Title
X-ray laser gain from Bragg reflection coupling in channeled relativistic beam systems

Journal
Applied Physics Letters, 52(11)

ISSN
00036951

Authors
Strauss, M.
Amendt, P.
Rostoker, N.
et al.

Publication Date
1988

DOI
10.1063/1.99256

Peer reviewed

X-ray laser gain from Bragg reflection coupling in channeled relativistic beam systems

M. Strauss
Physics Department, Nuclear Research Center-Negev, P. O. Box 9001, Beer-Sheva, Israel

P. Amendt[a] and N. Rostoker
Physics Department, University of California, Irvine, California 92717

A. Ron
Physics Department, Technion, Haifa, Israel

(Received 5 October 1987; accepted for publication 4 January 1988)

The application of distributed feedback by Bragg reflections in electron beam channeling x-ray lasers is investigated. Expressions for low threshold gain are derived for this cavity mirror structure in single crystals and are shown to have possible application in reducing beam high current requirements by many orders of magnitude.

Relativistic electrons propagating through axial and planar crystal channels may populate bound transverse energy eigenstates.[1-3] Spontaneous transitions between these states have been shown experimentally to yield narrow width, strongly forward-peaked, tunable x-ray radiation in excess of atomic bremsstrahlung radiation by an order of magnitude.[4-5] The possibility of using the channeling mechanism as a coherent x-ray source depends on future progress in three areas: (1) achieving significant population inversion, (2) increasing the coherence length for channeling particles, and (3) creating sufficient gain from induced emission. This letter addresses the third issue relating to the identification of an efficient mechanism for gain optimization in crystal channeling. Recent estimates suggest that even modest gains for short coherence length systems ($10\,\mu$) may require currents of the order of $\mathrm{MA/cm^2}$ range for energies near 10 MeV[6-8]; this requirement is in the limit of present capabilities of high current technology. Our aim here is to suggest a scheme to reduce the necessary currents by many orders of magnitude, thereby bringing one aspect of the channeling x-ray laser closer to experimental reach.

The concept of a distributed feedback laser in the optical range for atomic emitters has been proposed by Kogelnik and Sank[9] (KS) and was extended later on to the x-ray range.[10,11] The feedback mechanism is supplied by multiple Bragg reflections from periodic perturbations of the crystalline refractive index. Some advantages in using distributed feedback (DFB) include the intrinsic compactness and high degree of spectral selectivity available without the need for cavity mirrors.[9-11] Our purpose here is to apply DFB techniques to significantly reduce the spatial gain requirements in projected coherent x-ray channeling experiments.

The use of DFB x-ray lasers in channeled relativistic beam systems differs in two respects from atomic DFB lasers. First, the radiation in the forward direction emitted by the relativistic beam is Doppler up-shifted to the x-ray range, while the backward reflected radiation is Doppler down-shifted relative to the beam. Thus, amplification occurs only in the beam direction, leaving an asymmetric x-ray intensity distribution. This feature represents the main difference between our channeling DFB analysis and the atomic coupled wave theory of DFB lasers by SK.[9] Second, the channeling DFB has a significant advantage in radiation tunability. By adjusting the electron beam energy, the Doppler up-shifted radiation can be tuned with high precision ($\lesssim 1\%$) onto a line in the DFB mode spectrum near the Bragg reflection frequency.

We begin by characterizing the set of channeling transverse eigenstates as a two-level system with states $|1\rangle$ and $|2\rangle$, W and $\hbar\omega_0 = \epsilon_2 - \epsilon_1$ are the population and energy differences, respectively. The directions of beam channeling and Bragg reflections are taken in the z direction. The electric E and polarization P fields are taken in the transverse x direction and are defined in terms of forward and backward traveling waves:

$$E(z,t) = \epsilon_+(z,t)e^{-i\omega(t-z/c)} + \epsilon_-(z,t)e^{-i\omega(t+z/c)} + \text{c.c.},$$
$$P(z,t) = p_+(z,t)e^{-i\omega(t-z/c)} + p_-(z,t)e^{-i\omega(t+z/c)} + \text{c.c.},$$

where ω is the electromagnetic wave frequency, c is the speed of light, $\epsilon_\pm$ and $p_\pm$ are slowly varying complex amplitudes, and transverse field effects are not considered. The behavior of $p_\pm$ is readily determined from a density matrix approach and obeys the Bloch equation[7,12]:

$$\frac{\partial}{\partial t}p_\pm + v\frac{\partial}{\partial z}p_\pm = i\Delta_\pm p_\pm - i(1 \mp v/c)d^2 n_b W\epsilon_\pm/\hbar - \Gamma p_\pm, \quad (1)$$

where v is the channeling electron speed in the z direction, d is the electric dipole moment $e\langle 1|x|2\rangle$, n_b is the beam number density, $\hbar$ is Planck's constant, Γ is a phenomenological damping constant related to the channeling coherence length v/Γ, $\Delta_\pm = \omega(1 \mp v/c) - \omega_0$ is a detuning frequency, and the factor v/c represents a magnetic dipole interaction correction.[7] We note that $\Delta_\pm = 0$ defines the channeling resonance condition, giving a Doppler up-shifted frequency $\omega = \omega_0/(1 - v/c) \simeq 2\gamma^2\omega_0$ in the forward direction (Δ_+) and a reduced frequency $\omega = \omega_0/(1 + v/c) \simeq \omega_0/2$ in the backward direction (Δ_-). Typically $\hbar\omega_0$ is a few electron volts in the laboratory frame so that for the relativistic factor γ on the order of 20, $\hbar\omega$ is on the order of several keV. In this range of energies, ω may be chosen to closely match the first order Bragg frequency $\omega_B \equiv \pi c/a$,

