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ABSTRACT

The physical and mathematical basis for the direct-interaction model of nuclear reactions is reviewed.

IMPLICATIONS OF THE DIRECT-INTERACTION MODEL FOR NUCLEAR STRUCTURE

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I. INTRODUCTION

The physical basis for the direct-interaction model of nuclear reactions was given by Serber ten years ago.[1] In the intervening time we have seen the scope and usefulness of this model greatly extended. At the same time several developments in technique of scattering theory have made it possible to put this model into a quantitative form. It is this quantitative form that I should particularly like to emphasize now.

By "direct-interaction model" one means (essentially) the attempt to describe the scattering of a particle by an atomic nucleus in terms of collisions (one at a time) of that particle with nuclear protons and neutrons. Furthermore, such binary collisions are considered as resulting from the same forces as cause scattering from a free proton or neutron. At high energies the scattering cross section from a bound nucleon is actually considered to be the same as that from a free nucleon.

The fundamental requirement for the correctness of the direct-interaction model is the condition that the interaction energy of the given particle with the nucleus be of the form

$$V = \sum_{i=1}^{A} V_i \quad . \tag{1}$$

Here V_i is the interaction energy of the particle with the *i*th nucleon when that nucleon is removed from the nucleus. Aside from the condition of Eq. (1), the practical applicability of the direct-interaction (or Serber) model depends upon the complexity of nuclear structure and upon the energy of the scattered particle. It is, of course, because of this dependence on nuclear structure that we can hope to use the model to learn something about nuclear properties. How one does this is the second point that I should like to describe. The third point perhaps worth mentioning is the possibility of using nuclear interaction to learn something about the forces between nucleons and "strange particles."

The Serber model has been sufficiently successful that one can feel some confidence in at least the approximate validity of Eq. (1). This makes it reasonable to assume that Eq. (1) is strictly correct and then to develop the model as completely as possible. As we shall argue, the model is susceptible of a much more quantitative development than has been made. Also, comparisons with experiment seem to have been less precise than is justifiable. In other words, the limits on the accuracy of the direct-interaction model raise quantitative questions to which we are only beginning to find some answers.

Before going further, we mention that the Serber model must be handled quite differently in different energy ranges. It is much simpler at high than at intermediate and low energies. The possible applicability of the model at low energies has been discussed by Brueckner and his collaborators.[2] Dr. Brueckner has just described this work, which incorporates the physical basis of the direct-interaction model into a dynamical description of nuclear structure.

II. THE DIRECT-INTERACTION MODEL FOR HIGH-ENERGY INTERACTIONS

Developments of technique[3,4,5] in the quantum mechanical theory of scattering have been important for handling the direct-interaction model. There are several reasons for this: The concise notation has helped us to develop an intuitive feeling for physical processes that might otherwise be lost in a mass of unessential detail. Again, there are many problems in quantum mechanics that are really simple but that are not easily handled by conventional perturbation methods. In this connection one might mention the conservation of probability, the description of a sequence of single events, and the treatment of many problems for which classical mechanics is almost valid. Such phenomena are relatively easily handled by the algebraic techniques of scattering theory.[3,4,5]

Also important for the development of the Serber model is the class of techniques introduced by Wick[6] and Placzek[7] to handle sums over many states of excitation of the scattering medium.[8] This permits one to express all quantities appearing in the scattering cross section in terms of averages over the ground-state nuclear wave function. Furthermore these averages appear as quantities having a direct physical interpretation. The most important of such averages are:

$\rho(\underset{\sim}{x}) = A\, P(\underset{\sim}{x})$ = density of nucleons in nucleus;[9]

$P_2(x, x') \equiv P(\underset{\sim}{x})\, P(\underset{\sim}{x}')\left[1 + G(\underset{\sim}{x} - \underset{\sim}{x}')\right]$

= joint probability of finding one nucleon at $\underset{\sim}{x}$ and another at $\underset{\sim}{x}'$;

$\mathcal{P}(p)$ = momentum distribution of nucleons. (2)

Here $P(\underset{\sim}{x})$ is the probability of finding a nucleon at $\underset{\sim}{x}$. The quantity $G(\underset{\sim}{x} - \underset{\sim}{x}')$, as defined above, is the so-called "pair correlation function." It provides a measure of the amount of short-range order in nuclei.

