cloaking mechanisms, among which is the transformation optics approach [1–13]. The fundamental idea is the invariance of Maxwell's equations under a space-deforming transformation if the material properties are

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altered accordingly. In many real world applications, the direction of incoming wave is known or fixed, and it would be of great interest to investigate the cloaking effect for a fixed incoming wave, i.e., directional cloaking. A one-directional diamond-shape invisibility cloak is presented in [14], based on the fact that there will be no scattering when transverse magnetic (TM) polarized waves are incident onto an infinitesimally thin perfectly electric conducting (PEC) surface, with the electric fields perpendicular to the surface. The parameters of cloak layer are homogeneously anisotropic and relatively easy for practical realizations. The proposed model has been successfully applied to ground plane cloaking for large objects [15]. In [16], an elliptical cylindrical cloak with a perfect electric conducting lining at the interior surface achieves perfect invisibility along the direction parallel to the cloak's major axis when it is illuminated by a TM wave. Using quasi-conformal mapping, [17] designs and experimentally tests a directional cloak that can work very well with broadband and low-loss performance, even though the incident wave is not perfectly aligned with the major axis of the elliptical cloak. It is obvious that the aforementioned directional cloaks can find a lot of industrial and civil applications, such as wireless communication, air flight, and space exploration. In addition, as presented in [16], one possible application of using directional cloak is to reduce the scattering by struts in an antenna system.

Most directional cloaking devices published so far are two-dimensional (2D) ones. Although [14, 16] extend their design principles into three-dimensional (3D) scenarios, the performance of cloaking depends on polarization of incoming electromagnetic waves. However, in many applications, it is desirable to have a directional cloak that is independent of the polarization of incoming electromagnetic waves. In this paper, we present a 3D polarization-invariant directional cloak. The performance of the cloak under different polarizations of incoming waves are numerically investigated, and the case when the direction of incoming wave is perturbed off the ideal direction is also quantitatively studied.

2. DIRECTIONAL CLOAKING BY TRANSFORMATION OPTICS

The directional cloaking for arbitrary polarization of incidence electromagnetic wave is motivated by the fact that there will be negligible scattering when the direction of impinging wave coincides with the axial direction of a very thin and elongated PEC scatterer, regardless of the polarization of the incoming electromagnetic wave.

We consider that the electromagnetic wave impinges along the x direction. In designing the directional cloak, we implement a coordinate transform from the virtual space (x', y', z') to the physical space (x, y, z). In the virtual space, a very elongated cuboid with dimension $2 \times 2\rho \times 2\rho$ is placed along the x-axis, with the center being at the origin. Note that $\rho \ll 1$. The surface of the cuboid is PEC and the surrounding medium is free space. The to-be-transformed domain, which will be transformed to the cloaking layer in the physical space, lies in between two concentric cuboids, with the inner one being the aforementioned PEC coated cuboid and the outer one having the dimension $(2r_2/\rho) \times 2r_2 \times 2r_2$. The corresponding cloaking layer in the physical space lies also in between two concentric cuboids, with the inner one having the dimension $(2r_1/\rho) \times 2r_1 \times 2r_1$ and the outer one being the same as the one in the virtual space. The inner boundary is automatically PEC and objects can be hidden within it. As shown in Fig. 1, the cloaking layer in the physical space, as well as its pre-image in the virtual space, consists of six quadrilateral frustums. For each to-be-transformed quadrilateral frustum in the virtual space, define w to be the coordinate axis that is perpendicular to the inner and outer boundaries.

The transform from the virtual space to the physical space is given by

$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'}, \tag{1}$$

$$\frac{w - w_i}{w' - w'_i} = \frac{w_o - w_i}{w_o - w'_i}, \tag{2}$$

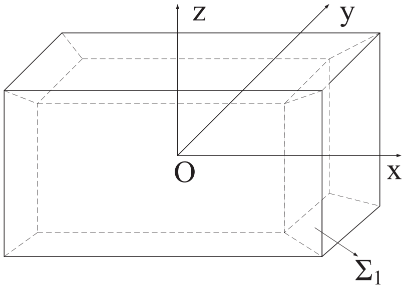


Figure 1. The directional cloaking device in the physical space exhibits cloaking effect for impinging waves along the x direction. The cloaking layer is an anisotropic material lying in between two cuboids, with the inner and outer one having the dimension $(2r_1/\rho) \times 2r_1 \times 2r_1$ and $(2r_2/\rho) \times 2r_2 \times 2r_2$, respectively, and the inner cuboid is coated with PEC within which objects can be hidden.

where w_i and w_o are the coordinates of the inner and outer boundaries of the quadrilateral frustum and the counterparts are denoted in primed coordinates. Note that the outer boundaries in physical and virtual spaces are identical, $w'_o = w_o$. For later convenience, (2) can be written in a compact format,

$$w = w'K + Q, \tag{3}$$

where $K = \frac{w_o - w_i}{w_o - w'_i}$ and $Q = w_i - w'_i K$.

