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Author
Feinberg, B.

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Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

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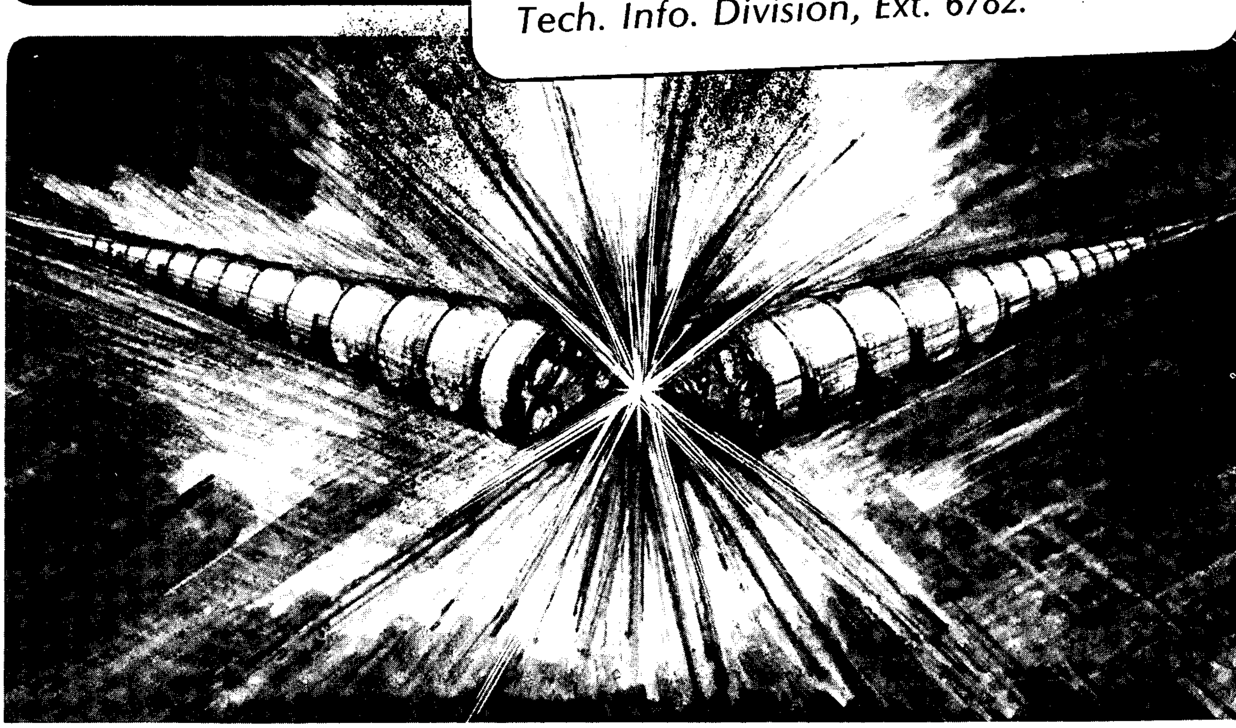

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B. Feinberg

Lawrence Berkeley Laboratory
University of California
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Abstract

A method is proposed to measure the dip in potential in the thermal-barrier region of a tandem mirror. Plasma waves at a frequency near the plasma frequency of the cold electron component of a two-temperature plasma are enhanced by the plasma Cerenkov effect, and can thus be used to scatter a laser beam. The measurements of cold plasma density as a function of position provide a sensitive measurement of the potential depression.

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A non-Maxwellian electron distribution, for example a two-temperature distribution, excites plasma waves. Fast electrons moving at a velocity larger than the phase velocity of electron plasma waves can increase the amplitude of these waves over the thermal level. This effect is analogous to the Cerenkov effect, where particles moving faster than the speed of light in a dielectric produce a wake of light waves. The enhanced spectrum of plasma waves can more easily scatter a laser beam, thereby enabling their detection. Since one resonance of the plasma wave is at approximately the plasma frequency of the cold component, a measurement of the cold electron density is possible, and thus, assuming a Boltzmann distribution, the depression in potential in the thermal-barrier region of a tandem mirror can be measured.