One must also be able to handle multiple interactions and the coherent interference of waves scattered from different neutrons and protons. This is accomplished by use of multiple-scattering theory.[10] For systematic evaluation of the quantities appearing in the multiple-scattering description one uses the Placzek-Wick[6,7] method. The application of this method within the framework of multiple-scattering theory has recently been discussed.[11]

We are now ready to piece together the direct-interaction model at high energies. First, we wish to describe the scattering of the given particle by a bound neutron or proton in terms of the scattering from a free neutron or proton. At high energies one may actually use the <u>scattered amplitude</u> $f_{free}(\theta)$ for free protons and neutrons even for bound nucleons. This has been called the "impulse approximation" by Chew.[12,13,4] The criterion for the validity of this approximation may be written in the form[12,13,4]

$$f_{bound} = f_{free}\left[1 + \mathcal{O}\left(\frac{V_{Av}^2}{\epsilon_0^2} \quad \frac{f_{free}}{\lambda\!\!\!^{-}}\right)\right] \quad . \qquad (3)$$

Here f_{bound} is the amplitude for scattering from a bound nucleon with a binding potential energy V_{Av}. ϵ_0 is the energy of the scattered particle, and $\lambda\!\!\!^{-}$ is its de Broglie wavelength.

It is important to be able to set $f_{bound} = f_{free}$, since this means that we may use observed free-nucleon cross sections in discussing nuclear scattering. When ϵ_0 is sufficiently large, the impulse approximation is valid.

In order to use observable free-nucleon scattering amplitudes we must also suppose the scattering mean free path in nuclear matter to be significantly larger than the particle wavelength λ; that is,

$$\lambda >> \lambda\!\!\!^{-} ,$$

$$\frac{1}{\lambda} \equiv \rho \sigma , \qquad (4)$$

where σ is the scattering cross section from a single nucleon and ρ is the nuclear density. Condition (4) assures us that the scattering will be "on the energy shell."

We henceforth assume that f_{free} may be used for scattering from a bound nucleon and drop the subscript "free" on f. Then the elastic scattering by the nucleus is described by the "optical model" potential. This potential is most simply expressed in momentum space:[10]

$$(\underset{\sim}{q}' \mid V_0 \mid \underset{\sim}{q}) = -\frac{2\pi}{\mu} \bar{f}(\hat{q}'\cdot\hat{q}) \left[1 + \Delta\right] (2\pi)^3 \int \rho(r) e^{-i(\underset{\sim}{q}'-\underset{\sim}{q})\cdot\underset{\sim}{r}} d^3r. \qquad (5)$$

Here $\underset{\sim}{q}$ and $\underset{\sim}{q}'$ are the momentum variables of the scattered particle and μ is the reduced mass for the scattered particle and a nucleon.[14] We have written the scattering amplitude f as a function of the cosine of the scattering angle θ. The quantity Δ is a correction term, depending on nuclear structure, which is discussed below.

Equation (5) is usually approximated by setting $\underset{\sim}{q}' = \underset{\sim}{q}$ and transforming V_0 to coordinate space. If we also neglect the dependence of f and Δ on q, there results

$$V_0(\underset{\sim}{r}) = -\frac{2\pi}{\mu} f(1) \left[1 + \Delta\right] \rho(\underset{\sim}{r}) . \qquad (6)$$

The appropriate Schrödinger equation for obtaining the elastic nuclear scattering is finally

$$\left[K + V_0\right] \psi = \epsilon \psi . \tag{7}$$

Here ϵ is the energy and K the kinetic energy operator of the scattered particle. When this particle has a spin there may also be a spin-orbit interaction in V_0.[10,15] The amplitude $\overline{f}$ is $\overline{f} \cong \frac{1}{A}\left[Z f_P + (A - Z) f_N\right]$ in terms of the amplitudes for protons and neutrons (Z is the atomic number of the nucleus).

