## Title
Supersymmetric unification requires extra dimensions



## Journal
AIP Conference Proceedings, 1534(1)

## ISSN
0094-243X

## ISBN
9780735411586

## Authors
Chen, Mu-Chun
Fallbacher, Maximilian
Ratz, Michael

## Publication Date
2013-05-23

## DOI
10.1063/1.4807363

# Supersymmetric unification requires extra dimensions

Mu-Chun Chen*, Maximilian Fallbacher† and Michael Ratz†

*Department of Physics and Astronomy, University of California, Irvine, California 92697–4575, USA
†Physik Department T30, Technische Universität München, James–Franck–Straße, 85748 Garching, Germany

**Abstract.** We discuss settings that predict precision gauge unification in the minimal supersymmetric standard model. We show that, if one requires anomaly freedom and fermion masses while demanding that unification is not an accident, only $R$ symmetries can forbid the supersymmetric Higgs mass term $\mu$. We then review the proof that $R$ symmetries are not available in conventional grand unified theories (GUTs) and argue that this prevents natural solutions to the doublet–triplet splitting problem in four dimensions. On the other hand, higher–dimensional GUTs do not suffer from this problem. We briefly comment on an explicit string–derived model in which the $\mu$ and dimension five proton decay problems are solved by an order four discrete $R$ symmetry, and comment on the higher–dimensional origin of this symmetry.

**Keywords:** Supersymmetry, Grand Unified Theory, Extra Dimensions
**PACS:** 12.10.Dm,12.60.Jv,

## INTRODUCTION & OUTLINE

The minimal supersymmetric standard model (MSSM) provides an attractive scheme for physics beyond the standard model (SM) of particle physics. The MSSM has the following, attractive features:

- it is based on supersymmetry, which is the, under certain modest assumptions, maximal extension of the Poincaré symmetry of our four–dimensional Minkowski space–time;
- it provides automatically a dark matter candidate, which is stable due to the $Z_2^{\mathscr{M}}$ matter parity;
- supersymmetry allows us to stabilize the gauge hierarchy.

In the context of unification, the perhaps most important property of the MSSM is that it provides us with the very compelling picture of precision gauge coupling unification [1]. That is, if one assumes that the superpartners have masses of the order TeV and extrapolates the gauge couplings $g_i$ of the SM gauge factors SU(3), SU(2) and U(1) to higher energies, one finds that they meet with a high precision at the scale of a few $\times\, 10^{16}$ GeV. This property of the MSSM represents, given the lack of evidence for superpartners at the LHC, perhaps the greatest motivation for supersymmetry. Arguably, the most compelling explanation of this fact arises if the SM gauge group is embedded in a simple gauge group, specifically

$$G_{\mathrm{SM}} \;=\; \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \;\subset\; \mathrm{SU}(5) \tag{1}$$

or a group containing SU(5).

This brings us to the scheme of grand unified theories (GUTs). Specifically, GUTs based on the gauge groups SU(5) and SO(10) have many appealing features (for a review see, e.g., [2]):

1. GUTs explain charge quantization;
2. they simplify the matter content. The five irreducible representations (irreps) forming one generation of SM matter can be grouped into two SU(5) irreps [3],

$$\text{SM generation} \;=\; \mathbf{10}+\overline{\mathbf{5}}\,. \tag{2}$$

A further simplification of the matter sector happens in SO(10) [4], where

$$\begin{aligned}\mathbf{16} &= \mathbf{10}\oplus\overline{\mathbf{5}}\oplus\mathbf{1}\\ &= \text{SM generation with `right–handed' neutrino}\,.\end{aligned} \tag{3}$$

One of the main assumptions in these proceedings is that these features are not by accident.

*Workshop on Dark Matter, Unification and Neutrino Physics*
AIP Conf. Proc. 1534, 244-252 (2013); doi: 10.1063/1.4807363

We will specifically discuss the role of (discrete) $R$ symmetries in supersymmetric models of unification. After reviewing some of the issues of the MSSM, we will discuss the importance of anomaly constraints and in particular “anomaly universality” for their resolution. Using these techniques, we will show that only $R$ symmetries can forbid the $\mu$ term in the MSSM. Furthermore, as we will then argue, these $R$ symmetries are already almost uniquely determined by the anomaly universality conditions. However, given certain general assumptions which we will specify, $R$ symmetries are not available in four–dimensional models of grand unification. On the other hand, $R$ symmetries are available in higher–dimensional and, in particular, in stringy settings, where they arise as discrete remnants of the Lorentz symmetry of compact space. We will comment on explicit models where precisely the phenomenologically desired symmetries arise this way. Finally, we will provide a short summary.

