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Publication Date
2005-03-01

Rational Information Choice
in Financial Market Equilibrium*

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March 18, 2005

Abstract

Adding a stage of signal acquisition to the expected utility model shows that Bayesian updating results in a well defined law of demand for financial information when asset return distributions are conjugate priors to signals such as in the gamma-Poisson case. Signals have a positive marginal utility value that falls in their number if and only if investors are risk averse, asset markets large, and variance-mean ratios of asset returns high in fully revealing rational expectations equilibrium. Expected asset price increases in the number of signals so that expected excess return drops. The diminishing excess return prevents Bayesian investors from unbounded information demand even if signals are costless, unless the riskfree asset is removed. Signals mutually benefit homogeneous investors because revealing asset price permits updating so that a Pareto criterion judges competitive equilibrium as not sufficiently informative. However, asset price responses make incentives for signal acquisition dependent on portfolios so that welfare and distributional consequences become intricately linked when investors are heterogeneous.   JEL D81, D83, G14

The role and value of information in financial markets figures prominently in theoretical and empirical research. Many approaches nevertheless treat financial information as if it were exogenously available to investors. Literatures on learning and experimentation, on the other hand, analyze incentives for information acquisition but frequently treat the asset or commodity markets on which information is obtained in abstract terms and tend to disregard equilibrium price effects. This paper pays close attention to both rational incentives for information acquisition in financial markets and to the effects of market conditions and heterogeneous portfolio positions on incentives for information acquisition.

The approach pursues four main objectives. First, this paper aims to derive a rational law of demand demand for financial information based on the marginal utility benefit of signals in an expected utility framework. For this purpose, the paper draws on Raiffa and Schlaifer's (1961) decision model under conjugate prior distributions, which provides a natural extension of the expected utility model of portfolio choice. A second objective is to move beyond the abstract experimentation view of information acquisition and tie the marginal utility benefit of financial information to the specific asset market environment and investors' portfolio positions. A third objective is to account for the impact of information acquisition on an asset's expected excess return when evaluating information in rational utility terms. A fourth goal is to apply a Pareto criterion to the assessment of informational efficiency of rational expectations equilibrium (REE).

Whereas Wald's (1947) prominent experimentation paradigm gives rise to a law of demand for information in a dynamic setting of repeated sampling with risk neutral agents (Moscarini and Smith 2001), the Raiffa and Schlaifer (1961) decision framework lends itself to a well defined demand function for financial information in a standard Walrasian REE with risk averse investors. In contrast to conjectures of non-convexities in the value of information (e.g. Chade and Schlee 2002), the law of demand for financial information passes three intuitive litmus tests in this natural extension of the expected utility model: signals have a well defined and positive value in terms of marginal expected utility for risk averse investors; when positive, the marginal expected utility benefit of signals falls monotonically in the number of signals; information has no value for risk neutral investors because they do not expect a change to their portfolio composition by the Law of Iterated Expectations so that information is *ex ante* irrelevant for their expected consumption path.

The present Bayesian model of information choice in financial markets establishes that the marginal utility benefit of a signal is low if relatively few risky assets are in the market, or if investors are little risk averse, or if prior expectations of the mean-variance ratio of the asset return are relatively high so that

uncertainty matters little compared to expected returns. Information alters beliefs about the expected asset price. This changes the expected value of investors' initial portfolios and intricately affects their individual marginal utility value of information.

In REE, asset price serves a double role as asset allocator and information aggregator. Rational investors only value signals *ante notitias* (before realizations become known) if they anticipate to act on signal realizations *post notitias* (after realizations are revealed to them). Otherwise information has no rational value. However, by acting on information, investors leak information to all others since an asset price that plays an allocative role at least partly reveals a statistic of aggregate information. This makes other investors' beliefs about asset return less uncertain, raises demand and price of the risky asset, and diminishes the expected excess return—an effect known for both partially revealing REE (Easley and O'Hara 2004, Veldkamp 2004) and fully revealing REE (Muendler 2004). Easley, Hvidkjaer and O'Hara (2002) confirm the diminishing effect of public information on the expected excess return empirically. They find for a set of NYSE listed stocks between 1983 and 1998 that assets exhibit a lower excess return if public information matters relatively more for their valuation (so that trades are less frequently private-information based). The present paper shows for the benchmark case of fully revealing REE that this price impact of privately obtained but publicly transmitted information prevents rational Bayesian investors from unbounded information acquisition even when signals cost nothing. Only if the riskless bond turns useless (destroys the principal with certainty) does unbounded information acquisition become rational for costless signals and investors turn a risky asset into the riskless bond by removing all uncertainty with infinitely many signals.

Informational efficiency of REE is often defined as the expected deviation of market price from the benchmark price that incorporates a statistic of all investors' beliefs (Fama 1970) or as the precision of price in that statistic (e.g. Wang 1993). Endogenous information acquisition in the present framework gives rise to a natural Pareto criterion based on individual *ante notitias* utilities, similar to the Samuelson (1954) condition for public goods. If information is valuable under given market conditions, a social planner wants more information to be allocated to homogeneous investors than markets provide, as is the case with public goods in other economic contexts. Individual investors do not account for the positive externality of their information acquisition on other investors who can update their beliefs through revealing price. If, on the other hand, market conditions are such that no investor acquires a signal in equilibrium then signals must have zero or negative value and a benevolent social planner agrees with the market outcome. However, the effect of signals on expected asset price has imme-

diate distributional consequences when investors' endowments are heterogeneous which makes it difficult to separate purely allocative effects of information from redistributive endowment revaluation effects.

In deriving a rational law of demand for financial information, the present approach resorts to the benchmark case of fully revealing REE in which a sufficient statistic of all investors' information becomes publicly known through price. Benchmark scenarios (such as perfect foresight, perfect competition, complete markets or the conditions of the welfare theorems) have proven to be useful tools for many fields of economics in elucidating key relationships between rational behavior and market outcomes. The benchmark case of fully revealing asset price serves this purpose. Some markets such as that for foreign exchange may indeed come close to the fully revealing benchmark. As Federal Reserve chairman Alan Greenspan remarked at the 21st Annual Monetary Conference in Washington D.C. on November 20, 2003: "My experience is that exchange markets have become so efficient that virtually all relevant information is embedded almost instantaneously in exchange rates to the point that anticipating movements in major currencies is rarely possible." Crucial benefits of the fully revealing REE are its tractability in closed form and its clear-cut predictions of asset price responses. The key impact of information acquisition is its diminishing effect on the asset's expected excess return in fully revealing REE. A non-revealing price, at the other extreme, would require an infinite variance of exogenous noise in price and preclude any allocative role of price. So, unless price loses its entire allocative function, the diminishing effect of information on the expected excess return will carry over in mitigated form to less than fully revealing REE.

Financial information often comes in discrete levels such as Standard & Poor's or Moody's investment grades, or on a three-level buy-hold-sell scale. It therefore not only seems convenient but realistic to consider discrete signals. Poisson distributed signals in particular exhibit several useful statistical properties. For many draws, Poisson probabilities approximate binomial signal distributions. In other words, a single Poisson signal approximates many individual thumbs-up, thumbs-down signals. A gamma distribution of the asset return is the unique conjugate prior distribution to Poisson signals so that closed-form solutions of the financial market equilibrium are guaranteed for all levels of information. Davis (1993) presents an earlier model in finance that employs the gamma distribution. Special cases of the gamma distribution are the chi-squared, the Erlang, and the exponential distribution, for instance. The prominence and success of the Nelson (1991) exponential ARCH model in empirical finance suggests that this is a particularly relevant family of return distributions. Realistically, gamma distributed gross returns cannot be negative so that investors can never lose more than their principal. Variants of the results with the Poisson-gamma signal-return distri-

bution carry over to other conjugate prior distributions. Under a normal-normal pair of signal-return distributions, for instance, asset price is fully revealing and information acquisition occurs in the presence of an endowment revaluation effect but unrealistically not in its absence (Muendler 2004, Theorem 4).

The joint equilibrium in signal and asset markets is called a rational information choice equilibrium (RICE) and builds on common equilibrium definitions: a Walrasian REE for assets and a Samuelson (1954) style public-goods equilibrium for signals. A Walrasian asset market REE is standard in literatures on information acquisition (Grossman and Stiglitz 1980), on delegated portfolio management (Bhattacharya and Pfleiderer 1985), and currency attacks (Morris and Shin 1998), for instance. When asset price fully reveals a sufficient statistic of all investors' signal realizations, as will be the case in this paper, signals are pure public goods *ante notitias*. Such a fully revealing REE can be viewed as the limit of a sequence of partially revealing auctions (Reny and Perry 2003). The public-goods character of signals is also common in models of experimentation (Bolton and Harris 1999, Cripps, Keller and Rady 2005). A finite number of investors assures that a RICE exists under fully revealing asset price—as it does in the Grossman and Stiglitz (1980) model (Muendler 2004). So, despite its intentional limitation to the fully-revealing benchmark equilibrium, the present model shares key features with partially revealing REE models and gives rise to empirically confirmed predictions.

The remainder of this paper is organized as follows. Section 1 presents a Bayesian model of rational investors' information and portfolio choice along with general implications. Section 2 imposes constant absolute risk aversion and Poisson distributed (sufficient) signals for tractable, non-trivial solutions. Section 3 shows that a unique financial market equilibrium results. Every investor can buy signals prior to portfolio choice. Section 4 analyzes this signal choice for investors in the absence of wealth effects, shows that a unique equilibrium exists in the market for signals too, and discusses under what market conditions information acquisition occurs in equilibrium. Section 5 discusses the implied efficiency properties of asset price under a Pareto welfare criterion. Section 6 introduces heterogeneity in investors' endowments of the risky assets. Section 7 relates the findings to the prior literature, and section 8 concludes. Some proofs are relegated to the appendix.

1 Bayesian Information and Portfolio Choice

The rational Bayesian model of asset and financial information choice adds a prior stage of signal acquisition to the standard expected utility model of portfolio choice. There are two periods, today and tomorrow, and there are two assets:

One riskless bond b and one risky stock x. Assets are perfectly divisible. The riskless bond sells at a price of unity today and pays a real interest rate $r \in (-1, \infty)$ tomorrow so that the gross interest factor is $R \equiv 1 + r \in (0, \infty)$. The risky asset sells at a price P today and pays a gross return $\theta \in \Theta \subseteq \mathbb{R}$ tomorrow, where Θ denotes the range of possible values.

Investors hold prior beliefs about the distribution of the risky asset return and can acquire signals to update their beliefs. To create an image for the abstract concept of information, one can liken signals in this framework to private detectives and signal realizations to detectives' reports. In a strict general-equilibrium sense, of course, there is only one class of agents in this model (investors) and a signal is an investor's costly effort to update beliefs.

Markets for private detectives (signals) S_n^i open at 9am today. Detective n, hired by investor i, reports back exclusively to investor i with a signal realization s_n^i before 10am. How many different private detectives N^i should investor i hire? Each investor knows that she will base her portfolio decision, to be taken at 10am today, on the information that she is about to receive from her N^i private detectives. She also knows the statistical distribution of the signals conditional on the unobserved asset return. Naturally, she does not know the content of the private detective's report when she takes her decision on information acquisition (she does not know signal realization s_n^i). Otherwise she would not pay for the information (the signal S_n^i).

Assumption 1 (Risk and conditionally independent signals). *The risky asset return θ distribution is non-degenerate. All signals $\{S_1^i, ..., S_{N^i}^i\}_{i=1}^{I}$ are conditionally independent given the realization of the asset return, $S_n^i | \theta \overset{i.i.d.}{\sim} f(s_n^i | \theta)$.*

While assets are assumed to be perfectly divisible, signals have to be acquired in discrete numbers.[1]

The asset price at 10am will contain information. The reason is that each investor chooses her portfolio given her observations of signal realizations $(\{s_n^i\}_{n=1}^{N^i})$, and the Walrasian auctioneer at Wall Street clears the market by calling an equilibrium price. In the benchmark case of a *fully revealing* equilibrium, the asset price is invertible in a sufficient statistic of all investors' *posterior* beliefs and hence permits the rational extraction of all relevant market information. This is the case of analysis in the present paper.

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[1]An informative signal distribution is invertible in θ. So, a continuum of signals would a.s. reveal the realization of θ to informed investor. For markets to clear, P must equal θ/R in this case, otherwise informed investors want to reshuffle their portfolio. But then the price fully reveals θ itself and removes all uncertainty—an unrealistic case of little interest. The inadmissability of a continuum of signals also clarifies that there is a fundamental difference between the precision of signals and their number.
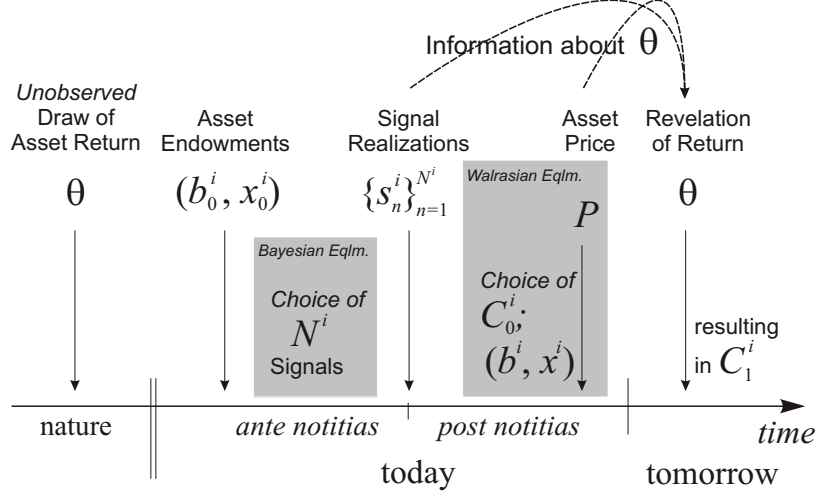
Figure 1: **Timing of decisions and information revelation**

The timing of decisions is illustrated in Figure 1. Every investor i is endowed with initial wealth $W_0^i \equiv b_0^i + P x_0^i$. At 9am, investors choose the number of signals (private detectives) N^i. To do so, they maximize *ante notitias* expected utility based on their prior beliefs before signal realizations become known (*ante notitias*). Investors then receive the realizations $\{s_1^i, ..., s_{N^i}^i\}$ of these N^i signals (they get to know the content of the private detectives' reports) and update their beliefs. When Wall Street opens at 10am today, investors choose consumption today and tomorrow, C_0^i and C_1^i, and decide how much of the risky asset to hold. At this stage, they maximize *post notitias* expected utility based on their posterior beliefs.[2] The Walrasian auctioneer in the financial market sets the price P for the risky asset such that the stock market clears. The bond market clears given the interest factor R.

