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MONOPOLY AND MARKET ORGANIZATION

by

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The Justice Department brought a suit against the International Business Machine Corporation (IBM) in 1954 and won a decision which forced IBM to sell its machines, not rent them. There is no doubt that this decision injured IBM, easily the largest and most successful manufacturer of computers. Had it not been in its interests, IBM would not have rented its machines. But it is not clear that public welfare was increased by forcing IBM to sell its machines. An analysis of the problem sufficiently complete to make a judgment as to the correctness of the court's decision would be long and highly detailed. However, it might be more useful to pick out certain features of the IBM case and analyze these features in order to extend our knowledge of the general economic role of this kind of contractual arrangement. One feature of the IBM case is the rapid rate of technological obsolescence in the computer industry. Yet Xerox clearly prefers renting its 914 copier to selling it (the implicit discount rate is far below market rates of interest), and there has been little change in the 914, technological innovations in the copier industry coming in the form of alterations in the size and flexibility of copiers. Rental receipts from the 914 are overwhelmingly the principal source of revenue for Xerox, so the rental arrangement occurs in contexts in which technological innovations do not happen at the fast rate which is characteristic of the computer industry.

An imaginative and powerful line of analysis based upon neoclassical capital theory was applied to what is essentially the same problem by M.

I am deeply grateful to my colleague, S. N. Goldman, for helpful discussions on this topic.

Friedman.[1] The purpose of this note is to extend Friedman's analysis and examine the normative implications of the altered analysis. Friedman's analysis begins with the idea that if machines are sold, not rented, a new set of competitive firms will arise. These institutions will buy and sell used machines and rent new and used machines at competitive prices. The rentals arrangement keeps the machine brokers from coming into existence. The substantive analysis consists of comparing the course of the stock of machines and rental prices when the manufacturer rents with the course of the stock of machines and rental prices when the manufacturer sells machines to firms and machine brokers. At the purely formal level, the analysis has many parts and is quite complicated, although the different problems which arise are, from the point of view of capital theory, one problem with alternative assumptions. One set of assumptions leads to an analysis of the possibility of extending the life of machines by repairing old machines; a second set of assumptions leads to analysis of the inability of the machine brokers to predict the actions of the manufacturer. The first problem was dealt with by Friedman and is the problem discussed here.

The manufacturer has a monopoly on the manufacture of a machine which lasts $\hat{X}$ years and is as good as new until it falls apart completely. With a fixed demand curve, it is possible to calculate the profit maximizing stationary stock of machines under the selling and rentals arrangements. Under conditions of perfect foresight and perfect capital markets (the interest rate is assumed constant for computational simplicity) the size of the profit

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[1] M. Friedman, "Monopoly and the Second Hand Market," Price Theory: A Provisional Text (Chicago: Aldine Publishing Company, 1962).

maximizing stationary stock of machines provides a simple and valid index of the public's welfare. The smaller the stock of machines, the worse (in the public's view) the market organization. But if there is no possibility of extending the life of the machines through repair, the size of the net revenue maximizing stock of machines is the same for both the renter and the seller.

Let Q be the number of machines in existence and q the rate of manufacture. It is possible to go from one to the other if the stock is stationary since

$$(1) \qquad q = \frac{Q}{\hat{X}}$$

It is easy to calculate the optimum stock as a function of the optimum rate of manufacture using (1). The demand curve for the services of machines will depend upon the demand curve for the stock. $R(Q)$ is demand curve; $R' < 0$ and $\frac{d}{dQ}(MR) < 0$, and $MR > 0$ for $Q > 0$. The cost curve must be specified as a function of q, say $g(q)$, with $g' > 0$, $g'' > 0$. The revenue of each new machine is $R(Q)$ and if r is the interest rate, the renter will maximize

$$(2) \qquad V_r = \sum_{i=0}^{\hat{X}} \frac{R(Q)\,q}{(1+r)^i} - g(q)$$

Substituting (1) into (2) and maximizing (2) with respect to Q

$$(3) \qquad \frac{dV_r}{dQ} = \sum_{i=0}^{\hat{X}} \frac{R'Q+R}{(1+r)^i} - g'\left(\frac{Q}{\hat{X}}\right) = 0$$

rearranging terms, the optimal stock equates

$$(4) \qquad (MR) \sum_{i=0}^{\hat{X}} \frac{1}{(1+r)^i} = g'\left(\frac{Q}{\hat{X}}\right)$$

The manufacturer who sells must maximize

$$(5) \qquad V_s = Pq - g(q)$$

P being the price of a new machine. The price of a new machine under perfect competition will be

$$(6) \qquad P = \sum_{i=0}^{\hat{X}} \frac{R(Q)}{(1+r)^i}$$

So that V_s as a function of Q is

$$(7) \qquad V_s = \sum_{i=0}^{\hat{X}} \frac{R(Q)\left(\frac{Q}{\hat{X}}\right)}{(1+r)^i} - g\left(\frac{Q}{\hat{X}}\right)$$

Maximizing V_s with respect to Q will lead to equation (4). Thus, a perfectly competitive secondhand market will not affect the size of the stationary stock.