For example, in region Σ_1, the x axis is perpendicular to two boundaries, $w'_i = 1$ and $w'_o = r_2/\rho$ in virtual space and $w_i = r_1/\rho$ and $w_o = w'_o$ in physical space. Thus, we have $w = x$ and the Jacobian matrix is given by

$$\bar{\bar{J}} = \begin{pmatrix} K & 0 & 0 \\ -\frac{y'}{x'^2}Q & \frac{x}{x'} & 0 \\ -\frac{z'}{x'^2}Q & 0 & \frac{x}{x'} \end{pmatrix}. \tag{4}$$

Thus, the relative permittivity and permeability tensors, $\bar{\bar{\epsilon}}/\epsilon_0$ and $\bar{\bar{\mu}}/\mu_0$, can be both expressed as [1]

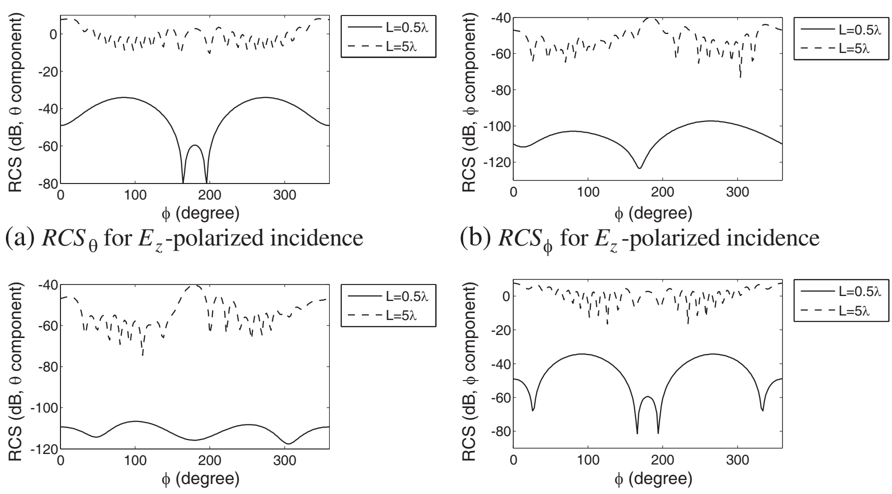
$$\bar{\bar{\epsilon}}_r = \bar{\bar{\mu}}_r = \frac{\bar{\bar{J}} \cdot \bar{\bar{J}}^{\mathrm{T}}}{\det \bar{\bar{J}}} = \begin{pmatrix} K\frac{x'^2}{x^2} & -\frac{y'}{x^2}Q & -\frac{z'}{x^2}Q \\ -\frac{y'}{x^2}Q & \frac{1}{K} + \frac{y'^2}{Kx^2x'^2}Q^2 & \frac{y'z'}{Kx^2x'^2}Q^2 \\ -\frac{z'}{x^2}Q & \frac{y'z'}{Kx^2x'^2}Q^2 & \frac{1}{K} + \frac{y'^2}{Kx^2x'^2}Q^2 \end{pmatrix}. \tag{5}$$

It can be seen that the permittivity and permeability tensors are spatially variant, which is different from the cloaking model presented in [14]. The relative permittivity and permeability tensors in other five quadrilateral frustums can be obtained similarly.

Due to the invariance of Maxwell's equations under a space transformation, the scattering in the physical space is equal to that in the virtual space. In the virtual space, when a plane wave impinges along the x direction, regardless of the polarization, the electric field is always perpendicular to the longitudinal direction of the inner cuboid, which is an elongated PEC coated structure. Thus, the induced electric currents are mainly oriented perpendicular to the longitudinal direction [18]. Consequently, the magnitude of the induced electric currents are inevitably small due to the small dimension of the PEC in the transverse direction. Therefore, the scattering will be negligible when the direction of impinging wave coincides with the longitudinal direction, regardless of the polarization of the incoming electromagnetic wave.