Each signal has a cost of c. So, the intertemporal budget constraint of investor i becomes

$$b^i + P x^i = b_0^i + P x_0^i - C_0^i - c N^i \tag{1}$$

today, and

$$C_1^i = R b^i + \theta x^i \tag{2}$$

will be available for consumption tomorrow.

---

[2]To clarify the timing of signal realizations, I distinguish between *ante notitias* and *post notitias* expected utility. *Ante notitias* expected utility is different from *prior* expected utility in that the arrival of N^i signals is rationally incorporated in *ante notitias* expected utility. Raiffa and Schlaifer (1961) favored the terms "prior analysis," "pre-posterior analysis" and "posterior analysis."

Assumption 2 (Expected utility). *Investors $i = 1, \ldots, I$ evaluate consumption with additively separable utility U^i at constant individual discount rates ρ^i:*

$$U^i = \mathbb{E}\left[u(C_0^i) + \rho^i u(C_1^i) \,\big|\, \mathcal{F}^i\right], \tag{3}$$

where $u(C)$ is a strictly increasing and weakly concave function of C, and $\mathcal{F}^i$ denotes investor i's information set.

Investor i maximizes expected utility (3) with respect to consumption C_0^i today and C_1^i tomorrow given (1) and (2). For ease of notation, abbreviate investor i's conditional expectations with $\mathbb{E}^i[\cdot] \equiv \mathbb{E}[\cdot|\mathcal{F}^i]$ when they are based on *post notitias* beliefs, and with $\mathbb{E}^i_{ante}[\cdot] \equiv \mathbb{E}[\cdot|\mathcal{F}^i_{ante}]$ for *ante notitias* beliefs in anticipation of N^i signal receipts. *Post notitias* expectations coincide for all investors under fully revealing price.

On the second stage, after having received the realizations of her N^i signals $\{s_j^i\}_{j=1}^{N^i}$ and updated beliefs to *post notitias* beliefs, each investor decides on asset holdings and consumption given the asset price P. The two first-order conditions at this stage can be summarized with the opportunity cost RP of holding the risky asset in terms of holding the riskless asset (portfolio composition), and the marginal rate of substitution between consumption today and tomorrow (portfolio size):

$$RP = \frac{\mathbb{E}^i\left[\theta\, u'(C_1^{i*})\right]}{\mathbb{E}^i\left[u'(C_1^{i*})\right]}, \quad \text{and} \quad \frac{1}{R} = \rho^i \frac{\mathbb{E}^i\left[u'(C_1^{i*})\right]}{u'(C_0^{i*})}, \tag{4}$$

where expectations $\mathbb{E}^i[\cdot]$ are conditional on the realizations of the signals and the asset price. The optimal choices $C_0^{i*}(R, P, \sum_{k=1}^I N^k, \mathcal{F}^i)$, $b^{i*}(R, P, \sum_{k=1}^I N^k, \mathcal{F}^i)$ and $x^{i*}(R, P, \sum_{k=1}^I N^k, \mathcal{F}^i)$ (where any two choices determine the third) are decision rules that depend on the opportunity cost RP, on the observed number of signals $\sum_{k=1}^I N^k$ that investors acquired, and on the information transmitted through the signal realizations and price P given $\sum_{k=1}^I N^k$. The choices of C_0^{i*}, b^{i*} and x^{i*} imply a level of *post notitias* indirect utility, which I denote with $U^{i*} = u(C_0^{i*}) + \rho^i \mathbb{E}^i\left[u(C_1^{i*})\right]$.

On the first stage, the investor chooses the number of signals she wants to receive. She does this by maximizing *ante notitias* utility given her beliefs before the realizations of the signals arrive. At this time she cannot know more than the prior parameters of the respective distributions, but she builds her *ante notitias* beliefs by taking into account how signals will likely change beliefs at 10am. *Ante notitias* utility is $\mathbb{E}^i_{ante}[U^{i*}] = \mathbb{E}^i_{ante}[u(C_0^{i*})] + \rho^i \mathbb{E}^i_{ante}[u(C_1^{i*})]$ by the Law of Iterated Expectations. The optimal number of signals $N^{i*} \in \mathbb{N}_0$ maximizes *ante notitias* utility $\mathbb{E}^i_{ante}[U^{i*}]$.

In the spirit of competitive equilibrium, a rational expectations equilibrium (REE) that clears both the asset market and the market for signals can be defined

as a Walrasian equilibrium at Wall Street preceded by a Bayesian public-goods equilibrium in the market for detective services. This extension of REE to a sequence of a rational Bayesian public-goods equilibrium in the signal market and a subsequent Walrasian asset market equilibrium is called a Rational Information Choice Equilibrium, or RICE.

Definition 1 (RICE). *A rational information choice equilibrium (RICE) is an allocation of x^{i*} risky assets, b^{i*} riskless bonds, and N^{i*} signals to investors $i = 1, ..., I$ and an asset price P along with consistent beliefs such that*

- *the portfolio (x^{i*}, b^{i*}) is optimal given RP and investors' post notitias beliefs for $i = 1, ..., I$,*

- *the market for the risky asset clears, $\sum_{i=1}^{I} x^{i*} = I\bar{x}$, and*

- *the choice of signals N^{i*} is optimal for investors $i = 1, ..., I$ given the sum of all other investors' signal choices $\sum_{k \neq i} N^{k,*}$ and a marginal signal cost c.*

$\bar{x}$ denotes the average risky asset supply per investor.

Rational Bayesian investors choose their demand for signals given the expected asset market REE at Wall Street under anticipated information revelation. The equilibrium in the market for signals is the benchmark public-goods equilibrium following Samuelson's (1954) definition, where agents know other agents' total demand for the public good at the time of their decision.

This expected utility framework immediately implies that, irrespective of whether asset price is fully revealing or not, demand for signals is strictly positive only if investors care about risk and information affects risk in a broad sense.

Lemma 1 (Necessary conditions for utility benefit of signals). *Suppose signals are costly, $c > 0$. Then an investor acquires a signal in a RICE only if she is not risk neutral under assumptions 1 and 2.*

Proof. Suppose the investor is risk neutral. Then *ante notitias* utility degenerates to $C_0^{i*} + \rho^i \mathbb{E}_{ante}^i [C_1^{i*}]$. For a risk neutral investor to neither demand a positively nor a negatively infinite number of assets, $\mathbb{E}^i [\theta] = RP$ and $R = 1/\rho^i$ in a *RICE*. Thus, *ante notitias* utility becomes $C_0^{i*} + \rho^i \mathbb{E}_{ante}^i [C_1^{i*}] = W_0^i - cN^i$ by (1) and (2). *Ex ante* utility of a risk neutral investor is independent of the portfolio composition. As a result, signals only cause costs, but do not have a benefit, which proves the first statement. ∎

A risk neutral investor is indifferent whether she holds a risky stock or a riskless bond in her portfolio. Hence, she expects her actions upon signal realizations to yield the same return *ex ante* as yields no action at all, which makes

signals useless to her. Lemma 1 casts doubt on the generality of information acquisition models with risk neutral investors (e.g. Jackson 1991, Barlevy and Veronesi 2000) and suggests that results in the literature on optimal experimentation, where agents are risk neutral too, are limited to non-financial markets. Lemma 1 also highlights that information is not a good or bad in its own right. It has a utility benefit only if it affects decisions. This suggests that market conditions will matter for the value of signals—a theme to be investigated in detail.

2 Risk Aversion and Conjugate Updating

Constant absolute risk aversion relates utility closely to properties of the return distribution.

Assumption 3 (CARA). *Investors have constant absolute risk aversion.*

Under CARA, period utility becomes $u(C) = -\exp\{-AC\} < 0$, where $A > 0$ is the Pratt-Arrow measure of absolute risk aversion. For expected CARA utility to exist, the return distribution must have a moment generating function (MGF). The MGF of a random variable Z is defined as $M_{Z|\mathcal{F}^i}(t) \equiv \mathbb{E}\left[\exp\{tZ\}|\mathcal{F}^i\right] \in (0, \infty)$. So, a CARA investor's expected utility can be recast in terms of MGFs where

$$\mathbb{E}^i\left[U^i\right] = -\exp\{-AC_0^i\} - \rho^i M_{\theta|\mathcal{F}^i}(-AC_1^i).$$

2.1 Common priors and risk aversion

To analyze the utility benefit of signals, given expected price responses to signal realizations in general equilibrium, it is instructive to consider the case of investors who are identical in beliefs and risk aversion. This homogeneity will make price fully revealing.

Assumption 4 (Common priors and risk aversion). *Investors have identical prior beliefs about the joint signal-return distribution and share identical parameters of risk aversion.*

So, assumption 4 limits possible differences in $\mathcal{F}^i$ across investors to *post notitias* differences.

The first-order conditions (4) become

$$RP = \frac{M'_{\theta|\mathcal{F}^i}(-A\,x^{i*})}{M_{\theta|\mathcal{F}^i}(-A\,x^{i*})} \quad \text{and} \quad \frac{1}{R} = \rho^i H^i\, M_{\theta|\mathcal{F}^i}\left(-A\,x^{i*}\right), \tag{5}$$

where the prime in $M'_\theta(t)$ denotes the first derivative of the MGF with respect to its argument t and $H^i \equiv \exp\{-A[(1+R)b^i + Px^{i*} - W_0^i - cN^i]\}$. The first relationship in (5) can also be viewed as the inverse demand function for the risky asset x^{i*}. The inverse demand function intersects with the price axis at $RP = \mathbb{E}[\theta]$ for $x^{i*} = 0$, strictly falls in x^{i*} by the second-order conditions (appendix A), and is independent of W_0^i by CARA. Bond demand $b^i \in \mathbb{R}$ varies to satisfy the wealth constraint.

Using (5) in utility (3) for CARA yields *post notitias* expected utility

$$
\mathbb{E}^i[U^i] = -\delta^i \exp\{-AR(b_0^i - cN^i)\}^{\frac{1}{1+R}} \tag{6}
$$
$$
\times \exp\left\{-AR\, \frac{M'_{\theta|\mathcal{F}^i}(-Ax^{i*})}{M_{\theta|\mathcal{F}^i}(-Ax^{i*})}(x_0^i - x^{i*})\right\}^{\frac{1}{1+R}} M_{\theta|\mathcal{F}^i}\big(-Ax^{i*}\big)^{\frac{1}{1+R}}
$$

after a round of simplifications, where $\delta^i \equiv \frac{1+R}{R}(\rho^i R)^{\frac{1}{1+R}}$. For the choice of the number of signals N^i, investors evaluate *ante notitias* expected utility, which becomes

$$
\mathbb{E}_{ante}^i[U^i] = -\delta^i \exp\{-AR(b_0^i - cN^i)\}^{\frac{1}{1+R}} \tag{7}
$$
$$
\times \mathbb{E}_{ante}^i\left[\exp\left\{-AR\, \frac{M'_\theta(-Ax^*)}{M_\theta(-Ax^*)}(x_0^i - x^*)\right\}^{\frac{1}{1+R}} M_\theta(-Ax^*)^{\frac{1}{1+R}}\right]
$$

for given R. For common priors and CARA by assumption 4, a symmetric equilibrium implies that all investors expect an identical asset demand x^* *ante notitias*.

The second-order conditions (appendix A) and general properties of MGFs impose little structure on (7). An instructive closed-form analysis of information acquisition demands specific distributional assumptions.

2.2 Financial information and conjugate updating

Financial information often comes in discrete levels such as Standard & Poor's or Moody's investment grades, or on a three-level buy-hold-sell scale.[3] It therefore appears not only convenient but realistic to consider discrete signals. Poisson distributed signals in particular exhibit several useful statistical properties. The sum of N^i conditionally independent Poisson signals, for instance, is itself Poisson distributed with mean and variance $N^i\theta$ (appendix B). For a large number of draws and small probabilities, Poisson probabilities approximate binomial signal distributions (Casella and Berger 1990, Example 2.3.6).

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[3]The amount of words to describe investment prospects is discrete and finite. Even quotes of asset prices at Wall Street used to be reported as common fractions, and decimals continue to make price quotes discrete in a strict sense.

Assumption 5 (Poisson distributed signals and conjugate updating). *Signals are Poisson distributed and update the prior distribution of the asset return θ to a posterior distribution from the same family.*

A gamma distribution of the asset return, $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$, uniquely satisfies assumption 5 (Robert 1994, Proposition 3.3). The distribution parameters α^i and β^i are specific to investors' beliefs in principle. The parameter α^i is sometimes referred to as the shape parameter and $1/\beta^i$ as the scale parameter.

Distributions that are closed under sampling so that prior and posterior distributions belong to the same family are called conjugate prior distributions. The gamma distribution is a conjugate prior to the Poisson distribution.[4] A gamma distributed asset return exhibits the additional advantage that its support $\Theta \subseteq \mathbb{R}^+$ is strictly positive so that, realistically, negative returns cannot occur. In contrast, a normal asset return would imply that stock holders must cover losses beyond the principal ($\theta < -P$) with a strictly positive probability. Moreover, the gamma-Poisson pair of distributions does not have an additive signal-return structure (Muendler 2004)—contrary to the normal-normal pair of signal-return distributions—so that signals can raise utility even in the absence of endowment revaluation effects of information (section 4).

Useful properties of the Poisson and gamma distributions are reported in appendix B. The most important property relates to the updating of beliefs.

Fact 1 (Conjugate updating). *Suppose the prior distribution of θ is a gamma distribution with parameters $\bar{\alpha} > 0$ and $\bar{\beta} > 0$. Signals $S_1^i, ..., S_{N^i}^i$ are independently drawn from a Poisson distribution with the realization of θ as parameter. Then the* post notitias *distribution of θ, given realizations $s_1^i, ..., s_{N^i}^i$ of the signals, is a gamma distribution with parameters $\alpha^i = \bar{\alpha} + \sum_{n=1}^{N^i} s_n^i$ and $\beta^i = \bar{\beta} + N^i$.*

Proof. See Robert (1994, Proposition 3.3). ∎

The MGF of a gamma distributed return is $M_{\theta|\alpha^i,\beta^i}(t) = [\beta^i/(\beta^i - t)]^{\alpha^i}$ (appendix B). So, the mean of a gamma distributed return θ is α^i/β^i, and its variance $\alpha^i/(\beta^i)^2$. The mean-variance ratio will play a key role in particular: $\mathbb{E}^i[\theta]/\mathbb{V}^i(\theta) = \beta^i$.