## THE MSSM AND GRAND UNIFICATION

### Problems of the MSSM

As is well known, the MSSM also has certain shortcomings. Some of them are associated to operators which are consistent with all symmetries of the MSSM but which have phenomenologically undesired effects. The gauge invariant superpotential terms up to order four include

$$\begin{aligned}\mathscr{W} \;=\;& \mu\, H_d H_u + \kappa_i\, L_i H_u \\ & + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\ & + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\ & + \kappa_{ij}^{(0)}\, H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\;. \end{aligned} \tag{4}$$

Here, in an obvious notation, $H_u$ and $H_d$ denote the MSSM Higgs doublets, and $Q_i$, $\overline{U}_i$, $\overline{D}_i$, $L_i$ and $\overline{E}_i$ the three generations of MSSM matter. The $\mu$ term in the first line has, for phenomenological reasons, to be of order TeV and Yukawa couplings $Y_e^{ij}$, $Y_u^{ij}$ and $Y_d^{ij}$ ($2^{\text{nd}}$ line) are required in order to describe fermion masses. Furthermore, a non–trivial $\kappa_{ij}^{(0)}$ of the order $(10^{14}\,\text{GeV})^{-1}$ is required in order to describe see–saw suppressed neutrino masses.

On the other hand, there are various problematic terms. First of all, the so–called $R$ parity violating couplings $\kappa_i$, $\lambda_{ijk}$, $\lambda'_{ijk}$ and $\lambda''_{ijk}$ ($1^{\text{st}}$ & $3^{\text{rd}}$ lines) have to be strongly suppressed, or absent (cf. e.g. [5]). Second, there are strong constraints on the coefficients $\kappa_{ijk\ell}^{(1,2)}$ of so–called dimension five proton decay operators (last line). It is hence clear that supersymmetry alone is not enough.

### Traditional cure of the proton decay problems

Of course, these problems are well known and there are some standard solutions. Let us specifically discuss the traditional cure of proton decay problems. The $R$ parity violating terms can be forbidden by $R$ or matter parity $\mathbb{Z}_2^{\mathscr{M}}$ [6, 7]. Formally, these two symmetries differ by the transformation of the superpartners. However, there is an ambiguity in any supersymmetric theory to send the superspace coordinate $\theta$ to minus itself. In view of this ambiguity, both symmetries are equivalent. The dimension five proton decay operators can be forbidden by baryon triality $B_3$ [8] (see table 0.1 for the charge assignment).

**TABLE 0.1.** Matter parity $\mathbb{Z}_2^{\mathscr{M}}$, baryon triality $B_3$ and proton hexality $P_6$.

| | $Q$ | $\bar{U}$ | $\bar{D}$ | $L$ | $\bar{E}$ | $H_u$ | $H_d$ | $\bar{\nu}$ |
|---|---|---|---|---|---|---|---|---|
| $\mathbb{Z}_2^{\mathscr{M}}$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| $B_3$ | 0 | −1 | 1 | −1 | 2 | 1 | −1 | 0 |
| $P_6$ | 0 | 1 | −1 | −2 | 1 | −1 | 1 | 3 |

The combination of $\mathbb{Z}_2^{\mathscr{M}}$ and $B_3$ goes under the name “proton hexality” $P_6$ [8, 9, 10]. $P_6$ comes with various very appealing features:

☺ it forbids dimension four and five proton decay operators;

☺ it allows the usual Yukawa couplings of the MSSM as well as Weinberg's neutrino operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$;

☺ it is the unique anomaly–free symmetry with the above features.

However, $P_6$ has also some disturbing aspects:

☹ it is not consistent with unification of matter, i.e. inconsistent with universal discrete charges for all matter fields (cf. [11]);

☹ it does not address the $\mu$ problem, i.e. does not provide us with a solution to all the above–mentioned problems of the MSSM.

In what follows, we will discuss alternative discrete symmetries which do not suffer from these shortcomings. Specifically, we will identify anomaly–free discrete symmetries which are consistent with (precision) gauge unification and allow us to control the $\mu$ term.