The important point in connection with Eqs. (5) and (6) is that (when Δ is negligible) one may obtain the potential V_0 directly from observable cross sections from free nucleons.[16]

As we have heard discussed several times here, the density $\rho(\underset{\sim}{r})$ as deduced from nucleon and pion scattering (in the energy range from zero to a few hundred Mev) does not seem to agree very well with that obtained from electron scattering. If this discrepancy is real, this presents a very serious difficulty for the direct interaction model. In attempting to resolve this discrepancy, however, one must use the exact Eq. (5) rather than the approximate Eq. (6). When transformed to corrdinate space Eq. (5) is nonlocal, giving a potential of the form $(\underset{\sim}{r}' \mid V_0 \mid \underset{\sim}{r})$. These nonlocal effects tend to "smear out" the nuclear boundary, which is in the direction of removing the discrepancy with the electron scattering. Unfortunately, no quantitative study of this point has been made, however.

In first approximation the quantity Δ is[10]

$$\Delta = - i \frac{3}{2} \frac{\overline{f}(1) \left[\int_0^\infty G(r) dr \right]}{q r_0^3} , \tag{8}$$

where $G(r)$ is the "pair correlation function" of Eqs. (2), and r_o is defined by the condition

$$\frac{4\pi}{3} r_0^3 A = \text{nuclear volume}.$$

For a degenerate Fermi gas model of the nucleus we would expect

$$\int_0^\infty G(r) dr \simeq -\frac{1}{4} r_0 .$$

In this case we expect Δ to give a rather small contribution to V_0 for nucleons of energy $\epsilon >$ 100 Mev. For π mesons the values of Δ are plotted in Fig. 1. Again fairly small corrections are found. Actually, very little is known concerning G. It may be calculated, for example, within the framework of Brueckner's[2] theory of nuclear structure.[17] This value would not seem to greatly alter the above conclusions concerning the importance of Δ.

Using the impulse approximation, we may relate the differential cross section $\sigma_b(\theta)$ for scattering from a bound nucleon to that from a free nucleon, $\sigma_f(\theta)$. The relation is[11]

$$\sigma_b(\theta) = \sigma_f(\theta) \left\{ 1 + \rho \int G(r) e^{i\Delta \underset{\sim}{q} \cdot \underset{\sim}{r}} d^3r + \frac{1}{3}\frac{m}{M} \frac{K_{Av}}{\epsilon_0} + \mathcal{O}\left(\frac{1}{3}\frac{m}{M} \frac{K_{Av} \; V_{Av}^2}{\epsilon_0^3} \right) \right\} . \tag{9}$$

Here $\Delta \underset{\sim}{q}$ is the momentum transferred to the scattered particle and ρ is the density of nucleons in the nucleus, m is the mass of the scattered particle, M is the nucleon mass. K_{Av} and V_{Av} are the respective average kinetic and potential energies of the bound nucleon in the nucleus. (These quantities are expected to be about 30 Mev.) As before, ϵ_0 is the energy of the particle to be scattered. The last two terms are Placzek-Wick[6,7] corrections. The second term represents a contribution due to interference of waves scattered from neighboring nucleons. Experimental observation of the effect of this term would provide information concerning the pair-correlation function.

With the cross section from Eq. (9) one may use Goldberger's transport theory[18] to calculate the inelastic scattering (at high energies) from nuclei. Goldberger's theory has recently been derived directly from a quantum mechanical theory.[11] Quantum mechanical corrections to the classical Goldberger description are of relative order Δ (see Eq. (8)) when the condition of Eq. (4) is satisfied.