The existence of enhanced plasma density fluctuations due to nonthermal electrons has been shown previously. Pines and Bohm[1] discuss this plasma Cerenkov effect, while Perkins and Salpeter[2], in a generalization of Salpeter's calculation of density fluctuations[3], discuss the enhancement for both collisionless and collisional plasmas. The enhanced fluctuation spectra have been observed in radar backscattering by the ionosphere, where high temperature photo-electrons provide the enhancement.[4] As the radar frequency is varied to detect fluctuations with higher phase velocities the waves are no longer enhanced by the plasma Cerenkov effect and the amplitude of the fluctuation decreases to the thermal level, in accord with theory.

An interesting feature of the spectra of plasma fluctuations produced in a two-temperature plasma is the existence of a resonance near the plasma frequency of the cold electron component.[5] This resonance is enhanced by the presence of the hot electrons, but the frequency, to a first approximation, is determined only by the cold plasma component. Numerical work by

Kegel[6] detected the presence of this resonance, but its interpretation was not understood.

The plasma dispersion relation can be evaluated using a straightforward generalization of the standard method used for ion acoustic waves.[7] Here the two plasma components, one with a thermal velocity much lower than the wave phase velocity ($T_C/m \ll v_\phi^2$), and the other with $T_H/m \gg v_\phi^2$ refer to the cold and hot electrons, respectively. For the high frequency plasma waves considered the ions may be assumed to be infinitely massive, and thus do not contribute to dispersion. Following Krall and Trivelpiece[7], for $\mathrm{Im}(\omega) \ll \mathrm{Re}(\omega)$, the dielectric constant $\varepsilon = \varepsilon_r + i\varepsilon_i$ is given by

$$\varepsilon_r = 1 - \frac{\omega_{p_C}^2}{k^2} ⨍_{-\infty}^{\infty} \frac{\partial F_C/\partial u}{u-\omega_r/k}\, du - \frac{\omega_{p_H}^2}{k^2} ⨍_{-\infty}^{\infty} \frac{\partial F_H/\partial u}{u-\omega_r/k}\, du$$

$$\varepsilon_i = -\pi \left(\frac{\omega_{p_C}^2}{k^2} \frac{\partial F_C}{\partial u} + \frac{\omega_{p_H}^2}{k^2} \frac{\partial F_H}{\partial u} \right) \Bigg|_{u = \omega_r/k}$$

where ω_{p_C} (ω_{p_H}) is the plasma frequency of the cold (hot) electron component, ω_r is the real part of the wave frequency, k is the wave number (real), F(u) is the appropriate one dimensional distribution function, and $⨍$ refers to the Cauchy principal value. The solution is approximated by assuming separate Boltzmann distributions for the cold and hot electrons, and separately expanding the denominators of the integrals, where $\partial F_C/\partial u$ is appreciable only for $u \ll \omega_r/k$, and $\partial F_H/\partial u$ is appreciable only for $u \gg \omega_r/k$.[7] The dielectric constant is therefore given by

$$\varepsilon_r = 1 - \frac{\omega_{p_C}^2}{\omega_r^2}\left[1 + \frac{3k^2}{\omega_r^2}\ \frac{T_C}{m}\right] + \frac{1}{k^2\ \lambda_{D_H}^2}$$

$$\varepsilon_i = \sqrt{\frac{\pi}{2}}\ \frac{\omega_r}{k}\left[\left(\frac{m}{T_C}\right)^{\frac{1}{2}} \frac{1}{k^2\ \lambda_{D_C}^2}\ e^{-m\omega_r^2/2k^2T_C} + \left(\frac{m}{T_H}\right)^{\frac{1}{2}} \frac{1}{k^2\lambda_{D_H}^2}\right]$$