## ANOMALY–FREE DISCRETE SYMMETRIES & UNIFICATION

In this section, we will first discuss (discrete) anomaly cancellation and will then focus on symmetries that are consistent with unification and forbid the $\mu$ term. Unlike in the traditional discussion, we will allow for the possibility of Green–Schwarz (GS) anomaly cancellation [12].

### Anomaly universality

In particular, we will argue that requiring anomaly freedom and consistency with grand unification amounts to demanding "anomaly universality", i.e. common anomaly coefficients of the SM gauge factors $G_i$. Let us explain what that means in practice. Consider, for example, the mixed $G_i - G_i - \mathbb{Z}_N$ anomaly coefficient for a $\mathbb{Z}_N$ symmetry,

$$A_{G_i-G_i-\mathbb{Z}_N} \;=\; \sum_f \ell(r^{(f)}) \cdot q^{(f)} \;. \tag{5}$$

Here the sums extend over all fermion representations $r^{(f)}$, while $\ell^{(f)}$ denotes the Dynkin index of the fermions $f$ w.r.t. the gauge group $G$ and $q^{(f)}$ are the discrete $\mathbb{Z}_N$ charges. The traditional anomaly constraints [13, 14] consist in the statement that the $A_{G^2-\mathbb{Z}_N}$ coefficients[1] have to vanish for all $G_i$,

$$A_{G_i-G_i-\mathbb{Z}_N} \;=\; 0 \mod \eta \quad \forall\, G_i \;, \tag{6}$$

where

$$\eta \;:=\; \begin{cases} N & \text{for } N \text{ odd}\;, \\ N/2 & \text{for } N \text{ even}\;. \end{cases} \tag{7}$$

On the other hand, "anomaly universality" is the requirement that they are universal,

$$A_{G_i-G_i-\mathbb{Z}_N} \;=\; \rho \mod \eta \quad \forall\, G_i \;, \tag{8}$$

but that they do not necessarily have to vanish. Here $\rho$ can be thought of as the contribution of a GS axion $a$, whose shift transformation under the $\mathbb{Z}_N$ symmetry cancels the anomaly.

Where does "anomaly universality" come from? Although universality of anomaly coefficients is empirically found to be a property of most heterotic string models [16, 15], it is, as correctly pointed out in [17], in general not a necessary condition for anomaly freedom. To see this, let us recall how the GS mechanism works in the path integral formalism [18, 19]. The crucial ingredient is the coupling of the GS axion $a$ to the $F\widetilde{F}$ term of the gauge group $G$. The GS axion $a$ is contained in the superfield $S$, $S|_{\theta=0} = s + \mathrm{i}\,a$, and shifts under the symmetry transformation. The GS anomaly cancellation requires the coupling

$$\int \mathrm{d}^2\theta\, f_S\, S W_\alpha W^\alpha \;\supset\; \mathscr{L} \;. \tag{9}$$

[1] Note that there are no meaningful $\mathbb{Z}_N^3$ anomaly constraints. This has been first shown in [14] and can be seen more directly in the path integral approach [15].

Therefore $s = \mathrm{Re}\, S|_{\theta=0}$ contributes to $1/g^2$, see [20, 21] for more details. In general, different couplings of $a$ to different SM gauge factors $G_i$ will give rise to the possibility of allowing for different $\rho$ constants for the different gauge factors of the SM. However, in general, the "saxion" $s$ has a non–trivial vacuum expectation value (VEV), such that non–universal couplings imply non–universal contributions to $1/g_i^2$. This will then spoil precision gauge unification. Since we assume that precision gauge unification is not an accident, we will require anomaly universality in the rest of our discussion.

### Non–$R$ symmetries cannot forbid the $\mu$ term in the MSSM

Let us now look at discrete anomalies of non–$R$ symmetries in the MSSM. After imposing $\mathrm{SU}(5)$ relations for the matter charges, the relevant anomaly coefficients read

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{10}^g + q_{\overline{5}}^g\right) , \tag{10}$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{10}^g + q_{\overline{5}}^g\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) . \tag{11}$$

Here, in an obvious notation, $q_{10}^g$ and $q_{\overline{5}}^g$ denote the discrete charges of the $g^{\mathrm{th}}$ 10– and $\overline{5}$–plet, respectively, with $g$ playing the role of a generation index while $q_{H_u}$ and $q_{H_d}$ are the charges of the Higgs doublets. Now, imposing anomaly universality, i.e. demanding that

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0 \mod \eta , \tag{12}$$

leads to a condition on the Higgs charges:

$$\frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} , \\ N/2 & \text{for } N \text{ even} . \end{cases} \tag{13}$$

It is easy to see that this implies that the $\mathbb{Z}_N$ symmetry does not forbid the Higgs bilinear. We hence see that ordinary, i.e. non–$R$, $\mathbb{Z}_N$ symmetries cannot forbid the $\mu$ term.

