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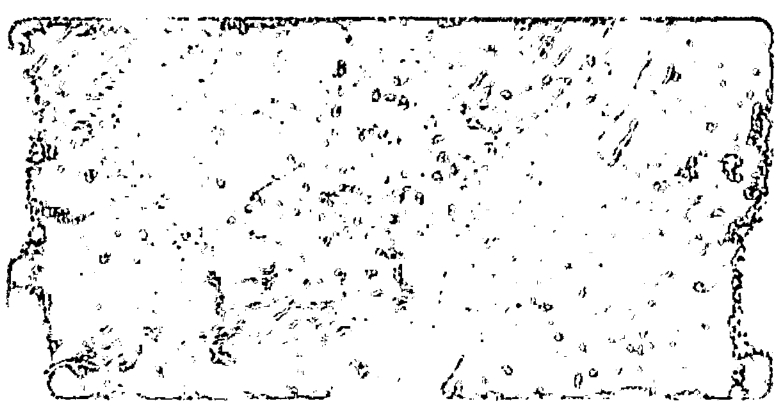

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ABSTRACT

Solutions of the τ-θ problem are sought whereby one could avoid the conclusion that parity is violated also in weak interactions not involving neutrinos. It is shown that no such solutions can be constructed that are theoretically acceptable, and experimental tests are indicated for a definite disproof of such models. The discussion is limited to the usual description of neutrino interactions such that the known neutrino processes occur at first order in the neutrino coupling constants.

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INTRODUCTION

It was pointed out by Dalitz that, in the absence of parity-nonconserving interactions, the $K^+_{2\pi}$ and the $K^+_{3\pi}$ decays could not be attributed to the same particle[1]--this conclusion is rigorous for spin zero. On the other hand, if two different K mesons were assumed, it was difficult to understand, in the absence of parity-nonconserving interactions, the apparent equality of the masses, cross sections, and lifetimes for such two particles. Recently, experimental evidence has been reported for parity nonconservation in β decay,[2] in $\pi \rightarrow \mu + \nu$ decay,[3] and in $\mu \rightarrow e + \nu + \nu$ decay.[3] Moreover, a theory of the neutrino has been proposed which provides a simple model for parity nonconservation in neutrino interactions.[4] Such a theory describes the physical neutrino as a screwon. This description is possible only if the

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†On leave of absence from Istituto di Fisico dell' Universita di Roma.

[1] R. H. Dalitz, Proc. Rochester Conf. 1956 8, 19.

[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957).

[3] Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415 (1957); Friedman and Telegdi, Phys. Rev. 105 1681 (1957).

[4] T. D. Lee and C. N. Yang, Phys. Rev. 105, 1671 (1957); A. Salam, Nuovo Cimento 5, 299 (1957). L. Landau, Nuclear Physics 3, 127 (1957).

physical neutrino has mass zero—for a particle of finite mass, in the center-of-mass system, there would be no way of defining a screw sense. No models for parity nonconservation in interactions not involving neutrinos have been proposed so far--however, the absence of simple models does not, of course, imply a difficulty. It is the purpose of this paper to examine the possible role in the τ-θ problem of the established parity nonconservation in neutrino interactions, under the conservative assumption that the other interactions do not violate parity. We shall show that it is difficult to account for all the present evidence on strange-particle decays and interactions on this basis. We shall, however, explicitly restrict our discussion to the assumption that the neutrinos are coupled only by weak interactions, such that, for instance, β decay or μ decay occurs at first order in the neutrino interactions. The possibility that the known weak interactions are effectively second-order processes in the neutrino interactions is now being investigated by Lee and Yang.

If the strong parity-conserving Hamiltonian has only one eigenstate describing a K meson (at rest), such a state can decay according to one of the two decay modes, $K^+_{2\pi}$ and $K^+_{3\pi}$, by a parity-conserving interaction, and according to the other mode through virtual neutrino states. The probability for the latter transition, however--involving two virtual steps due to weak interactions--would be extremely small. Therefore a solution of the τ-θ problem with only one K meson and with parity nonconservation only for neutrino interactions does not appear to be possible, at least under the restriction imposed in our discussion to the neutrino interactions. If the strong Hamiltonian has two independent eigenstates describing a K meson, these two states must have opposite parity and the same mass in order that, after the degeneracy has been removed by turning on weak interactions, the resulting physical states contain appreciable amplitudes for both parities. The relevant difference between the case with only one K eigenstate and this case with two eigenstates is that, whereas in the former case only the first-order perturbed state could have amplitudes for both parities, in the latter case the zero-order perturbed state contains amplitudes for both parities, as soon as some of the perturbing weak interactions do not conserve parity. The possibility of such solution for the τ-θ problem has been pointed out by Treiman and Wyld.[5] The required equality of mass for the two independent K-meson states almost necessarily leads to parity-doublet structures. The strong argument against nonparity-doublet theories for such models would be the impossibility of concerning two different K mesons with same mass (in the absence of weak interactions) without a symmetry principle responsible for the degeneracy--of course there would be essentially no parity mixing at all if an original mass difference, much larger than the decay rates, were already present when weak interactions are turned off.