Following Professor Friedman, suppose that the life of a machine can be increased by repairing it and that the repairing can be obtained competitively. If Q machines are being repaired, let $\xi(Q)$ be the cost of repairing a single machine, and let $\xi' > 0$ so that the supply curve is upward sloping. The monopolist who rents will maximize

(8) $$V_r = \sum_{i=0}^{\hat{X}} \frac{R(Q)q}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{R(Q) - \xi(Q)q}{(1+r)^i} - g(q)$$

subject to $$q = \frac{Q}{X^*}$$

Maximizing (8) with respect to Q and the constraint,

(9) $$\frac{dV_r}{dQ} = \sum_{i=0}^{\hat{X}} \frac{R'Q + R}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{(R'Q - \xi'Q - \xi)}{(1+r)^i} - g'\left(\frac{Q}{X^*}\right) = 0$$

Rearranging terms,

(10) $$MR \sum_{i=0}^{X^*} \frac{1}{(1+r)^i} = \sum_{i=\hat{X}+1}^{X^*} \frac{(\xi'Q + \xi)}{(1+r)^i} + g'\left(\frac{Q}{X^*}\right)$$

There is no reason for X^* to be finite so that the stationary stock of machines might consist of machines which will be repaired indefinitely. The monopolist who sells will sell to competitive purchasers, each of whom will be insensitive to the effect each has upon the rental price or the cost of repairing a machine. If this is the case, each purchaser of a new machine will repair his machine forever if $R > \xi$. Hence, the stationary stock of machines will always be growing if new machines are being manufactured and $R > \xi$. If the seller finds it profitable to produce when the stock of machines is stationary, then $R \leq \xi$. The stock of machines which equates R and ξ must be larger than the stock which equates $\frac{d}{dQ}(RQ)$ and $\frac{d}{dQ}(\xi Q)$, and the stock of machines selected by the renter must be at least this small and might be still smaller. Hence, if the seller is producing machines once the stock is stationary, there must be more machines than the renter has chosen. With this larger stock of machines, the manufacturer may no longer be able to equate marginal revenue to marginal cost, but as long as price is greater than average cost, he will find it worthwhile to manufacture new machines.[2]

Without further qualification it is possible to state that if the monopolist who sells machines is manufacturing machines once the stock of machines is stationary, then the stock of machines is larger under the selling arrangement than under the renting arrangement. This substantiates Professor Friedman's conclusion that the stock of machines will definitely be larger under one of the arrangements than under the other (the text does not say which one), and that the value of the monopoly will be greater under the arrangement which has the smaller number of machines. This result, though formally correct, is not sufficiently general to be an economically correct analysis of the possibility of repair and the effect it will have upon the stock of machines in conjunction with the selling or renting market arrangement.

A different kind of increasing cost of repairing is as relevant to this problem as that implied by the upward rising supply curve of the repairing industry. The longer a machine has been in existence, the more it costs to get another year of life out of the machine. For a machine to last $\hat{X}+1$ years, it will cost ξ in repairing costs; to last $\hat{X}+2$ years,

[1]Provided average cost computes fixed costs as the highest value the fixed factors of the monopoly could earn being rented to other firms.

it will cost $(\xi_1 + \xi_2)$, and so on. The cost of extending the life of a machine will be represented by a (not necessarily finite) sequence $\{\xi_1, \xi_2, \ldots \xi_j, \ldots\}$, which has the property $\xi_{j+1} > \xi_j$. It would be easy to include the scale effect (the more machines are repaired, the more it costs to repair each machine) by considering sequences $\{\xi_1(Q), \ldots, \xi_j(Q), \ldots\}$ with the properties that $\frac{\partial \xi_j}{\partial Q} > 0$ for $j = 1, 2, \ldots$ and $\xi_{j+1} > \xi_j$ for $Q > 0$. However, it is easier to establish the qualitative nature of the combined effect indirectly. First, consider the stationary stock offered by the monopolist who rents. This monopolist will maximize V_r

$$V_r = \sum_{i=0}^{\hat{X}} \frac{R(Q)q}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{(R(Q) - \xi_i)q}{(1+r)^i} - g(q) \tag{11}$$