We have shown how to obtain a consistent, precise description of nuclear reactions at high energies, if the direct-interaction model is accepted. This includes both elastic and inelastic scattering. Also, first-order corrections to the most simple form of the theory are given. These corrections would seem to be rather small for $\epsilon_0 > 100$ Mev, unless nuclear structure effects (G(r), for example) are much more important than we think they are.

There are several reasons for a careful study of the direct-interaction model at high energies. First, it can provide information concerning the correctness of the basic assumption of Eq. (1). We may then hope to use the model to determine experimentally such quantities as $\rho(r)$, $G(r)$, $\mathcal{P}(p)$, etc. As mentioned above, it also provides a means for studying strange-particle interactions.

III. INTERMEDIATE-ENERGY REACTIONS

At intermediate energies (that is, at energies comparable to nuclear binding energies) the importance of simple direct-interaction effects is evidently a difficult question and at present poorly understood.[19] One sees here aspects of both the compound- (and statistical-) nucleus model and the direct-interaction model. For example, it appears that direct-interaction effects appear in both the energy and angular distribution of reaction products. Indeed, in a careful study of the reaction for $C^{12}(p, p')C^{12}$, Levinson and Banerjee[20] have obtained rather strong evidence for the applicability of a simple version of the Serber model even at intermediate energies.

IV. COMPARISON WITH EXPERIMENT AT HIGH ENERGIES

The most detailed study of inelastic scattering at high energies seems to be that by Bernardini, Booth, and Lindenbaum, who scattered protons of 300 to 400 Mev energy in emulsions.[21] The distribution in number, angle, and energy of the nucleons emitted from these reactions was studied and the result compared with the Goldberger transport theory.[18] The agreement with theory was very good and gives strong support for the usefulness of the Serber model.

More recently, there have been a number of interesting experiments[22] of a type that will undoubtedly prove to be quite important in connection with applications of the direct-interaction model to the study of nuclear structure. In these experiments inelastic cross sections associated with the excitation of specific nuclear levels have been measured.

The optical-model potential has been evaluated from Eq. (6) for π mesons, by use of the dispersion relations of Goldberger.[14,23] The result seems to be in agreement with the rather sparse experimental information concerning this.[24] There is possibly an experimental discrepancy for 1-Bev pions, however.[25]

Equation (6) may be used to obtain the optical-model potential for elastically scattered nucleons if one accepts a set of nucleon-nucleon scattering phase shifts. This has been done by Riesenfeld and Watson[15] and by Bethe.[26] In Fig. 2 we compare the real part of the potential with existing experimental knowledge of this quantity.[27]

Our survey has of necessity been rather hurried and far from exhaustive. The purpose has been, however, to argue first that there is very good experimental evidence for the usefulness of the Serber model. We then assert that the model is susceptible of a precise dynamical formulation which puts it on a quantitative basis. Finally, existing comparisons with experiment seem to be less precise than the model seems to warrant.