Assuming the validity of present experimental evidence--which clearly indicates that only single particles are observed with single lifetimes and fixed branching ratios and not doublets--we shall see that essentially only one solution can be constructed that is able to simulate such evidence. A detailed discussion of this solution shows, however, that its consistency would require rather unphysical conditions for the weak interactions, and moreover it would lead to specific difficulties for neutral K^0, which would fit the scheme only accidentally. The model would lead to up-down asymmetries in a way qualitatively similar to that for parity nonconservation. It would lead, however, to the associated production in nuclear interactions, with a cross section equal to that for production of the observed strange particles, of hyperons and K mesons which immediately decay into leptons; evidence against such processes would constitute a direct disproof.

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[5] S. B. Treiman and H. W. Wyld, Phys. Rev. (to be published).

I

We discuss here the decay of particles that exist in parity doublets. In order to be definite we limit the discussion to K^+ mesons. The treatment is similar to that given by Lee, Oehme, and Yang for K^0 decay.[6] We derive both the decay matrix and the mass matrix from the Schrödinger equation at lowest order in the weak interactions. The weak decay interactions produce at the same time a level width (finite lifetime) and a level shift (mass difference) for the states of a K^+ particle. The total Hamiltonian is H + h, where h contains only the weak interactions. The states $|K^+\rangle$ of a K^+ particle in the absence of weak interactions satisfy the equation $H|K^+\rangle = \mu|K^+\rangle$, where μ is the mass eigenvalue. After h is turned on, a state $|K^+\rangle$ goes into a time-dependent state $|t\rangle$, which satisfies

$$(H + h)\,|t\rangle = i(d/dt)|t\rangle \tag{1}$$

and the initial condition $|0\rangle = |K^+\rangle$. We solve Eq. (1) to lowest order in h with the ansatz[6]

$$|t\rangle = [c_1|K_1^+\rangle e^{-(\lambda_1/2)t} + c_2|K_2^+\rangle e^{-(\lambda_2/2)t}]e^{-i\mu t}$$
$$+ \sum_{f\epsilon} v_{f\epsilon}(t)e^{-i\epsilon t}|f,\epsilon\rangle , \tag{2}$$

where $|f,\epsilon\rangle$ satisfy $H|f,\epsilon\rangle = \epsilon|f,\epsilon\rangle$, $|K_1^+\rangle$, $|K_2^+\rangle$, λ_1 and λ_2 are determined from Eq. (1) and from the initial conditions. If Eq. (2) is substituted into Eq. (1) the states $|K_1^+\rangle$ and $|K_2^+\rangle$ and the amplitudes $v_{f\epsilon}$ are found to satisfy

$$-i\lambda|K^+\rangle e^{-(\lambda_1/2)t} = 2\sum_{f\epsilon} v_{f\epsilon}(t)e^{-i(\epsilon-\mu)t}h|f,\epsilon\rangle , \tag{3}$$

$$i\dot{v}_{f\epsilon}(t) = \langle f\epsilon|h|K^+\rangle e^{-(\lambda_1/2)t + i(\epsilon-\mu)t} \tag{3'}$$

Integrating Eq. (3') and substituting in Eq. (3), one finds that the states $|K_1^+\rangle$, $|K_2^+\rangle$ must satisfy the non-Hermitian eigenvalue problem

$$\Lambda|K^+\rangle = \lambda|K^+\rangle, \tag{4}$$

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[6] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. (in press)

where $\Lambda = h\,T\,h$ and

$$T = -2i \sum_{f\epsilon} \left| f\epsilon \right\rangle \frac{1-e^{-i(\epsilon-\mu+i\lambda/2)t}}{\epsilon-\mu+i\lambda/2} \left\langle f\epsilon \right| = 2\sum_{f} \{\pi \left| f\,\mu \right\rangle \rho_f(\mu) \left\langle f\mu \right| - iP\int d\epsilon \left| f\epsilon \right\rangle \rho_f(\epsilon) \frac{1}{\epsilon-\mu} \left\langle f\epsilon \right| \}. \tag{5}$$

The last equality holds provided the variation of the matrix elements in an energy interval on the order of the energy uncertainty of the initial state can be neglected;[6] $\rho_f(\epsilon)$ is the density of states f, and P denotes the principal part. Inserting Eq. (5) into Eq. (4) and putting $\Lambda = \Gamma + i\Delta$, we find

$$\Gamma = 2\pi h \left(\sum_f \left| f\,\mu \right\rangle \rho_f(\mu) \left\langle f\mu \right| \right) h, \tag{6}$$

$$\Delta = 2h \left(\sum_f P\int d\,\epsilon \left| f\,\epsilon \right\rangle \rho_f(\epsilon) \frac{1}{\mu-\epsilon} \left\langle f\epsilon \right| \right) h \;. \tag{6'}$$

