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Asset Class Diversification and Delegation of Responsibilities

between Central Banks and Sovereign Wealth Funds *

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Abstract:

This paper presents a model comparing the optimal degree of asset class diversification abroad by a central bank and a sovereign wealth fund. We show that if the central bank manages its foreign asset holdings in order to meet balance of payments needs, particularly in reducing the probability of sudden stops in foreign capital inflows, it will place a high weight on holding safer foreign assets. In contrast, if the sovereign wealth fund, acting on behalf of the Treasury, maximizes the expected utility of a representative domestic agent, it will opt for relatively greater holding of more risky foreign assets. We also show how the diversification differences between the strategies of the bank and SWF is affected by the government's delegation of responsibilities and by various parameters of the economy, such as the volatility of equity returns and the total amount of public foreign assets available for management.

1. Introduction

In the aftermath of the financial crises of the late 1990s, many countries, particularly in emerging markets, have focused on the accumulation of official reserve assets. More recently, authorities have chosen to entrust more of their public foreign assets to sovereign wealth funds (SWFs). Private analysts put current sovereign wealth fund assets in the range of $2 to 3 trillion or even higher. This amount is projected to grow to as much as $13 trillion in the next ten years, an amount larger than the current global stock of foreign reserves of about $7 trillion at year-end 2009.[1]

There are several reasons for the growth of sovereign wealth funds. First, commodity price increases have swelled the sovereign asset holdings of commodity-exporting countries where the public sector controls commodity exports or heavily taxes the revenues earned by private commodity exporters. In some cases, the assets accumulated by SWFs are used as a financial stabilizer if commodity prices fall and depress tax revenue declines. In other cases, the establishment of SWFs, such as Norway's Government Pension Fund and Chile's Pension Reserve Fund, was motivated by the desire to transform concentrated exposure of public assets to volatile commodity prices into a more balanced and diversified global exposure, thereby protecting the income of future generations. A second factor behind the growth of SWFs is a byproduct of efforts by many emerging market countries to accumulate large stockpiles of international reserves by running persistent current account surpluses, efforts that in many cases have lead reserve holdings to exceed what is needed for purely prudential reasons (see Aizenman 2008; Jeanne, 2007). The goal of diversifying these reserves into potentially higher-yielding assets has been pursued by transferring them from the control of the central bank to a SWF with the mandate to pursue financial strategies aiming at higher long-run returns. For example, China set up the China Investment Corporation (CIC) in 2007 to manage more aggressively a portion of the country's more than $2 trillion in public foreign assets.[2]

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[1] Setser and Ziemba (2009) estimate that assets for major sovereign wealth funds totaled $2.1 trillion at year-end 2008 (including $700 billion in non-reserve assets of Russia and Saudi Arabia, but excluding roughly $300 billion in non-reserve assets held by Hong Kong and other smaller funds). Truman (2008) estimates that SWFs managed just under $3 trillion at the end of 2007 (he also identifies another $2.3 billion under management by sovereign pension funds). His higher figure is partly attributable to higher estimates for funds managed by Abu Dhabi. Forecasts of future global SWF assets depend on a myriad of factors, including the magnitude of current account imbalances, central bank demand for greater liquidity, and changes in commodity prices and asset returns. See Aizenman and Glick (2009).

[2] Aizenman and Glick (2009) present statistical analysis supporting stylized facts about sovereign wealth funds (SWFs). Evidence is provided about the association of SWFs with a country's fuel export performance and current account surpluses as well as the degree to which measures of SWF governance and transparency compare with national governance standards.

The optimal portfolio management of public foreign assets necessitates a balancing of the goals and investment strategies of the central bank with those of the SWF. In most countries, the SWF operates in a fundamentally different manner than the central bank which primarily holds official foreign reserves for precautionary reasons, as liquidity concerns necessitate a short investment horizon and low risk tolerance. Consequently, central banks generally invest their foreign exchange reserves conservatively in safe and marketable instruments that are readily available to monetary authorities to meet balance of payments needs, particularly in responding to sudden stops of foreign capital inflows. In contrast, SWFs typically seek to diversify foreign exchange assets and earn a higher return by investing in a broader range of asset classes, including longer-term government bonds, agency and asset-backed securities, corporate bonds, equities, commodities, real estate, derivatives, and foreign direct investment.[3]

The governance structure of central banks and sovereign wealth funds also differ. Central banks typically operate with some degree of independence, subject to achievement of specific monetary policy mandates, such as achieving an inflation or exchange rate target or maintaining financial stability. In contrast, SWFs usually operate with more explicit portfolio management goals, with their performance monitored more directly by government authorities. In Norway, for example, the Ministry of Finance is responsible for the management of the Government Pension Fund and has delegated responsibility for the operational management of the Fund's foreign assets to Norges Bank Investment Management (NBIM), a separate part of Norway's central bank. Presently, the Chilean central bank appoints members of a Financial Committee which is responsible for making investment decisions and for the day-to-day running of the country's sovereign wealth fund. The Financial Committee reports to the Finance Minister who in turn reports to the country's President.

The growth of assets under foreign sovereign management by both central banks and sovereign wealth funds has prompted concerns about their motivations, particularly as they have diversified more into equity and other privately-issued assets. In this paper we analyze the optimal patterns of portfolio diversification by a central bank and a sovereign wealth fund, given their specific policy mandates assigned by the government and the government's mode of delegating these policy responsibilities.

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[3] There are exceptions to this division of goals. In some countries the central bank itself actively manages a portfolio with risky as well as safe assets. For example, Saudi Arabia's Monetary Authority manages the government's oil surpluses as well as riskier foreign investments without the vehicle of a standalone SWF. In China, the State Administration of Foreign Exchange (SAFE) has shifted its holdings towards riskier foreign investments, even with the establishment of the CIC.

We start with a benchmark model with which we compare the optimal degree of diversification between safe and risky foreign assets depending on the authorities' utility function. We assume that the goal of the central bank is to maintain financial stability by solely focusing on the goal of reducing the probability of sudden capital stops. In contrast, we presume that the SWF, acting on behalf of the Treasury, maximizes the expected utility of a representative domestic agent.[4] In this framework we show that the central bank's focus on financial stability implies that it will place a higher weight on limiting the downside risk of sudden stops than will the SWF. Consequently, the bank tends to bias its portfolio strategy towards holding more of the safe foreign asset, whereas the SWF opts for greater diversification towards riskier foreign assets. We also show how the diversification gap between the strategies of the bank and SWF is affected by various parameters of the economy, such as the volatility of equity returns and the total amount of public foreign assets available for management.