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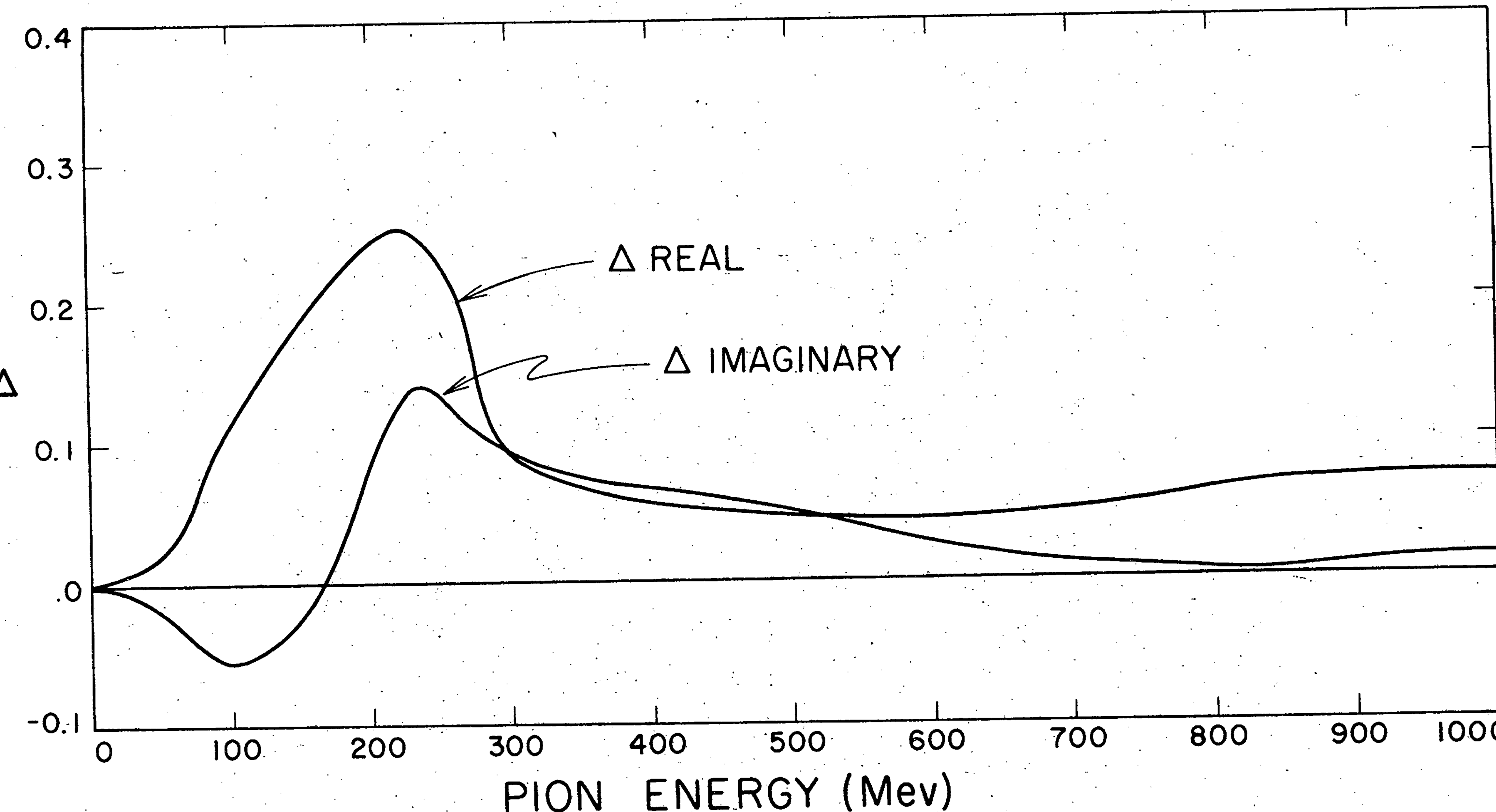
FIGURE CAPTIONS

Figure 1: The real and imaginary parts of Δ (Eq. (8)) are shown for pions. The function G used is that appropriate for a degenerate Fermi gas.

Figure 2: The negative of the real well depth for elastic scattering of nucleons is shown as a function of energy. The two dashed curves represent "limits" on the experimentally determined value. The solid curve is obtained from Eq. (6) by use of the Feshbach-Lomon phase shifts (Phys. Rev. 102, 891 (1956)).