[a] Present address: Sissa, Strada Costiera 11, Trieste, Italy.

where a is the periodic reflection plane spacing. Consequently, the channeling electron energy may be tuned to satisfy the Bragg reflection condition and induce distributed feedback in the channeling crystal.

Equation (1) must be supplemented by Maxwell's wave equation for the electric field:

$$\frac{\partial^2}{\delta z^2}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\mathbf{P} + c\nabla\times\mathbf{M} + \mathbf{J}\right), \quad (2)$$

where the polarization $\mathbf{P}$ and magnetization $\mathbf{M} = \mathbf{P}\times\mathbf{v}/c$ are due to the beam electrons.[7] In Eq. (2) $\mathbf{J}$ is the crystal-induced current of the bound electrons. We approximate $\mathbf{J} = n_e e \mathbf{v}_e$ and $(\partial/\partial t)\mathbf{J} = e^2 n_e \mathbf{E}/m_e$, where the spatially modulated atomic electron density $n_e(z) = n_0\cos(2\omega_B z/c)$ with n_0 on the order of the crystal bound electron density provides coupling between the forward and backward propagating waves.[9,10] In the slowly varying envelope approximation Eq. (2) can be written as

$$\frac{1}{c}\frac{\partial}{\partial t}\epsilon_\pm \pm \frac{\partial}{\partial z}\epsilon_\pm = \frac{2\pi i}{c\omega}\left[\omega^2\left(1 \mp \frac{v}{c}\right)p_\pm - \frac{n_0 e^2}{2m}\epsilon_\mp e^{\pm 2i(\omega_B - \omega)z/c}\right]. \quad (3)$$

Decoupling of Eqs. (1) and (3) can be accomplished as follows. In the limit of small coherence lengths v/Γ, i.e., $\Gamma \gg v[(\partial/\partial z)p_\pm]/p_\pm$, $[(\partial/\partial t)p_\pm]/p_\pm$ and Eq. (1) simplifies: $p_\pm \simeq i d n_b W(d\epsilon_\pm/\hbar)(1 \mp v/c)/(i\Delta_\pm - \Gamma)$. Near resonance, i.e., $\omega \sim 2\gamma^2\omega_0$ and $\Delta_+/\Gamma \ll 1$, giving

$$p_+ = -i d^2 n_b W \epsilon_+ (1 - v/c)\hbar\Gamma. \quad (4)$$

In this limit $\Delta_- \sim \omega$, $\Delta_- \gg \Gamma$ and in the case of low gain p_- can be ignored in Eq. (3). We now define the scalar gain $g = 2\pi\omega d_1^2 n_b W/\hbar c\Gamma$, where $d_1 = d(1 - v/c)$. Substituting Eq. (4) in Eq. (3) and redefining $\epsilon_\pm$ as $\epsilon_\pm \times \exp[\pm iz(\omega_B - \omega)/c]$ we obtain

$$\pm\frac{\partial}{\partial z}\epsilon_\pm + \frac{1}{c}\frac{\partial}{\partial t}\epsilon_\pm + i\delta\epsilon_\pm + i\kappa\epsilon_\mp = g_\pm\epsilon_\pm, \quad (5)$$

where $g_+ = g$, $g_- = 0$, $\kappa = \pi n_0 e^2/c m_e\omega$, and $\delta = (\omega_B - \omega)/c$.

We now find steady-state solutions appropriate for the system at threshold.[9] Using Eq. (5),

$$\frac{d}{dz}\epsilon_+ - (g_+ - i\delta)\epsilon_+ + i\kappa\epsilon_- = 0, \quad (6)$$

$$-\frac{d}{dz}\epsilon_- - (g_- - i\delta)\epsilon_- + i\kappa\epsilon_+ = 0, \quad (7)$$

where g is identified as a threshold gain. To include radiation losses in Eqs. (6) and (7) $g_\pm \to g_\pm - g_{\rm los}$, where $g_{\rm los}$ is the radiation loss factor, and for simplicity is ignored in the following. Notice that Eqs. (6) and (7) differ from the corresponding equations of KS due to the fact that beam electrons produce gain only in the forward direction. The anisotropy $(g_+ \neq g_-)$ forces p_- to be negligibly small.