Both Γ and Δ are Hermitian; Γ is the decay matrix (contributions from the energy shell) and Δ is the mass matrix (off energy-shell contributions). In the representation with basic vectors $|K_+^+\rangle$ and $|K_-^+\rangle$ $(P|K_+^+\rangle = |K_+^+\rangle$, $P|K_-^+\rangle = -|K_-^+\rangle)$, in which P is the parity operator, we shall write:

$$\langle K_+^+ | \Lambda | K_+^+ \rangle = \Gamma_+ + i\Delta_+ = c_+ ,$$

$$\langle K_-^+ | \Lambda | K_-^+ \rangle = \Gamma_- + i\Delta_- = c_- ,$$

$$\langle K_+^+ | \Lambda | K_-^+ \rangle = \Gamma_x + i\Delta_x = c \;.$$

In this representation the states $|f\epsilon\rangle$ contributing to Λ_+ and Λ_- are those which can be reached from K_+^+ and K_-^+ respectively by the weak interaction; the states contributing to Λ_x are those which can be reached from both K_+^+ and K_-^+. If parity is assumed to be violated only by neutrino interactions, the states $|f\epsilon\rangle$ contributing to Λ_x must contain neutrinos, while the states contributing to Λ_+ and to Λ_- are the states with neutrinos and the states without neutrinos having parity + and - respectively. If time reversal is satisfied, the matrix elements can essentially be taken as real, and the matrix is symmetric in such a case. Such a synmetry also follows independently of time reversal if it is assumed that the amplitudes for a final state with neutrinos from K_+^+ and K_-^+ are the same--such an assumption will appear in the following to be the most consistent one. From Eq. (4) it follows that for a normalized $|K^+\rangle$ satisfying Eq. (4) the corresponding λ satisfies

$$\langle K^+ | \Gamma | K^+ \rangle = \text{Re}(\lambda), \quad \langle K^+ | \Delta | K^+ \rangle = \text{Im}(\lambda), \tag{7}$$

where Re and Im denote real and imaginary parts respectively. Comparing with Eqs. (6) and (6'), we find that Re(λ) is the inverse lifetime and $\mu + (1/2)\,\text{Im}(\lambda)$ is the mass of $|K^+\rangle$. The scalar product of two eigen-solutions $|K_1^+\rangle$ and $|K_2^+\rangle$ satisfies

$$(\lambda_1 + \lambda_2^*) \langle K_2^+ | K_1^+ \rangle = 2 \langle K_2^+ | \Gamma | K_1^+ \rangle \tag{8}$$

Since Γ must be positive definite, from Eq. (8) we obtain the inequality

$$\left| \langle K_1^+ | K_2^+ \rangle \right|^2 < 4\,\text{Re}(\lambda_1)\,\text{Re}(\lambda_2) / [(\text{Re}(\lambda_1 + \lambda_2))^2 + (\text{Im}(\lambda_1 - \lambda_2))^2]. \tag{9}$$

In the following we shall write simply K^+ for a state $|K^+\rangle$. From the assumed symmetry of Λ, the normalized solutions of Eq. (5) are of the form

$$K_1^+ = \frac{1}{\sqrt{N}} (q K_+^+ + K_-^+), \quad K_2^+ = \frac{1}{\sqrt{N}} (K_+^+ - q K_-^+), \tag{10}$$

where $N = |q|^2 + 1$, and q is a complex number,

$$q = \frac{c_+ - c_- + \sqrt{(c_+ - c_-)^2 + 4c^2}}{2c} . \tag{11}$$

The eignevalues corresponding to K_1 and K_2 are

$$\lambda_{1,2} = 1/2 \left[(c_+ + c_-) \pm \sqrt{(c_+ - c_-)^2 + 4c^2} \right] . \tag{12}$$

The two solutions are not, in general, orthogonal:

$$\langle K_1^+ | K_2^+ \rangle = 2i N^{-1}\,\text{Im}(q) . \tag{13}$$

In Eq. (12) Im(q) is zero if Δ is negligible in comparison with Γ. The states that at t = 0 represent a K_+^+ and a K_-^+ respectively are

$$\phi_+(t) = \frac{1}{q^2+1}\left[(q^2 e^{-(\lambda_1/2)t} + e^{-(\lambda_2/2)t})K_+^+ + q(e^{-(\lambda_1/2)t} - e^{-(\lambda_2/2)t})K_-^+\right]e^{-i\mu t}, \tag{14}$$

$$\phi_-(t) = \frac{1}{q^2+1}\left[q(e^{-(\lambda_1/2)t} - e^{-(\lambda_2/2)t})K_+^+ + (e^{-(\lambda_1/2)t} + q^2 e^{-(\lambda_2/2)t})K_-^+\right]e^{-i\mu t}. \tag{14'}$$