### Only discrete $R$ symmetries may do the job

It is also obvious that, if anomaly–free discrete non–$R$ symmetries cannot forbid the $\mu$ term, this also applies to continuous non–$R$ symmetries, for which the anomaly constraints are stronger. We are hence left with $R$ symmetries. Recalling that there are no anomaly–free continuous $R$ symmetries in the MSSM [22], the only remaining option is given by discrete $R$ symmetries.

### Interlude: 't Hooft anomaly matching for $R$ symmetries

As is well known, a powerful tool for analyzing symmetries is provided by the method of anomaly matching [23], which can also be used for discrete symmetries [24]. Let us spell this out for the case of discrete $R$ symmetries in the MSSM, still assuming unification [21]. At the $\mathrm{SU}(5)$ level, there is only one anomaly coefficient,

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta , \tag{14}$$

which we have decomposed into the contribution from matter $A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}$, extra states $A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}$ and gauginos $5q_\theta$ with $q_\theta$ denoting the $R$ charge of the superspace coordinate.[2] $M$ is the order of the $R$ symmetry transformation, which

[2] Note that there exists some confusion in the literature. It is often assumed that the superpotential $\mathscr{W}$ has $R$ charge 2, corresponding to $R$ charge 1 of the superspace coordinate, $q_\theta = 1$. However, as pointed out in [21], one cannot, in general, make this choice and, at the same time, demand that all discrete charges are integer. We follow the convention that all discrete charges are integer and keep $q_\theta$ variable.

might be part of a larger symmetry. Consider now the SU(3) and SU(2) subgroups. The anomaly coefficients read

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta \;, \tag{15a}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \frac{1}{2}\cdot 2\cdot 3\cdot q_\theta \;. \tag{15b}$$

Here we have decomposed the gaugino contributions into their SU(3) and SU(2) parts, respectively, and in the contributions from $\mathrm{SU}(5)/G_{\mathrm{SM}}$. Assume now that some mechanism eliminates the extra gauginos. This will lead to a non–universality, which will, given our assumption that matter charges commute with SU(5), have to be compensated for by the extra states. That is, the extra states have to come in split multiplets. In other words, 't Hooft anomaly matching for (discrete) $R$ symmetries implies the presence of split multiplets below the GUT scale. The arguably simplest possibility to "repair" the gaugino mismatch is to assume that there is a pair of massless weak doublets, which is chiral w.r.t. $\mathbb{Z}_M^R$, but no corresponding triplets. From this one infers that, in the presence of an $R$ symmetry, the same mechanism that breaks the GUT symmetry will also provide a mechanism for doublet–triplet splitting. However, as we shall see later, one cannot have $R$ symmetries in four–dimensional models of grand unification, which is consistent with the observation that natural (in 't Hooft's sense) solutions to the doublet–triplet splitting problem do not exist in such schemes.

## Unique $\mathbb{Z}_4^R$ symmetry

Let us now impose, instead of SU(5) relations, stronger SO(10) relations, i.e. universal charges $q$ for matter fields. That is, consider a $\mathbb{Z}_M^R$ symmetry under which quarks and leptons have universal charge $q$. As we shall demonstrate, this implies a unique symmetry [20, 21]. In the first step, we require the existence of $u$– and $d$–type Yukawas, implying that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M \;. \tag{16}$$

Subtracting these equations from each other leads to

$$q_{H_u} - q_{H_d} = 0 \mod M \;. \tag{17}$$

The conditions for the presence of $u$–type Yukawa couplings and the Weinberg operator are

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M \;, \tag{18}$$

implying that $q_{H_u} = 0 \mod M$. Altogether we see that

$$q_{H_u} = q_{H_d} = 0 \mod M \quad \text{and} \quad q = q_\theta \mod M \;. \tag{19}$$

