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Dual equilibrium in a finite aspect ratio tokamak

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ABSTRACT

A new approach to high pressure magnetically-confined plasmas is necessary to design efficient fusion devices. This Letter presents a new sort of equilibrium combining two solutions of the Grad–Shafranov equation, which describes the magnetohydrodynamic equilibrium in toroidal geometry. The outer equilibrium is paramagnetic and confines the inner equilibrium, whose strong diamagnetism permits to balance large pressure gradients. The existence of both equilibria in the same volume yields a dual equilibrium structure. This combination improves free-boundary mode stability.

The most promising candidate to a large-scale fusion reactor is the tokamak concept, a closed magnetic topology confining a hot ionized gas or plasma, where electrons and ions are not bound together due to energetic collisions. To reduce particle loss, a strong toroidal magnetic field B_ϕ (ϕ denotes the toroidal axisymmetric direction) is used and effectively locks both charged species on magnetic field lines. This results in relative thermal insulation. However, turbulence and collisions between particles degrade confinement. Whilst the plasma core is hot, the edge remains relatively cold and a pressure gradient exists across the plasma section. In order to obtain a magnetohydrodynamic (MHD) equilibrium, a toroidal current density J_ϕ runs inside the plasma and generates a poloidal field B_P and the resulting inward Lorentz force balances the pressure gradient. While the toroidal field B_ϕ is the main cost of the reactor, it does not play any role in the macroscopic MHD equilibrium. However, it does limit the maximum value of J_ϕ [1], in turn controlling the maximum allowable pressure. As a consequence, the fusion power follows the scaling law given in Eq. (1),

$$P_{\text{fusion}} \propto \langle\beta\rangle^2 B^4 a^3 A. \tag{1}$$

a is the plasma minor radius, R is the major radius and A is the aspect ratio given by R/a. B is the total field inside the plasma and β measures the efficiency of kinetic pressure confinement by magnetic fields, i.e.

$$\beta = 2\mu_0 \frac{p}{B^2} \quad \text{and} \quad \langle\beta\rangle = 2\mu_0 \left\langle \frac{p}{B^2} \right\rangle, \tag{2}$$

where $\langle\cdot\rangle$ denotes volume average quantities and p the plasma kinetic pressure. To obtain an attractive fusion reactor design, Eq. (1) shows that β has to be maximized, while reactor costs force lower B_ϕ. Unfortunately, the Troyon limit [1] restricts the allowable plasma $\langle\beta\rangle$ to a few percents. Beyond a critical value β_c, MHD disturbances, or modes, perturb the axisymmetry of the plasma, leading to loss in confinement and, ultimately, plasma disruptions. The normal β, defined by

$$\beta_N = \frac{\langle\beta\rangle(\%)a(m)B_\phi(T)}{I_p(MA)} \tag{3}$$

is a relative measurement of plasma stability. Here, I_P is the total toroidal plasma current. Instabilities typically occur for β_N above 2.5 or 3 (the Troyon limit). This requirement is found to be quite robust in any experiment running with conventional current profiles. However reactor economics requires pressures larger than presently achievable in conventional tokamaks. Previous research has demonstrated that high pressure equilibria exist and are stable to fixed boundary modes $n = 1$, 2 and 3 [2], internal instabilities typically leading to confinement degradation or plasma disruptions. Unfortunately free-boundary modes, instabilities developing on the plasma outer boundary, remained a serious issue. Their stabilization would require a perfectly conducting vacuum vessel wall next to the plasma edge, a solution which is not realistic. This Letter presents a new type of equilibrium where such internal and external instabilities are suppressed, even at large plasma pressures.

The extended energy principle [3] assesses the nature of free-boundary mode stability by studying the perturbed plasma and vacuum energies caused by infinitesimal displacements. These displacements generate a total perturbed energy δW_{Total}, which is a