where λ_{D_C} (λ_{D_H}) is the cold (hot) component Debye length. The wave frequency is given by[7] $\omega_r = \omega_r\Big|_{\varepsilon_r = 0}$ and $\omega_i = -\varepsilon_i/(\partial\varepsilon_r/\partial\omega_r)$. Using the approximations $k^2\lambda_{D_C}^2 << 1$ and $k^2\lambda_{D_H}^2 >> 1$ one finds

$$\omega_r^2 \approx \omega_{p_C}^2\ \frac{\left[1 + 3k^2\lambda_{D_C}^2\right]}{\left[1 + 1/k^2\lambda_{D_H}^2\right]} \tag{1}$$

$$\frac{\omega_i}{\omega_r} \approx -\sqrt{\frac{\pi}{8}}\ \frac{1}{k^3\lambda_{D_C}^3}\left[\exp\left\{-\left(\frac{1}{2k^2\lambda_{D_C}^2} + \frac{3}{2}\right)\left(\frac{1}{1 + 1/k^2\lambda_{D_H}^2}\right)\right\} + \left(\frac{T_C}{T_H}\right)^{3/2} \frac{n_H}{n_C}\right] \tag{2}$$

Equation (1) therefore shows that the real part of the resonant wave frequency is equal to the cold component plasma frequency with some small correction terms.

The power spectrum, $I(\omega)$, near the resonance is given by[2]

$$I(\omega) = \frac{\omega_r^2 F(u)}{4k\left\{(\omega-\omega_r)^2 + \left[\frac{\pi}{2}\ \frac{\omega_r\omega_p^2}{k^2}\ \frac{\partial F}{\partial u}\right]^2\right\}}\Bigg|_{u = \omega_r/k} \tag{3}$$

This is normalized so that $\int I(\omega)d\omega = 1$ for completely randomly distributed electrons. Integrating the power spectrum about the resonant frequency yields[2]

$$I_p = \frac{\omega_r k}{2\omega_p^2} \left[\frac{F(u)}{\partial F/\partial u}\right]\Bigg|_{u = \omega_r/k} \tag{4}$$

If there is only one electron component, equation (4) yields the standard result, $I_p = 1/2\alpha^2$, where $\alpha^2 = 1/k^2\lambda_D^2$. When a hot electron component is present, and $k^2\lambda_{D_C}^2 << 1 << k^2\lambda_{D_H}^2$, then the distribution function $F(u) = F_C(u)+F_H(u)$ is greatly increased at $u = \omega_r/k$, while there is a much smaller change in $\partial F/\partial u$. It should be emphasized that this enhancement depends on the two-temperature electron distribution. If the cold electrons develop a tail and "fill in" the region between the cold distribution function and the resonant velocity (ω_r/k), $\partial F/\partial u$ will increase and the resonance will disappear. Another important point is that these plasma waves are longitudinal waves, and therefore the hot electron temparature (T_H) along the direction of interest, parallel to B, is the relevant temperature in the case of an anisotropic hot electron distribution. With a two-temperature distribution equation (4) yields

$$I_p \approx \frac{f}{2\alpha_C^2}\left[\frac{f\ \exp -\left\{K^2/2k^2\lambda_{D_C}^2\right\} + (1-f)\ (T_C/T_H)^{1/2}}{f\ \exp -\left\{K^2/2k^2\lambda_{D_C}^2\right\} + (1-f)(T_C/T_H)^{3/2}}\right]$$

$$\approx \frac{(1-f)}{2\ \alpha_C^2}\frac{(T_C/T_H)^{1/2}}{\left[e^{-\left(1/2k^2\lambda_{D_C}^2\right)} + \frac{(1-f)}{f}\left(\frac{T_C}{T_H}\right)^{3/2}\right]} \tag{5}$$

where $\alpha_C^2 = 1/k^2\lambda_{D_C}^2$, $f = n_C/(n_C + n_H)$, and $K^2 = \dfrac{1 + 3\,k^2\,\lambda_{D_C}^2}{1 + 1/k^2\,\lambda_{D_H}^2}$. Using Eq. (3) the half width is determined by letting

$$\omega - \omega_r = \frac{\pi}{2}\,\omega_r\,\frac{\omega_p^2}{k^2}\,\frac{\partial F}{\partial u}\Bigg|_{u = \omega_r/k}$$