Of course, $g = \frac{Q}{X^*}$, and X^* is determined by implicit condition that $\xi_{X^*} = MR = R'Q + R$. Substituting (1) into (11),

$$V_r = \sum_{i=0}^{\hat{X}} \frac{R(Q)(\frac{Q}{X^*}}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{(R(Q) - \xi_i(\frac{Q}{X^*})}{(1+r)^i} - g(\frac{Q}{X^*}) \tag{12}$$

$$\frac{dV_r}{dQ} = \sum_{i=0}^{\hat{X}} \frac{MR}{(i+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{(MR - \xi_i)}{(1+r)^i} - g'(\frac{Q}{X^*}) = 0 \tag{13}$$

rearranging terms,

$$\sum_{i=0}^{X^*} \frac{MR}{(1+r)^i} = g'(\frac{Q}{X^*}) + \sum_{i=\hat{X}+1}^{X^*} \frac{\xi_i}{(1+r)^i} \tag{14}$$

The price of a new machine sold to a group of purchasers who face a perfectly competitive secondhand market and perfect capital markets will be:

$$(15) \qquad P = \sum_{i=0}^{\hat{X}} \frac{R(Q)}{(1+r)^i} + \sum_{i=\hat{X}+1}^{\overline{X}} \frac{(R - \xi_i)}{(1+r)^i}$$

$\overline{X}$ being determined by the condition that $\xi_{\overline{X}} = R$. The seller will again maximize (5),

$$V_s = P_q - g(q)$$

Substituting (15) into (5) and using the equality $q = \frac{Q}{\overline{X}}$ leads to (16):

$$(16) \qquad V_s = \sum_{i=0}^{\hat{X}} \frac{R(\frac{Q}{\overline{X}})}{(1+r)^i} + \sum_{i=\hat{X}+1}^{\overline{X}} \frac{(MR - \xi_i)}{(1+r)^i} - g\ (\frac{Q}{\overline{X}}) = 0$$

Rearranging terms once again,

$$(17) \qquad \sum_{i=0}^{\overline{X}} \frac{MR}{(1+r)^i} = \sum_{i=\hat{X}+1}^{\overline{X}} \frac{\xi_i}{(1+r)^i} + g'\ (\frac{Q}{\overline{X}})$$

It is possible to show that the stock of machines may be smaller under the selling arrangement than under the renting arrangement. To compare (17) and (14), write them in the form:

It is possible to show that the stock of machines may be smaller under the selling arrangement than under the renting arrangement. To compare (17) and (14), write them in the form

$$\text{Rental} \quad \sum_{i=0}^{\hat{X}} \frac{MR}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{MR - \xi_i}{(1+r)^i} = g'\left(\frac{Q}{X^*}\right)$$

$$\text{Sales} \quad \sum_{i=0}^{\hat{X}} \frac{MR}{(1+r)^i} + \sum_{i=\hat{X}+1}^{\overline{X}} \frac{MR - \xi_i}{(1+r)^i} = g'\left(\frac{Q}{\overline{X}}\right)$$

First, $\overline{X} > X^*$ for the same stationary value of Q, for $\xi_{\overline{X}} = R > MR = \xi_{X^*}$,[3] hence, $\overline{X} > X^*$ for the same stationary value of Q. Therefore, the cost curve of the production of new machines must be lower for the seller than for the renter for the same stationary value of Q. Labeling the marginal cost curve of the manufacture of new machines by the renter as $(g')^*$, and of the seller curve as $(g')^-$, their respective positions are shown in Figure 1, with stationary values of Q on the horizontal axis.

