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ANALYSIS OF INJECTION TESTING OF GEOTHERMAL RESERVOIRS

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ABSTRACT

By introducing a similarity variable $r/\sqrt{t}$ a quasi-analytical method can be used to calculate the flow induced by the injection of cold water into a hot water or boiling geothermal reservoir. The results obtained are compared with those produced by the reservoir simulator SHAFT79 and show good agreement.

INTRODUCTION

When water is injected into a groundwater reservoir the pressure build-up follows the well known Theis curve. The Theis curve analysis assumes that the flow is isothermal and therefore the physical properties of the injected fluid are the same as the reservoir fluid. In a geothermal reservoir the fluid may be hot water at temperatures in the range 150°C - 250°C or a mixture of steam and water at even higher temperatures. In either case the physical properties of the reservoir fluid are significantly different from those of the injected fluid with a temperature of 10° - 100°C. For example the density, viscosity and compressibility of 100°C water are approximately 25%, 100%, and 50% different, respectively, from the 250°C values. For a two-phase mixture the contrast is even greater with order of magnitude differences between the liquid and two-phase values (see Grant, 1978 for example).

The present work compares two methods for analyzing the pressure build-up produced by the injection of water into a geothermal reservoir. The first procedure is the integrated finite difference method as implemented in the SHAFT79 program and the second is a quasi-analytical method based on the similarity variable technique used previously by one of the authors to investigate constant flow rate production tests (O'Sullivan, 1980).

The injection problem is a particularly difficult one for numerical simulators to handle since it involves the propagation of sharp fronts in the reservoir. There is a 'hydrodynamic' front where the fluid in the reservoir first starts to move significantly and, trailing behind it, there is a 'thermal front' where the fluid cools down to the injection temperature. Earlier work (Pruess and Schroeder, 1979) has demonstrated the ability of the SHAFT79 program to model injection tests. The purpose of the present work is to demonstrate the usefulness of the similarity method for analyzing injection tests and to confirm the accuracy of SHAFT79 by comparing results obtained with the two methods.

BASIC EQUATIONS

In radial coordinates, the equations governing the conservation of mass and energy in a geothermal system are:

$$\frac{\partial A_m}{\partial t} - \frac{1}{r}\frac{\partial Q_m}{\partial r} = 0 , \qquad (1)$$

and

$$\frac{\partial A_e}{\partial t} - \frac{1}{r}\frac{\partial Q_e}{\partial r} = 0 . \qquad (2)$$

Here the mass accumulation term A_m and energy accumulation term A_e are defined by:

$$A_m = \phi\rho , \qquad (3)$$

$$A_e = (1-\phi)\ \rho_r C_r T + \phi(\rho h - p), \qquad (4)$$

where ϕ is the porosity of the rock matrix, ρ_r is its density, and C_r its specific heat.

The mixture density ρ is given by

$$\rho = \rho_\ell S_\ell + \rho_v S_v \qquad (5)$$

and the mixture enthalpy h by

$$h = (\rho_\ell h_\ell S_\ell + \rho_v h_v S_v)/\rho . \qquad (6)$$

For the fluid S_v and S_ℓ are the vapor and liquid saturations, respectively; ρ_v and ρ_ℓ are densities; h_v and h_ℓ enthalpies and T is the temperature. The mass flux Q_m and energy flux Q_e, toward the origin (r = 0), are given by (after normalizing with a factor of 2π):

$$Q_m = \frac{k}{\nu_t} r \frac{\partial p}{\partial r} , \qquad (7)$$

and

$$Q_e = h_f Q_m + K_m r \frac{\partial T}{\partial r} \quad , \tag{8}$$

where k is the permeability of the rock matrix and K_m is the effective conductivity of the saturated porous medium. The formulas (7) and (8) assume that Darcy's law gives the volume flow rate of the liquid and vapor phases, V_ℓ and V_v, respectively, in terms of the pressure gradient as

$$V_\ell = \frac{kk_{r\ell}}{\mu_\ell} \frac{\partial p}{\partial r} \quad ,$$

$$V_v = \frac{kk_{rv}}{\mu_v} \frac{\partial p}{\partial r} \quad ,$$

where μ_ℓ and μ_v are viscosities. The permeability reduction factors $k_{r\ell}$ and k_{rv} are commonly assumed to have the form suggested by Corey (1954):