If K_+^+ and K_-^+ are produced incoherently, the rate of decay into a final state f is given by

$$R_f(t) = \frac{2\pi\rho_f(\mu)}{\left|q^2+1\right|^2}\Big[N_+\Big|(q^2 e^{-(\lambda_1/2)t} + e^{-(\lambda_2/2)t})\langle f|h|K_+^+\rangle + q\,(e^{-(\lambda_1/2)t} - e^{-(\lambda_2/2)t})\langle f|h|K_-^+\rangle\Big|^2 + N_-\Big|q(e^{-(\lambda_1/2)t} - e^{-(\lambda_2/2)t})\langle f|h|K_+^+\rangle + (e^{-(\lambda_1/2)t} + q^2 e^{-(\lambda_2/2)t})\langle f|h|K_-^+\rangle\Big|^2, \tag{15}$$

where N_+ and N_- are the numbers of K_+^+ and K_-^+ respectively that are originally produced. For the 2π and the 3π decays we find from Eq. (14) for $N_+ = N_-$ (parity-conjugation theory),[7]

$$R_{2\pi}(t) = w(K_+^+|2\pi)\left[(1+|q|^2)(|q|^2 e^{-\gamma_1 t} + e^{-\gamma_2 t}) + 2e^{-(\gamma_1+\gamma_2)t/2}\,\mathrm{Re}[(q^2-|q|^2)e^{i\delta t}\right], \tag{16}$$

$$R_{3\pi}(t) = w(K_-^+|3\pi)\left[(1+|q|^2)(e^{-\gamma_1 t} + |q|^2 e^{-\gamma_2 t}) + 2e^{-(\gamma_1+\gamma_2)t/2}\,\mathrm{Re}[(q^2-|q|^2)e^{-i\delta t}\right]. \tag{16'}$$

We have put $\lambda_i = \gamma_i + i\delta_i$ and introduced $\delta = \frac{1}{2}(\delta_2 - \delta_1)$, and we denote with w the transition probabilities -- it is assumed in this model that only K_+^+ contributes to the 2π mode and only K_-^+ to the 3π mode. The relevant feature is the appearance of the oscillating interference term (last term in Eqs. (16) and (16')).[5] This term comes from the interference between the K_1^+ and the K_2^+ amplitudes. This term is small, however, if the lifetimes are widely different. We can find a limitation for the interference term by using the condition in Eq. (9) that follows from the requirement that Γ be positive definite:

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[7] T. D. Lee and C. N. Yang, Phys. Rev. 102, 290, 1956.

$$\left| \text{interference term} \right| \leq 2 \sqrt{\frac{\tau_S}{\tau_L}} \text{ (direct term)}. \qquad (17)$$

We see from Eqs. (14) and (14') that after a time t from its production, an original K_+^+ becomes a coherent superposition of K_+^+ and K_-^+. In the parity-doublet model, for production processes in which no pseudoscalars are observed, the K^+ beam originally produced is an incoherent superposition of K_+^+ and K_-^+. Parity nonconservation in neutrino processes transforms each of the incoherent amplitudes into a coherent superposition of K_+^+ and K_-^+ after a time comparable to the lifetimes. However, in a process such as (for instance) $K^- + p \rightarrow \Sigma + \pi$ occurring at rest, the beam again behaves as an incoherent mixture of K_+^- and K_-^-. In the Lee-Yang parity-conjugation model, the predicted forward-backward asymmetry in the angular distribution of the pions from Σ decay is still expected to occur essentially in the same way as for absolute parity conservation.

Present evidence clearly indicates only single particles with definite lifetimes and definite branching ratios. We shall now examine, using the results derived in the last section, whether a solution can be constructed that can simulate a situation in which only single particles instead of doublets are observed. In particular, a necessary requirement will be that the two K lifetimes be widely different, so that in most experiments only the long-lived component is observed. In experiments in which only the long-lived component is observed the decay rates would exhibit a single exponential time dependence consistent with the same lifetime--that of the long-lived component--and the relative abundances of the various decay modes would always be the same. Both the 2π and the 3π decay modes would be permitted for this component. In general, however, the long-lived K, after scattering, would also generate short-lived K. The short-lived K that is produced would be easily detected, for instance, in K^+-scattering experiments in nuclear emulsion--if the lifetime is very short, so that the produced short-lived K decays before being observed, the process would appear in most cases to violate strangeness if the energy of the incident K^+ is known; moreover, associated production experiments such as $\pi^+ + p \rightarrow \Sigma^+ + K^+$ would also appear in some cases to violate strangeness. To be definite let us assume that K_1 is the short-lived component and K_2 the long-lived. In the parity-doublet model we find, for the scattered amplitude from an initial

K^+ beam that has lived enough so that the short-lived component is already extinguished,

$$(\text{scattered amplitude}) \sim (1 - q^2) f_- K_1^+ + [(1 + q^2) f_+ - 2q f_-] \; K_2^+ \; , \quad (18)$$

where f_+ and f_- are the two independent scattering amplitudes (energy- and angle-dependent) in the parity-conjugation model. It is seen from Eq. (18) that only if q is close to ± 1 the scattered amplitude never contains a short-lived component. From Eq. (11), q = ± 1 is possible only if $c_+ = c_-$, in which case q = +1. However c_+ contains the parity-conserving amplitudes with P = + 1 (such as those for $K_+ \to 2\pi$) and c_- the P = -1 parity-conserving amplitudes (such as for $K_- \to 3\pi$). These amplitudes are completely unrelated, and, further, there is no general way of relating them to the amplitudes for the neutrino decay modes. Therefore the only nonaccidental explanation of $c_+ \cong c_-$ would be the predominance of the neutrino contributions in both c_+ and c_-, since such contributions could be related one to the other. The simplest way of relating the neutrino amplitudes so that, in the absence of nonneutrino interactions we have $c_+ = c_-$, is to require the invariance of the Hamiltonian under the composite transformation (the signs are not necessarily related),