The net revenue curves (the graphs of the left hand sides of the above equations) are also easy to order since $\overline{X} > X^*$ for the same stationary values of Q. The first terms are identical, and the net revenue curve of the renter will lie above that of the seller if

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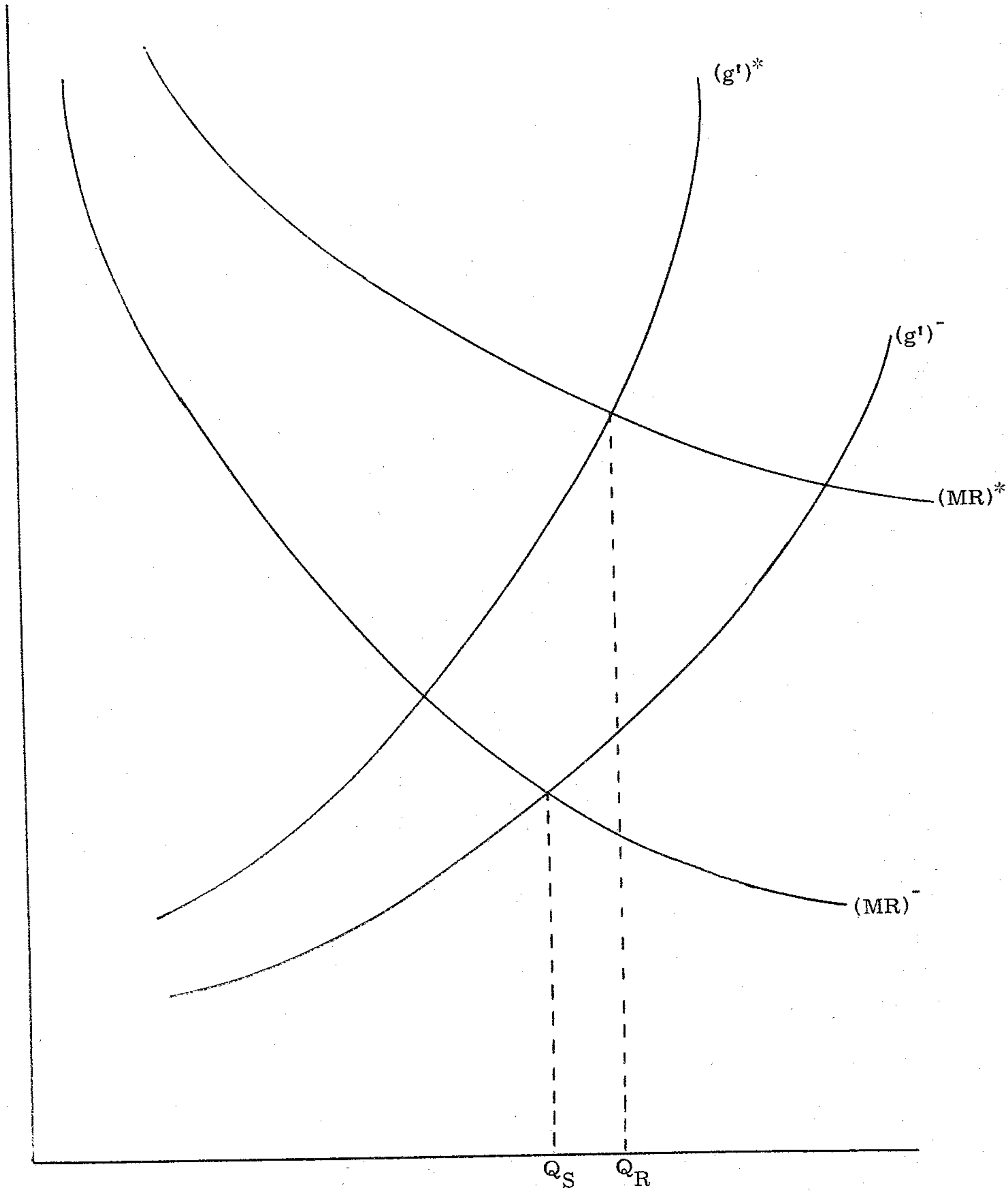
[3]More precisely, $\xi_{\overline{X}} \leq R > MR \geq \xi_{X^*}$, since ξ_j is not a continuous function of j; hence, all that can strictly be deduced is that $\overline{X} \geq X^*$; the possibility of equality seems unlikely enough so that explicit investigation may be avoided.