$$k_{r\ell} = S_\ell^{*4} \quad ,$$

$$k_{rv} = (1-S_\ell^*)^2 \cdot (1-S_\ell^{*2}) \quad ,$$

where $S_\ell^* = (S_\ell - S_{\ell r}) \,/\, (1-S_{\ell r} - S_{vr})$. Here $S_{\ell r}$ and S_{vr} are the irreducible liquid and vapor saturations often taken as 0.30 and 0.05, respectively, in geothermal reservoir modeling.

The total kinematic viscosity ν_t is defined by

$$\frac{1}{\nu_t} = \frac{k_{r\ell}}{\nu_\ell} + \frac{k_{rv}}{\nu_v} \tag{9}$$

and the flowing enthalpy h_f by

$$h_f = \nu_t \left(\frac{h_\ell k_{r\ell}}{\nu_\ell} + \frac{h_v k_{rv}}{\nu_v}\right) \quad . \tag{10}$$

A more complete discussion of the basic equations presented here is given in the work on reservoir modeling by Pruess and Schroeder (1979). The most important assumptions made are that Darcy's law applies, the fluid and rock are in local thermal equilibrium, and capillary pressure is negligible.

The SHAFT79 program can be used for solving problems in any coordinate system. It approximates the general three-dimensional equations corresponding to (1), (2), (7) and (8) by using an integrated finite difference form for all spatial derivatives and fully implicit differencing of the time derivatives. More details are included in the SHAFT78 user's manual (see Pruess, et al., 1979).

The equations above apply to the flow of any single-phase or two-phase fluid in a porous medium. To complete the formulation thermodynamic data defining ρ_ℓ, ρ_v, h_ℓ, h_v, ν_ℓ, $\bar{\nu}_v$ and T in terms of p and h (or ρ and mixture energy for SHAFT79) must be supplied. Suitable approximating formulas are given by O'Sullivan (1980).

Also boundary conditions and initial conditions are required.

The initial conditions are

$$Q_m = 0 \; , \quad p = p_o \; , \quad h = h_o \quad \text{at} \quad t = 0. \tag{11}$$

The boundary conditions at the well, approximated by r = 0 as for the Theis solution, are

$$Q_m \to Q_o \; , \; h \to h_1 \quad \text{as } r \to 0 \quad . \tag{12}$$

Far from the well the reservoir is unchanged from its initial state; that is

$$p \to p_o \; , \; h \to h_o \quad \text{as } r \to \infty \quad . \tag{13}$$

SIMILARITY METHOD

Following the standard similarity procedure, the variable $\eta = r/\sqrt{t}$ is introduced and then (1), (2), (7), and (8) can be rewritten as

$$\frac{dQ_m}{d\eta} + \frac{\eta^2}{2} \frac{dA_m}{d\eta} = 0 \; , \tag{14}$$

$$\frac{dQ_e}{d\eta} + \frac{\eta^2}{2} \frac{dA_m}{d\eta} = 0 \quad , \tag{15}$$

$$Q_m = T_m \, \eta \, \frac{dp}{d\eta} \quad , \tag{16}$$

$$Q_e = h_f Q_m + K_m \, \eta \, \frac{dT}{d\eta} \quad , \tag{17}$$

where $T_m = k/\nu_t$.

The idea of using the similarity technique for solving geothermal injection problems was also suggested by Tsang and Tsang (1978), but they considered single-phase flow only and used simplified thermodynamics in their formulation. They also assumed an approximate form for k/μ_ℓ in terms of η in order to obtain an analytic solution.