$$\psi_+ \to \pm \psi_- \; , \qquad \psi_\nu \to \pm \gamma_5 \psi_\nu \; , \qquad (19)$$

where ψ_+, ψ_- are the field operators for the two components of a parity doublet, ψ_ν is the neutrino field, and the first transformation is carried out for each particle existing in parity doublet (particles of odd strangeness in the parity-conjugation model). The $K \to \mu + \nu$ decay interaction, for instance, would possibly be written as

$$q_s K_s \overline{\psi}_\mu (1 \pm \gamma_5) \psi_\nu + q_A K_A \overline{\psi}_\mu (1 \mp \gamma_5) \psi_\nu,$$

giving equal total decay rates from K_+ and from K_-. A further requirement of separate invariance under $\psi_\nu \to \gamma_5 \psi_\nu$ (or $\psi_\nu \to -\gamma_5 \psi_\nu$) would be equivalent to postulating equal or opposite amplitudes for the neutrino modes, and in this case all elements of the Λ matrix would be equal in the absence of nonneutrino interactions. We shall not use this particular assumption; however, we shall be led in the following to adopt it for the consistency of the model. In any case, it follows from the more general assumption Eq. (19), and from the assumption of the preponderance of the contributions from neutrino interactions that the physical particles in a parity doublet (i.e., the eigensolutions of the equations of the form of Eq. (4)) are approximately described by states of the form (Eqs. 10)

$$s_S = \frac{1}{\sqrt{2}} \; (s_+ + s_-), \qquad s_A = \frac{1}{\sqrt{2}} \; (s_+ - s_-), \tag{20}$$

where s_S and s_A are the two states of opposite parity for a particle of odd strangeness in the parity-conjugation theory. (The case of the neutral K mesons is more complicated and is discussed later.) The states (20) are eigenstates of the parity-conjugation operator c_P with eigenvalue + 1 and - 1 respectively, therefore these states are conserved in scattering processes. The corresponding eigenvalues λ are $\lambda_S = c_+ + c$ and $\lambda_A = c_+ - c$. If the real part of c is positive, s_S is short-lived and s_A long-lived, and the opposite holds if the real part of c is negative. It can be seen that, if the sign for each s in the first of the two relations (19) is chosen appropriately, it is possible to construct a situation such that, for instance, s_S is always short-lived and s_A always long-lived. In particular, if the more stringent assumption is adopted of separate invariance under $\psi_\nu \to \pm\gamma_5 \psi_\nu$, the choice of the sign in the first of Eq. (20) would be the same for all s. If so, for each doublet the short-lived omponent would have, for instance, $c_P = +1$ and the long-lived $c_P = -1$. Let us now consider a production experiment, to be definite $\Sigma + K^+$ production from ordinary particles. Conservation of c_P restricts the final amplitude to the form

$$c_S \, (\Sigma_S K^+{}_S) \; + \; c_A (\Sigma_A K^+{}_A), \tag{21}$$

where c_S and c_A are complex amplitudes. This final amplitude gives zero probability for the production of a short-lived Σ with a long-lived K^+, and for the production of a long-lived Σ with a short-lived K^+. Therefore, if only the long-lived particles are observed, no apparent violation of the strangeness selection rule will be noticed. Moreover, a beam of long-lived K^+ or K^- in subsequent strong interactions with ordinary nuclear matter will always produce long-lived particles (the case in which K^0 are produced will be examined later). Therefore experiments such as (N stands for nucleon), $K^+ + N \rightarrow K^+ + N$, $K^- + N \rightarrow \Sigma^{\pm,0} + \pi$ (including the subsequent Σ^0 decay and possible subsequent interactions of the $\Sigma^{\pm}$) will fail to reveal a short-lived component. This conclusion does not hold at higher energies if more than one strange particle is produced by the long-lived K. Production of more than one strange particle from the K^- beam can occur, for instance, in the reaction $K^- + p \rightarrow \Xi^- + K^+$, and in various other reactions at still higher energies. The Ξ^- has a given c_P (= ± 1) and exhibits only one lifetime, namely the observed one. If c_P of Ξ^- is -1, the emitted K^+ is short-lived. In this case the reaction would appear to violate strangeness. If c_P of Ξ^- is +1 the emitted K^+ is long-lived. In this case the final K amplitude in the production process

$$(\text{ordinary particles}) \rightarrow \Xi^- + K^+ + K^+$$

has the form $c_S(K^+_S K^+_S) + c_A(K^+_A K^+_A)$, and it would sometimes lead to apparent violation of the strangeness rule. If c_P of Ξ^- is -1 the final K amplitude has the form $c_{SA}(K^+_S K^+_A)$, and it would practically always lead to apparent violation of the strangeness. From present evidence[8] we would be led to assume $c_P = +1$ for Ξ^-. In the decay of Ξ^- the produced Λ^0 could be sometimes long-lived and sometimes short-lived. If the shorter lifetime were such as to give no visible track the event would simulate $\Xi^- \rightarrow \pi^- + (\Lambda^0$ decay products).