0.4
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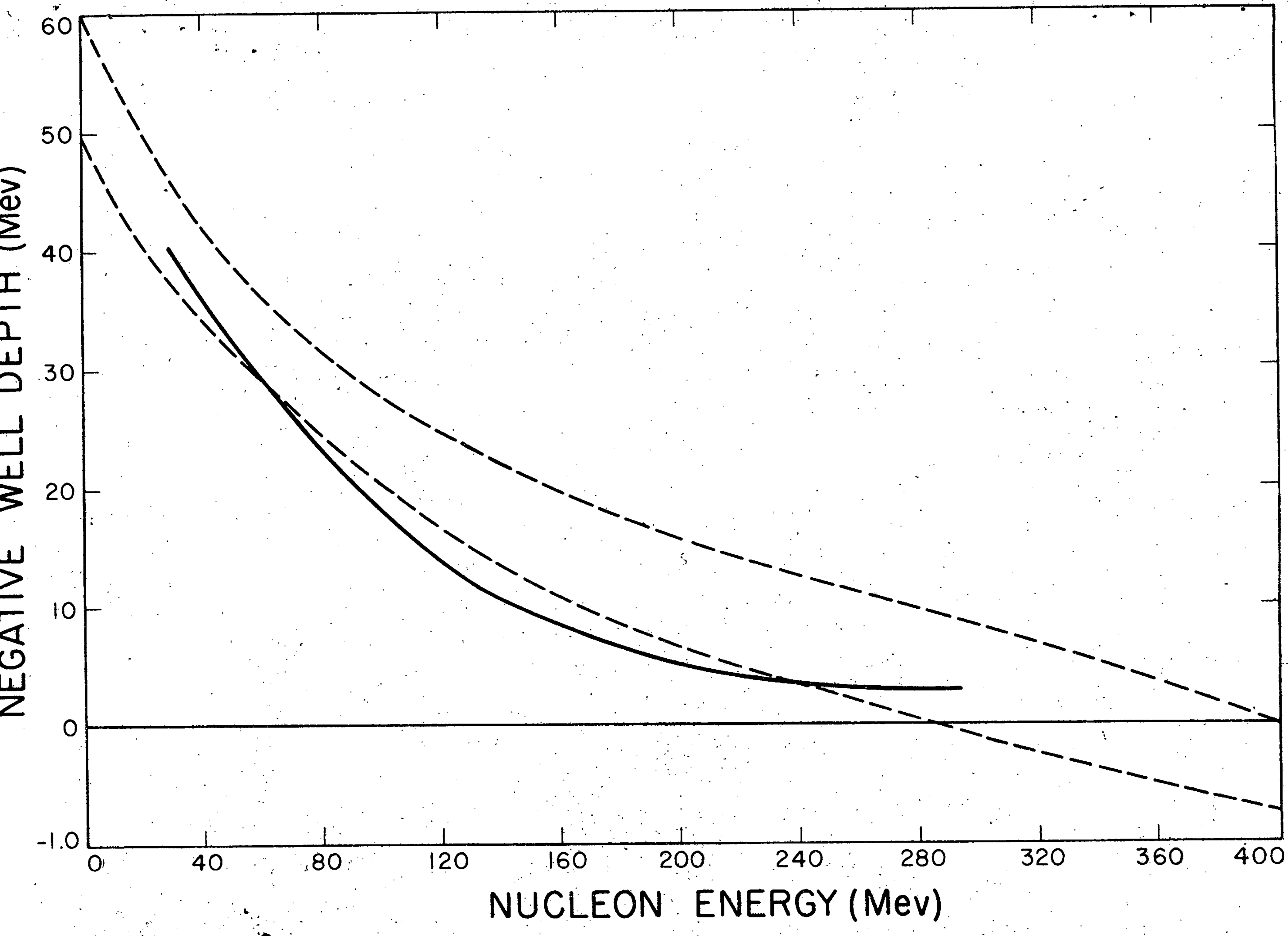
600
700
800
900
1000
PION ENERGY (Mev)

50,583-1


NEGATIVE WELL DEPTH (Mev)
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50
40
30
20
10
0
-1.0
0
40
80
120
160
200
240
280
320
360
400
NUCLEON ENERGY (Mev)

50,584-1