From the conditions that the symmetry must be an $R$ symmetry,

$$q_\theta \neq 0 \mod \eta \;, \tag{20}$$

and that it is "anomaly universal" in the MSSM,

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = 3q_\theta \mod \eta \stackrel{!}{=} q_\theta \mod \eta = A_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} \;, \tag{21}$$

it follows that $\eta$ is even which in turn implies that the order $M$ of the symmetry is a multiple of 4,

$$M = 4m, \quad m \in \mathbb{N} \;. \tag{22}$$

Furthermore, given the ambiguity discussed previously, equations (0.20) and (0.21) fix the $R$ charge of the superspace coordinate $\theta$ to $q_\theta = m$. As a result, the simplest non–trivial possibility is $M = 4$ and $q = q_\theta = 1$, i.e. a $\mathbb{Z}_4^R$ symmetry. As is straightforward to see, the extensions to $\mathbb{Z}_{4m}^R$ symmetries, $m > 1$, are trivial extensions as far as the MSSM is concerned. While it appears interesting to study such symmetries in the context of (singlet) extensions of the MSSM, we can conclude that there is a unique symmetry for the MSSM: a $\mathbb{Z}_4^R$ with $q = q_\theta = 1$ and $q_{H_u} = q_{H_d} = 0$.

This symmetry was first discussed in [25]. A version of the uniqueness proof appeared in [26]. However, there it was assumed that the superpotential has charge 2 in a normalization in which all discrete charges are integer, which is, in general, not a valid assumption (cf. footnote 2). The uniqueness proof has been completed in [21].

The $\mathbb{Z}_4^R$ anomaly coefficients are

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_4^R} \;=\; 6q-3q_\theta \;=\; q_\theta \;=\; 1 \mod 4/2\,, \tag{23a}$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_4^R} \;=\; 6q+\frac{1}{2}\left(q_{H_u}+q_{H_d}\right)-5q_\theta \;=\; q_\theta \;=\; 1 \mod 4/2\,, \tag{23b}$$

The fact that the coefficients are non–trivial implies that the $\mathbb{Z}_4^R$ is anomaly–free only via a non–trivial GS mechanism.

## Implications of GS anomaly cancellation

Let us briefly comment on the implication of GS anomaly cancellation. As discussed above, the GS axion $a$ is contained in a superfield $S$, $S|_{\theta=0}=s+\mathrm{i}\,a$. Since $a=\mathrm{Im}\,S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, $R$ non–covariant superpotential terms can be made invariant by multiplying them with $\mathrm{e}^{-bS}$. To be specific, consider, as an example, the Higgs bilinear. The $\mu$ term is obviously forbidden by the $\mathbb{Z}_4^R$ symmetry, but the term

$$B\,\mathrm{e}^{-bS}\,H_u H_d \tag{24}$$

will be allowed for appropriate values of $b$. In other words, the holomorphic $\mathrm{e}^{-bS}$ terms appear to violate the $\mathbb{Z}_M^R$ symmetry. Such terms have a well–known interpretation. Given the coupling (0.9), $s=\mathrm{Re}\,S|_{\theta=0}$ contributes to $1/g^2$, and the holomorphic $B\,\mathrm{e}^{-bS}$ terms can be interpreted as non–perturbative effects (cf. the “retrofitting” discussion [27]). Altogether we see that there is a unique symmetry of the MSSM that (i) forbids the $\mu$ term, (ii) is compatible with $\mathrm{SO}(10)$ and (iii) is anomaly–free; this symmetry has the feature that the $\mu$ term appears non–perturbatively, and is naturally suppressed.

## Further implications of $\mathbb{Z}_4^R$

The $\mathbb{Z}_4^R$ symmetry has important implications for the MSSM. Among the gauge invariant terms (0.4), the $\mu$ term, the $R$ parity violating terms and the dimension five proton decay operators are forbidden at the perturbative level while, by construction, the Yukawa couplings and the Weinberg operator are allowed. While $\mu$ and the dimension five proton decay operators appear at the non–perturbative level, the $R$ parity violating terms are also forbidden at the non–perturbative level by a “non–anomalous” $\mathbb{Z}_2$ subgroup which is equivalent to matter parity. What is the size of the non–perturbative terms? The order parameter for $R$ symmetry breaking is the superpotential VEV $\langle\mathscr{W}\rangle$, or, in other words, the gravitino mass $m_{3/2}$. Hence

$$\mu \;\sim\; m_{3/2} \;\simeq\; \langle\mathscr{W}\rangle/M_\mathrm{P}^2 \tag{25}$$

with $M_\mathrm{P}$ denoting the Planck scale. The non–perturbatively generated dimension five proton decay operators are phenomenologically harmless,

$$\kappa_{ijk\ell}^{(1,2)} \;\sim\; m_{3/2}/M_\mathrm{P}^2 \;\ll\; 10^{-8}/M_\mathrm{P}\,, \tag{26}$$

where we compare the theoretical expectation with the experimental constraints [28].