The experimental upper limits on the shorter K^+ lifetime will depend of course on the assumed abundance of short-lived K^+. However, in this theory short-lived and long-lived particles are produced in the same abundance. To see this point in a simple way, consider the production of K^+ without observing the Σ. The K^+_+ and the K^+_- are produced incoherently. Therefore there are two amplitudes, one for K^+_+ and one for K^+_-, which are incoherent. Both amplitudes contain the same number of $K_S{}^+$ and $K_A{}^+$. From present data it would seem that the theory could be made consistent with a K lifetime $\leq 10^{-14}$ to 10^{-13} sec, but it is difficult to obtain reliable estimates. The couplings responsible for such decays would therefore be

[8] G. H. Trilling and A. Neugenbauer, Phys. Rev. 104, 1688 (1956).

rather larger than generally assumed for the weak decay interactions, and this could constitute a theoretical difficult. Moreover, we must discuss the mathematical conditions for the consistency of the solution. We shall see that in order for the solution to be consistent the weak interactions must satisfy rather strange conditions, and we shall derive the relevant result that the short-lived particles must always decay into neutrino modes. We decompose the diagonal matrix elements of Λ into two parts, separating the contributions due to neutrino interactions from the remaining contributions due to the parity-conserving interactions. We thus write

$$\Gamma_+ + i\,\Delta_+ = (V + W_+) + i\,(M + N_+),$$

$$\Gamma_- + i\,\Delta_- = (V + W_-) + i\,(M + N_-),$$

where V and M are the contributions from neutrino interactions. The nondiagonal matrix element $\Gamma_x + i\,\Delta_x$ is entirely due to neutrino interactions--it must be noted that $V \geqslant \Gamma_x$. For the two physical states, $s_1 = N^{-1/2}(q\,s_+ + s)$ and $s_2 = N^{-1/2}(s_+ - q\,s_-)$, we can calculate the total decay rate for decay into nonneutrino final states with parity +, which we call $w(s\,|\,+)$, and the total decay rate for decay into nonneutrino final states with parity -, $w(s\,|\,-)$. These rates are given by

$$w\,(s\,|\,+) = N^{-1}\,|q|^2\,W_+ ,$$

$$w\,(s\,|\,-) = N^{-1}\,W_- ,$$

$$w\,(s\,|\,\nu) = N^{-1}\left[(1 + |q|^2)\,V + 2\,\mathrm{Re}\,(q)\,\Gamma_x\right],$$

and by

$$w\,(s_2\,|\,+) = N^{-1}\,W_+$$

$$w\,(s_2\,|\,-) = N^{-1}\,|q|^2\,W_-$$

$$W\,(s_2\,|\,\nu) = N^{-1}\left[(1 + |q|^2)\,V - 2\,\mathrm{Re}\,(q)\,\Gamma_x\right].$$

From these quantities we can obtain the quantities that are relevant for our discussion, namely the branching ratios between neutrino modes and parity-conserving modes for s_1 and s_2, B_1 and B_2 respectively, and the lifetime ratio τ_2/τ_1 :

$$B_1 = \frac{(1 + |q|^2)\, V + 2\,\mathrm{Re}\,(q)\,\Gamma_x}{|q|^2\, W_+ + W_-} , \tag{22}$$

$$B_2 = \frac{(1 + |q|^2)\, V - 2\,\mathrm{Re}\,(q)\,\Gamma_x}{W_+ + |q|^2\, W_-} \tag{22'}$$

$$\frac{\tau_2}{\tau_1} = \frac{\mathrm{Re}(\lambda_1)}{\mathrm{Re}(\lambda_2)} = \frac{W_+ + |q|^2\, W_- + (1 + |q|^2)V - 2\,\mathrm{Re}(q)\,\Gamma_x}{|q|^2\, W_+ + W_- + (1 + |q|^2)V + 2\,\mathrm{Re}(q)\,\Gamma_x} \tag{23}$$

The mixing parameter q can be expanded as

$$q = 1 + \frac{c_+ - c_-}{2c} + 0\left(\left(\frac{c_+ - c_-}{2c}\right)^2 \right), \tag{24}$$

and the condition $q \cong 1$ thus means $\left| \frac{c_+ - c_-}{2c} \right|^2 << 1$. To be definite we assume s_2 long-lived and s_1 short-lived. Let us first discuss the solution for the hyperons (Λ, Σ). We must simultaneously satisfy

$$\left| \frac{c_+ - c_-}{2c} \right| << 1, \tag{25}$$

which guarantees $q \cong 1$, and

$$\frac{\tau_2}{\tau_1} = \frac{W_+ + W_- + 2\,(V + \Gamma_x)}{W_+ + W_- + 2(V - \Gamma_x)} >> 1, \tag{25'}$$