We follow with a more elaborate model of joint decision making in which we treat the Treasury as the dominant player and assume that it sets the total level of foreign assets managed by the central bank, but delegates to the central bank the responsibility for maintaining financial stability through the composition of its portfolio. Conditional on the actions of the central bank, the Treasury then decides what level of public foreign assets to entrust to the central bank for management as well as the composition of the residual share of the country's public foreign assets managed through the SWF. In these circumstances, we find that the assignment of the financial stability objective to the central bank tends to increase the gap between the optimal diversification patterns of the bank and the SWF, with the central bank specializing even more in holding safe assets so as to minimize the downside risk of sudden stop crises, while the SWF specializes more in holding foreign equity assets in its portfolio.

The plan of the paper is as follows. Section 2 presents the basic model of asset management by a central bank and sovereign wealth fund. Section 3 analyzes the implications of joint management through delegation of policy goals. Section 4 considers an extension to the intertemporal case. Section 5 presents conclusions.

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[4] We abstract from the possibility that SWF investments may be driven by noneconomic considerations. Concerns that they may do has prompted a range of responses in potential recipient countries (see Aizenman and Glick, 2007).

2. The Base Model

Central banks and sovereign wealth funds hold and manage public foreign assets typically accumulated through commodity export revenues or foreign exchange intervention in the face of current account surpluses. In this section we present a base model that explains how the authorities – either a central bank (CB) or a sovereign wealth fund (SWF) -- determine the diversification of a given initial level of public foreign asset holdings, A, between risk-free international reserves, R, and risky foreign equities F, such that $A = R + F$. We start by treating the stock of foreign assets A as exogenous, but later show how it can be determined endogenously. International reserves yield a *gross* risk-free interest rate r, whereas equity yields a stochastic return $r + e$, where e is the equity premium. The realized value of public foreign assets at the end of the period, A_1, is given by

$$A_1 = rR + (r + e)F = rA + eF; \quad 0 \le F \le A.$$ (1)

The timing of the model is such that, *after* (i) determination of the portfolio composition $\{R, F\}$ at the beginning of the period and (ii) the realization of the equity return shock e at the end of the period, there is a probability ϕ of a sudden stop crisis. The probability of a sudden stop depends negatively on the public sector's end-of-period total foreign assets, A_1, and positively on the exogenously given level of the country's aggregate foreign liabilities, B, such that $\phi = \phi[A_1/B]; \quad \phi' < 0$.[5] (In our notation we denote arguments of a function within square brackets.) Note that greater holdings of foreign risky asset can raise the probability of a sudden stop in the case of *negative* equity return shocks, i.e. $d\phi/dF = \phi'[A_1/B](e/B) > 0$ if $e < 0$. In these circumstances, greater diversification towards foreign equities entails growing balance sheet vulnerability because adverse equity return shocks *reduce* the value of foreign asset holdings, thereby raising the probability of sudden stops.

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[5] Foreign liabilities may be sovereign or private if the government serves as a lender of last resort in the event of a sudden stop crisis. We can motivate this setup by assuming in the background of the model that agents finance long-term investment via banks, while banks intermediate short-term deposits into longer term investment a la Diamond and Dybvig (1983). Financial integration implies exposure to sudden stops and capital flight shocks, which may drain the liquidity of the domestic banking system. A sudden stop entails costly pre-mature liquidation of long term investments, reducing thereby the output associated with these projects. The sudden stop probability can be derived endogenously by allowing for liquidity shocks z to foreign borrowing (see Aizenman and Lee, 2007). If $z < A_1$, the liquidity shock is financed by A_1, without disrupting long-term investment. If $z > A_1$, it leads to a sudden stop crisis associated with premature and costly liquidation of $z - A_1$ units of capital.

In the absence of a crisis, the foreign debt B is repaid and (exogenous) end-of period output Y_1 is received.[6] With the occurrence of a crisis, however, the debt is not repaid and there is a penalty P associated with premature liquidation costs and possibly punitive actions by lenders that is assumed to be proportionate to end-of-period output and foreign assets[7]:

$$P = \tau(Y_1 + A_1), \ 1 \geq \tau > 0.$$

Hence consumption without ("ns) and with ("ss") a sudden stop are, respectively,

$$C_1^{ns} = Y_1 + A_1 - B,$$
$$C_1^{ss} = Y_1 + A_1 - P.$$

Agents are assumed to be risk averse, with utility function U. The SWF determines the portfolio allocation in order to maximize the expected utility of a representative domestic agent, V_{sw}:

$$V_{sw} = E\left[\left(1 - \phi[A_1 / B]\right) U\left[C_1^{ns}\right] + \left(\phi[A_1 / B]\right) U\left[C_1^{ss}\right] \right], \tag{2}$$

subject to the foreign asset accumulation relation (1), where E is the expectation operator. Assuming an interior solution with diversification, the first-order condition determining F for a given level of A is:

$$dV_{sw}/dF = E\left[(1 - \phi)U'\left[C_1^{ns}\right](e) + \phi U'\left[C_1^{ss}\right]e(1 - \tau) \right]$$
$$\qquad - E\left[\phi'[A_1 / B](e / B)\left\{ U\left[C_1^{ns}\right] - U\left[C_1^{ss}\right] \right\} \right] = 0. \tag{3}$$

Rearranging terms,

$$E[dU/dF] = E\left[\left(\phi'[A_1 / B]\right)(e / B)\left\{ U\left[C_1^{ns}\right] - U\left[C_1^{ss}\right] \right\} \right], \tag{4}$$

where

$$E[dU/dF] \equiv E\left[(1 - \phi)U'\left[C_1^{ns}\right](e) + \phi U'\left[C_1^{ss}\right]e(1 - \tau) \right].$$

The left hand size of (4), $E[dU/dF]$, is the expected marginal benefit of diversification, i.e., the expected marginal utility valuation of the equity return premium associated with higher F,

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[6] The model can be generalized by endogenizing the determination of foreign borrowing B. For example, in a two-period economy, where second-period output is subject to productivity shocks, the possibility that the country may default in the second period with a bad enough productivity shock implies that foreign borrowing is constrained by the condition that the cost of funds equals the expected return to lenders (see Aizenman and Marion, 2004). Y_1 might be expressed as a function of borrowing B in this more general framework.

[7] This is a reduced form of the resources left following the premature liquidation of investments needed to finance deposits and capital flight. To simplify, we assume that that A_1 and Y_1 shrink at the same rate. The logic of our analysis continues to hold with different shrinkage rates of A_1 and Y_1, where the resources left would be $A_1(1 - \tau_A) + Y_1(1 - \tau_Y)$, $1 \geq \tau_A > 0, 1 \geq \tau_Y > 0$.