The fractional peak width, which is determined by Landau damping in the collisionless case, is therefore

$$\frac{\Delta\omega}{\omega} = \frac{\omega-\omega_r}{\omega_r} \approx \frac{K}{f}\,\sqrt{\frac{\pi}{8}}\,\frac{1}{(k\,\lambda_{D_C})^3}\left[f\,\exp -\left\{K^2/2k^2\lambda_{D_C}^2\right\} + (1-f)(T_C/T_H)^{3/2}\right] \qquad (6)$$

making use of the approximation

$$k^2\lambda_{D_C}^2 << 1 << k^2\lambda_{D_H}^2 \quad .$$

Figure 1 shows the axial potential and density profiles expected in TMX-U.[8] If one looks at $z \leqslant 5.5$ m, there are only the hot and cold electron components. Letting $f \sim 20\,\%$, $n_C + n_H \sim 5 \times 10^{12}$ cm^{-3}, and $T_H/T_C \sim$ 50 keV/0.1 keV = 500, the enhancement over the thermal level is 300 times. Since forward scattering can easily detect the thermal level associated with the so-called ion component of the fluctuation spectrum, where $I_p \sim 1$,[9] this enhancement brings the electron plasma signal up to $I_p \sim 1.9$ for $k\lambda_{D_C} = 0.25$. These parameters give $k^2\lambda_{D_C}^2 \sim .06 << 1$ and $k^2\lambda_{D_H}^2 \sim 8 >> 1$, so that the approximations made in deriving these results are justified. The resonant frequency given by equation (1) (which includes the correction terms) is therefore $\omega_r = \omega_{p_C}$ (1.03), yielding an error of only 3%. The peak half width given by equation (6) is $\Delta\omega/\omega \sim 2\%$, making identification of the peak frequency quite simple. Since the cold electron density given by Fig. 1 varies by more than a factor of 10, the better than 10% accuracy of the measurement should suffice. For f = 20%, an enhancement of I_p such that $I_p \sim 0.5$ would be found for a temperature ratio as small as 125. For the expected $T_H \sim 50$ keV, this method could therefore be used with $T_C \leqslant 400$ eV for an accuracy of about 10% in frequency with $20\% \leqslant f \leqslant 80\%$. Equivalently, using Eqs. (5) and (6), the same accuracy would be obtainable for $T_C \leqslant$ 100 eV and $T_H \geqslant 12.5$ keV.

A technique has been described for using the plasma Cerenkov effect, which enhances electron plasma waves, to measure the density of the cold component of a two-temperature electron distribution. This diagnostic is directly applicable to the thermal-barrier region of a tandem mirror, such as TMX-U. Since the cold electron temperature is of the same order as (or less than) the thermal-barrier-potential dip, its density should provide a sensitive measurement of the potential.