3. NUMERICAL SIMULATIONS

This section considers numerical simulations to test the performance of the cloak under different polarizations of incoming waves. We also consider the case when the direction of incoming wave is perturbed off the ideal direction. The following parameters are used in the numerical simulations. The unit of length that is used in Section 2 is chosen as 0.25λ, where $\lambda = 1\,\text{m}$. ρ, r_1, and r_2 are chosen as 0.1, 1, and 1.1, respectively. Thus, the actual dimension of the cuboid PEC scatterer in the virtual space and the physical space is $0.5\lambda \times 0.05\lambda \times 0.05\lambda$ and $5\lambda \times 0.5\lambda \times 0.5\lambda$, respectively. The far field scattered fields are expressed as the radar cross section (RCS), and two RCSs (RCS_θ and RCS_ϕ) are considered for the θ and ϕ components of the electric field, where θ and ϕ are respectively the inclination angle measured from the z-axis and the azimuth angle in the spherical coordinate system. The scattered fields are measured in the xy plane ($\theta = \pi/2$). The method of moment is used in numerical simulations [19]. We compare the RCSs of bare PEC scatterer and cloaked one for different directions and polarizations of incoming waves.



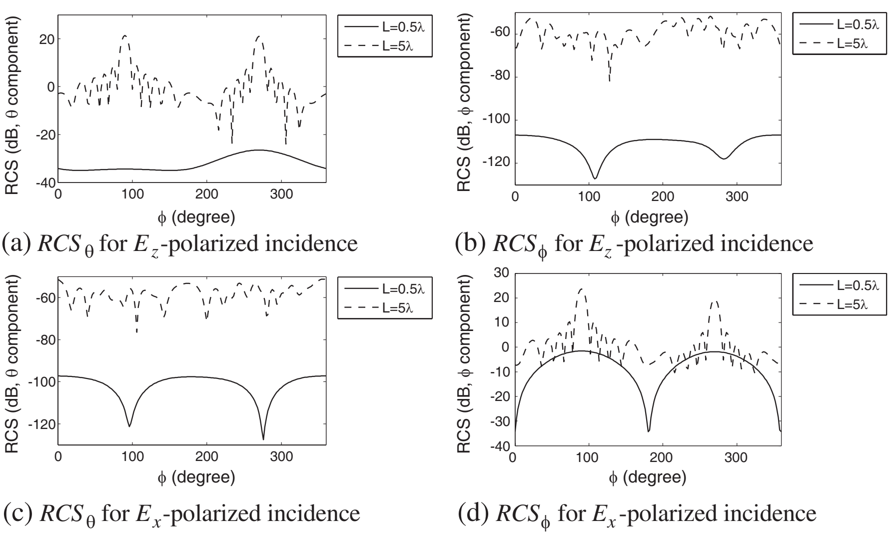
(a) RCS_θ for E_z-polarized incidence (b) RCS_ϕ for E_z-polarized incidence

(c) RCS_θ for E_y-polarized incidence (d) RCS_ϕ for E_y-polarized incidence

Figure 2. The radar cross sections for the x-axis incidence direction: two polarization of incoming electric field are considered. The scattering of bare PEC scatterer (the curve of $L = 5\lambda$) is large, whereas the scattering of the cloaked PEC scatterer (the curve of $L = 0.5\lambda$) is very small, independent of the polarization of the incoming electric field.

First, let the electromagnetic wave impinge along the x direction. The comparison between the RCSs of bare PEC scatterer and cloaked one for E_z and E_y polarization is shown in Fig. 2. We observe that the scattering is quite small for cloaked PEC scatterer for both polarizations. Consequently, for an arbitrary polarization, which can be expressed as a linear superposition of the aforementioned two polarizations, the scattering from the cloaked PEC scatterer is negligibly small. The results are expected since the scattering of the cloaked PEC cuboid in the physical space is equivalent to that of a very thin and elongated PEC cuboid embedded in air in the virtual space. In comparison, when the cloaking layer is absent, i.e., for the bare PEC scatterer case, the scattering is much larger.