$dA_1 / dF = e$. The right hand side of (4) is the expected marginal cost of diversification in the presence of a negative equity premium, i.e. the marginal increase in the probability of a sudden stop, $d\phi / dF = \phi'[A_1 / B](e / B)$, times the utility cost of lost consumption associated with a sudden stop, $U\left[C_1^{ns}\right] - U\left[C_1^{ss}\right]$.[8] Thus, the SWF increases its holdings of foreign equity assets to the level that balances the marginal increase in returns with the marginal cost of a rise in the probability of a sudden stop.

In contrast to a SWF, CB management is assumed to focus solely on maintaining financial stability, ignoring the effects of its behavior on the utility of domestic agents.[9] As long as a crisis is avoided, the CB manager enjoys a constant consumption level of C_{cb}, and utility $U[C_{cb}]$. We assume a financial crisis entails a penalty to the central bank at rate τ_{cb}; $1 \geq \tau_{cb} > 0$ (possibly involving replacement of the central bank manager), reducing the central bank's utility to $U[C_{cb}(1-\tau_{cb})]$. In these circumstances, the CB's expected utility is

$$V_{cb} = E\left[\left(1 - \phi[A_1 / B]\right)U[C_{cb}] + \left(\phi[A_1 / B]\right)U[C_{cb}(1-\tau_{cb})]\right]. \tag{5}$$

Consequently, the first-order condition from maximizing (5) with respect to the CB's level of foreign equities F, while subject to (1), is

$$dV_{cb} / dF = E\left[-\phi'[A_1 / B](e / B)\right] = 0 . \tag{6}$$

That is, the central bank increases its foreign equity assets to a level that minimizes the probability of a sudden stop by setting its marginal impact on the probability to zero.

Comparing the first-order condition for optimal portfolio diversification by the SWF, (4), to that for the CB, (6), reveals that the CB ignores the possible expected gains to domestic agents induced by diversification in the presence of a positive equity premium. Instead, the CB minimizes the downside risk associated with adverse equity shocks, recognizing that such shocks would increase the leverage ratio B / A_1 at a rate proportionate to the equity portfolio share (since (1) implies $A_1 / A = r + e(F / A)$), thereby increasing the probability of sudden stops in bad

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[8] Note also that the utility cost is positive as long as $(A_1 + Y_1)\tau > B$, i.e. the penalty costs are greater than the foregone foreign borrowing obligation.

[9] We abstract from other possible central bank objectives, such targeting inflation under flexible exchange rate regime. We also do not model deeper reasons for our assignment of objectives, such as a possible bias towards nominating a conservative central bank governor with a narrower policy focus than the representative agent (see Rogoff, 1985; Walsh, 1995; Obstfeld, 1996; and Aizenman and Glick, 2008).

times.[10] This suggests that the CB will opt for much less diversification into foreign equities than the SWF.

Further insight is gained by considering a simple example, where the equity premium e equals an expected equity premium ε plus an equity-return shock δ that takes on two values, either $\overline{\delta}$ or $-\overline{\delta}$, each with probability one half, implying mean value 0 and standard deviation $\sigma_\delta = \overline{\delta}$. The left panel of Figure 1 plots the optimal share of foreign equities in a portfolio managed by the SWF (top curve) and the CB (bottom curve), each as a function of volatility $\overline{\delta}$, for the case where the initial public asset position is $A = 1$.[11] With the initial foreign public asset level A normalized to 1, F also measures the equity share of the portfolio. We assume CRRA utility with relative risk aversion parameter β, $\beta = 2$, and a power function specification for the probability of a sudden stop, $\phi = a\left(1 + (A_1 / B)\right)^{-h}$, with parameter values $h = 12$ and $a = 4 \cdot 10^3$. Other calibrated values include setting $\tau = 0.26$ and $\tau_{cb} = 0.9$.

Observe in Figure 1 that lower return volatility $\overline{\delta}$ raises the optimal share of foreign holdings, *F/A,* for both a central bank and a sovereign wealth fund. In addition, because of the difference in preferences, the sovereign wealth fund holds more equities for any given level of volatility than does the central bank. To illustrate, for $\overline{\delta} = 0.25$, the SWF's equity share in its portfolio, *F/A,* is 79% (at point a in Figure 1), whereas the CB opts for a much lesser equity share, about 5% (at point b). Had the anticipated volatility in equity returns been higher, say $\overline{\delta} = 0.4$, both institutions would reduce their foreign equity holding. The SWF would reduce its equity investment from 79% of its portfolio to about 30%, whereas the more conservative CB would reduce the portfolio share of equities from about 5% to near 0%. A lower degree of risk aversion, i.e. lower β, shifts both schedules up (not shown), implying the share of equity assets held is higher for any given level of volatility.

What is the effect of a higher level of initial public asset holdings on portfolio decisions? The right panel of Figure 1 illustrates the effects of the size of the asset base on diversification choices, by tripling A from 1 to 3. As can be observed, the higher level of public assets reduces relative demand for foreign equities, i.e., the curves plotting the association between *F/A* and volatility both shift down and to the left. In this example, higher A has a negligible effect on the CB's demand for equities: for equity return volatility $\overline{\delta} = 0.25$, and $A = 1$, equity demand was 5%

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[10] Note B/A_1 is proportional to A/A_1 since $B/A_1 = (B/A)(A/A_1)$ and B/A is constant.

[11] More specifically, these curves reflect the locus of points satisfying (3) and (6), respectively

of A (at point b in the left hand panel), while for $A = 3$, this demand is imperceptibly closer to 0 (at point b' in the right hand panel. In contrast, a higher A induces a drop in the SWF's equity demand by about 20% of A, from a portfolio share of 79% (at point a in the left panel of Figure 1) to 58% (at point a' in the right panel in Figure 1)   Therefore, higher A induces an increase in the absolute demand for equities by the SWF, though at a rate smaller than the rate of increase in A, implying a decline in the foreign equity share of the SWF's portfolio. This suggests that for a large enough asset base A, the opportunity cost associated with the limited portfolio diversification of the CB would be high enough to induce the authorities to establish a SWF, acting as an agent for the Treasury in pursuit of greater returns.