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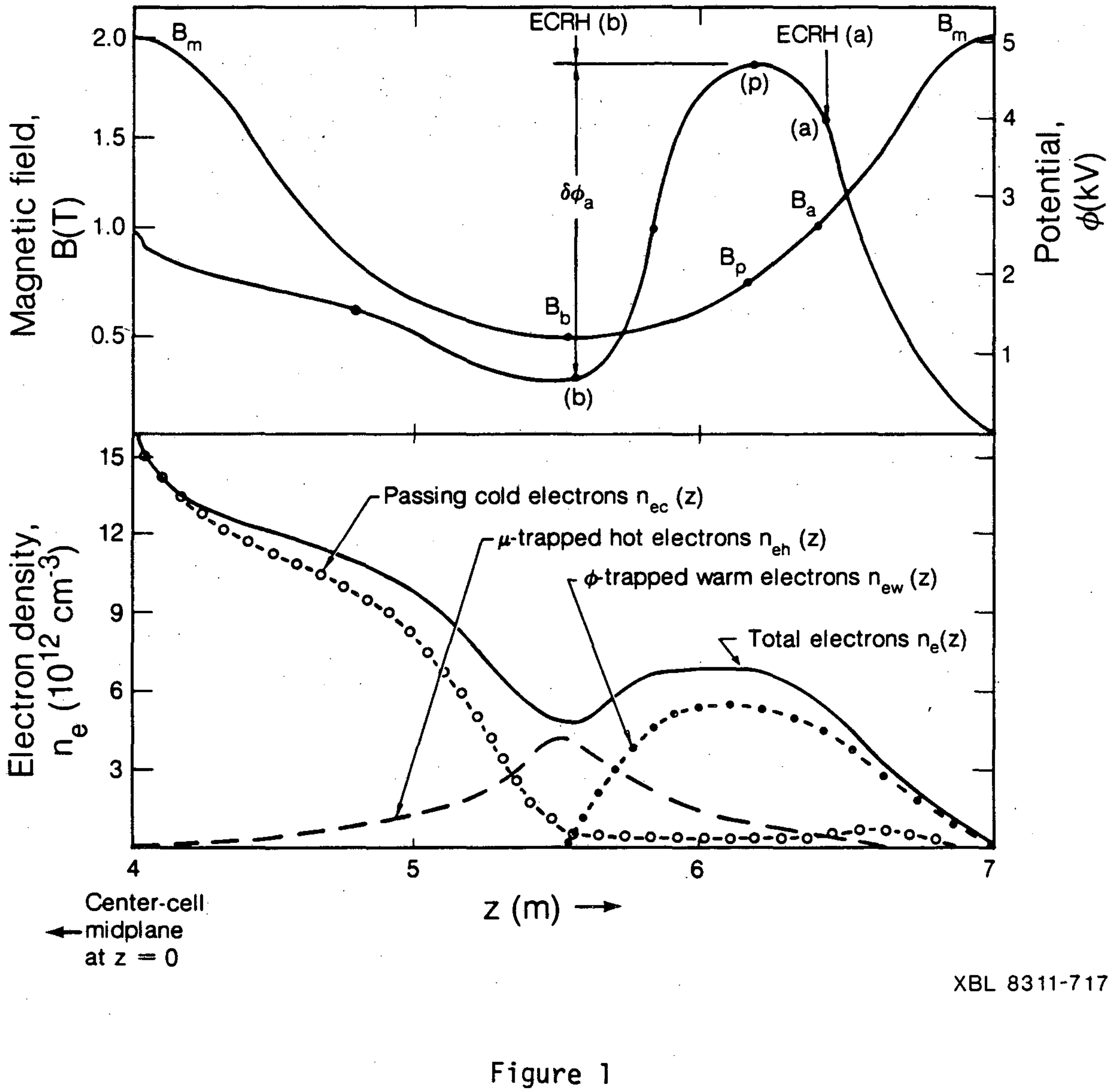
Figure Caption

Figure 1: Profiles of magnetic field, potential, and electron density in the end plugs of TMX Upgrade. The thermal-barrier is at about 5.5 m, and the proposed measurements would be made for $z < 5.5$ m. (From F. H. Coensgen, T. C. Simonen, A. K. Chargin, and B. G. Logan, LLNL Report LLL-PROP-172, 1980 (unpublished)).



Magnetic field, B(T)
Potential, φ(kV)
ECRH (b)
ECRH (a)
B_m
B_m
(p)
(a)
$\delta\phi_a$
B_a
B_p
B_b
(b)
2.0
1.5
1.0
0.5
5
4
3
2
1
Electron density, n_e (10^{12} cm^{-3})
15
12
9
6
3
Passing cold electrons n_{ec} (z)
μ-trapped hot electrons n_{eh} (z)
φ-trapped warm electrons n_{ew} (z)
Total electrons n_e(z)
4
5
6
7
Center-cell midplane at z = 0
z (m)
XBL 8311-717

Figure 1

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