For risk averse investors to have an incentive for signal acquisition, it is important that the *ante notitias* variance falls in the number of signals N^i. Indeed,

$$\frac{\partial}{\partial N^i}\mathbb{E}_{ante}^i\left[\mathbb{V}\left[\theta\,\big|\,\alpha^i,\beta^i\right]\right] = \frac{\partial}{\partial N^i}\left(\frac{\bar{\alpha} + \frac{\bar{\alpha}}{\bar{\beta}}N^i}{(\bar{\beta} + N^i)^2}\right) = -\frac{\bar{\alpha} + \frac{\bar{\alpha}}{\bar{\beta}}N^i}{(\bar{\beta} + N^i)^3} < 0$$

---

[4]The gamma distribution is also a conjugate prior distribution to itself and a normal distribution, for instance.

(by fact 3, appendix B). Risk averse investors not only expect a lower variance of the risky asset return but also look forward to making a more educated portfolio choice at 10am *post notitias*. Anticipating this improved portfolio choice at 9am, investors consider information acquisition a means of reducing the *ante notitias* variance of tomorrow's consumption.

Both the mean and the variance of a Poisson distributed signal S^i with parameter θ are equal to θ. Thus, the precision of a signal $\mathbb{E}^i_{ante}\left[\mathbb{V}^i\left(s^i|\theta\right)\right]^{-1} = \mathbb{E}^i_{ante}\left[\theta\right]^{-1} = \beta^i_{ante}/\alpha^i_{ante} = \bar{\beta}/\bar{\alpha}$ depends solely on individual priors and is common to all investors as assumption 4 requires.

For a gamma distributed asset return, demand for the risky asset becomes

$$x^{i*} = \frac{\beta^i}{A}\frac{\mathbb{E}^i\left[\theta\right] - RP}{RP} \equiv \frac{\beta^i}{A} \cdot \xi^i \qquad (8)$$

by first-order condition (5) and the MGF of the gamma distribution (fact 4 in appendix B). Demand for the risky asset decreases in price and the riskless asset's return; demand is the higher the less risk averse investors become (lower A) or the higher the expected mean-variance ratio β^i of the asset is. Investors go short in the risky asset whenever their return expectations fall short of opportunity cost, $\mathbb{E}^i\left[\theta\right] < RP$, and go long otherwise. Under CARA, demand for the risky asset is independent of wealth W^i_0.

The term $\mathbb{E}^i\left[\theta - RP\right]/RP$ is an individual investor i's *expected relative excess return* over opportunity cost. Risk averse investors demand this premium.[5] For later reference, define the *expected relative excess return* as

$$\xi^i \equiv \frac{\mathbb{E}^i\left[\theta\right] - RP}{RP}. \qquad (9)$$

The *expected relative excess return* ξ^i has important informational properties that crucially affect incentives for information acquisition.

3   Financial Market Equilibrium

The utility benefit of signals depends on the expected equilibrium at Wall Street and asset price responses to signal realizations in that equilibrium. To solve for a RICE backwards, restrict attention to the partial REE at Wall Street first, given any market equilibrium for private detectives. Investors $i = 1, ..., I$ have received the realizations of their conditionally independent $N^i \geq 0$ signals. It is 10am,

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[5]The (absolute) *expected relative excess return* $\mathbb{E}^i\left[\theta - RP\right]$ is unrelated to the *expected relative excess market value* $\mathbb{E}^i\left[RP - \mathbb{E}^i\left[RP\right]\right]$ in the sense of (Fama 1970). Financial markets are strong-form efficient in this model so that $\mathbb{E}^i\left[RP - \mathbb{E}^i\left[RP\right]\right] = 0$.

and investors choose portfolios (x^{i*}, b^{i*}) given their *post notitias* information sets $\mathcal{F}^i$.

In REE, rational investors not only consider their own signal realizations. They extract information also from price so that $\mathcal{F}^i = \{\sum_{n=1}^{N^i} s_n^i, RP\}$. Since RP and $\sum_{n=1}^{N^i} s_n^i$ are correlated in equilibrium, the *post notitias* distribution of the asset return, based on this information set, can be complicated. If price P is fully revealing, however, the information sets of all investors coincide: $\mathcal{F}^i = \mathcal{F}$ for all i. This gives the rational beliefs in REE a closed and linear form analogous to fact 1.

Investors are identical in their degree of risk aversion and in their prior beliefs by assumption 4. If they also know market size, asset price becomes fully revealing.

Assumption 6 (Known market size). *The total supply of the risky asset $\bar{x}$ and the total number of investors I are certain and known.*

Under these assumptions and by definition 1 of RICE, financial market equilibrium takes the following closed form.

Proposition 1 (Unique asset market REE). *Under assumptions 1 through 6, the asset market REE in RICE is unique and symmetric with*

$$\alpha^i = \bar{\alpha} + \sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k \equiv \alpha, \tag{10}$$

$$\beta^i = \bar{\beta} + \sum_{k=1}^{I} N^k \equiv \beta, \tag{11}$$

$$RP = \frac{\alpha}{\beta} \frac{1}{1+\xi}, \tag{12}$$

where $x^{i*} = \bar{x}$ and $\xi^i = \xi \equiv A\bar{x}/\beta$.

Proof. By (8) and for beliefs (10) and (11), $x^{i*} = \alpha/(A\,RP) - \beta/A$ for all i. So, market clearing $x^{i*} = \bar{x}$ under definition 1 of RICE implies (12).

Uniqueness of beliefs (10) and (11) follows by construction. By (8) and market clearing, RP can always be written as $RP = T_0 + T_1(\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k)$ for an appropriate choice of constants $T_0, T_1 > 0$ because risk aversion A is common to all investors. But then, every investor i can infer $\sum_{k \neq i} \sum_{n=1}^{N^k} s_n^k = (RP - T_0)/T_1 - \sum_{n=1}^{N^i} s_n^i$ from her knowledge of own signal realizations. Since the random variables $\sum_{k \neq i} \sum_{n=1}^{N^k} s_n^k$ and $\sum_{n=1}^{N^i} s_n^i$ are Poisson distributed by fact 3

(appendix B) and conditionally independent given θ, a rational investor must apply Bayesian updating following fact 1. Hence, $\alpha^i = \bar{\alpha} + \sum_{n=1}^{N^i} s_n^i + \sum_{k \neq i}^{I} \sum_{n=1}^{N^k} s_n^k$ and $\beta^i = \bar{\beta} + N^i + \sum_{k \neq i}^{I} N^k$. $\sum_{k \neq i}^{I} N^k$ is known by definition 1 of RICE.

Finally, no less than $\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$ signals can get revealed in REE. Suppose one signal s_n^i is received by some investor i but does not enter price. Then, investor i cannot have based demand x^{i*} on that signal since market clearing $\sum_{k=1}^{I} x^{k,*} = I\bar{x}$ would have transmitted s_n^i to price. However, if α^i does not include s_n^i, Bayesian updating following fact 1 is violated, which is ruled out in an REE. ∎

The equilibrium price P fully reveals aggregate information of all market participants. Formally, aggregate information is the total of all signals received: $\sum_{i=1}^{I} \sum_{n=1}^{N^i} s_n^i$. This is a sufficient statistic for every moment of θ given $\sum_{i=1}^{I} N^i$ (which is known by definition 1 of RICE). In general, the equilibrium price is fully revealing if and only if assumptions 1 through 6 are satisfied (corollary 1.1 in appendix C restates this formally).

In fully revealing REE, investors' information sets $\mathcal{F} = \{\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k\}$ coincide by (10) and (11). Consequently, the *expected relative excess return* $\xi^i = \xi$ (9) coincides. It becomes

$$\xi = \frac{\mathbb{E}[\theta] - RP}{RP} = \frac{A\bar{x}}{\beta} = \frac{A\bar{x}}{\bar{\beta} + \sum_{k=1}^{I} N^k} \in (0, \bar{\xi}] \quad \text{where} \quad \bar{\xi} \equiv \frac{A\bar{x}}{\bar{\beta}}. \quad (13)$$

The *expected relative excess return* over opportunity cost $\mathbb{E}^i[\theta - RP]/RP$ is crucial for individual incentives to acquire information. Information acquisition diminishes the expected relative excess return. Equilibrium price P will reveal signal realizations. So, private information will become publicly known to investors through informative price and risk averse investors will value the risky asset more, thus bidding up price. Therefore, investors expect higher opportunity cost of the risky asset $\mathbb{E}_{ante}[RP]$ in the face of reduced uncertainty. The diminishing effect of public information on the expected relative excess return also occurs in additive signal-return models for any distribution with a moment-generating function (Muendler 2004) and when price is partially revealing (Easley and O'Hara 2004, Veldkamp 2004).

Proposition 2 (Diminishing expected excess return). *Under assumptions 1 through 6, the* expected relative excess return ξ *in asset market REE strictly falls in the number of signals, while the expected opportunity cost of the risky asset* $\mathbb{E}_{ante}[RP]$ *strictly increases in the number of signals* ante notitiam.

Proof. Note that $\xi = \mathbb{E}_{ante}[\xi]$ by (13). The number of signals $\bar{N} = \sum_{k=1}^{I} N^k$ strictly diminishes ξ by (13). The number of signals strictly raises $\mathbb{E}_{ante}[RP] = (\bar{\alpha} + \bar{\alpha}\bar{N}/\bar{\beta})/(A\bar{x} + \beta)$ since $\partial\mathbb{E}_{ante}[RP]/\partial\bar{N} = \bar{\xi}/\beta^2(1+\xi)^2 > 0$. ∎

4 Information Market Equilibrium in the Absence of Endowment Revaluation

Given the expected financial market equilibrium, how much information do investors acquire in RICE? Investors dislike the diminishing effect of information on the expected relative excess return ξ but anticipate a more educated portfolio choice if they can receive signal realizations. In their *ante notitias* choice of the optimal number signals, risk averse investors weigh the diminishing excess return and the marginal cost of a signal against the benefit of a more informed intertemporal consumption allocation.

The acquisition of signals changes asset price *ante notitias* by proposition 2. So, investors can affect the value of their endowments $W_0^i = b_0^i + Px_0^i$ by buying signals. To investigate incentives for information acquisition in the absence of the wealth effect, this section considers homogeneous investors with $x_0^i = 0$. Section 6 will present the general case. For now, there is a sole (foreign) agent who offers the risky asset ($x_0^I = I\bar{x}$) and is considered irrelevant for information acquisition. Section 6 will show that the sole owner of the risky asset may indeed not acquire any signal herself.

Investors evaluate *ante notitias* expected utility for their signal choice. However, *ante notitias* expected utility (7) has no closed form unless R is constant. Assumption 7 assures this.

Assumption 7 (Single-price responses to signal realizations). *The equilibrium price of an asset only responds to signal realizations on its own return.*

The assumption is equivalent to the limiting case where markets for single risky assets are small relative to the overall market for riskless bonds so that single signal realizations alter R negligibly little (see appendix D for a formal derivation). Economies with large safe forms of debt such as government debt and small open economies are examples.

For Poisson-gamma signal-return distributions and homogeneous investors with $x_0^i = 0$, *ante notitias* expected utility (7) becomes

$$\mathbb{E}_{ante}^i\left[U^i\right] = -\delta^i \exp\left\{-A\tfrac{R}{1+R}(W_0^i - cN^i)\right\} \tag{14}$$

$$\times \left[1 + \left(\left[(1+\xi)\exp\left\{-\tfrac{\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\tfrac{\bar{\xi}}{\xi}\right]^{-\bar{\alpha}}$$

(see appendix E). The cost of signals cN^i enters (14) in the form of an initial wealth reduction. The last factor in (14) captures the effect of the relative excess return $\xi \in (0, \bar{\xi}]$ on utility. The term $(1 + \xi) \exp(-\xi/(1 + \xi))$ strictly exceeds unity since, by (13), $\xi > 0$ for arbitrarily large but finite numbers of signals $\sum_{k=1}^{I} N^k \geq 0$. Hence, the last factor in (14) is well defined.

Although the number of signals must be discrete, one can take the derivative of *ante notitias* utility with respect to N^i to describe the optimal signal choice. Strict monotonicity of the first-order condition in the relevant range will prove this to be admissible. Differentiating (14) with respect to the number of signals yields the incentive to purchase information. As long as $\partial \mathbb{E}_{ante}^i [U^{i*}] / \partial N^i > 0$, investor i will generically purchase more signals. If $\partial \mathbb{E}_{ante}^i [U^{i*}] / \partial N^i \leq 0$ for all N^i, she purchases no information at all. Taking the derivative of (14) with respect to N^i, and dividing by $-\mathbb{E}_{ante}^i [U^{i*}] > 0$ for clarity, yields

$$-\frac{1}{\mathbb{E}_{ante}^i [U^{i*}]} \frac{\partial \mathbb{E}_{ante}^i [U^{i*}]}{\partial N^i} = -A \frac{R}{1+R} c \tag{15}$$

$$+ \frac{\bar{\alpha}}{\bar{\beta}} \frac{\left[(1 + \xi) \exp\left\{ \frac{-\xi}{1+\xi} \right\} \right]^{\frac{1}{1+R}} \left(1 - \frac{1}{1+R} \frac{\xi^2}{(1+\xi)^2} \right) - 1}{1 + \left(\left[(1 + \xi) \exp\left\{ \frac{-\xi}{1+\xi} \right\} \right]^{\frac{1}{1+R}} - 1 \right) \frac{\bar{\xi}}{\xi}}.$$

The first term on the right hand side of (15) is negative and represents the *marginal cost* of a signal (MC). The second term expresses the *potential marginal benefit* of a signal (MB) and can be positive or negative. The incentive for information acquisition does not depend on an investor's patience.

Rational investors view the choice of the total number of signals $\sum_{k=1}^{I} N^k$ as the converse of a choice of the *expected relative excess return* ξ because ξ strictly monotonically falls in the number of signals by proposition 2. This constitutes a fundamental trade-off behind the *potential marginal benefit MB* of a signal. In fact, an additional signal can diminish the *expected relative excess return* ξ so strongly that this negative effect more than outweighs the benefits of information. In the case of a normal-normal pair of signal-return distributions, the diminishing effect of information on the expected excess return can be shown to always outweigh the benefit unless there is an endowment revaluation effect (a corollary of Muendler 2004, Theorem 4). For a Poisson-gamma pair of distributions, however, the numerator of the MB term in (15) can take either a negative or a positive sign while the denominator is always positive.