$$B_2 = \frac{2\,(V - \Gamma)}{W_+ + W_-} << 1. \tag{25''}$$

The last condition ensures that the branching ratio for neutrino modes of the observed component is small, as known experimentally. After Eq. (25") has been substituted into (25'), (25') can be written as

$$\frac{\tau_2}{\tau_1} = \frac{2\,(V + \Gamma_x)}{W_+ + W_-} >> 1, \qquad (26)$$

and, comparing with (22), we find the important result

$$B_1 = \frac{\tau_2}{\tau_1} >> 1, \qquad (27)$$

which shows that the short-lived component essentially decays into neutrino modes. However, from Eqs. (25") and (26) we find

$$\frac{V - \Gamma}{V + \Gamma} \doteq B_2 \frac{\tau_1}{\tau_2} ,$$

which would strongly suggest $V = \Gamma$ (or $V = -\Gamma$ if the long-lived is s_1). This equality, as shown by Eq. (6), would strongly suggest the equality of the neutrino-mode amplitudes from s_+ and from s_- -- we have already discussed this possible symmetry requirement. The conditions to be satisfied are now the inequality (26), in the form

$$\frac{\tau_2}{\tau_1} = \frac{4\,V}{W_+ + W_-} >> 1, \qquad (28)$$

and (25), which can be written as

$$\left| \frac{c_+ - c_-}{2c} \right| = \left| \frac{(W_+ - W_-) + i\,(N_+ - N_-)}{2\,(V + i\,\Delta)} \right| << 1 . \qquad (29)$$

These conditions are consistent, and by taking--for instance-- $V \sim \Delta$, $W_+ \sim W_- \sim N_+ \sim N_-$, we find $\left|(c_+ - c_-)/2c\right| \sim \tau_2/\tau_1$, which would mean that the physical particles are very closely eigenstates of $\mathcal{C}_P$, in agreement with the experimental conditions, as already discussed. We then find that for $V = \Gamma$ the branching ratio in neutrino modes for the long-lived component would be, from Eq. (22') and (24) and the last result on $\left|(c_+ - c_-)/2c\right|$

$$B_2 \sim \frac{\tau_1}{\tau_2} ,$$

which would agree very well with the experimental results. We therefore see that for hyperons a rather consistent scheme can be constructed--at the following price, however: first we must admit an inequality like (28), which is not easy to understand theoretically and sounds rather unphysical; second, the short-lived component, according to (27), would mostly decay into neutrino modes, a situation that could give rise to peculiar observational consequences--because of conservation of nucleons another lepton (μ, e or ν) must be emitted in these decay modes.

For $K^{\pm}$ mesons we want again to satisfy conditions (25) and (25'); however, condition (25") must be substituted by

$$B_2 \sim 1, \tag{30}$$

as known experimentally. From (30) and (25') we are led to

$$\frac{V - \Gamma}{V + \Gamma} = \frac{1}{2} \frac{\tau_1}{\tau_2} ,$$

which again strongly indicates $V = \Gamma$. We therefore write for this case the conditions corresponding to (25') and (25") in the hyperon case in the form

$$\frac{\tau_2}{\tau_1} = \frac{W_+ + W_- + 4\Gamma}{W_+ + W_- + \left|\frac{c_+ - c_-}{2c}\right|^2} \gg 1, \tag{31}$$

$$B_2 = \frac{\left|\frac{c_+ - c_-}{2c}\right|^2 \Gamma}{W_+ + W_-} \sim 1 . \tag{31'}$$

If Eq (31') is substituted into (31), (31) can be written as

$$\frac{\tau_2}{\tau_1} = \frac{2}{W_+ + W_-} \gg 1. \tag{32}$$

Again, comparing with (22), we have

$$B_1 = 2 \frac{\tau_2}{\tau_1} \gg 1,$$

which implies that the short-lived component must practically always decay into neutrino modes--this result follows in any case even if we do not assume $V = \Gamma$. The independent conditions to be satisfied are now the conditions similar to (29), (31'), and (32). The last two give

$$\left| \frac{c_+ - c_-}{2c} \right| \cong \sqrt{2 \frac{\tau_1}{\tau_2}} \,, \tag{33}$$

which is sufficient to guarantee that K_1 and K_2 are close to eigenstates of $\mathcal{C}_P$. Equation (32), like (28), seems very unphysical. Moreover, we note that (33) is different from the corresponding condition for the hyperons, essentially because of the square root in (33)--this difference is because we have assumed for the K mesons $B_2 \sim 1$ instead of $B_2 \ll 1$ as for the hyperons. This means that Eq. (33) cannot be satisfied with the same choice $W \sim N$ as for the hyperons--in which case, using Eq. (34), one obtains $|(c_+ - c_-)/2c| < \sim (\tau_1/\tau_2)$. It can be shown from Eq. (34) that (33) can be satisifed only provided we have

$$\frac{|N_+| + |N_-|}{W_+ + W_-} \gg 2 \frac{\tau_2}{\tau_1} \,,$$

a condition which again is rather unphysical. An alternate possibility could be to assume that only some neutrino decay interactions, not observed for the long-lived K, are predominant. In any case, according to conservation of statistics and conservation of heavy particles, the predominantly neutrino decay modes of the short-lived particles must lead to at least two final leptons. One of these leptons would be expect to be a muon or an electron, at least in some cases. No cases of μ mesons or electrons coming out from stars have been reported. Direct mass measurements on the secondary tracks emerging from high-energy stars would generally fail to identify μ mesons and electrons.[9] A systematic investigation carried out at Bristol on the nature of the particles emerging from stars[10] was not in an appropriate condition for identifying such mesons or electrons because of the low limit imposed on the velocity of the particles examined.