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volume integral over the whole plasma–vacuum system. This integral can be split into two volume integrals: one over the plasma, yielding the perturbed energy $\delta W_{\mathrm{Plasma}}$, and one over the whole vacuum region, yielding $\delta W_{\mathrm{Vacuum}}$. We will assume here that no currents run on the plasma edge. The system is stable if and only if the total perturbed energy is positive for *any* infinitesimal displacement. For displacements $\boldsymbol{\xi}_\perp$ locally perpendicular to the magnetic field, we can express the perturbed energy using the following form [4]:

$$\delta W_{\mathrm{Plasma}} = \frac{1}{2} \int\limits_{\mathrm{Plasma}} \left[\frac{\mathbf{Q}_\perp^2}{\mu_0} + \frac{B^2}{\mu_0} |\nabla \cdot \boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp \cdot \boldsymbol{\kappa}|^2 - 2[\boldsymbol{\xi}_\perp \cdot \nabla p][\boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp] - J_\parallel [\boldsymbol{\xi}_\perp \times \mathbf{b}] \cdot \mathbf{Q}_\perp \right] d\tau. \tag{4}$$

Here $\mathbf{Q}_\perp = \nabla \times (\boldsymbol{\xi}_\perp \times \mathbf{B})$ is the perturbed plasma magnetic field, $\mathbf{b}$ correspond to the magnetic field direction and the curvature of the magnetic field lines is given by $\boldsymbol{\kappa} = \mathbf{b} \cdot \nabla \mathbf{b}$. While it is evident from Eq. (4) that large pressures will lead to negative perturbed energies for some infinitesimal displacements, finite aspect ratio tokamaks also suffer from a handicapping side effect at high pressure, namely diamagnetism [5]. The loss in magnetic field compressibility (second term in Eq. (4)) is an inconvenient by-product of high pressure plasmas. Stability is sensitive to this term since the magnetic field strength has a quadratic contribution. While diamagnetic plasmas can be stable to fixed boundary modes, this side-effect is at the origin of the free-boundary mode instability in unity beta plasmas. Stabilizing such modes is *the* major problem of magnetic fusion confinement. One possible solution naturally comes to mind when looking at Eq. (4). If diamagnetism forces a negative value of the perturbed plasma energy, it seems possible to increase the overall plasma perturbed energy by adding an extra outer layer of plasma which carries a positive value of perturbed energy. This "rim" needs to contain enough positive perturbed energy so that the total volume integral of Eq. (4) becomes positive. For instance, positive perturbed energy could come from the magnetic field compressibility, pointing to a paramagnetic plasma rim.

Starting from a typical unity beta equilibrium [2,5], plasma stability can be restored by "grafting" a supplemental paramagnetic layer onto this equilibrium. We will use the term "dual equilibrium" herein to differentiate this combination from standard plasma equilibria. Fig. 1(a) shows the pressure and toroidal function profiles of a dual equilibrium. The toroidal function F corresponds to the amount of poloidal currents inside the plasma (these currents run in the vertical plane). Since they generate a local toroidal magnetic field, F also corresponds to the amount of total toroidal field at a radius R via the simple relation $F = RB_\phi$. Fig. 1 shows both profiles as a function of ψ, the normalized flux of the poloidal magnetic field on the plasma mid-plane. The central equilibrium has a value of F smaller than F_{edge}, the value of F at the plasma edge and it is de facto diamagnetic ($0 < \psi < 0.37$). The outer part of the equilibrium has a value of F larger than F_{edge} characterizing paramagnetism ($0.37 < \psi < 1$). Fig. 1(b) gives the profile of the safety factor. It is an important stability parameter which accounts for the ratio of the number of turns a field line executes in the toroidal (axisymmetric) direction while going once around the plasma cross-section. The safety factor should always be larger than 1. The dual equilibrium shown in Fig. 1 was computed using the free-boundary equilibrium code CUBE [6], with the following parameters. The plasma major radius R is 6 m, the plasma minor radius a is 2 m. The plasma elongation factor is 2, triangularity is 0.6 and squareness is 0.1. The toroidal field is 2.5 T at $R = 6$ m. These values are comparable to the ITER [7] design, except for the magnetic field which is half that of ITER. Fig. 2 shows the spatial distribution of this dual equilibrium. Its current profile shares