The *potential marginal benefit MB* of a signal turns negative when the *expected relative excess return* $\xi \equiv (\mathbb{E}[\theta] - RP)/RP$ drops too low. The *potential benefit MB* does not constitute a benefit but a cost in this range. Note that a low ξ means that investors currently hold relatively many signals given market

size and the expected mean-variance ratio of the asset. The negative MB for low ξ reflects that, given a relatively large number of available signals, the negative effect of an additional signal on the expected relative excess return ξ outweighs the benefit from a more informed expected portfolio choice *ante notitias*. The diminishing effect of an additional signal on the expected relative excess return is particularly strong for investors with no endowment of the risky asset ($x_0^i = 0$) since the increase in the opportunity cost RP is not mitigated by any positive wealth effect of asset price on their endowments. As a consequence, every additional signal lowers an investor's *ante notitias* utility once the available amount of information has driven ξ below a certain level ($\underline{\xi}$).

Proposition 3 (Potential marginal signal benefit in the absence of endowment revaluation). *Under assumptions 1 through 7 and in the absence of endowment revaluation, the following is true for the potential marginal benefit* $\mathrm{MB}(\xi)$ *of a signal.*

- *The potential marginal benefit* $\mathrm{MB}(\xi)$ *attains strictly positive values if and only if* $\xi > \underline{\xi}$, *where* $\underline{\xi} \in (0, \infty)$ *is independent of* $\bar{\xi}$ *and uniquely solves* $\mathrm{MB}(\underline{\xi}) = 0$ *given* $R \in (0, \infty)$.

- *If* $\underline{\xi} < \bar{\xi}$ *then, in the range* $\xi \in [\underline{\xi}, \bar{\xi}]$, *the marginal signal benefit* $\mathrm{MB}(\xi)$ *strictly monotonically increases in* ξ *and is unbounded for arbitrarily large* ξ.

Proof. See appendix F for the general case and set $x_0^i = 0$. ∎

Figures 2 through 4 depict the marginal signal cost (MC) and potential marginal benefit of a signal (MB) under varying parameters.[6]

For a strictly positive interest factor R, MB turns positive at one unique point $\underline{\xi} > 0$ and subsequently increases unboundedly in ξ. The unique zero point $\underline{\xi}$ solves $MB(\underline{\xi}) = 0$ and is independent of $\bar{\xi}$ (and $\bar{\alpha}, \bar{\beta}$). In contrast to examples of non-convexities (Chade and Schlee 2002), the value of information is well behaved in the rational Bayesian model of financial information acquisition. In the range where signals have positive utility value, the marginal benefit MB of an additional signal strictly monotonically falls. *Ante notitias* expected utility is thus strictly concave in signals in the relevant range.

Under what conditions do investors acquire information? Figure 2 shows a case. As investors acquire signals, ξ moves away from $\bar{\xi}$ and to the west. The

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[6]Parameters underlying the benefit curves in Figures 2 through 5 are $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, and $R = 1.1$. The level of $\bar{\xi}$ depends on average asset supply, which is $\bar{x} = 7$ in Figures 2, 6, 7 and 5; $\bar{x} = 3$ in Figure 3; and $\bar{x} = 1$ in Figure 4. Marginal cost is given by $c = .1$, A, and R in Figures 2, 3 and 7.
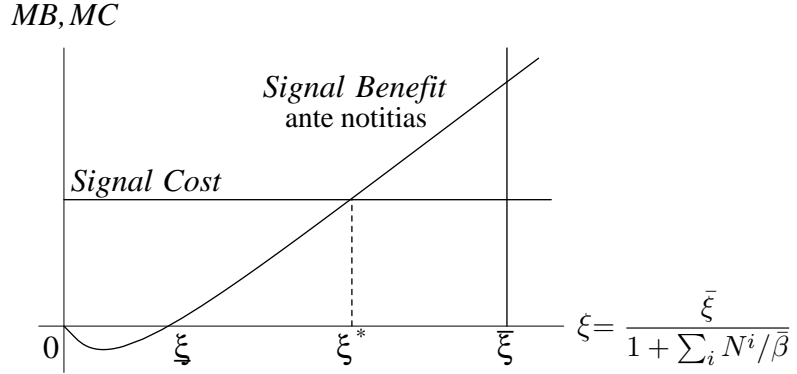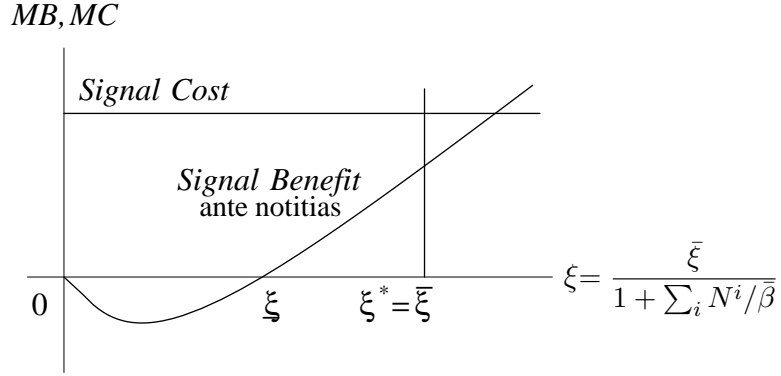
$$\xi = \frac{\bar{\xi}}{1 + \sum_i N^i / \bar{\beta}}$$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 2: **Information acquisition in equilibrium**

potential marginal benefit MB curve has a long arm in the positive range that slopes strictly upward by proposition 3. So, as long as $\bar{\xi}$ is large enough, there is a strictly positive expected relative excess return ξ^* at which the marginal benefit MB of a signal equals marginal cost MC. Although the relative excess return could attain any real value in principle, signals are not perfectly divisible. As a consequence, the precise optimal number of signals will yield an expected relative excess return in an open interval around ξ^*.

Since the expected relative excess return ξ cannot exceed $\bar{\xi}$, such an interior equilibrium can only occur if $\bar{\xi}$ is sufficiently large. Hence, investors will acquire a strictly positive amount of information only if the financial market meets the following two conditions. First, supply of the risky assets needs to be strong so that $\bar{x}$ is high. Then investors anticipate that they will invest a relatively large portion of their savings in the risky asset, and information about the risky asset return becomes relatively important to them. Second, investors need to be sufficiently risk averse relative to their prior beliefs about the mean-variance ratio of the risky asset so that $A/\bar{\beta}$ is high. Since the benefit of information stems from lowering the prior variance of the portfolio, information matters more for investors who are more risk averse.

So, the market environment determines whether information is valuable to investors indeed. Information is not a good in itself. When $\bar{\xi}$ drops too low, the *potential* marginal benefit MB of a signal cannot reach the point where it would meet or exceed marginal cost, and nobody will acquire a signal so that $\xi^* = \bar{\xi}$. This case is depicted in Figure 3 (risky asset supply is reduced by more than

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MB, MC

Signal Cost

Signal Benefit
ante notitias

0 $\underline{\xi}$ $\xi^* = \overline{\xi}$ $\xi = \dfrac{\bar{\xi}}{1 + \sum_i N^i / \bar{\beta}}$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 3$, $c = .1$.

Figure 3: **No information in equilibrium due to high signal cost**

half as compared to Figure 2). $\overline{\xi}$ is low if relatively few risky assets are supplied
to the market (low $\bar{x}$), or if investors are little risk averse (low A), or when the
prior mean-variance ratio of the asset return is relatively high (high $\bar{\beta}$) so that
risk matters little compared to payoff. Then investors do not value information
enough to acquire it.

What if signal cost drops to zero? Even then, there are market conditions in
which information has zero or negative value. Figure 4 depicts a case in which
the price of a signal c is zero but information would not be acquired (risky asset
supply is reduced to a seventh of the level in Figure 2). When the amount
of available information is large already, the price externality that diminishes
expected relative excess return ξ weighs more heavily than any positive effects
of more information on higher moments of the return distribution. The *potential*
marginal signal benefit MB is strictly negative and investors find information
undesirable even at zero cost.

The potential benefit MB vanishes as ξ goes to zero. In this limit, no investor
wants to purchase a costly signal. But every investor would accept signals for
free. The limiting level of $\xi = 0$ is reached when no risky assets are supplied
to the market ($\bar{x} \rightarrow 0$). Similarly, when investors become risk neutral ($A \rightarrow 0$),
or when the prior variance tends to zero ($\bar{\beta} \rightarrow \infty$), then there is no benefit
of holding information but also no harm done. Finally, if investors were given
infinitely many signals for free, ξ would reach zero but the return realization θ
would become known with certainty and the previously risky asset would turn
into a perfect substitute to the bond. The common cause for information to lose
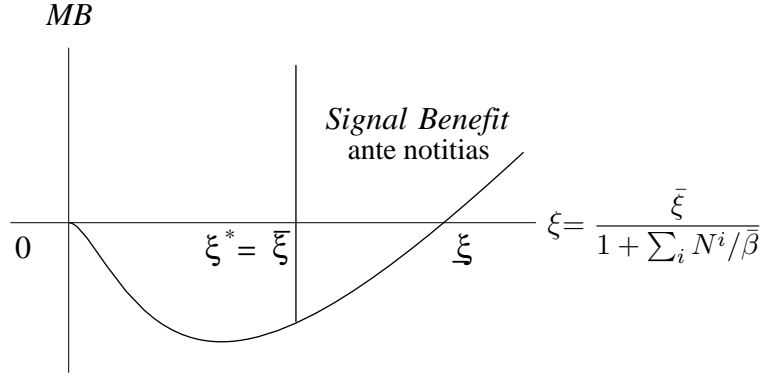
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The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 1$.

Figure 4: **No information in equilibrium due to market environment**

its value in all these cases is that the relative excess return ξ is driven down to zero so that no investor chooses to hold any risky asset. In this limit, information does not have a negative value either. Investors are simply unaffected. If investors don't think at 9am that they will be holding a risky asset at 10am, they know they will never need to act upon information. An infinite amount of information makes investors indifferent to it in the presence of a riskless alternative asset.

Proposition 4 (RICE in the absence of endowment revaluation). *Under assumptions 1 through 7 and in the absence of endowment revaluation, a RICE has the following properties for any $R \in (0, \infty)$.*

- *Investors acquire a strictly positive and finite number of signals in signal market equilibrium if and only if the asset market environment satisfies $\bar{\xi} > \underline{\xi}$, where $\underline{\xi} > \sqrt{1 + 1/R}$ solves $\mathrm{MB}(\underline{\xi}) = 0$.*

- *If the cost of a signal is strictly positive, then the market equilibrium for signals is unique up to a permutation of the signal allocation.*

- *If the cost of a signal is nil but $R > 0$, then there are two signal market equilibria, one of which involves an infinite amount of freely received signals.*

Proof. Under assumptions 1 through 7, investors acquire a strictly positive amount of signals if and only if $\bar{\xi}$ falls in the range of ξ where marginal benefit

MB is strictly positive (proposition 3). So, $\bar{\xi} > \underline{\xi} > \sqrt{1 + 1/R}$, where $\underline{\xi}$ solves $MB(\underline{\xi}) = 0$.

For $c > 0$, the equilibrium number of $\sum_{k=1}^{I} N^{k,*}$ must be unique because the positive arm of the marginal benefit MB in (15) strictly monotonically increases in ξ by proposition 3. If $\xi^* \geq \bar{\xi}$, the unique information equilibrium entails no information acquisition and $\xi^* = \bar{\xi}$. As $\bar{\xi}$ increases, there will be a unique information equilibrium with exactly one acquired signal since the marginal benefit MB in (15) strictly monotonically increases in ξ (proposition 3). As $\bar{\xi}$ moves further up, there will be a new and unique information equilibrium with exactly two acquired signals for the same reason, and so forth. Only the sum $\sum_k N^k$ is unique but the equilibrium assignment of signals to investors is not.

If $c = 0$, there is a second equilibrium at $\xi = 0$, in which $\sum_{k=1}^{I} N^k \to \infty$ while another equilibrium continues to exist for $R > 0$. ∎

The equilibrium does not determine how many signals a single investor holds. In equilibrium, one investor may acquire all $\sum_i N^i$ signals while nobody else buys any signal, or all investors may hold the same number of signals. Signals are public goods and therefore perfect strategic substitutes under fully revealing price because any fellow investors' signal is as useful (or detrimental) as an own signal.

By propositions 3 and 4, there is always a market size $\bar{x}$, or a degree of risk aversion A, or a level of the prior mean-variance ratio of the risky asset $\bar{\beta}$ behind $\bar{\xi}$ so that a costly signal becomes worthwhile to acquire in equilibrium.

Corollary 4.1 points to the degenerate limiting case where the interest rate of the bond becomes infinite $R \to \infty$ (and the potential benefit curve coincides with the horizontal axis). Then investors are completely indifferent to free information since they would never hold a risky asset but investors do not demand costly signals. When, at the other extreme, the bond becomes entirely worthless and eliminates the principal for sure ($r = -1, R = 0$), investors do not want to hold the bond in their portfolio. In this extreme case, they would choose to acquire an infinite amount of information about the risky asset as signal costs fall to zero. So, if there is no riskless asset in the economy yet, investors desire to create the riskless asset by acquiring infinitely much information about a risky asset in RICE. In this sense, the riskless bond is the basic asset in financial markets.

Corollary 4.1 (RICE responses to riskless returns in the absence of endowment revaluation). *Under the conditions of proposition 4, the following is true for a RICE.*

- *In the limit when $R \to \infty$, an information market equilibrium involves no information acquisition if signals are costly ($c > 0$).*

- *For $R \to 0$, the marginal benefit MB of a signal is strictly positive at any $\xi > 0$ and zero at $\xi = 0$. Then, if $c = 0$, there is a unique information market equilibrium which involves infinite information acquisition.*

Proof. The numerator of the marginal benefit *MB* term in (15) vanishes for $R \to \infty$, which proves the first statement. For $R \to 0$, the marginal information benefit cannot drop below zero by claim 2 in appendix F. So, there is only one equilibrium if $c = 0$, proving the second statement. ∎
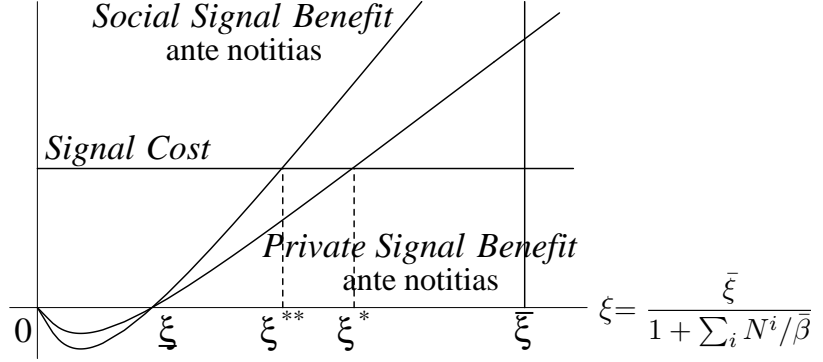
A finite number of investors has well defined incentives to acquire signals in a rational Bayesian model under fully revealing price, as does a finite number of investors in additive signal-return frameworks such as the Grossman and Stiglitz (1980) model (Muendler 2004). However, information need not be desirable. Proposition 4 and corollary 4.1 clarify that signals can turn from a public good into a public bad as market conditions change. These market conditions are captured by $\overline{\xi}$ and can be affected through R. In financial markets, information is a tertiary commodity. Investors are concerned about consumption, the primary good. Assets are mere means to the end of consumption, or secondary commodities. Information, finally, has value only if it helps investors make better portfolio decisions with regard to these assets. In this sense, information is a tertiary commodity. Consequently, the utility benefit of signals changes with market conditions.