## NO–GO FOR *R* SYMMETRIES IN 4D GUTS

We have seen that only $R$ symmetries can forbid the $\mu$ term in the MSSM. However, as we shall show now, $R$ symmetries are not available in four–dimensional GUTs [29]. More specifically, assuming

(i) a GUT model in four dimensions based on $G\supset\mathrm{SU}(5)$,

(ii) that the GUT symmetry breaking is spontaneous and

(iii) that there is only a finite number of fields, one can prove that it is impossible to get a low–energy effective theory with both

1. just the MSSM field content and
2. residual $R$ symmetries.

For the purposes of these proceedings, we will restrict ourselves to presenting the basic argument. Consider an SU(5) model with an (arbitrary) $R$ symmetry and a chiral 24–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$. Recall the branching rule

$$24 \;\to\; (8,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-5/6} \oplus (\overline{3},2)_{5/6} \oplus (1,1)_0 \;. \tag{27}$$

Since the 24–plet attains a VEV but may not break the $R$ symmetry, it has to have $R$ charge 0. The multiplets $(3,2)_{-5/6} \oplus (\overline{3},2)_{5/6}$ get eaten by the extra gauge bosons from $\mathrm{SU}(5)/G_{\mathrm{SM}}$. We are hence left with the extra massless states $(8,1)_0 \oplus (1,3)_0$, whose masses are forbidden by the $R$ symmetry.

Can one get rid of these unwanted states? Introducing extra 24–plets with $R$ charge 2 does not help because this would lead to massless $(3,2)_{-5/6} \oplus (\overline{3},2)_{5/6}$ representations. Iterating this argument shows that with a finite number of 24–plets one will always have massless exotics. Let us mention that there is a loophole for infinitely many 24–plets.

It is possible to generalize the basic argument to

- arbitrary SU(5) representations;
- larger GUT groups $G \supset \mathrm{SU}(5)$;
- singlet extensions of the MSSM.

The proof can be found in [29]. Here we shall only discuss the implications of these statements. A 'natural' solution of the $\mu$ and/or doublet–triplet splitting problem requires a symmetry that forbids $\mu$. So far we have learned that:

1. only $R$ symmetries can forbid the $\mu$ term;
2. anomaly matching requires the existence of split multiplets;
3. $R$ symmetries are not available in 4D GUTs.

This implies that 'natural' solutions to the $\mu$ and/or doublet–triplet splitting problems are not available in four dimensions! This might be interpreted as the necessity to go to models with extra dimensions, such as string compactifications.

## STRING MODELS

In this section, we will discuss how going to extra dimensions allows us to evade the no–go theorem. In such settings one can answer the question of the origin of the $R$ symmetries and one has better control over the higher–dimensional operators such as the effective $\mu$ term.

### Grand unification in higher dimensions

It is often stated that higher–dimensional GUTs appear more "appealing". This is because new possibilities of symmetry breaking arise [30, 31]. In particular, the KK towers provide us with, from a four–dimensional point of view, infinitely many states (cf. the discussion in [32]), thus allowing us to to evade the no–go theorem.

Even more, $R$ symmetries have a clear geometric interpretation. They originate from the Lorentz symmetry of compact dimensions (cf. e.g. the discussion in [33]) and are arguably on the same footing as the fundamental symmetries $C$, $P$ and $T$.