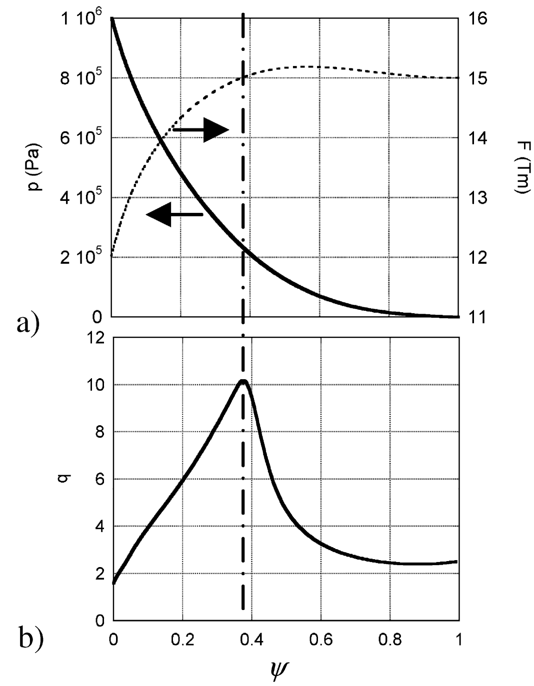

Fig. 1. (a) Pressure, toroidal function and (b) safety factor profiles versus the normalized poloidal field flux ψ. The dot-dash line marks the interface between the paramagnetic and diamagnetic equilibria.

strong similarities with experimental current holes [8,9] except for the distinctive asymmetry in the current wing heights (Fig. 2(a)). The two different p and F profiles (Fig. 2(a)) and the two different flux surface and current distributions (Fig. 2(b) and Fig. 2(c)) clearly show the paramagnetic edge and diamagnetic core. For this particular equilibrium, the peak β (at the location where pressure is maximum) is 100% and $\langle\beta\rangle$ is 12% for a total plasma current I_P of 13 MA. Thus the fusion power computed from Eq. (1) is similar to ITER ($\langle\beta\rangle \sim 3\%$, $B_\phi \sim 5$ T). The peak pressure is 1 MPa, also on the order of ITER's. The major advantage of the dual equilibrium is primarily in the lowering of the magnetic field, reducing significantly the cost of the device.

To finalize the viability of the dual equilibrium, its stability has been investigated numerically with the DCON code [10]. Fig. 3(a) shows high-n ballooning [11,12] as well as Mercier [13] stability. Fig. 3(b) focuses on the stability of the toroidal mode number $n = 1$ for both fixed and free boundary modes. We have included in this study all the poloidal harmonics spanning $m = -30$ to $m = 30$. Fig. 3(b) also shows free-boundary mode stability. While the criterion behavior changes near the interface location, fixed boundary mode stability is present in both equilibria. It is interesting to dwell on the free-boundary mode stability since this is the major issue such high pressure plasmas face. To understand the stabilizing mechanisms, we have moved the numerical last closed flux surface of the plasma, assuming vacuum beyond, from the plasma core all the way to the edge. As we cross the interface between both core and rim equilibria, the change in plasma energy evolution is clearly observable. The presence of the paramagnetic padding changes the evolution of the plasma energy. As the numerical last closed flux surface is moved outwards, the plasma energy rises rapidly. After we pass the optimum in F, located at $\psi = 0.56$, the increase in plasma energy slows down, demonstrating the strong influence of the magnetic field on free-boundary mode stability. The plasma energy at the edge is marginally positive. When the vacuum energy is added to the plasma energy, the total perturbed energy becomes positive for $\psi > 0.75$, guar-