5   Informational Efficiency in Absence of Endowment Revaluation

The rational Bayesian framework permits the application of a Pareto criterion to judge information allocation in financial markets. To investigate the informational efficiency of RICE in its pure form, this section continues to consider the absence of wealth effects of information and homogeneous investors with $x_0^i = 0$. Put differently, the social planner of the present section ignores in the welfare judgement of RICE the single investor who is the sole owner of the risky project with $x_0^i = I\bar{x}$.

Definition 2 (Informational Pareto efficiency) *An allocation of x^{i**} risky assets, b^{i**} riskless bonds, and N^{i**} signals to investors $i = 1, ..., I$ is called informationally Pareto efficient in a given market environment $(\overline{\xi}, R)$ if there is no other allocation such that all investors are at least as well off and at least one investor is strictly better off.*

SB, MB, MC

Social Signal Benefit
ante notitias

Signal Cost

Private Signal Benefit
ante notitias

0 $\underline{\xi}$ ξ^{**} ξ^{*} $\bar{\xi}$ $\xi = \dfrac{\bar{\xi}}{1 + \sum_i N^i / \bar{\beta}}$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 5: **Socially desirable information choice**

It does not matter for this Pareto criterion that information can change from a public good into a public bad. The criterion is conditional on a given market environment. To investigate whether the RICE in section 4 is Pareto efficient, imagine a benevolent social planner who can dictate every consumer j to buy exactly N^{j**} signals. This social planner maximizes $\sum_{j=1}^{I} \mathbb{E}_{ante}^{i}[U^j]$ with respect to $\{N^1, ..., N^I\}$. Thus, similar to Samuelson's (1954) condition for public good provision, a benevolent social planner's first-order conditions for information allocation are not (15) but instead

$$-\frac{1}{\mathbb{E}_{ante}^{j}[U^{j**}]} \frac{\partial \sum_{k=1}^{I} \mathbb{E}_{ante}^{k**}[U^{k**}]}{\partial N^k} = -A\frac{R}{1+R}\,c \tag{16}$$

$$+\frac{\bar{\alpha}}{\bar{\beta}} \frac{\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}}\left(1 - \frac{1}{1+R}\frac{\xi^2}{(1+\xi)^2}\right) - 1}{1 + \left(\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\frac{\bar{\xi}}{\xi}} \left(1 + \sum_{k\neq j}^{I} \frac{\mathbb{E}_{ante}^{k}[U^{k**}]}{\mathbb{E}_{ante}^{j}[U^{j**}]}\right)$$

for any $j \in 1, ..., I$, written in terms of that investor j's utility. Thus, compared to the privately perceived benefits, the potential social benefits SB that a social planner considers scale up the private benefits MB by a factor of $1 + (1/\mathbb{E}_{ante}^{j}[U^{j**}]) \cdot \sum_{k\neq j}^{I} \mathbb{E}_{ante}^{k}[U^{k**}] > 1$. Therefore, if information is a public bad, a benevolent social planner wants to implement an even smaller amount of information than the private market. However, since no information is acquired in

private markets in that case, the market equilibrium is informationally efficient when information is a public bad.

On the other hand, if information is a public good under given market conditions, a social planner wants (weakly) more information to be allocated than markets provide. Individual investors do not take into account that their signal acquisition also benefits other investors through fully revealing price. In this case, markets allocate (weakly) less information than desirable. However, signals are not divisible and one cannot infer from condition (16) that a social planner wants to implement strictly more information. It can happen that an additional signal diminishes relative excess return ξ so strongly that all investors are worse and not better off. So, discreteness of the number of signals only permits a conditional efficiency statement up to discrete tolerance. In Figure 5, a social planner wants to allocate information so that relative excess return is brought down from around ξ^* to around ξ^{**}. However, if an additional signal makes the implementable level of ξ drop far below ξ^{**}, investors are better off if relative excess return ξ remains at the market equilibrium level around ξ^*.

Proposition 5 *Under assumptions 1 through 7 and in the absence of endowment revaluation, the following is true in a RICE.*

- *If $\overline{\xi} \leq \underline{\xi}$, then the equilibrium is informationally Pareto efficient.*

- *If $c > 0$ and at least one signal is acquired in equilibrium, then the equilibrium is not informationally Pareto efficient up to discrete tolerance.*

- *If $c = 0$, then the equilibrium with finite information is informationally Pareto efficient for $R > 0$, whereas the equilibrium with infinitely much information is not Pareto efficient.*

Proof. To prove the first statement note that, if $\overline{\xi} \leq \underline{\xi}$, information benefits are weakly negative by proposition 3 and a social planner would not allocate any signal. For the second statement, if $c > 0$ and at least one signal is acquired in equilibrium, then the equilibrium level of ξ (around ξ^*) must be strictly lower than $\overline{\xi}$, and the marginal benefit term in (15) must be strictly positive. Then the augmented marginal benefit term of the social planner in (16) must strictly exceed marginal cost at the equilibrium level of ξ^*. Up to discrete tolerance, increasing the number of signals by one augments the sum of investors' *ante notitias* utilities.

For the third statement note that, if $c = 0$, the marginal benefit term in (15) must be as close to zero in equilibrium as possible because investors must have chosen a discrete number of signals such that ξ is as close to zero or $\underline{\xi}$ as discretely possible. The equilibrium with finite information always yields higher utility for

all investors than the equilibrium with infinite information since utility losses are incurred as ξ falls from $\underline{\xi}$ to zero by proposition 3. ∎

Even if signals are free with $c = 0$, only the market outcome with finite information is efficient but not the one with infinite information. In other words, as long as the bond is valuable ($R > 0$), neither markets nor the social planner want to remove uncertainty. The reason is that investors in incomplete markets prefer having a second asset around that is not a perfect substitute to the bond. Risk-averse investors want to hold risky assets that yield a positive excess return ξ over opportunity cost. Only if the bond becomes useless and $R \to 0$ ($r \to -1$), unbounded information is Pareto efficient.

Most commonly, the informational efficiency of financial markets is judged with criteria that do not relate to welfare but to the degree of information transmission through asset price. Fama (1970) discerns three degrees of market efficiency in this welfare-independent sense: Strong, semi-strong, and weak. Prices are fully revealing in RICE under assumptions 1, 4 and 6 (corollary 1.1). So, $\mathbb{E}^i[RP - \mathbb{E}^i[RP]] = 0$ and RICE satisfies strong-form efficiency. An alternative statistically well defined measure of the informativeness of a signal is its precision, the inverse of the *ante notitias* variance. Informational efficiency in this non-welfare sense relates to price as a source of information.

The precision of price, a Poisson variable by (8) and fact 3 (appendix B), is

$$\frac{1}{\mathbb{E}^i_{ante}[\mathbb{V}^i(P|\theta)]} = \frac{\bar{\beta}^2}{\bar{\alpha}} \frac{\left((1+\overline{\xi}) + \sum_{i=1}^I N^{i*}/\bar{\beta}\right)^2}{\sum_{i=1}^I N^{i*}/\bar{\beta}}.$$

So, the precision of price can fall with the number of signals purchased. For

$$\frac{\partial}{\partial N^i}\left(\frac{1}{\mathbb{E}^i_{ante}[\mathbb{V}^i(P|\theta)]}\right) = \frac{\bar{\beta}}{\bar{\alpha}} \frac{(\sum_{i=1}^I N^{i*}/\bar{\beta})^2 - (1+\overline{\xi})^2}{(\sum_{i=1}^I N^{i*}/\bar{\beta})^2},$$

each additional signal reduces the precision of the market clearing price if the amount of pre-existing information $\sum_{k=1}^I N^{k,*}$ is small.

Proposition 6 (Precision loss of the price system). *In asset market REE under assumptions 1 through 6, the* ante notitias *precision of the price system decreases with every additional signal if and only if* $\sum_{k=1}^I N^{k,*}/\bar{\beta} < 1 + \overline{\xi}$.

The fact that precision of price can fall with the number of signals purchased may seem surprising at first. However, each investor anticipates that she and all others will respond to signals in their portfolio choice. From an *ante notitias*

perspective, asset demand (8) can become more volatile with the anticipated arrival of information. The expected variance of asset demand is

$$\mathbb{E}^i_{ante} \left[\mathbb{V}^i \left(x^{i*} | \theta \right) | RP \right] = \frac{\bar{\alpha}}{\bar{\beta} A^2 (RP)^2} \left(\sum_{k=1}^{I} N^k \right)$$

by fact 3. Financial markets need to clear. So, every investor ends up holding $\bar{x}$ risky assets in equilibrium by proposition 4, irrespective of information. Hence, market price has to fully absorb any demand moves that stem from information revelation. As a consequence, the variance of price can increase with more information acquisition. When there is relatively little pre-existing information $\sum_{k=1}^{I} N^{k,*}$, an additional signal will affect individual demands strongly and thus add to the price's variance. If, on the other hand, a lot of information is available already, an additional signal that gets fully revealed through price will move investors' demands little. If investors receive many signals, an additional piece of information is likely to confirm previous observations and tends to stabilize demand. So, equilibrium price is expected to become less volatile with an additional signal if the pre-existing information level $\sum_{k=1}^{I} N^{k,*}$ is high.

Rational investors completely internalize this change in price volatility when they maximize their *ante notitias* utility. In that sense, the precision of price is irrelevant for the Pareto efficiency of RICE.

6 Information Market Equilibrium with Heterogeneous Investors

Signals raise asset price $\mathbb{E}^i_{ante}[P]$ *ante notitias* by proposition 2. So, investors who are endowed with the risky asset x_0^i experience a positive endowment revaluation effect of signal acquisition *ante notitias*. To distinguish between the diminishing effect of information on the expected relative excess return ξ and the positive endowment revaluation effect of information, it is instructive to define the *relative risky asset endowment* of investor i as

$$\omega^i \equiv \frac{x_0^i}{\bar{x}} \in [0, I].$$

In the extreme that sections 4 and 5 considered, one investor j owned all assets $\omega^j = I$ in the risky project, while all other investors $i \neq j$ did not own any asset initially ($\omega^i = 0$).

For heterogeneous investors with arbitrary endowments $x_0^i = \omega^i \bar{x}$ and Poisson-
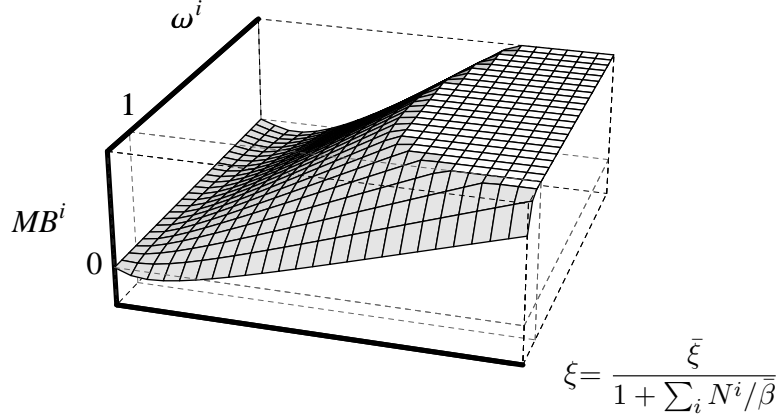
The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$.

Figure 6: **Information benefits for investors with heterogeneous endowments**

gamma signal-return distributions, *ante notitias* expected utility (7) becomes

$$
\begin{aligned}
\mathbb{E}^i_{ante}\left[U^i\right] \;=\; & -\delta^i \exp\left\{-A\tfrac{R}{1+R}(W^i_0 - cN^i)\right\} \\
& \times \left[1 + \left(\left[(1+\xi)\exp\left\{\tfrac{\xi}{1+\xi}(\omega^i-1)\right\}\right]^{\frac{1}{1+R}} - 1\right)\tfrac{\bar{\xi}}{\xi}\right]^{-\bar{\alpha}}
\end{aligned}
\tag{17}
$$

(see appendix E). Differentiating (17) with respect to the number of signals yields

$$
-\frac{1}{\mathbb{E}^i_{ante}[U^{i*}]}\frac{\partial \mathbb{E}^i_{ante}[U^{i*}]}{\partial N^i} \;=\; -A\tfrac{R}{1+R}\,c
\tag{18}
$$

$$
+\frac{\bar{\alpha}}{\bar{\beta}}\frac{\left[(1+\xi)\exp\left\{\tfrac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}}\left(1 - \tfrac{1}{1+R}\tfrac{\xi(\xi+\omega^i)}{(1+\xi)^2}\right) - 1}{1 + \left(\left[(1+\xi)\exp\left\{\tfrac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\tfrac{\bar{\xi}}{\xi}}.
$$

So, the potential marginal benefit MB^i of a signal depends on ω^i and is investor specific.

Figure 6 depicts the range of individual marginal benefit schedules MB^i by relative risky asset endowment $\omega^i \in [0, I]$. The schedule in the $\omega^i = 0$ plane is identical to that in Figure 2 (section 4). As the graph in Figure 6 shows for

varying levels of ω^i, the basic monotinicity properties of the individual marginal benefit schedules MB^i resemble those of Figure 2 when investors had zero endowments of the risky asset (proposition 3). For every investor, there is an endowment-specific cutoff level of the expected relative excess return $\underline{\xi}^i$ beyond which the individual marginal benefit schedule MB^i turns strictly positive and increases in ξ unboundedly.

Proposition 7 (Potential marginal signal benefit). *Under assumptions 1 to 7, the following is true for the potential marginal benefit* $\mathrm{MB}(\xi, \omega^i)$.

- *The potential marginal benefit* $\mathrm{MB}(\xi, \omega^i)$ *of a signal attains strictly positive values if and only if* $\xi > \underline{\xi}^i$, *where* $\underline{\xi}^i \in (0, \infty)$ *is investor specific in* ω^i *but independent of* $\overline{\xi}$ *and uniquely solves* $\mathrm{MB}(\underline{\xi}^i, \omega^i) = 0$ *given* $R \in (0, \infty)$.