How do these two different diversification strategies affect the probability of a sudden stop? Figure 2 plots the association between the equity shock realization δ and the sudden stop probability ϕ for the CB (dotted line) and SWF (bold line), given the optimal equity share F chosen when $\bar{\delta} = 0.25$. That is, we first determine the optimal F conditional on $\bar{\delta} = 0.25$, call it F^* (which equals 79% for the SWF and 5% for the CB), and then calculate the implied values of $A_1 = rA + eF^*$ and hence $\phi = \phi[A_1/B]$ for different realized levels of δ, not necessarily equal to $\bar{\delta}$ or $-\bar{\delta}$, and hence realized equity premium $e = \varepsilon + \delta$. The negative slopes of the curves plotted in figure 2 indicate that more positive return realizations imply higher end-of-period asset holdings and a lower probability of sudden stop for both the CB and SWF. Observe as well that the limited diversification strategy pursued by the CB results in a relatively low probability of sudden stops, less than 0.05, for the entire range of equity return shocks shown (-0.4 to 0.4). In contrast, the greater holdings of risky foreign equities by the SWF expose the economy to the greater downside risk of a sudden stop in bad times, e.g. for a δ realization of -0.4, the probability of a crisis (at point c) is almost 0.3, eight times higher than the probability 0.04 for the CB (at point c').

This analysis implies that the central bank's diversification strategy tends to be relatively more resilient to unanticipated returns by maintaining a relatively low probability of sudden stop for a wide range of possible shocks. This suggests why it can be advantageous to assign portfolio management independence to the CB in order to limit the probability of sudden stops. However, these gains come at the opportunity cost of lost returns on foreign assets, which rise with the volume of finds under management.

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To summarize, the central bank's focus on financial stability implies that its portfolio share of risky foreign assets is lower and is less sensitive to changes in return volatility and in size than is a sovereign wealth fund.

3. Delegation of Financial Stability Objective to Central Bank

This section extends the base model, recognizing the possibility that the central bank has a comparative advantage in avoiding sudden stops and maintaining financial stability, whereas the SWF has a comparative advantage in managing risky portfolios. We continue to assume that the initial total foreign assets of the public sector A, and aggregate foreign borrowings B are exogenously determined. But we now treat the Treasury as the dominant decision maker, determining the split of the given level of public foreign assets A between the management of the SWF and the CB.

Each entity manages its own portfolio of foreign reserves and equities. Denote by A_{cb}, A_{sw} the initial assets designated for management by the SWF and CB, respectively, such that $A = A_{cb} + A_{sw}$. Note that if A is given and the Treasury determines A_{cb}, then A_{sw} is determined as well.

The CB diversifies its portfolio between foreign reserves (R_{cb}) and foreign equities (F_{cb}), given its total initial asset holdings $A_{cb} = R_{cb} + F_{cb}$, implying that the realized value of assets under the CB's management at the end of the period, $A_{1,cb}$, is

$$A_{1,cb} = rR_{cb} + (r + e)F_{cb} = rA_{cb} + eF_{cb}; \quad 0 \leq F_{cb} \leq A_{cb} . \tag{1'}$$

As before, r is the (gross) risk-free rate and $e = \varepsilon + \delta$ is the stochastic equity premium for investment in risky assets by the central bank, with ε the expected equity premium and δ a mean-zero equity-return shock.

The SWF is assumed to have a comparative advantage in active management of the external portfolio, resulting in a higher expected equity premium ε_{sw}, $\varepsilon_{sw} \geq \varepsilon$, for any given volatility δ. This reflects the possibility that the CB diversifies by passive portfolio investment channels (such as investing in broad and relatively liquid foreign stock indexes), whereas the SWF diversifies by more active portfolio management, including foreign direct investment, etc. possibly offering higher expected returns. Accordingly, the realized value of assets under SWF management at the end of the period is $A_{1,sw}$:

$$A_{1,sw} = r(A - A_{cb}) + e_{sw}F_{sw}; \quad 0 \leq F_{sw} \leq A_{sw} \leq A \tag{1''}$$

As the CB deals directly with monitoring and supervising financial intermediation, we presume that the CB has a comparative advantage in reducing the probability of a financial crisis triggered by sudden stops. This may reflect both the lower liquidity of the SWF's investments as well as institutional frictions that may slow down the SWF's ability to mobilize resources. Specifically, for a given ratio of assets to foreign borrowing A_1 / B, the CB's management results in a lower probability of sudden stops than would occur under SWF management, i.e. $\phi_{cb}[A_1 / B] < \phi_{sw}[A_1 / B]$ for all A_1 / B, where $\phi_{cb}[.], \phi_{sw}[.]$ denote the sudden stop probability if assets are managed by the CB or a Treasury-controlled SWF, respectively.

The cost of delegating financial stability management to the CB is that, in case of need, only a fraction q of the SWF's assets A_{sw} would be available to the CB ($0 \leq q < 1$). Hence, under CB management $\phi_{cb}[.] = \phi_{cb}\left[\left(A_{1,cb} + qA_{1,sw}\right) / B\right]$. For simplicity, we assume that $q = 0$, implying $\phi_{cb}[.] = \phi_{cb}\left[A_{1,cb} / B\right]$. As before, the probability of a sudden stop is assumed to depend negatively on the ratio of the public sector's foreign assets to foreign liabilities, implying $\phi'_{cb} < 0$.

3.1. Delegation and Diversification

The diversification problem facing the CB is identical to the one described in Section 2, i.e. choose the level of foreign equity assets F_{cb} that maximizes the objective function

$$V_{cb} = E\left[\left(1 - \phi_{cb}\left[A_{1,cb} / B\right]\right)U[C_{cb}] + \left(\phi_{cb}\left[A_{1,cb} / B\right]\right)U[C_{cb}(1 - \tau_{cb})]\right] \tag{2'}$$

subject to the asset accumulation relation (1'), where ϕ is replaced with ϕ_{cb}. We continue to treat the foreign debt overhang B as given by assumption and note that A_{cb} is exogenous to the CB's actions, since it is determined by the Treasury. An analogue to the first-order condition (6) holds:

$$dV_{cb} / dF_{cb} = E\left[\left(-\phi'_{cb}\left[A_{1,cb} / B\right]\right)(e / B)\right] = 0. \tag{6'}$$

As in the case without delegation, the CB sets policies that minimize the probability of sudden stops. Note that its optimal level of reserves, determined by condition (6'), implies that the expected equity premium evaluated by its impact on the probability of a sudden stop is zero.