FIGURE 1



$$(18) \qquad \sum_{i=\hat{X}+1}^{X^*} \frac{MR - \xi_i}{(1+r)^i} - \sum_{i=\hat{X}+1}^{\overline{X}} \frac{MR - \xi_i}{(1+r)^i} > 0$$

But,

$$(19) \qquad \sum_{i=\hat{X}+1}^{X^*} \frac{MR - \xi_i}{(1+r)^i} - \sum_{i=\hat{X}+1}^{\overline{X}} \frac{MR - \xi_i}{(1+r)^i} = - \sum_{i=X^*+1}^{\overline{X}} \frac{MR - \xi_i}{(1+r)^i}$$

The R.H.S. of (19) is positive since $\xi_{\overline{X}} > \xi_{\overline{X}-1} > \xi_{\overline{X}-2} \ldots > \xi_{X^*} = MR$. The net marginal revenue curves for stationary values of Q are labeled as $(MR)^*$ for the renter and $(MR)^-$ for the seller and shown in Figure 1. The intersection of the net marginal revenue curve and of the marginal cost curve give the stationary stock for each market arrangement. As drawn in Figure 1, the stationary stock under the selling arrangement is smaller than under the rental arrangement; this is possible mathematically, and economically meaningful. However, certain possibilities are ruled out as being economically meaningless. Clearly, the rental monopoly must be of greater value than the selling monopoly, since the renter may do anything that the seller and the competitive purchasers do jointly. He will never do what they do together, even if by some quirk $Q_S = Q_R$; the purchasers of the machines will always extend the life of the machines too far.

Where goods are durable, it need not be true that the value of a monopoly lies solely in the ability of a monopolist to restrict output and raise price. The monopolist who rents may have more machines and larger profits than the monopolist who sells. However, to complete the analysis it is necessary to show than an increasing cost to lengthening the life of capital is economically relevant; certainly it is formally possible. Increasing the life of a machine through repair is simply a means of slowing down the rate of depreciation, and with smooth depreciation schemes it is much more plausible. Smooth depreciation rates are common in automobiles, houses, boats, and tractors, and most of the durable goods which are commonly owned by households. With a smooth rate of depreciation, a piece of equipment may be worth more at every instant of its life through careful maintenance and repair. With a smooth depreciation schema one might think that an increasing cost of the kind used in the one-hoss shay problem would imply that as the length of life of the equipment is increased, it costs more to slow down the rate of depreciation by the same amount. This is not the case. Slowing down the rate of depreciation by the same amount at every instant in the life of a machine is a perfect analogue to the $\{\xi_j\}$ sequence used in the above analysis, provided that the slower the rate of depreciation, the more it costs; the length of time that each piece of capital has been in existence need not be an issue at all with a smooth rate of depreciation. However, with a one-hoss-shay type of depreciation scheme, there is only one way in which to have an increasing cost of slowing down the rate of depreciation and that is by dating the year in which the repairs are made, and having them increase at later dates. The importance of this consideration can be illustrated with the case analyzed by Professor Friedman, the aluminum industry at a time when Alcoa was virtually the only manufacturer of primary aluminum, but there was lively competition in the recovery of scrapped aluminum and its subsequent refabrication.

The recovery and refabrication correspond to the repair operations. Without the scrapping, manufactured items with high aluminum content simply become worthless after a period of time. With the scrapping, some of the aluminum is recovered and recovered aluminum can then be refabricated into new products. An important factor which is easily overlooked is that there is an important distinction between the amount of aluminum recovered and the percentage of aluminum recovered. The increase in costs accompanying the increase in the total amount recovered is analogous to the rising supply curve of the repairing industry; the increase in costs accompanying the increase in the percentage of aluminum recovered corresponds to the increasing cost of supplying a year of machine life as the machine ages. An individual firm may not be sensitive to this distinction. But no matter what the rate of output of the primary manufacturer, the scrapping industry will only recover a fraction of the total output. An increase in the percentage of aluminum recovered is a decrease in the net rate of depreciation of the stock of aluminum. And significant increases in the percentage of aluminum recovered will be costly. At the scrapping end of the operation, recovery of those aluminum products lost at sea would be extremely costly, but a certain number of aluminum products are lost at sea. At the refabrication end, the processes used to recover scrap become more costly as they become more efficient in recovering the aluminum in the scrap.

Summary

(1) Friedman's theorem is only partially valid. It is true that with perfect foresight and perfect capital markets, the rentals arrangement

is more profitable than the selling arrangement when the life of machines can be extended through repair and servicing. However,

(2) It is not true that the more profitable arrangement is the arrangement with the smaller number of machines. It is quite possible that rentals arrangement will result in a larger stock of machines and a lower rentals price than the selling arrangement. Forcing a monopolist to sell his products will lower his net revenue, but it might lower public welfare.