- *If* $\underline{\xi}^i < \overline{\xi}$ *then, in the range* $\xi \in [\underline{\xi}^i, \overline{\xi}]$, *the marginal benefit* $\mathrm{MB}(\xi, \omega^i)$ *strictly monotonically increases in* ξ *and is unbounded for arbitrarily large* ξ.

Proof. See appendix F. ∎

Similar to proposition 3, proposition 7 shows that there is always a market size $\bar{x}$, or a degree of risk aversion A, or a level of the prior mean-variance ratio of the risky asset $\bar{\beta}$ behind $\overline{\xi}$ so that, for any investor i with endowment ω^i, at least one costly signal becomes worthwhile to acquire in equilibrium. However, incentives for information acquisition vary with risky asset endowments.

Figure 7 depicts four sections of the graph in Figure 6, along individual marginal benefit schedules MB^i, for four relative risky asset endowments ω^i. These sections could represent an economy with eleven investors, for instance, where eight investors hold $\omega^i = 1/8$ and one investor each holds $\omega^i = 0$, $\omega^i = 1$ and $\omega^i = 8$.

Information demand is intricately tied to investors' risky asset endowments in RICE. As proposition 8 below will confirm formally, there is a single *dominant investor* with an above-average endowment of the risky asset. This dominant investor's marginal valuation of signals dominates everyone else's valuation so that she single-handedly determines the information market outcome. In the sample economy of Figure 7, the average investor κ with $\omega^\kappa = 1$ has the strongest incentive for information acquisition among eleven investors and continues to acquire signals until the expected relative excess return ξ is diminished into a neighborhood around ξ_κ^*. All other investors would stop acquiring signals earlier: at some expected relative excess returns $\xi_\omega^* > \xi_\kappa^*$. The dominant investor κ's endowment revaluation effect is so strong that the individual marginal signal benefit MB^κ never turns negative for any level of the expected relative excess

29
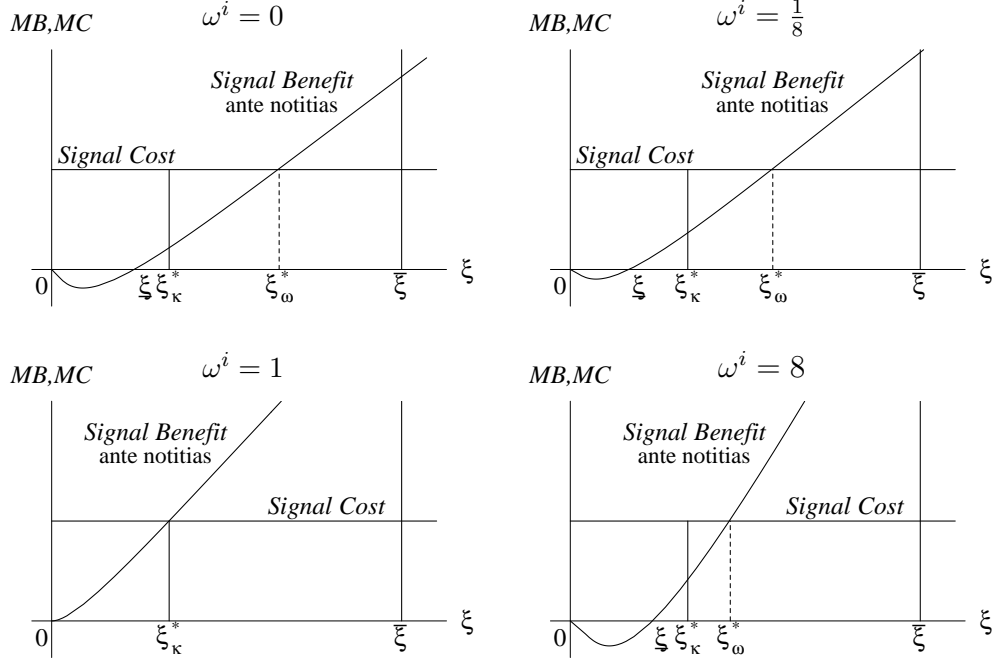
The expected relative excess return $\xi = \bar{\xi}/(1 + \sum_i N^i/\bar{\beta})$ strictly decreases in $\sum_i N^i$. Parameters: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 7: **Information equilibrium for investors with heterogeneous endowments**

return ξ. Proposition 8 will show that the dominant investor is to be found in an open set of investors with endowments of $\omega^\kappa = 1$ and above.

However, for investors outside the open set of endowments of $\omega^\kappa = 1$ and above, the individual marginal signal benefit MB^i can turn strictly negative even in the presence of the endowment revaluation effect. Figure 7 shows for $\omega^i = 1/8$ and $\omega^i = 8$, for instance, that the individual marginal benefit schedules MB^i dip into the strictly negative range below some minimal expected relative excess return $\underline{\xi}^i$. For these investors, the endowment revaluation effect of information does not generally outweigh the utility loss from a diminishing excess return. So, when the distribution of risky asset endowments is very unequal so that many investors hold risky asset endowments far from average, the dominant investor's information choice may inflict a strict negative externality on a majority of investors. If, on the other hand, investors' risky asset endowments are distributed closely around the market average, the endowment revaluation effect makes signals similarly valuable to all investors.

The preceding sections 4 and 5 considered a sole owner j of the risky project with $\omega^j = I$ but ignored her incentives for information acquisition. The upper left and lower right graphs in figure 2 exemplify (for a sample economy with $I = 8$ investors, seven of whom hold $\omega^i = 0$ while one owns $\omega^j = I = 8$) that a sole owner may not value signals. In fact, the marginal signal benefit approaches negative infinity for the sole owner of a risky project as her relative risky asset endowment ω^j (the project size $I\bar{x}$) increases for a given average endowment $\bar{x}$ (claim 4 in appendix G).

The individual marginal benefit of a signal in equation (18) involves the expected relative excess return ξ and investor i's relative risky asset endowment ω^i in non-algebraic ways. Accordingly, proposition 8 can only state results for intervals of endowments. Characteristics of the individual marginal benefit $MB(\xi, \omega^\kappa)$ in these intervals determine key properties of RICE when risky asset endowments are heterogeneous.

Proposition 8 (Dominant Investor Valuation of Signals). *Under assumptions 1 through 7, a RICE has the following properties for any $R \in (0, \infty)$.*

- *For any investor i with relative risky asset endowment $\omega^i \in [0, I]$, there exists a market environment $\overline{\xi} > \underline{\xi}^i$ so that investor i acquires at least one costly signal in equilibrium ($c > 0$), where $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ solves $\mathrm{MB}(\underline{\xi}^i, \omega^i) = 0$.*

- *The individual marginal signal benefit $\mathrm{MB}(\xi^*, \omega^\kappa)$ is maximal in equilibrium for a unique dominant investor κ with relative risky asset endowment $\omega^\kappa_{\max MB} \in (1, 1 + R(1 + \overline{\xi}))$. This investor determines the total number of signals $\sum_{k=1}^{I} N^{k,*}$ in equilibrium and diminishes expected relative excess return to ξ^*.*

- *The individual marginal benefit $\mathrm{MB}(\xi^*, \omega^i)$ at expected relative excess return ξ^* is strictly positive in an open interval Ω^+ of risky asset endowments that includes $[1, \omega^\kappa] \subset \Omega^+$.*

- *If the cost of a signal is strictly positive, then the market equilibrium for signals is unique up to a permutation of the signal allocation.*

- *If the cost of a signal is nil but $R > 0$, and if there is at least one investor with a risky asset endowment $\omega^i \in [1, \omega^\kappa]$, then the unique signal market equilibrium involves an infinite amount of freely received signals.*

Proof. See appendix G. ∎

There is a unique investor, with relative risky asset endowment $\omega^{\kappa}_{\max MB} \in (1, 1 + R(1 + \bar{\xi}))$ for whom the incentives to acquire information strictly exceed those of any other investor. For investors with relative risky asset endowments below or above $\omega^{\kappa}_{\max MB}$, the diminishing effect of signals on the expected excess return ξ weighs more heavily and the endowment revaluation effect does not provide as strong an incentive for information acquisition. So, the investor with relative risky asset endowment $\omega^{\kappa}_{\max MB}$ determines the information market outcome. This investor κ will continue acquiring signals and diminish the expected relative excess return ξ until the total number of signals $\sum_{k=1}^{I} N^{k,*}$ satisfies her first-order condition (18) for signal demand.

Investors with endowments in an open interval around $[1, \omega^{\kappa}]$ strictly benefit from investor κ's additional information choice since their marginal utility benefit of signals is strictly positive and they do not have to pay for the public good. However, information acquisition creates a two-group society of investors. The endowment revaluation effect of more signals strictly outweighs the diminishing effect on the expected excess return ξ for a first group of investors in an open set Ω^+ of relative risky asset endowments (which includes $[1, \omega^{\kappa}] \subset \Omega^+$). Given the choice of *free* signals, they would remove all uncertainty from the market—just to enjoy the endowment revaluation. It remains a question for further research how the cost of information acquisition would have to relate to endowment effects to prevent unbounded information acquisition. For the second group of investors, endowments are either too small or too large so that the diminishing effect on the expected excess return starts to outweigh the endowment revaluation effect at some small enough ξ. This second group suffers a strict negative externality on their *ante notitias* utility from the rush to information of the first group of investors.

7   Related Literature

Radner (1979) and Allen (1981) lay the grounds for REE under fully or partly revealing prices. These papers and a series of further contributions establish that a fully revealing rational expectations equilibrium at Wall Street generically exists for real assets (Jordan 1982, Citanna and Villanacci 2000a) in the absence of information acquisition (but not necessarily for nominal assets, Rahi 1995). Wang (1993) and several other authors (e.g. Einy, Moreno and Shitovitz 2000, Citanna and Villanacci 2000b) investigate the informational properties of REE—that is, how partly or fully revealing prices aggregate exogenously available information. Easley and O'Hara (2004) analyze how differential information affects asset prices. However, these papers stop short of investigating the resulting incentives for investors to acquire information in the first place.

Grossman and Stiglitz (1980) introduce the acquisition of financial information into Walrasian REE. While they refute the existence of a joint equilibrium in signal and asset markets when there is a continuum of investors, a well defined joint signal and asset market equilibrium does exist in their model, as in any model with additive signal-return distributions under CARA, for an arbitrarily large but finite number of investors (Muendler 2004). Similarly, the present model of rational Bayesian information choice under conjugate prior updating has a well defined fully revealing equilibrium for a finite number of investors.

In a setting of market makers, rather than in Walrasian REE, Foster and Viswanathan (1993) and Holden and Subrahmanyam (1996) give investors a choice of information. Their equilibrium concept resembles the one of Grossman and Stiglitz (1980) in that investors have only a binary choice of becoming informed or remaining uninformed. While disregarding the market making process and returning to REE for tractability, the present model gives investors the choice of a number of signals and thus allows for the derivation of a well defined law of demand for financial information based on the marginal utility benefit of signals.

Closely related recent papers of information acquisition in financial market REE are Calvo and Mendoza (2000), who show that larger markets diminish gains from information acquisition in the presence of short-selling constraints, Popper and Montgomery (2001), who derive utility benefits from information sharing among investors, and Veldkamp (2004), who shows that fixed costs of information acquisition can cause alternating low-price equilibria with little information and high-price equilibria with much information. Related also are earlier models by Jackson (1991) with risk neutral investors who set price, by Jackson and Peck (1999) with risk neutral investors who submit demand functions, and by Barlevy and Veronesi (2000) with risk neutral investors in a Walrasian REE. However, the marginal utility benefit of signals is zero for risk neutral Bayesian investors by the Law of Iterated Expectations. This casts some doubt on the generality of results in models with risk neutral investors.

The present expected utility model of financial information choice under conjugate prior signal-return distributions lends itself to revisiting four issues that were the subject of prior approaches: Conditions for unbounded information acquisition, the strategic complementarity of signals, the relationship between information acquisition and market size, and the response of price precision to information acquisition.

- Contrary to the Burguet and Vives (2000) result for risk neutral investors that unbounded information acquisition is prevented if and only if the marginal cost of information is positive, the present model yields an equilibrium with a finite amount of information even if signals cost nothing.

So, unbounded information acquisition is prevented *if* the marginal cost of information is positive, but not only in this case. Moreover, bounded information strictly Pareto dominates unbounded information in the present rational Bayesian model with incomplete markets. Only once the riskfree bond becomes useless, wiping out the principal with certainty, do investors prefer to remove all risk from a risky asset by receiving infinitely much costless information.

- Being public goods (or bads depending on market conditions), signals are perfect strategic substitutes in the present rational Bayesian model of information choice. This is in accordance with the Grossman and Stiglitz (1980) and the Burguet and Vives (2000) frameworks. Considering values of subsequent signals, rather than the strategic interaction of players, Admati and Pfleiderer (1987) call two signals complements (substitutes) if the value of the second signal increases (decreases) after acquisition of the first signal. Given partially revealing price Admati and Pfleiderer (1987) find conditions for complementarity because more private signals improve the precision of the posterior belief about a statistic of everyone else's information in asset price. Under fully revealing price as in the present paper, acquired signals strictly reduce the marginal utility value of subsequent signals.

- Grossman and Stiglitz (1980, conjecture 7) state that markets are thinner in cases of very little or extremely much information. Foster and Viswanathan (1993) consider a similar issue. In the present setup, one can turn the question around and ask how the aggregate amount of information changes with market size. The thinner markets are in the present rational information choice model, that is the lower the average asset holdings per investor, the less aggregate information is available in equilibrium.

- The response of the *ante notitias* precision of price to private information varies in earlier models. Grossman and Stiglitz (1980) conjecture that "the more individuals who are informed, the more informative is the price system" but cannot confirm this conjecture because positive and negative effects offset each other in their model, and informativeness of price remains constant. Verrecchia (1982) confirms the conjecture in a competitive REE under partially revealing price. The present rational information choice model shows that the *ante notitias* precision of the price may rise or fall with more information, depending on the amount of prior information. From an *ante notitias* perspective, asset demand can become more volatile with the anticipated arrival of information so that the expected allocative

response of price may outweigh the expected informational role and a larger volume of information may cause a loss of precision.