The SWF's expected utility is akin to (2), modified by delegating the financial stability objective to the CB:

$$V_{sw} = E\left[\left(1 - \hat{\phi}_{cb}\left[A_{1,cb}/B\right]\right)U\left[C_{1,sw}^{ns}\right] + \left(\hat{\phi}_{cb}\left[A_{1,cb}/B\right]\right)U\left[C_{1,sw}^{ss}\right]\right], \tag{2''}$$

where

$$C_{1,sw}^{ns} = Y_1 + A_{1,sw} - B,$$

$$C_{1,sw}^{ss} = \left(Y_1 + A_{1,sw}\right)(1-\tau)$$

and $\hat{\phi}_{cb}$ is the probability of a sudden stop, conditioned on optimal CB policy as determined by (6').[12]

The Treasury's problem is to determine, for a given initial level of total public foreign assets A, the optimal levels of the SWF's investment in equities F_{sw} and the CB's initial assets A_{cb} that maximize its expected utility (2''), subject to (1''). The first-order condition for optimal F_{sw} holdings by the SWF, assuming an internal equilibrium (i.e., $0 < F < A_{sw}$), is

$$E[dU_{sw}/dF_{sw}] \equiv E\left[\left[\left(1 - \hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right] + \left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right](e_{sw})\right] = 0. \tag{4''}$$

Comparing (4'') to (4), we infer that delegating financial stability policy to the CB implies that the SWF equates the expected marginal benefit of diversification, $E[dU_{sw}/dF_{sw}]$, to zero, instead of, as in the no delegation case, equating it to the expected cost of increasing the probability of a sudden stop. Note also that condition (4'') implies that in a SWF's equilibrium, the expected marginal utility associated with the equity premium is zero.

Figure 3 illustrates the effect of policy delegation on the SWF's degree of diversification. The bottom curve shows the SWF's diversification pattern in the absence of delegation, corresponding to the case depicted in the left panel of Figure 1 (with the same parameter values). The top curve (labeled "SWF, CB delegation") corresponds to the SWF's diversification pattern with the same asset base ($A_{sw} = 1$), in circumstances where the CB is assigned the objective of minimizing the probability of a sudden stop (for the case where $A_{cb} = 1$). With delegation, for

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[12] To derive $\hat{\phi}_{cb}$, first note that (6') yields an expression for F_{cb} as an implicit function of A_{cb}, as well as B and e:

$F_{cb} = \widehat{F}_{cb}\left[A_{cb}; B, \varepsilon, \sigma_e\right]$, where $\partial \widehat{F}_{cb}/\partial A_{cb} > 0$. Substituting $\widehat{F}_{cb}$ into ϕ gives $\phi_{cb} = \phi_{cb}[A_{1,cb}/B]$

$= \phi_{cb}[(rA_{cb} + \widehat{F}_{cb}[A_{cb}]e)/B] \equiv \hat{\phi}_{cb}[A_{cb}; B]$, where for large enough r and small enough σ_e^2, $E\left[\partial \hat{\phi}_{cb}/\partial A_{cb}\right] < 0$. Note

the latter follows from the approximation $E\left[\partial \hat{\phi}_{cb}/\partial A_{cb}\right] = E\left[\hat{\phi}_{cb}'(r + e\partial F_{cb}/\partial A_{cb})\right]$

$\cong E\left[\{\hat{\phi}_{cb}'[0] + \hat{\phi}_{cb}''[0]e\}(r + e(\partial F_{cb}/\partial A_{cb}))\right] = \hat{\phi}_{cb}'[0]r + \hat{\phi}_{cb}''[0]\sigma_e^2(\partial F_{cb}/\partial A_{cb}).$

any given volatility of returns, the SWF chooses to hold a higher level of foreign equity assets, implying greater diversification.

The Treasury also determines the CB's initial foreign assets, A_{cb}, with the SWF managing the remainder $A_{sw} = A - A_{cb}$. The discussion in the previous section suggests that an increase in A should lead to greater asset allocation by the Treasury to the SWF in order to secure higher returns. This holds trivially as the probability of sudden stop, $\hat{\phi}_{cb}$, approaches zero. More formally, the first-order condition determining A_{cb} is:

$$dV_{sw}/dA_{cb} = E\left[\left(\left(1-\hat{\phi}_{cb}\right)\left(-U'\left[C_{1,sw}^{ns}\right]\right) + \left(\hat{\phi}_{cb}\right)\left(-U'\left[C_{1,sw}^{ss}\right]\right)(1-\tau)\right)\left(r + e\widehat{F}'_{cb}\right)\right]$$
$$- E\left[\left(\hat{\phi}'_{cb}\left((r + e\widehat{F}'_{cb})/B\right)\left\{U\left[C_{1,sw}^{ns}\right] - U\left[C_{1,sw}^{ss}\right]\right\}\right)\right] = 0. \tag{7}$$

or

$$E\left[dU/dA_{cb}\right] = E\left[-\hat{\phi}'_{cb}\, e\left(\widehat{F}'_{cb}/B\right)\left\{U\left[C_{1,sw}^{ns}\right] - U\left[C_{1,sw}^{ss}\right]\right\}\right] = 0$$

where

$$E\left[dU/dA_{cb}\right] \equiv E\left[\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right] + \left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\left(r + e\widehat{F}'_{cb}\right)\right]$$

and $\widehat{F}'_{cb}$ is the marginal impact of an increase in initial foreign assets A_{cb} on the CB's optimal foreign equity holdings.[13] Thus the Treasury allocates more initial foreign assets to the central bank up to the point that the expected marginal benefit from doing so, $E\left[dU/dA_{cb}\right]$, equals the marginal increase in the probability of a sudden stop, $d\phi_{cb}/dF_{cb} = -\hat{\phi}'_{cb}\, e(\widehat{F}'_{cb}/B)$, times the utility cost of a sudden stop, $U\left[C_{1,sw}^{ns}\right] - U\left[C_{1,sw}^{ss}\right]$.

3.2. Delegation and Welfare

In this section we analyze the welfare gains to the policy of delegating foreign asset management to the CB and SWF as described above. The representative consumer cares only about utility associated with the net resources available for household consumption. (We presume that the compensation of the CB, C_{cb}, and of the SWF's managers are negligible portions of GDP, and hence too small to impact consumption per capita.) Therefore, for the

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[13] As noted in the previous footnote, (6') yields an expression for F_{cb} as an implicit function of A_{cb}, as well as B and e: $F_{cb} = \widehat{F}_{cb}\left[A_{cb}; B, e_\delta\right]$.

representative consumer, consumption is determined by the total sum of the assets accumulated by both agencies, $A = A_{sw} + A_{cb}$.

The consumer's expected utility is

$$V = E\left[(1-\phi)U\left[C^{ns} \right] + \phi U\left[C^{ss} \right] \right],$$ (8)

where

$$C_1^{ns} = Y_1 + A_{1,cb} + A_{1,sw} - B = Y_1 + rA + eF_{cb} + e_{sw}F_{sw} - B$$

$$C_1^{ss} = \left(Y_1 + A_{1,cb} + A_{1,sw} \right)(1-\tau) = \left(Y_1 + rA + eF_{cb} + e_{sw}F_{sw} \right)(1-\tau).$$

It is assumed that the default penalty is large enough such that a sudden stop is associated with lower consumption, i.e. $C^{ns} > C^{ss}$. The following claim follows:

Claim: Assume that (i) the CB has a comparative advantage in prudential regulation, as defined by the relation $\phi_{cb} = (1-b)\phi_{sw}, \quad 0 \le b \le 1$, where the parameter b measures the extent to which the probability of a sudden stop is lower with the CB than with the SWF, and (ii) the SWF has an advantage in portfolio management in the form of a higher expected return to foreign equity investment, i.e. $\varepsilon_{sw} > \varepsilon$. Then the direct gains from delegation increase with the comparative advantage of each agency.