### Stringy origin of $\mathbb{Z}_4^R$

String models offer a geometric explanation of discrete symmetries (for a recent review see e.g. [33]). Specifically, in stringy heterotic orbifolds, one obtains effective theories with residual discrete $R$ symmetries. In particular, one can determine the $R$ charges of the different states. Such models often have a $\mathbb{Z}_4^R$ symmetry, under which localized fields have odd $R$ charges while bulk fields have even $R$ charges. This harmonizes nicely with the scheme of "local grand unification" [34] where matter fields are localized in regions with $\mathrm{SO}(10)$ symmetry, and therefore come in complete $\mathrm{SO}(10)$ multiplets, while Higgs fields come from the bulk and therefore are split.[3]

Let us now discuss globally consistent string models with these features [38, 39]. These are $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models with the exact MSSM spectrum. They exhibit vacua, i.e. field configurations that preserve supersymmetry perturbatively, with various good features

- ✓ non–local GUT breaking;
- ✓ no 'fractionally charged exotics';
- ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$;
- ✓ non–trivial full–rank Yukawa couplings;
- ✓ gauge–top unification;
- ✓ SU(5) relation $y_\tau \simeq y_b$.

Note that these are, unfortunately, just toy models since they exhibit certain unrealistic features such as $\mathrm{SU}(5)$ Yukawa relations also for light generations. Nevertheless such models illustrate that a successful string embedding of the $\mathbb{Z}_4^R$ symmetry is possible.

## SUMMARY

In these proceedings, we discussed the role of $R$ symmetries in supersymmetric models that give rise to (precision) gauge unification. Specifically, we have made the following assumptions:

(i) anomaly freedom (allowing for GS anomaly cancellation);
(ii) $\mu$ term forbidden at the perturbative level;
(iii) Yukawa couplings and Weinberg neutrino mass operator allowed;
(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons.

We have then shown that

1. assuming (i) and $\mathrm{SU}(5)$ relations, only $R$ symmetries can forbid the $\mu$ term in the MSSM;
2. assuming (i)–(iii) and $\mathrm{SO}(10)$ relations, there is a unique $\mathbb{Z}_4^R$ symmetry;
3. $R$ symmetries are not available in 4D GUTs, implying that there is no 'natural' solution to doublet–triplet splitting in four dimensions.

The simple anomaly–free $\mathbb{Z}_4^R$ symmetry turns out to provide a solution to the $\mu$ problem and, as a bonus, automatically suppresses proton decay operators. Models with this symmetry predict that proton decay proceeds via dimension six operators, i.e. via gauge boson exchange. Yet, since such settings cannot be embedded into four–dimensional GUTs, one will have to analyze higher–dimensional models in order to make more detailed predictions.

Deriving the $\mathbb{Z}_4^R$ symmetry from string theory allowed us to understand where it comes from: it can arise as a discrete remnant of Lorentz symmetry in extra dimensions. Guided by this $\mathbb{Z}_4^R$ symmetry we have reported on a globally consistent string model with (i) the exact MSSM spectrum; (ii) non–local/Wilson line GUT breaking; (iii) non–trivial full–rank Yukawa couplings; (iv) exact matter parity; (v) $\mu \sim m_{3/2}$ and (vi) dimension five proton decay operators sufficiently suppressed.

## ACKNOWLEDGMENTS

M.-C.C. would like to thank TU München, where part of the work was done, for hospitality. M.R. would like to thank the UC Irvine, where part of this work was done, for hospitality. This work was partially supported by the DFG cluster of excellence "Origin and Structure of the Universe" and the Graduiertenkolleg "Particle Physics at the Energy

---

[3] In concrete models the third family comes partially from the bulk [35] (and is a so–called "patchwork family" [36], among other things giving rise to gauge–top unification [37]).

Frontier of New Phenomena" by Deutsche Forschungsgemeinschaft (DFG). The work of M.-C.C. was supported, in part, by the U.S. National Science Foundation under Grant No. PHY-0970173. M.-C.C. and M.R. would like to thank CETUP* for hospitality and support.