There is a large body of alternative approaches to investor behavior. Benabou and Laroque (1992) and Avery and Zemsky (1998), to name but two contributions, rationalize herding behavior in financial markets. Incentives for information acquisition have also been analyzed in abstract contexts of learning and experimentation (Moscarini and Smith 2001, Bergemann and Välimäki 2002, Cripps et al. 2005). However, none of those models can assign a rational marginal utility value to signals in a market context since there is no anticipated REE response of prices to signal realizations in these settings. Datta, Mirman and Schlee (2002) consider a generalized optimal experimentation model in which signal realizations are allowed to enter future payoffs directly, and not just through beliefs, but do not explicitly account for equilibrium price responses to information. In contrast, the present paper shows that both the asset market environment and the distribution of risky asset endowments are intricately linked to the marginal utility benefit of signals since signals alter *ex ante* expected equilibrium price and thus change expected excess returns and endowment values.

8 Conclusion

This paper has shown that a well defined rational information choice equilibrium (RICE) in asset markets exists for an extension of the standard expected utility model of portfolio choice to signal acquisition. While the equilibrium on the signal acquisition stage that precedes the asset market equilibrium involves variables in non-algebraic ways, key properties of RICE can nevertheless be derived. Most importantly, RICE establishes a law of demand for financial information by which the marginal utility benefit of an additional signal is strictly positive if and only if investors are risk averse, supply of the risky asset is sufficiently large, and the prior mean-variance ratio of the risky asset is sufficiently low. The positive marginal utility benefit strictly falls in the number of signals.

Financial information not only changes its utility benefit with market conditions. In reducing uncertainty, financial information raises expected asset price and thus also affects investors' portfolio positions *ante notitias*. So, the value of signals is intricately linked to the distribution of risky asset endowments across heterogeneous investors.

The rational Bayesian model of this paper can easily be generalized to a model with a finite number of assets and conditionally independent signals on individual asset returns. The realism and convenience of the Poisson-gamma pair of distributions notwithstanding, several results in this paper carry over to other

signal-return distributions such as the normal-normal pair. Any distribution in the exponential family possesses both a moment generating function and a conjugate prior distribution so that the framework of this paper generally applies to distributions in the exponential family. However, the marginal utility benefit of signals will depend on symmetry properties and higher moments of the asset and return distributions.

More substantive variations and extensions remain for future work. They include an analysis of information values in complete markets, an investigation of partially revealing equilibrium, and the consideration of investors who engage in strategic demand decisions to partly conceal their information. However, the key driving force behind results in the present benchmark with fully revealing asset price is the diminishing effect of information on an asset's excess return because a statistic of private signals is publicly inferrable from price. Neither complete markets nor partially revealing equilibrium nor strategic investors can make asset price completely uninformative, or else price would lose its entire allocative function, so that information will continue to diminish excess returns in those settings albeit in a mitigated manner.

Appendix

A   Optimality conditions and portfolio value

Define $t \equiv -Ax^i \in (-\infty, 0)$ for the moment generating function (MGF) $M_{\theta|\mathcal{F}^i}(t)$. Maximizing (3) over x^i and b^i for CARA (assumption 2 and 4) yields the first-order conditions

$$\frac{P}{\rho^i} = H^i \, M'_{\theta|\mathcal{F}^i}(t) \quad \text{and} \quad \frac{1}{\rho^i R} = H^i \, M_{\theta|\mathcal{F}^i}(t), \tag{A.1}$$

where $H^i \equiv \exp\{-A[(1+R)b^i + Px^i - W_0^i - cN^i]\}$. Dividing the latter by the former equation implies equation (5) in the text as a necessary condition. Note that H^i, W_0^i, C_1^i and C_0^i are functions of $\mathcal{F}^i$ since RP depends on $\mathcal{F}^i$.

With the definition of H^i, the optimal portfolio value can be written

$$
\begin{aligned}
b^i + Px^i &= \tfrac{1}{1+R} \left(W_0^i - cN^i + RP \, x^i - \tfrac{1}{A} \ln H^i \right) \\
&= \tfrac{1}{1+R} \left[b_0^i + RP(x_0^i/R + x^i) + \tfrac{1}{A} \ln \rho^i R M_{\theta|\mathcal{F}^i}(-Ax^i) - cN^i \right],
\end{aligned}
\tag{A.2}
$$

where the second line follows from the bond first-order condition in (A.1).

The matrix of cross-derivatives for the two assets b^i and x^i reflects the second-order conditions:

$$\mathbf{B} = -A^2 \rho^i \exp\{-ARb^i\} \begin{vmatrix} R(1+R)M_{\theta|\mathcal{F}^i}(t) & \cdot \\ (1+R)M'_{\theta|\mathcal{F}^i}(t) & PM'_{\theta|\mathcal{F}^i}(t) + M''_{\theta|\mathcal{F}^i}(t) \end{vmatrix} \tag{A.3}$$

by (A.1). If $\mathbf{B}$ is negative definite, a unique global utility maximum results. Equivalently, we require $-\mathbf{B}$ to be positive definite and all upper-left sub-matrices must have positive determinants. Since the upper-left entry in $\mathbf{B}$ is strictly positive, negative definiteness of $\mathbf{B}$ is equivalent to

$$\det(-\mathbf{B}) = A^4 (\rho^i)^2 \exp\{-2ARb^i\} R(1+R) \left[M''_{\theta|\mathcal{F}^i}(t) M_{\theta|\mathcal{F}^i}(t) - M'_{\theta|\mathcal{F}^i}(t)^2 \right] > 0,$$

which in turn is equivalent to

$$\frac{M''_{\theta|\mathcal{F}^i}(t)}{M_{\theta|\mathcal{F}^i}(t)} - \left(\frac{M'_{\theta|\mathcal{F}^i}(t)}{M_{\theta|\mathcal{F}^i}(t)} \right)^2 > 0 \tag{A.4}$$

since $M_{\theta|\mathcal{F}^i}(t) > 0$. This condition implies that $M'_{\theta|\mathcal{F}^i}(t)/M_{\theta|\mathcal{F}^i}(t)$ strictly monotonically increases in t, or strictly monotonically decreases in x^i for $t \equiv -Ax^i$.

B   Properties of the Poisson and gamma distributions

Fact 1 in the text states how Poisson signals update beliefs about gamma distributed returns. This appendix lists further useful properties of Poisson and gamma distributions

B.1   Poisson signals

Poisson distributed signals $S_n^i|\theta \overset{i.i.d.}{\sim} \mathcal{P}(\theta)$ have a density

$$f\left(s_n^i \mid \theta\right) = \begin{cases} \exp\{-\theta\}\,\theta^{s_n^i}/s_n^i! & \text{for } s_n^i > 0 \\ 0 & \text{for } s_n^i \leq 0 \end{cases}$$

Fact 2 (Poisson MGF). *The MGF of a Poisson signal is*

$$M_{S|\theta}(t) = \exp\{\theta(\exp\{t\} - 1)\}.$$

Proof. Casella and Berger (1990). ∎

Fact 3 (Sum of Poisson signals). *The sum of N independently Poisson distributed signals with a common mean and variance θ, $S_1+...+S_N$, has a Poisson distribution with parameter $N\theta$.*

Proof. The distribution of the sum of N independent Poisson variables is the product $\Pi_{n=1}^N f\left(s_n^i \mid \theta\right) = \exp\{-N\theta\}\,\theta^{\sum_{n=1}^N s_n^i}/\sum_{n=1}^N s_n^i!$, a Poisson distribution with parameter $N\theta$. ∎

B.2   Gamma returns

Given an individual investor i's information set $\{\alpha^i, \beta^i\}$, the risky asset return is distributed $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$ so that its density is

$$\pi\left(\theta \mid \alpha^i, \beta^i\right) = \begin{cases} (\beta^i)^{\alpha^i}\,\theta^{\alpha^i-1}\,\exp\{-\beta^i\theta\}/\Gamma(\alpha^i) & \text{for } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

where the gamma function is given by $\Gamma(\alpha^i) \equiv \int_0^\infty z^{\alpha^i-1}e^{-z}\,\mathrm{d}z$. The two parameters α^i and β^i must be positive.

Fact 4 (Gamma MGF). *The MGF of a gamma distributed return is*

$$M_{\theta|\alpha^i,\beta^i}(t) = \left(\frac{\beta^i}{\beta^i - t}\right)^{\alpha^i}.$$

Proof. Casella and Berger (1990). ∎

C Sufficient and necessary conditions for fully revealing price

Corollary 1.1 *Suppose expected utility is CARA (assumptions 2 and 3), signals are Poisson distributed and the asset return is gamma distributed (assumption 5). Then equilibrium price P fully reveals all market participants' information $\sum_{i=1}^{I} \sum_{n=1}^{N^i} s_n^i$ in RICE if and only if*

- *signals are conditionally independent (assumption 1),*

- *investors know average prior beliefs, share a common degree of risk aversion (assumption 4), know market size (assumption 6), and*

- *investors know the total number of all other investors' signals $\sum_{k=1}^{I} N^k$ at the time of portfolio choice.*

Proof. Proposition 1 establishes sufficiency. Necessity of assumptions 4 and 6 follows by inspection of the general solution for market price given individual beliefs, based on heterogeneous priors, $\alpha^i = \bar{\alpha}^i + \sum_{n=1}^{N^i} s_n^i$ and $\beta^i = \bar{\beta}^i + N^i$, and arbitrary degrees of risk aversion A^i:

$$RP = \frac{\frac{1}{I}\sum_{i=1}^{I} \frac{\alpha^i}{A^i}}{\bar{x} + \sum_{i=1}^{I} \frac{\beta^i}{A^i}} = \frac{\left(\frac{1}{I}\sum_{i=1}^{I} \frac{\bar{\alpha}^i}{A^i}\right) + \frac{1}{I}\sum_{i=1}^{I} \frac{1}{A^i}\sum_{n=1}^{N^i} s_n^i}{\bar{x} + \left(\frac{1}{I}\sum_{i=1}^{I} \frac{\bar{\beta}^i}{A^i}\right) + \frac{1}{I}\sum_{i=1}^{I} \frac{1}{A^i}N^i}.$$

If investors have a common degree of risk aversion $A^i = A$, only knowledge of the average prior beliefs $\frac{1}{I}\sum_{i=1}^{I} \bar{\alpha}^i$ and $\frac{1}{I}\sum_{i=1}^{I} \bar{\beta}^i$ is necessary to make price fully revealing.

Assumption 1 is necessary since investor i would not know the correlation between RP and her signals if perfect copies or correlated signals had been sent to other investors. If $\sum_{k=1}^{I} N^k$ were unknown to investor i, she would not be able to extract the sufficient statistic $\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$ from price. ∎

D Bond return response to stock return information

Taking logs of both sides of the bond first-order condition in (A.1) yields

$$A(1+R)b^i - Ab_0^i + AP(x^i - x_0^i) = \ln[\rho^i RM_{\theta|\mathcal{F}^i}(-A\,x^i)] + AcN^i,$$

a permissible operation since $\rho^i, R, M_{\theta|\mathcal{F}^i}(\cdot) > 0$ by their definitions. Summing up both sides over investors i and dividing by their total number yields

$$AR\bar{b} - \ln \rho^i R - \ln M_{\theta|\mathcal{F}^i}(t) - Ac\sum_{k=1}^I N^k/I = 0 \qquad (D.1)$$

where $\bar{b} \equiv \sum_{i=1}^I b_0^i/I$ is the average initial bond endowment per investor and $t \equiv -A\bar{x}$. Equation (D.1) implicitly determines the gross bond return R. *Post notitias*, $M_{\theta|\mathcal{F}^i}(t)$ and R respond to the signal realization. Define $\bar{s} \equiv \sum_{k=1}^I \sum_{n=1}^{N^k} s_n^k$.

Applying the implicit function theorem to (D.1) for the MGF of the gamma distribution $M_{\theta|\alpha,\beta}(t) = [\beta/(\beta - t)]^\alpha$ yields

$$\frac{dR}{d\bar{s}} = -\frac{\ln(1 + \xi)}{A\bar{b} - 1/R}$$

for $\alpha = \bar{\alpha} + \bar{s}, \beta = \bar{\beta} + \sum_{k=1}^I N^k$ by (10) and $\xi = A\bar{x}/\beta$ given $\sum_{k=1}^I N^k$. The bond return falls in response to a favorable signal realization $\bar{s}$ iff $\bar{b} > 1/(AR)$. So, in principle, R too is a function of the signal realization $\bar{s}$. For large bond endowments $\bar{b}$, however,

$$\lim_{\bar{b}\to\infty} dR/d\bar{s} = 0.$$

Similarly, $dR/d\bar{s} = 0$ for $\xi = \bar{x} = 0$.

E   *Ante notitias* expected indirect utility

The following property of the Poisson-gamma signal-return distributions proves useful for the derivation of *ante notitias* expected indirect utility.

Fact 5 (Expected signal effect on utility). *For two arbitrary constants B and ξ, $\bar{N}$ Poisson distributed signals $S_1, ..., S_{\bar{N}}$ and a conjugate prior gamma distribution of their common mean θ, the following is true:*

$$\mathbb{E}_{ante}\left[(1+\xi)^{-B\cdot\sum_{n=1}^{\bar{N}} s_n} \cdot \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\cdot\sum_{n=1}^{\bar{N}} s_n\right\}\right]$$

$$= (1+\xi)^{\bar{\alpha}B}\exp\left\{\bar{\alpha}\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\left(1 + \left[(1+\xi)^B\exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}B\right\} - 1\right]\frac{\beta}{\bar{\beta}}\right)^{-\bar{\alpha}},$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the parameters of the prior gamma distribution of θ, and $\beta = \bar{\beta} + \bar{N}$ is the according parameter of the post notitias *distribution.*

Proof. By the Law of Iterated Expectations $\mathbb{E}_{ante}\left[\cdot\right] = \mathbb{E}_\theta\left[\mathbb{E}\left[\cdot\,|\theta\right]\right]$. The 'inner' expectation $\mathbb{E}\left[\cdot\,|\theta\right]$ is equal to

$$
\begin{aligned}
\mathbb{E}\left[\cdot\,|\theta\right] &= \sum_{(\sum_{n=1}^{\bar{N}} s_n)=0}^{\infty} (1+\xi)^{-B \sum_{n=1}^{\bar{N}} s_n} \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi} B \sum_{n=1}^{\bar{N}} s_n\right\} f\left(\sum_{n=1}^{\bar{N}} s_n\right) \\
&= \exp\left\{-\bar{N}\theta\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)\right\},
\end{aligned}
$$

because the sum $\sum_{n=1}^{\bar{N}} s_n$ is Poisson distributed with mean $\bar{N}\theta$ (fact 3). Thus, by the MGF of a gamma distribution (fact 4),

$$
\begin{aligned}
\mathbb{E}_{ante}\left[\cdot\right] &= \mathbb{E}_\theta\left[\exp\left\{-\theta\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta-\bar{\beta})\right\}\right] \\
&= (\bar{\beta})^{\bar{\alpha}}\left(\bar{\beta}+\left(1-(1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta-\bar{\beta})\right)^{-\bar{\alpha}}.
\end{aligned}
$$

since $\bar{N} = \beta - \bar{\beta}$ (fact 1). Simplifying the last term and factoring out $(1+\xi)^B \exp\{\frac{\xi(\omega^i-1)}{1+\xi}B\}$ proves fact 5. ∎

For a gamma distributed asset return, *post notitias* expected indirect utility (6) becomes

$$
\mathbb{E}^i\left[U^{i*}\right] = -\delta^i \exp\left\{-A\frac{R}{1+R}(W_0^i - cN^i)\right\} \exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}\right\}^{-\frac{\alpha^i}{1+R}}(1+\xi)^{-\frac{\alpha^i}{1+R}} \quad \text{(E.1)}
$$

where $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$ is the relative endowment of investors with the risky asset, and $\xi \equiv A\bar{x}/\beta$. With fact 5 at hand, one can set $B \equiv 1/(1+R)$ (by assumption 7) and obtains *ante notitias* expected utility (14) for $\omega^i = 0$ and (17) for arbitrary $\omega^i \in [0, I]$.