APPENDIX A

It is easy to show that a set of sequences $\{\xi_j(Q)\}$ with the properties that $\frac{\partial \xi_i}{\partial Q} > 0$ and $\xi_{j+1} > \xi_j$ will also result in an ambiguity as to the size of the capital stock. The renter will maximize

(1) $$V_r = \sum_{i=0}^{\hat{X}} \frac{R(Q)q}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{\{R(Q) - \xi_j(Q)\}q}{(1+r)^i} - g(q)$$

Subject to $q = \frac{Q}{X^*}$ and $\xi_{X^*} + \frac{\partial \xi_{X^*}}{\partial Q} = MR$

Maximizing V_r after substituting in the first constraint,

$$\frac{\partial V_r}{\partial Q} = \sum_{i=0}^{\hat{X}} \frac{MR}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X} \frac{\{MR - (\frac{\partial \xi_j}{\partial Q})\,Q + \xi_j)\}}{(1+r)^i} - g'\,(\frac{Q}{X^*}) = 0$$

or

(2) $$\sum_{i=0}^{\hat{X}} \frac{MR}{(1+r)^i} + \sum_{i=\hat{X}+1}^{X^*} \frac{\{MR - MC\}}{(1+r)^i} = g'\,(\frac{Q}{X^*})$$

Using the fact that the price of a new machine will be

$$P = \sum_{i=0}^{\hat{X}} \frac{R(Q)}{(1+r)^i} + \sum_{i=\hat{X}+1}^{\bar{X}} \frac{\{R(Q) - \xi_j(Q)\}}{(1+r)^i}$$

and that $q = \frac{Q}{\bar{X}}$, $\xi_{\bar{X}} = R$, the seller will select as his optimum the stock which equates

$$(3) \qquad \sum_{i=0}^{\hat{X}} \frac{MR}{(1+r)^i} + \sum_{i=\hat{X}+1}^{\overline{X}} \frac{\{MR - MC\}}{(1+r)^i} = g'\left(\frac{Q}{\overline{X}}\right)$$

Using the fact that $\overline{X} > X^*$, it is possible to show that the seller s stock of machines may be smaller than the renter's stock.

APPENDIX B

The optimum stock selected by calculus methods is also the stock which results from choosing the optimum rate of investment given the initial value of Q. The renter will choose the rate of investment which maximizes:

(1) $$V_r = \int_0^{\infty} \{RQ - g(q)\} e^{-rt}$$

subject to the same demand curve as the discrete case, and the same cost function. The rate of depreciation is a positive constant so that in the absence of investment,

$$\frac{\dot{Q}}{Q} = -\mu, \quad \mu > 0$$

and in general,

(2) $$\dot{Q} = q - \mu Q$$

Of course, $Q(0) = Qo$. The problem will be to select the nonnegative production plan which maximizes

1. First, form $H(Q, q, \psi)$

(3) $$H(Q, q, \psi) = \{RQ - g(q) + \psi(Q)\} e^{-rt}$$

Pontryagin's maximum principle states that a path which is an optimum has the property of giving a maximum value to H with respect to nonnegative values of Q, q, and ψ. So, substituting (2) into (3) and maximizing with respect to q,

$$\frac{\partial H}{\partial Q} = \psi - g' = 0$$

Along a maximum,

(4) $$\psi = g'(q)$$

ψ, the shadow value of changing the capital stock will obey (5) along an optimum,

(5) $$\frac{d}{dt}(\psi e^{-rt}) = \frac{\partial H}{\partial Q}$$

or

$$\dot{\psi} = (r + \mu)\,\psi - \{R'Q + R\}$$

Together with (4) and (2), (5) forms a pair of autonomous differential equations whose stationary point is a saddle point. More explicitly,

$$\begin{cases} \dot{\psi} = (r + \delta)\,\psi - \{R'Q + R\} \\ \dot{Q} = \pi(\psi) - \mu Q, \qquad \pi' = \dfrac{1}{g''} \end{cases}$$

The roots of the linear approximation to this first order system have opposite signs, hence there is a unique path satisfying the nonnegative conditions and the condition that

$$\lim_{t \to 0} \psi e^{-rt} = 0$$

which simply says that any path which is an optimum cannot blow up too fast. Note that at the stationary value of Q, $\dot{Q} = 0$ and $\dot{\psi} = 0$, so

$$\begin{cases} q = \delta Q \\ \psi = \dfrac{MR}{(r + \mu)} \\ g'(q) = \psi \end{cases}$$

or

$$g'(\delta Q) = \frac{MR}{(r + \mu)}$$

which is the continuous analogue to (4). At the cost of computational complexity, all of the discrete results may be established for the continuous case.