The boundary conditions and initial conditions combine to give the boundary conditions

$$Q_m \to Q_o, \; h \to h_1 \text{ as } \eta \to o$$

and

$$p \to p_o, \; h \to h_o \quad \text{as } \eta \to \infty \, .$$

The only difference between equations (14)-(17) and those derived by O'Sullivan (1980) for constant flow rate production problems is the inclusion of the conduction term in (17). The injection of cold water into a reservoir produces very large temperature gradients whereas the production of hot fluid out of a reservoir does not. Mathematically, the conduction term introduces considerable complexity. This can be best seen by rewriting (14) and (15) showing their dependence on Q_m, p, h and the enthalpy gradient, dh/dz, where $z = \log \eta$, more explicitly:

$$\frac{dQ_m}{dz} + \frac{\eta^2}{2}\left(\frac{\partial A_m}{\partial p}\frac{Q_m}{T_m} + \frac{\partial A_m}{\partial h}\frac{dh}{dz}\right) = 0, \quad (18)$$

$$h_f \frac{dQ_m}{dz} + Q_m \left(\frac{\partial h_f}{\partial p}\frac{Q_m}{T_m} + \frac{\partial h_f}{\partial h}\frac{dh}{dz}\right)$$

$$+ K_m \frac{d}{dz}\left(\frac{\partial T}{\partial p}\frac{Q_m}{T_m} + \frac{\partial T}{\partial h}\frac{dh}{dz}\right)$$

$$+ \frac{\eta^2}{2}\left(\frac{\partial A_e}{\partial p}\frac{Q_m}{T_m} + \frac{\partial A_e}{\partial h}\frac{dh}{dz}\right) = 0. \quad (19)$$

For the injection of cold water into a hot water reservoir the fluid remains single phase but for the injection of cold water into a two-phase or dry steam reservoir the spreading cold water heats up by extracting thermal energy from the rock and as it advances, the reservoir fluid condenses. In the two-phase region of flow, the term $\partial T/\partial h$ is zero and equation (19) reduces from a second order equation in h to first order, thus requiring only one boundary condition instead of two. This singular behavior of equation (19) in the two-phase region leads to considerable difficulty in its solution.

NUMERICAL PROCEDURE

For single-phase flow the numerical procedure follows that used by O'Sullivan (1980). A logarithmic scale is introduced by solving in terms of z where $z = \log \eta$. Then (18) is numerically integrated for Q_m starting from $Q_m = Q_o$ at $\eta = 0$ using estimated values for $p(\eta)$ and $h(\eta)$. Then (16) is used to update $p(\eta)$ using the newly calculated values for $Q_m(\eta)$ and the boundary condition $p \to p_o$ as $\eta \to \infty$. Finally, a difference approximation of (19) is solved, by inverting a tridiagonal matrix, using the latest values for $p(\eta)$ and $Q(\eta)$ and the boundary conditions $h \to h_1$ as $\eta \to o$ and $h \to h_o$ as $\eta \to \infty$. These steps are then repeated until convergence is obtained. For single-phase flow the process works very well with convergence to within a very small tolerance requiring only 5 - 10 iterations on crude intitial estimates. However, for two-phase flows the above process does not work. Instead, an inverse procedure is adopted where the position, say η_c, of the condensing front is specified and the corresponding injection rate Q_o required to produce it is calculated.

Once η_c is specified, a doubly iterative method is applied. For $\eta_c < \eta < \infty$ in the two-phase region (18), (16) and the two-phase version of (19) are solved iteratively, similar to above, with boundary conditions $Q_m(\eta_c) = Q_c$ and $p \to p_o$, $h \to h_o$ as $\eta \to \infty$. Q_c is adjusted until condensing just starts to occur at $\eta = \eta_c$. Using the calculated values of $p(\eta_c)$ and $h(\eta_c)$ the flow in the liquid region, $o < \eta < \eta_c$, is solved as above with Q_o adjusted until the value of $Q_m(\eta_c)$ matches the Q_c obtained from the two-phase region.

RESULTS

The reservoir parameters used here (see Table 1) were used by Sorey et al., (1979) and by O'Sullivan (1980) for a production well test problem. Two cases are reported here. The first is a single-phase problem for the injection of 100°C water into a 231°C reservoir. The second problem is for the injection of 100°C water into a reservoir of 233°C with an initial liquid saturation of 0.80. The pressure and temperature responses are shown in Figure 1 and the flow rate build-up and saturation profile are shown in Figure 2. As can be seen, the agreement with SHAFT79 results is good. Naturally, the discrete nature of the SHAFT79 simulator tends to smear the sharp thermal front but it gives a reasonable estimate of its position. There were some differences in the steam table approximations used between SHAFT79 and the similarity technique and therefore exact correspondence cannot be expected.