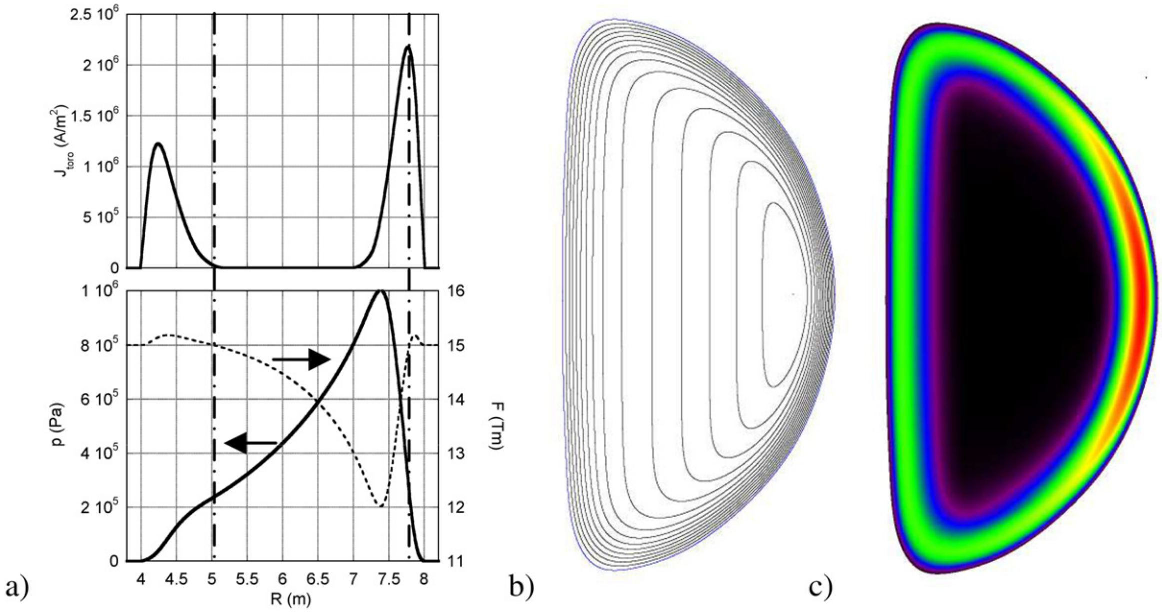

Fig. 2. (a) Current density, pressure and toroidal function profiles versus the major radius R. (b) Flux surface and (c) current density distributions in the (R, Z) plane. The vertical dot-dash lines mark the interface between both equilibria.

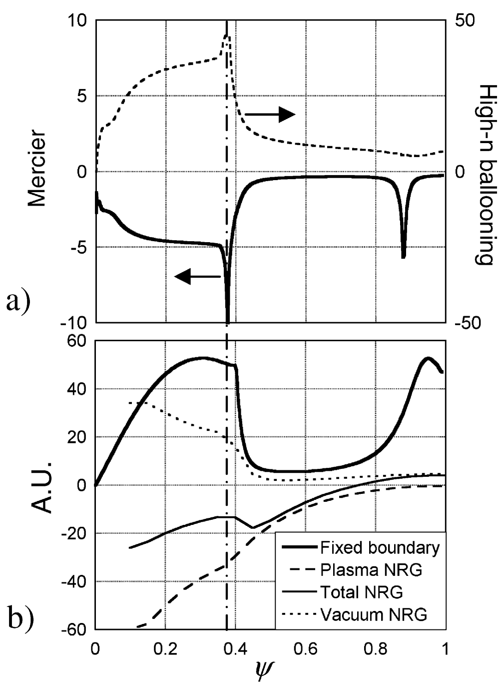

Fig. 3. (a) Mercier (stable when negative) and high-n ballooning (stable when positive) criteria. (b) Fixed boundary mode criterion (thick uninterrupted line) and free boundary energies (NRGs) for the toroidal mode number $n = 1$ (stable when positive).

antying free-boundary mode stability for the $n = 1$ external kink. This approach demonstrates the influence of the paramagnetic rim and asserts the plasma energy dependence with magnetic field. We have also found that fixed and free boundary modes for $n = 2$ and 3 are stable in DCON. This is not surprising since the β_N for this dual equilibrium is 4.6, a value yielding stable plasmas in some finite aspect ratio machines [14].

These numerical studies have demonstrated that the dual equilibrium is a configuration which combines successfully a high pressure diamagnetic equilibrium with a low pressure paramagnetic equilibrium. Numerous configurations can be obtained using this idea. The present Letter focused only on a single instance to highlight the interesting properties of such equilibria. However a thorough investigation needs undertaking to fully assess the experimental potentials of the dual equilibrium.

In conclusion, this Letter has presented a new type of unity β configuration called dual equilibrium. It is composed of a diamagnetic core, confining high plasma pressures, and an outer paramagnetic rim, stabilizing the free-boundary modes with toroidal mode numbers $n = 1$, 2 and 3. Hitherto stability results have to be carefully interpreted. The dual equilibrium has peculiar features such as large gradients, requiring high resolution of the computational grid, or flows, which tend to invalidate stability results. However the stability study presented in this Letter highlights the physical mechanisms reducing the impact of free boundary modes. Overall, the major asset of dual equilibria is the similarity they share with regular current holes [8,9]. Consequently, unity β plasmas seem attainable more easily when starting from a regular current hole configuration. Reaching unity β plasmas when starting from conventional current profiles has proven to be a rather complicated task [15]. Hence this new type of equilibrium shows great promise and the viability of economical low field fusion reactors now appears likely.

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