Next, let the electromagnetic wave impinge along the y direction. Note that this incidence direction is perpendicular to the ideal direction (the x axis), and thus we do not expect cloaking. Now, we provide numerical simulations to verify it. The comparison between the RCSs of the bare PEC scatterer and the cloaked one for E_z and E_x polarizations is shown in Fig. 3. Even if the direction of wave is perpendicular to the longitudinal direction, E_z polarization still



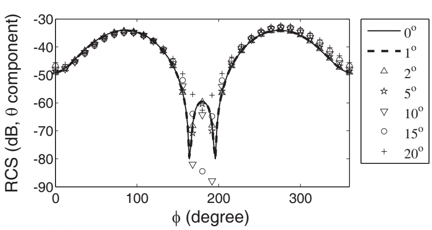
(a) RCS_θ for E_z-polarized incidence (b) RCS_ϕ for E_z-polarized incidence

(c) RCS_θ for E_x-polarized incidence (d) RCS_ϕ for E_x-polarized incidence

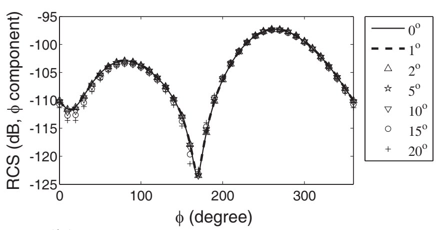
Figure 3. The radar cross sections for the y-axis incidence direction: two polarizations of incoming electric field are considered. The scattering of bare PEC scatterer (the curve of $L = 5\lambda$) is large, whereas the scattering of the cloaked PEC scatterer (the curve of $L = 0.5\lambda$) can be either large or small, depending on the polarization of the incoming electric field.

gives cloaking effect since the dimension of the PEC scatterer is small along the z-direction in the virtual space, thus inducing small electric current. On the other hand, E_x polarization performs poorly in cloaking because the electric fields along the longitudinal direction induce strong electric currents and result in large RCS. Once again, we observe large scattering when the cloaking layer is absent, i.e., for the bare PEC scatterer case. We also observe that despite poor cloaking performance of E_x polarization, it still performs better than the bare PEC.

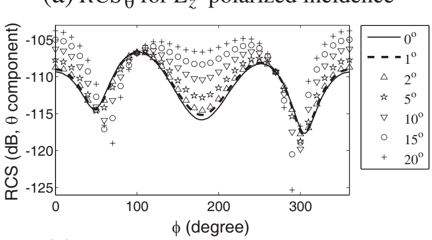
The above two numerical simulations show that the proposed cloaking device is indeed a directional cloaking one. It will be of great interest to test the performance of directional cloaking device for an impinging wave that is slightly off the ideal incidence direction. For this purpose, we let the direction of incidence wave be in the xy plane, with the angle of incidence with respect to the x-axis being 1, 2, 5, 10, 15, and 20 degree, respectively. Two polarizations are considered, E_z and E_{xy}, with the latter denoting an electric field that is in the xy plane and is perpendicular to the incidence direction. The RCSs of the cloaked PEC scatterer are displayed in Fig. 4. We see from Figs. 4(a) and (b) that the E_z polarized incidence is barely influenced by the tilted angle that the incidence direction makes with the x-axis and the scattering is negligibly small for all simulated cases. Figs. 4(c) and (d) display that the E_{xy} polarized incidence is prominently influenced by



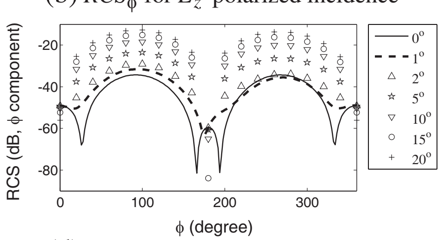
(a) RCS_θ for E_z-polarized incidence



(b) RCS_ϕ for E_z-polarized incidence



(c) RCS_θ for E_{xy}-polarized incidence



(d) RCS_ϕ for E_{xy}-polarized incidence

Figure 4. The radar cross sections of the cloaked PEC scatterer under oblique incidences: two polarizations of incoming electric field are considered.

the incidence angle. For this polarization, the induced current on the surface of PEC scatter is mainly parallel to the xy plane. Thus, there is nearly no scattering in the θ direction, as shown in Fig. 4(c), and there is dominant scattering in the ϕ direction, as shown in Fig. 4(d). We see that the RCSs are smaller than $-20\,\mathrm{dB}$ for incidence angles no larger than 10 degree, which means the directional cloaking device is practically useful since it tolerates a large range of tilted angles.

4. DISCUSSION AND CONCLUSION

A three-dimensional directional cloak for an arbitrary polarization of incoming electromagnetic wave is proposed in this paper. Physical justification in terms of induced current is given in the virtual space. The performance of the cloak under different polarizations of incoming waves is numerically investigated, and the case when the direction of incoming wave is perturbed off the ideal direction is also quantitatively studied. Numerical simulations show that the directional cloaking device is practically useful since it tolerates a large range of tilted angles of incoming waves.

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