The results show a dual straight line pressure drop corresponding to the Theis curves for cold and hot water, respectively. Also, the very high effective compressibility of the two-phase fluid is evident from the much later build up of pressure.

CONCLUSIONS

The similarity method described here enables the ready analysis of single-phase geothermal injection tests. With more effort, because of the trial and error procedure required, injection tests for two-phase or dry steam reservoirs can also be analyzed. The limited results obtained so far indicate that the SHAFT79 program is a useful tool for analyzing such tests and is much more flexible than the similarity method in terms of the types of tests, such as multiple rate tests, to which it can be applied.

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TABLE 1. Reservoir Data.

Porosity	ϕ = 0.15
Rock density	ρ_r = 2000 kg/m^3
Rock specific heat	C_r = 1.0 kJ/kg.K
Permeability	k = 0.24 x 10^{-12}m^2
Initial pressure	p_o = 3.0 MPa
Initial enthalpy	h_o = 1.0 MJ/kg or 1.0158MJ/kg
Injection rate	Q_o = 0.4036 kg/s
Injection enthalpy	h_1 = 0.4154 MJ/kg

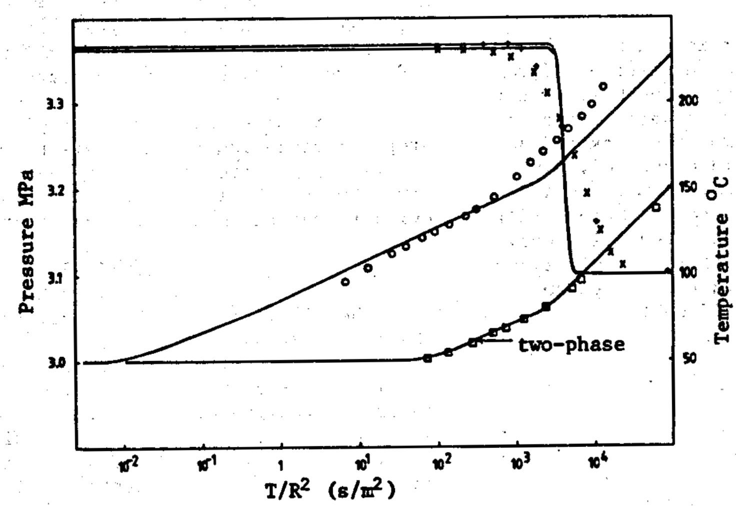

Figure 1. Pressure and temperature profiles. The temperature profiles for the hot-water and two-phase reservoirs are coincident after cooling commences. SHAFT79 results are shown as o for pressure and x for temperature for the hot-water reservoir and □ for pressure and + for temperature for the boiling reservoir

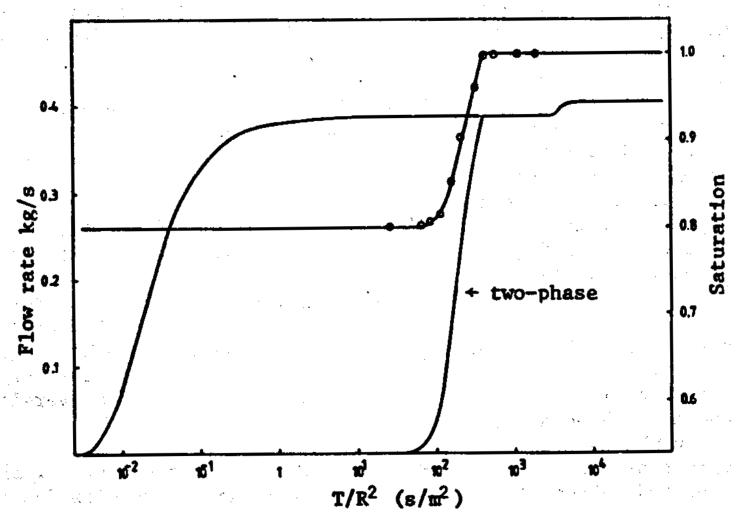

Figure 2. Flow rate and liquid saturation profiles. The flow rate curves coincide after cooling commences. SHAFT79 results are shown as o.

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