Proof: The claim flows from the observation that $dV/db > 0, \ dV/d(\varepsilon_{sw} - \varepsilon) > 0$, i.e. consumer utility is increasing in the magnitude of the CB's comparative advantage and in the SWF's investment equity premium.[14]

Intuitively, the case for delegation is stronger the greater are the efficiency gains associated with allowing each agency to focus on its comparative advantage. Quite trivially, in the absence of any distortions and gains from the proper division of management objective among the CB and the SWF, delegation could be welfare reducing, since the CB's preferences are biased towards "safety first" relative to that of the representative consumer. Henceforth we assume that the gain from the division of goals is powerful enough to induce higher expected utility under delegation.

We close this section with an example illustrating the welfare gains associated with delegation. The example is based on the simulation described in the left hand side panel of

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[14] These results can be established by noting that $dV/db = E\left[\phi_{sw} \left\{ U\left[C^{ns} \right] - U\left[C^{ss} \right] \right\} \right] > 0$ and

$dV/d\varepsilon_{sw} = E\left[(1-\phi)U'_{C^{ns}} + \phi(1-\tau)U'_{C^{ss}} + (d\phi/d\varepsilon_{sw})(1-\phi)\left\{ U\left[C^{ns} \right] - U\left[C^{ss} \right] \right\} \right] > 0$.

Figure 1, with $A = 1$ and $\varepsilon = 0.05$, where in the absence of delegation the optimal level of F was 0.79, i.e., the authorities allocated 79% of foreign assets into equities, and 21% into reserves. This amount of diversification implies that the equity return in the bad state is -20% (since $e_l = \varepsilon - \bar{\delta} = .05 - .25 = -0.2$, where e_l denotes the equity return in the low state), and the sudden stop probability, $\phi_{\delta = -\bar{\delta}}$, is 10.5%.

Suppose that delegation induces gains from the division of goals. In this spirit, we assume that delegating risk monitoring to the CB reduces the probability of sudden stop for any given amount of diversification by 40%, implying $\phi_{cb} = (1-b)\phi_{sw}$, with $b = 0.4$. We also assume that the expected equity premium earned by the SWF over the central bank is 2%, so that $\varepsilon_{sw} = \varepsilon + 0.02 = 0.07$. Lastly, assume that with the delegation of prudential management to the CB, the authorities (i) allocate to the CB the same amount of foreign assets that were put into reserves in the absence of delegation (i.e. $A_{cb} = 0.21$) and direct the CB to invest all these assets in reserves (i.e. $A_{cb} = R_{cb} = 0.21$) and (ii) allocate to the SWF the remaining assets, 0.79, all to be invested in equities (i.e. $F_{sw} = A - A_{cb} = 0.79$).[15] It can be shown that, in the presence of the gains from the division of management described above, delegation leads to welfare gains similar to increasing the initial endowment by about 0.5%, reducing the probability of sudden stop in the bad state of nature from 10.5% to 5.6%.[16]

4. Extending the Model to the Intertemporal Case

In this section we extend the model to the intertemporal case, where the authorities maximize the two-period expected utility function

$$V = U[C] + \frac{1}{1+\rho} E\left[U[C_1] \right] \tag{9}$$

with C, C_1 denoting consumption in the initial and subsequent periods, respectively, and ρ the intertemporal rate of time preference. This extension allows us to derive the total size of the public foreign asset portfolio (A) endogenously. Under delegation, policies are determined by the

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[15] It can be verified that for $A_{cb} = 0.21$, the CB has a strong bias towards foreign reserves. Thus, the CB would opt to invest more in reserves even in the absence of the authorities' directive.

[16] This result is obtained by calculating the expected utility gain, deflating it by the marginal utility evaluated for a zero equity return shock. This ratio provides the expected utility gain in terms of the equivalent rise in the initial endowment Y_1. Deflating this gain by the initial endowment translates the welfare gain to the equivalent percentage increase in the initial endowment, net of debt overhang, $Y_1 + rA + eF - B$.

CB and SWF as described in Section 3, where the representative consumer cares only about the net resources supporting consumption.

The initial period consumption, C, is output Y, net of "investment" in foreign reserves R and foreign equity assets F (we abstract from the effect of past foreign borrowing, the "debt overhang," on income flows in the initial period). Next-period consumption, C_1, depends on asset returns and the costs of a possible sudden stop as well as repayment of the debt overhang B. As before, foreign reserves earn a gross return of r and equity assets a return of $r + e$, where the equity premium e equals an expected equity premium ε plus a zero-mean equity-return shock δ. Hence, the intertemporal consumption conditions are:

$$C = Y - R - F$$
$$C_1 = \begin{cases} C_1^{ns} = Y_1 + rR + (r + \varepsilon + \delta)F - B \\ C_1^{ss} = Y_1 + rR + (r + \varepsilon + \delta)F(1 - \tau) \end{cases} \tag{10}$$

where next-period consumption without and with a sudden stop are denoted by "ns and "ss", respectively. We assume that the default penalty is large enough such that a sudden stop is associated with lower consumption, i.e. $C_1^{ns} > C_1^{ss}$. Accordingly, the representative consumer's expected utility is

$$V = U[Y - R - F] + \frac{1}{1 + \rho} E\Big[(1 - \phi)U\big[C_1^{ns}\big] + \phi U\big[C_1^{ss}\big] \Big]. \tag{11}$$

Applying the logic of section 3, with delegation such that $\phi = \phi_{cb}$ the CB acts to maximize consumer expected utility by minimizing the probability of sudden stops.[17] The optimal levels of reserves and foreign assets that achieve this goal are determined by the conditions[18]

$$d\phi / dR_{cb} = E\big[\phi'_{cb}\big[A_{1,cb} / B\big]\big](r / B) = 0 \tag{12a}$$

$$d\phi / dF_{cb} = E\big[\phi'_{cb}\big[A_{1,cb} / B\big](r + e)\big] = 0. \tag{12b}$$

Note that in a diversified equilibrium, (12a) and (12b) jointly imply that the expected equity premium evaluated by its impact on the probability of sudden stop is zero:

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[17] This follows trivially from the assumption that the CB's manager is paid a lump sum in good times, and only a fraction of it in sudden stops (possibly zero, losing his post).
[18] A sufficient condition for the central bank to hold both foreign equity as well as reserves is

$d\phi / dF_{cb} = E\big[\phi'_{cb}\big[A_{1,cb} / B\big](r + e) / B)\big]_{F=0} < 0$, i.e. adding foreign equity to the portfolio lowers the probability of a sudden stop.

$$E\left[\phi'_{cb}\left[A_{1,cb}/B\right](e/B)\right]=0. \tag{13}$$

The SWF takes as given $\hat{\phi}_{cb}$, the sudden stop probability conditioned on optimal CB behavior, and determines its equity position F in a way akin to Section 3. The optimality conditions for diversification by the SWF, i.e., holding positive levels of both F and R, are [19]

$$U'(C) = \frac{E\left[(r+e_{sw})\left[\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right]+\left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right]\right]}{1+\rho} \tag{14a}$$

$$U'(C) = \frac{rE\left[\left[\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right]+\left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right]\right]}{1+\rho}. \tag{14b}$$

Thus, in equilibrium the SWF equates current marginal utility to expected (discounted) future marginal utility. Note (14a) and (14b) imply that the expected marginal utility associated with the equity premium is zero:

$$E\left[e_{sw}\left[\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right]+\left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right]\right]=0. \tag{15}$$

The equilibrium conditions above pin down the optimal levels of $F_{cb}, R_{cb}, F_{sw}, R_{sw}$. To illustrate, suppose that the configuration of parameter values is such that there is full division of management goals, with the CB holding only international reserves, and the SWF holding only equities, i.e. $F_{cb}=0$, $R_{sw}=0$. This implies that $A_{cb}=R_{cb}$, $A_{sw}=F_{sw}$ and hence $A=R_{cb}+F_{sw}$. Hence the determination of R_{cb} and F_{sw} pins down A, the overall level of foreign assets under public management.

To gain further insight we assume that the probability of a sudden stop follows a power function as specified in section 2, and that the risk-free real interest rate equals the subjective discount factor ($r=1+\rho$). It can be shown that the central bank's demand for reserves increases with the debt overhang, equity return volatility, and the foreign equity held by the SWF, i.e.

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[19] A sufficient condition for the SWF to hold reserves as well as foreign equity is if doing so increases expected utility evaluated at the optimal F, $\hat{F}_{sw}$, i.e.

$$rE\left[\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right]+\left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right]/(1+\rho)\Big|_{\substack{F=\hat{F}_{SW}\\R_{SW}=0}} > U'(C)\Big|_{\substack{F=\hat{F}_{SW}\\R_{SW}=0}}$$

or, equivalently through use of the FOC for optimal F, (14a),

$$E\left[\left\{\left(1-\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ns}\right]+\left(\hat{\phi}_{cb}\right)U'\left[C_{1,sw}^{ss}\right](1-\tau)\right\}e_{sw}\right]/(1+\rho)\Big|_{\substack{F=\hat{F}_{SW}\\R_{SW}=0}} > 0.$$

$R_{cb} = R_{cb}[B, \overline{\delta}, F_{sw}]$; $R'_{cb,B} > 0$, $R'_{cb,\overline{\delta}} > 0$, $R'_{cb,F_{sw}} > 0$. The equity holdings of the SWF decline

with a higher debt overhang and greater volatility of equity returns, and increase with the

expected equity premium, i.e., $F_{sw} = F_{sw}[B, \overline{\delta}, \varepsilon_{sw}]$; $F'_{sw,B} < 0$, $F'_{sw,\overline{\delta}} < 0$, $F'_{sw,\varepsilon_{Sw}} > 0$.

What are the effects of changes in the exogenous parameters -- B, $\overline{\delta}$, ε_{sw} -- on *total* public

assets? In the case of an increase in ε_{sw}, we can infer that the effect is unambiguously positive:

an increase in the expected equity premium leads to higher F_{sw} (since $F'_{sw,\varepsilon_{Sw}} > 0$), which in turn

leads to higher R_{cb} (since $R'_{cb,F_{Sw}} > 0$) as well. Intuitively, if the foreign equity return is higher,

policymakers will want to increase the total stock of foreign assets ($A = R_{cb} + F_{sw}$) under public

management.

The effects of an increase in $\overline{\delta}$ or B are ambiguous. For example, an increase in volatility

$\overline{\delta}$ has the direct effect of reducing demand for holding risky foreign assets F_{sw} (since $F'_{sw,\overline{\delta}} < 0$)

which in turn indirectly reduces R_{cb} as well (since $R'_{cb,F_{sw}} > 0$). But there is also the direct effect

that leads to higher R_{cb} in order to provide more insurance against sudden stops (since $R'_{cb,\overline{\delta}} > 0$).

The overall sign depends on which effect dominates.

5. Conclusions

The optimal portfolio management of public foreign assets necessitates a balancing of

returns and risks, where the latter reflect the risks of financial instability as well as the volatility

of investment returns. In this paper we have presented a model with which we compare the

optimal degree of diversification into safe foreign reserve assets and other higher-yielding, but

risky, foreign assets by a central bank versus that of a sovereign wealth fund. We show that if

the central bank manages its foreign assets with the objective of reducing the probability of

sudden stops in foreign capital inflows, it will place a high weight on the downside risk of

holding risky assets abroad and will tend to hold primarily safe foreign assets for precautionary

purposes. In contrast, if the sovereign wealth fund, acting on behalf of the Treasury, operates

with the objective of maximizing the expected utility of a representative domestic agent, it will

opt for relatively greater holding of higher yielding, though riskier, foreign assets. We also show

how the diversification differences between the strategies of the bank and SWF is affected by

government's delegation of responsibilities and by various parameters of the economy, such as

the volatility of equity returns and the total amount of public foreign assets available for

management. In addition, we illustrate why it might be desirable for policymakers to delegate the objective of achieving financial stability to the central bank alone.

The global financial crisis of 2007-08 highlighted the importance of the precautionary purpose for holding international reserves by central banks. Moreover, the drop in global commodity prices and equity returns during the crisis reduced the supply of public foreign funds and dampened the relative appeal of investing abroad in risky assets by both central banks and sovereign wealth funds. If the volatility in financial markets remains high, monetary authorities may continue to place a high weight on holding more reserves as a means of minimizing the expected costs of sudden stop crises. Nevertheless, the resumption of global growth, particularly in emerging markets, appears to be restoring the attractiveness of foreign investment by sovereign wealth funds. Policymakers in these countries still must seek to balance the relative rewards to doing so against the gains to reducing the probability of financial instability.