## REFERENCES

1. S. Dimopoulos, S. Raby, and F. Wilczek, *Phys. Rev.* **D24**, 1681–1683 (1981).
2. S. Raby, *Eur. Phys. J.* **C59**, 223–247 (2009), 0807.4921.
3. H. Georgi, and S. L. Glashow, *Phys. Rev. Lett.* **32**, 438–441 (1974).
4. H. Fritzsch, and P. Minkowski, *Ann. Phys.* **93**, 193–266 (1975).
5. B. C. Allanach, A. Dedes, and H. K. Dreiner, *Phys. Rev.* **D60**, 075014 (1999), hep-ph/9906209.
6. G. R. Farrar, and P. Fayet, *Phys. Lett.* **B76**, 575–579 (1978).
7. S. Dimopoulos, S. Raby, and F. Wilczek, *Phys. Lett.* **B112**, 133 (1982).
8. L. E. Ibáñez, and G. G. Ross, *Nucl. Phys.* **B368**, 3–37 (1992).
9. K. S. Babu, I. Gogoladze, and K. Wang, *Phys. Lett.* **B570**, 32–38 (2003), hep-ph/0306003.
10. H. K. Dreiner, C. Luhn, and M. Thormeier, *Phys. Rev.* **D73**, 075007 (2006), hep-ph/0512163.
11. S. Förste, H. P. Nilles, S. Ramos-Sánchez, and P. K. S. Vaudrevange (2010), 1007.3915.
12. M. B. Green, and J. H. Schwarz, *Phys. Lett.* **B149**, 117–122 (1984).
13. L. E. Ibáñez, and G. G. Ross, *Phys. Lett.* **B260**, 291–295 (1991).
14. T. Banks, and M. Dine, *Phys. Rev.* **D45**, 1424–1427 (1992), hep-th/9109045.
15. T. Araki, et al., *Nucl. Phys.* **B805**, 124–147 (2008), 0805.0207.
16. M. Dine, and M. Graesser, *JHEP* **01**, 038 (2005), hep-th/0409209.
17. C. Lüdeling, F. Ruehle, and C. Wieck, *Phys.Rev.* **D85**, 106010 (2012), 1203.5789.
18. K. Fujikawa, *Phys. Rev. Lett.* **42**, 1195 (1979).
19. K. Fujikawa, *Phys. Rev.* **D21**, 2848 (1980).
20. H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. Vaudrevange, *Nucl.Phys.* **B850**, 1–30 (2011), 1102.3595.
21. M.-C. Chen, M. Ratz, C. Staudt, and P. K. Vaudrevange, *Nucl.Phys.* **B866**, 157–176 (2013), 1206.5375.
22. A. H. Chamseddine, and H. K. Dreiner, *Nucl.Phys.* **B458**, 65–89 (1996), hep-ph/9504337.
23. G. 't Hooft, *NATO Adv. Study Inst. Ser. B Phys.* **59**, 135 (1980).
24. C. Csáki, and H. Murayama, *Nucl. Phys.* **B515**, 114–162 (1998), hep-th/9710105.
25. K. S. Babu, I. Gogoladze, and K. Wang, *Nucl. Phys.* **B660**, 322–342 (2003), hep-ph/0212245.
26. H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. Vaudrevange, *Phys.Lett.* **B694**, 491–495 (2011), 1009.0905.
27. M. Dine, J. L. Feng, and E. Silverstein, *Phys. Rev.* **D74**, 095012 (2006), hep-th/0608159.
28. I. Hinchliffe, and T. Kaeding, *Phys. Rev.* **D47**, 279–284 (1993).
29. M. Fallbacher, M. Ratz, and P. K. Vaudrevange, *Phys.Lett.* **B705**, 503–506 (2011), 1109.4797.
30. E. Witten, *Nucl. Phys.* **B258**, 75 (1985).
31. J. D. Breit, B. A. Ovrut, and G. C. Segre, *Phys. Lett.* **B158**, 33 (1985).
32. M. W. Goodman, and E. Witten, *Nucl.Phys.* **B271**, 21 (1986).
33. H. P. Nilles, M. Ratz, and P. K. Vaudrevange (2012), 1204.2206.
34. W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz pp. 143–156 (2005), hep-ph/0512326.
35. O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, *Phys. Rev.* **D77**, 046013 (2007), arXiv:0708.2691[hep-th].
36. D. K. M. Pena, H. P. Nilles, and P.-K. Oehlmann (2012), 1209.6041.
37. P. Hosteins, R. Kappl, M. Ratz, and K. Schmidt-Hoberg, *JHEP* **07**, 029 (2009), 0905.3323.
38. M. Blaszczyk, S. G. Nibbelink, M. Ratz, F. Ruehle, M. Trapletti, et al., *Phys.Lett.* **B683**, 340–348 (2010), 0911.4905.
39. R. Kappl, B. Petersen, S. Raby, M. Ratz, R. Schieren, and P. K. Vaudrevange, *Nucl.Phys.* **B847**, 325–349 (2011), 1012.4574.