F   Monotone marginal signal benefit schedule (proof of propositions 3 and 7)

Define the relative endowment of investors with the risky asset as $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$. The expected relative excess return ξ is bounded by $\xi \in (0, \bar{\xi}]$. Under assumptions 1 through 7, the potential marginal benefit $MB(\xi, \omega^i)$ of a signal is

$MB(\xi, \omega^i) = g(\xi, \omega^i)/h(\xi, \omega^i)$ by (18) with

$$m(\xi, \omega^i) \equiv \left[(1+\xi) \exp\left\{ \frac{\xi(\omega^i - 1)}{1+\xi} \right\} \right]^{\frac{1}{1+R}}, \tag{F.1}$$

$$h(\xi, \omega^i) \equiv 1 + \left[m(\xi, \omega^i) - 1 \right] \frac{\overline{\xi}}{\xi}, \tag{F.2}$$

$$g(\xi, \omega^i) \equiv -\frac{\xi^2}{\overline{\xi}} \frac{\partial h(\xi, \omega^i)}{\partial \xi} = m(\xi, \omega^i) \left(1 - \frac{1}{1+R} \frac{\xi(\xi + \omega^i)}{(1+\xi)^2} \right) - 1. \tag{F.3}$$

Proposition 3 is a special case of proposition 7 for $\omega^i = 0$. The proof of proposition 7 proceeds in four steps.

First, claim 1 sates useful properties of $m(\xi, \omega^i)$ for the discussion of $g(\xi, \omega^i)$ and $h(\xi, \omega^i)$. Second, claim 2 establishes that the numerator $g(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that it is not bounded above. So, the numerator boosts the marginal benefit $MB(\xi, \omega^i)$ higher and higher as ξ rises. Third, claim 3 establishes that the denominator $h(\xi, \omega^i)$ is bounded below and above in the positive range, and that it strictly decreases in ξ iff the numerator is strictly positive. So, the denominator cannot explode and boosts the marginal benefit $MB(\xi, \omega^i)$ higher where the potential benefit $MB(\xi, \omega^i)$ is positive. The latter two claims imply that $MB(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that $MB(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, fourth and last, $MB(\xi, \omega^i)$ ultimately attains strictly positive values and continues to strictly increase in that positive range.

Claim 1 $m(\xi, \omega^i)$ *strictly increases in* ω^i; $m(0, \omega^i) = 1$; *and* $m(\xi, \omega^i) > 1$ *for any* $\xi > 0$, $\omega^i \geq 0$ *and* $R \in (0, \infty)$.

Proof. By (F.1), $\partial m(\xi, \omega^i)/\partial \xi = m(\xi, \omega^i)\xi/(1+\xi) > 0$, which establishes the first part of the claim.

Taking natural logs of both sides of (F.1) is permissible since $m(\xi, \omega^i) > 0$ and shows that $m(\xi, \omega^i) \geq 1$ iff $\ln(1+\xi) \geq -\xi(\omega^i - 1)/(1+\xi)$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, consider $\omega^i = 0$. So, $m(\xi, 0) \geq 1$ iff $\ln(1+\xi) \geq \xi/(1+\xi)$. Note that equality holds at $\xi = 0$ but $\ln(1+\xi)$ increases strictly faster in ξ than $\xi/(1+\xi)$ increases in ξ for any $\xi > 0$. So, $m(\xi, 0) \geq 1$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, $m(\xi, \omega^i) \geq 1$. ∎

Claim 2 $g(\xi, \omega^i)$ *strictly increases in* ξ *iff* $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$. *In addition,* $\lim_{\xi \to 0} g(\xi, \omega^i) = 0$ *and* $\lim_{\xi \to \infty} g(\xi, \omega^i) = +\infty$.

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ξ is

$$\frac{\partial g(\xi, \omega^i)}{\partial \xi} = \frac{\xi}{(1+R)^2(1+\xi)^4} \, m(\xi, \omega^i) \left[R(\xi + \omega^i)^2 - (1+R)(\omega^i - 1)^2 \right].$$

So, $\partial g(\xi, \omega^i)/\partial \xi = 0$ at $\xi = 0$ and at $\xi = |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ (the negative root is ruled out by $\xi \geq 0$). Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around the zero points shows that $g(\xi, \omega^i)$ strictly decreases in ξ if $\xi \in (0, |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i)$ and strictly increases if $\xi \in (|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i, \infty)$.

$\lim_{\xi \to 0} g(\xi, \omega^i) = m(0, \omega^i) - 1 = 0$ by claim 1. $\lim_{\xi \to \infty} g(\xi, \omega^i) = -1 + \lim_{\xi \to \infty} \exp\{\xi/(1 + R)\} = +\infty$ since $R \in (0, \infty)$. ∎

Claim 2 implies that there must be a $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ that uniquely solves $g(\underline{\xi}^i, \omega^i) = 0$ because $g(\xi, \omega^i)$ strictly decreases as long as $\xi < |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i)$ but subsequently strictly increases in ξ.

Claim 3 $h(\xi, \omega^i)$ *strictly decreases in ξ iff $g(\xi, \omega^i) > 0$. $h(\xi, \omega^i)$ is bounded in $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ for $\xi \in (0, \overline{\xi}]$ and $R \in (0, \infty)$, where $h(\underline{\xi}^i, \omega^i) > 1$, $\overline{\xi}$ is given by (13) and $\underline{\xi}^i$ solves $g(\underline{\xi}^i, \omega^i) = 0$.*

Proof. By (F.3), $\partial h(\xi, \omega^i)/\partial \xi < 0$ iff $g(\xi, \omega^i) > 0$. So, $h(\xi, \omega^i)$ attains its global maximum at $\underline{\xi}^i$, which solves $g(\underline{\xi}^i, \omega^i) = 0$, and $h(\xi, \omega^i)$ attains its global minimum either for $\xi \to 0$ or for $\xi \to \infty$. By L'Hôpital's rule, $\lim_{\xi \to 0} m(\xi, \omega^i)/\xi - 1/\xi = 0$ so $\lim_{\xi \to 0} h(\xi, \omega^i) = 1$. Similarly, for $R \in (0, \infty)$, $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1$ while $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1 + \overline{\xi} \exp\{\omega^i - 1\}$ for $R \to 0$. This establishes that $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ for $\xi \in (0, \overline{\xi}]$. ∎

Claims 2 and 3 imply that $MB(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that $MB(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, $MB(\xi, \omega^i)$ attains strictly positive values if and only if $\xi > \underline{\xi}^i$, where $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ solves $g(\underline{\xi}^i, \omega^i) = 0$, and $\underline{\xi}^i \in (0, \infty)$ is independent of $\overline{\xi}$ and unique given $R \in (0, \infty)$.

G   Dominant investor valuation of signals (proof of proposition 8)

Define the relative endowment of investors with the risky asset as $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$. The expected relative excess return ξ is bounded by $\xi \in (0, \overline{\xi}]$. Under assumptions 1 through 7, the potential marginal benefit $MB(\xi, \omega^i)$ of a signal is

$MB(\xi, \omega^i) = g(\xi, \omega^i)/h(\xi, \omega^i)$ by (18) with $h(\xi, \omega^i)$ and $g(\xi, \omega^i)$ given by (F.3) and (F.2).

The first statement of proposition 7 follows immediately from appendix G where a general proof of proposition 8 for any ω^i is given. Uniqueness is an implication of the second statement in proposition 7. The proof of the remainder of proposition 7 draws on properties of $g(\xi, \omega^i)$ and $h(\xi, \omega^i)$, which claims 4 and 5 establish. Claim 6 evaluates the potential marginal benefit $MB(\xi, \omega^i)$ of a signal at $\omega^i = 1$. Together, these insights give rise to the remaining statements in proposition 7.

Claim 4 $g(\xi, \omega^i)$ *strictly decreases in* ω^i *iff* $\omega^i > 1 + R(1 + \xi)$. *In addition,* $\lim_{\xi \to \infty} g(\xi, \omega^i) = -\infty$.

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ω^i is

$$\frac{\partial g(\xi, \omega^i)}{\partial \omega^i} = \frac{1}{1+R} \frac{\xi^2}{(1+\xi)^2} m(\xi, \omega^i) \left[R(1 + \xi) - (\omega^i - 1) \right],$$

where $m(\xi, \omega^i)$ is given by (F.1). So, $\partial g(\xi, \omega^i)/\partial \omega^i = 0$ at $\omega^i = 1 + R(1 + \xi)$. Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around this unique zero point shows that $g(\xi, \omega^i)$ strictly increases in ω^i if $\omega^i \in [0, 1 + R(1 + \xi))$ and strictly increases if $\omega^i \in (1 + R(1 + \xi), I]$. So, $\lim_{\omega^i \to \infty} g(\xi, \omega^i) = -\infty$ for $R \in (0, \infty)$ and $\xi \in (, \bar{\xi}]$. ∎

Claim 5 $h(\xi, \omega^i)$ *strictly increases in* ω^i *and is strictly convex in* ω^i *at any* $\xi > 0$.

Proof. The first and second derivatives of $h(\xi, \omega^i)$ with respect to ω^i are

$$\frac{\partial h(\xi, \omega^i)}{\partial \omega^i} = \frac{1}{1+R} \frac{\bar{\xi}}{1+\xi} m(\xi, \omega^i) > 0$$

and

$$\frac{\partial^2 h(\xi, \omega^i)}{\partial (\omega^i)^2} = \frac{1}{1+R} \frac{\xi}{1+\xi} \frac{\partial h(\xi, \omega^i)}{\partial \omega^i} > 0.$$

∎

Claim 6 *The potential marginal benefit* $\mathrm{MB}(\xi, 1)$ *is strictly positive at* $\omega^i = 1$ *for* $\xi > 0, R > 0$. *At* $\omega^i = 1$, *the potential marginal benefit* $\mathrm{MB}(\xi, 1)$ *strictly increases in* ω^i.

Proof. At $\omega^i = 1$, $MB(\xi, 1) > 0$ iff

$$\tfrac{1}{1+R} \ln(1 + \xi) > -\ln\left(1 - \tfrac{1}{1+R}\tfrac{\xi}{1+\xi}\right).$$

Note that equality holds at $\xi = 0$ but the left-hand side increases strictly faster in ξ (it increases by $1/(1+R)(1+\xi)$) than the right-hand side increases (which increases in ξ by $1/(1+\xi)^2[1+R - \xi/(1+\xi)]$) for any $\xi R > 0$. So, $MB(\xi, 1) > 0$.

The first derivative of $MB(\xi, \omega^i)$ with respect to ω^i is

$$\frac{\partial MB(\xi, \omega^i)}{\partial \omega^i} = \left(\frac{\partial g(\xi, \omega^i)/\partial \omega^i}{g(\xi, \omega^i)} - \frac{\partial h(\xi, \omega^i)/\partial \omega^i}{h(\xi, \omega^i)}\right) MB(\xi, \omega^i).$$

So, $\partial MB(\xi, \omega^i)/\partial \omega^i > 0$ at $\omega^i = 1$ iff

$$\left(\frac{\partial g(\xi, \omega^i)/\partial \omega^i}{g(\xi, \omega^i)} \bigg/ \frac{\partial h(\xi, \omega^i)/\partial \omega^i}{h(\xi, \omega^i)}\right)\bigg|_{\omega^i=1} > 1 \qquad\qquad (G.1)$$

since $h(\xi, \omega^i) > 1$ by claim 3 and $\partial h(\xi, \omega^i)/\partial \omega^i > 1$ by claim 5 for $\xi > 0$. A round of simplifications shows that inequality (G.1) is equivalent to

$$\xi(\bar{\xi} + \xi R) > \bar{\xi}(1+R)\left[(1+\xi)^{\frac{1}{1+R}} - 1\right].$$

Note that this condition holds with equality at $\xi = 0$ but the left-hand side increases strictly faster in ξ (it increases by $\bar{\xi}(1+2R\xi/\bar{\xi}) > \bar{\xi}$) than the right-hand side increases (which increases in ξ by $\bar{\xi}(1+\xi)^{-\frac{1}{1+R}} < \bar{\xi}$). So, $\partial MB(\xi, 1)/\partial \omega^i > 0$. ∎

These claims help establish the second and third statements of proposition 7. $g(\xi, \omega^i)$ attains its unique maximum in ω^i at $\omega^i = 1 + R(1 + \xi)$ by claim 4 while $h(\xi, \omega^i)$ strictly increases in ω^i but is convex. So, $MB(\xi, \omega^i)$ must attain its global maximum for some $\omega^\kappa < 1 + R(1 + \xi)$ given ξ. At $\omega^i = 1$, $MB(\xi, 1)$ strictly increases. This proves the second statement that $MB(\xi, \omega^i)$ must attain its unique global maximum for some $\omega^\kappa \in (1, 1 + R(1 + \bar{\xi}))$. The third statement that $MB(\xi, \omega^i) > 0$ in an open interval Ω^+ that includes $[1, \omega^\kappa] \subset \Omega^+$ follows because $MB(\xi, \omega^i)$ is strictly positive and strictly increases at $\omega^i = 1$ for any $\xi > 0$. So, $MB(\xi, \omega^i) > 0$ in an open interval around $\omega^i = 1$. $MB(\xi, \omega^\kappa)$ is maximal at ω^κ so that the open interval Ω^+ must in fact extend to $[1, \omega^\kappa] \subset \Omega^+$. These facts at hand, the fourth and fifth statements of proposition 7 become corollaries of proposition 3.

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