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**Figure 1.** Equity returns volatility, foreign equity holdings, and asset size

a. **A = 1**   b**. A = 3**
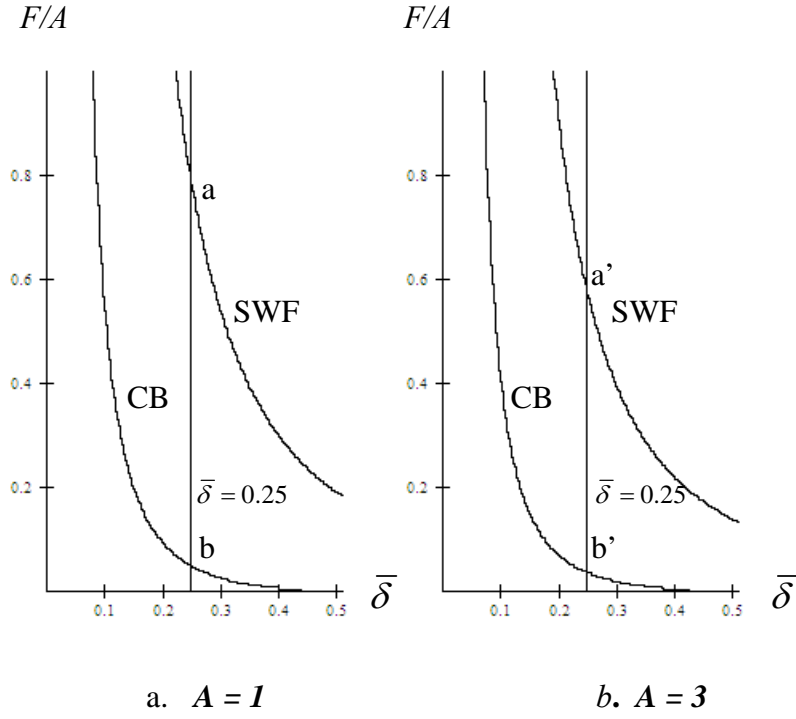
Note: The bottom (top) curve describes asset diversification of the CB (SWF). The simulation assumes $U[C] = C^{1-\beta}/(1-\beta)$, $\phi = MIN\left\{a\left(1+(A_1/B)\right)^{-h};1\right\}$, the equity shock δ takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$, and $r = 1$, $\beta = 2$, $Y_1 = 1.8$, $A_{sw} = 1$, $B = 0.6$, $e = 0.05$, $\tau = 0.26$, $\tau_{cb} = 0.9$, $h = 12$, $a = 4 \cdot 10^3$. The left-hand and right-hand panels assume initial asset size levels A of 1 and 3, respectively. The vertical line corresponds to $\bar{\delta} = 0.25$.

Figure 2. Probability of sudden stop for different equity return shock realizations.
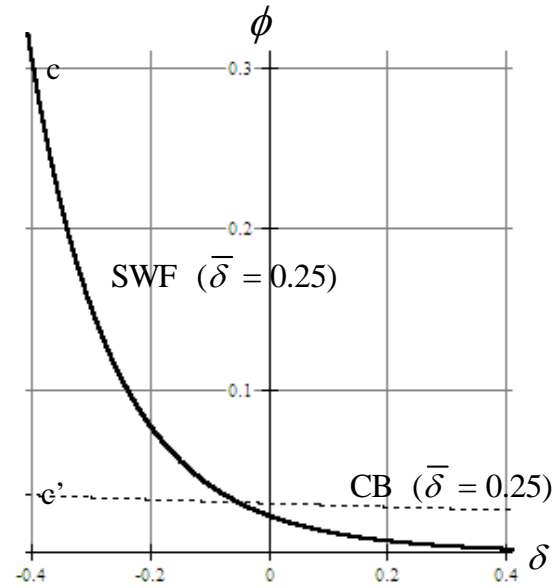
Note: The bold curve describes the sudden stop probability for $F = 0.79$ (the SWF equity share for a volatility level $\bar{\delta} = 0.25$), while the dotted curve shows the sudden stop probability for $F = 0.05$ (the CB equity share for $\bar{\delta} = 0.25$). The simulation assumes $F_{cb} = 0.05$; $F_{sw} = 0.79$; $A = 1$, and the parameters specified in Figure 1.

Figure 3. Equity return volatility and diversification by SWF, with and without CB delegation
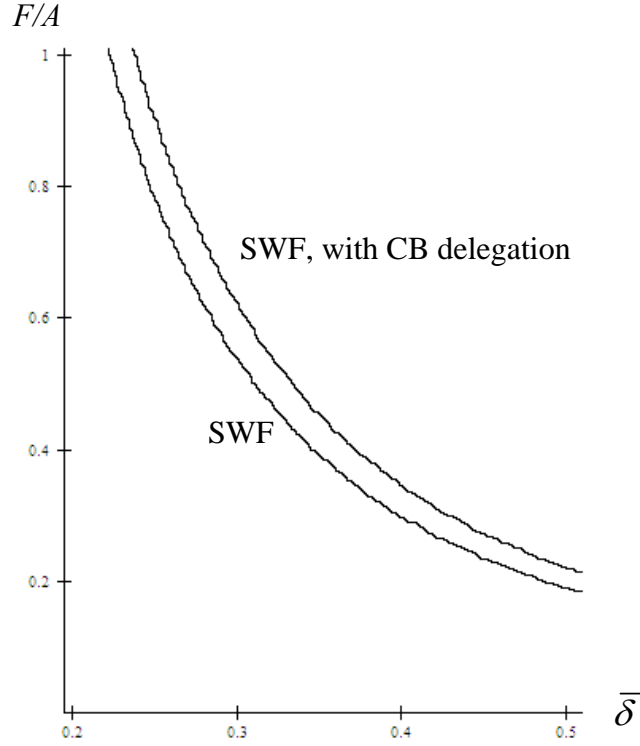
Note: Lower curve describes SWF diversification in the absence of delegation. The simulation assumes $\phi_{sw} = MIN\left\{a[1 + A_{1,sw} / B]^{-h}; 1\right\}$, the equity shock, δ, takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$, and $r = 1$, $\beta = 2$, $Y_1 = 1.8$, $B = 0.6$, $e = 0.05$, $\tau = 0.26$, $\tau_{cb} = 0.9$, $h = 12$, $a = 4 \cdot 10^3$. The upper curve describes SWF behavior with delegation, where $A_{sw} = 1$, $A_{cb} = 1$, and $\phi_{cb} = MIN\left\{a[1 + A_{1,cb} / B)]^{-h}; 1\right\}$.