The coupled wave Eqs. (6) and (7) describe the spatial variation of transmitted and reflected wave amplitudes in a beam channeling DFB medium. For a slab of length L centered at $z = 0$, the accompanying boundary conditions read: $\epsilon_+(-L/2) = \epsilon_-(L/2) = 0$ and no external radiation sources are assumed. The corresponding eigenvalue solutions to Eqs. (6) and (7) are found directly:

$$\epsilon_+(z) = e^{gz/2}\sinh[\lambda(z + L/2)],$$
$$\epsilon_-(z) = \pm e^{gz/2}\sinh[\lambda(z - L/2)], \quad (8)$$

where $\lambda = [(g/2 - i\delta)^2 + \kappa^2]^{1/2}$ and the dispersion relation is

$$(\lambda - g/2 + i\delta) + (\lambda + g/2 - i\delta)e^{-2\lambda L} = 0. \quad (9)$$

The allowed resonance frequencies δ and threshold values g can be obtained from Eq. (9). A formal solution of Eq. (9) is

$$\lambda = \pm i\kappa\sinh(\lambda L), \quad (10)$$

$$g/2 - i\delta = \mp i\kappa\cosh(\lambda L). \quad (11)$$

Equation (10) determines λ for given κ and L. Substitution of λ into Eq. (11) and equating real and imaginary parts yields the allowed δ and g.

Equations (10) and (11) are transcendental equations requiring numerical solution in general.[9] Approximate formulas can be obtained in the limit of strong reflections: $(\kappa L)^2 \gg (gL)^2 + 1$ and $\lambda L \ll 1$. Upon expanding Eq. (10) in this limit and using the expression for λ we find for the first resonance:

$$\delta \simeq \kappa, \quad (12)$$

and the threshold gain condition g_t is

$$g_t \simeq 6/\kappa^2 L^3. \quad (13)$$

Typically $\kappa \sim \pi n_0 e^2/m_e c\omega_B$ is on the order of 5×10^3 cm^{-1} in a number of crystalline samples used in channeling studies, e.g., silicon, diamond, where n_0 is approximately the crystal bound electron density. For $L \sim 0.1$ cm, then $g_t = 2\times10^{-4}$ cm^{-1} and low threshold values can be obtained in beam channeled DFB techniques. For the case the gain g is larger than the threshold gain g_t the radiation fields $\epsilon_\pm$ increase with time as $\exp[(g - g_t)ct/2]$ in the linear range. Thus an amplification factor $(g - g_t)ct/2 \sim 1$ is obtained for a beam pulse duration of 50 ns in $L = 0.1$ cm for $g \sim 10^{-3}$ cm^{-1}. This result should be compared to the gain $gL \sim 1$ obtained in a one passage amplification (with no reflections), wherefore $L = 0.1$ cm, $g \sim 10$ cm^{-1}. In terms of beam current requirements the DFB mechanism in beam channeling has possible application in reducing current requirements by many orders of magnitude.

In the present treatment we have neglected radiation losses ($g_{\rm los}$). It is possible making use of Borman anomalous transmission[13-16] that x-ray losses can be negligibly small. In an actual experiment the requirement of the Bragg condition would generate standing waves with nodes on the atomic sites, so that the condition for the Borman effect is fulfilled. Threshold conditions of the combined effects of the DFB mechanism and the Borman effect require further study.

The authors acknowledge useful discussions with Professor Amnon Fisher.

[1]J. U. Andersen, E. Bonderup, and R. H. Pantell, Ann. Rev. Nucl. Part. Sci. 33, 453 (1983).
[2]V. V. Beloshitsky and F. F. Komarov, Phys. Rep. 93, 117 (1982).
[3]G. Kurizki and J. K. McIver, Phys. Rev. B 32, 4358 (1985).
[4]R. K. Klein, J. O. Kephart, R. H. Pantell, H. Park, B. L. Berman, R. L. Swent, S. Datz, and R. W. Fearick, Phys. Rev. B 31, 68 (1985).

[5]B. L. Berman, B. A. Dahling, S. Datz, J. O. Kephart, R. K. Klein, R. H. Pantell, and H. Park, Nucl. Instrum. Methods B **10/11**, 611 (1985).
[6]V. V. Beloshitsky and M. A. Kumakhov, Phys. Lett. A **69**, 247 (1978).
[7]G. Kurizki, M. Strauss, J. Oreg, and N. Rostoker, Phys. Rev. A **35**, 3424 (1987).
[8]Y. H. Ohtsuki, Nucl. Instrum. Methods B **2**, 80 (1984).
[9]H. Kogelnik and C. V. Shank, Appl. Phys. Lett. **18**, 152 (1971); J. Appl. Phys. **43**, 2327 (1972).
[10]A. Yariv, Appl. Phys. Lett. **25**, 105 (1974).
[11]R. A. Fisher, Appl. Phys. Lett. **24**, 598 (1974).
[12]M. Strauss, P. Amendt, H. U. Rahman, and N. Rostoker, Phys. Rev. Lett. **55**, 406 (1985).
[13]B. W. Batterman, Rev. Mod. Phys. **36**, 681 (1964).
[14]B. Borie, Acta Crystallogr. **21**, 470 (1966).
[15]E. J. Saccocio and A. Zajac, Phys. Rev. **139**, 225 (1965).
[16]J. P. Hannon and G. T. Trammell, Opt. Commun. **15**, 330 (1975).