Other difficulties for this solution occur in connection with neutral K mesons. A parity-doublet theory would lead in general to four different states with four different lifetimes for the neutral K mesons. In the representation in which the strangeness S and the parity P are diagonal K^0_+, K^0_-, K^0_+, K^0_- are the four basic states--these states do not in general have a definite lifetime. Since weak interactions do not conserve strangeness, the physical particles are linear combinations of K^0 and $\bar{K}^0$ states. States of both parities are present in these combinations because of the mixing produced by neutrino interactions. It will be convenient to use the representation with basic vectors

$$K^0_{++} = \frac{1}{\sqrt{2}}(K^0_+ + \bar{K}^0_+), \quad K^0_+ = \frac{1}{2}(K^0_+ - \bar{K}^0_+),$$

$$K^0_{-+} = \frac{1}{\sqrt{2}}(K^0_- + \bar{K}^0_-), \quad K^0_- = \frac{1}{2}(K^0_- - \bar{K}^0_-). \tag{34}$$

On the basis of Lüders theorem, the assumption of time-reversal invariance is equivalent to the assumption of conservation of the quantum number L=CP. If a single K^0 is assumed, L conservation alone uniquely determines the physical particles in terms of K^0 and $\bar{K}^0$. The physical particles are in this case the two eigenstates of L,

$$\frac{1}{\sqrt{2}}(K^0 + \bar{K}^0), \quad \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0).$$

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[9] Daniel, Davies, Mulvey, and Perkins, Phil. Mag. 43, 753 (1952).

[10] Dahanake, Francois, Fujimoto, Iredale, Waddington, Yasin, Phil. Mag. 45, 885 (1954).

These states are the eigenstates of L, with eigenvalue + 1 and - 1 respectively. In fact these states are eigenstates of L when there are no weak interactions, and as long as weak interactions are assumed to conserve L, they remain eigenstates of L, with eigenvalues +1 and - 1 respectively, also after turning on the weak interactions. A system of two pions has L=+1, and therefore only the state with L = + 1 can decay into two pions. By the same argument it follows that of the states (34), K_{++}^{0} and K_{--}^{0} are eigenstates of L with L = + 1, and K_{+-}^{0} and K_{-+}^{0} are eigenstates of L with eigenvalue L = - 1. Of the states (34) only K_{++}^{0} and K_{--}^{0} can decay into two pions. Conservation of L requires the physical states to be of the form

$$u_{++} K_{++}^{0} + u_{--} K_{--}^{0}, \qquad \text{with } L = 1, \tag{35}$$

$$u_{-+} K_{-+}^{0} + u_{+-} K_{+-}^{0} \qquad \text{with } L = -1. \tag{35'}$$

There is no choice of the coefficients u, which makes the states (35) and (35') eigenstates of $\mathcal{C}_P$. The eigenstates of $\mathcal{C}_P$ can be expressed in terms of the basic vectors

$$K_S^0 = \frac{1}{\sqrt{2}} (K_+^0 + K_-^0), \quad K_A^0 = \frac{1}{\sqrt{2}} (K_+^0 - K_-^0), \quad \bar{K}_S^0 = \frac{1}{\sqrt{2}} (\bar{K}_+^0 + \bar{K}_-^0),$$

$$\bar{K}_A^0 = \frac{1}{\sqrt{2}} (\bar{K}_+^0 - \bar{K}_-^0) \tag{36}$$

as

$$c_S K_S^0 + \bar{c}_S \bar{K}_S^0, \qquad \text{with } \mathcal{C}_P = +1, \tag{37}$$

$$c_A K_A^0 + \bar{c}_A \bar{K}_A^0, \qquad \text{with } \mathcal{C}_P = -1. \tag{37'}$$

A superposition of the form (35) or (35') cannot be expressed in the form (37) or (37'), unless all the coefficients are zero. This result merely reflects the circumstance that L and $\mathcal{C}_P$ anticommute for strangeness odd. In fact, from the definition of the parity conjugation operator $\mathcal{C}_P$, it follows that $\mathcal{C}_P L - (-)^S L \mathcal{C}_P = 0$. In a Wigner-Weisskopf treatment the physical states (35) and (35') are determined as eigensolutions of a non-Hermitian matrix $\Lambda = \Gamma + i\,\Delta$, where Γ is the contribution from the energy shell and Δ the contribution from outside the energy shell. In the representation with basic vectors (34) there are no matrix elements of Λ connecting the two subspaces with L = + 1 and with L = - 1. This follows from the assumed L conservation for the total Hamiltonian (which is equivalent, according to Lüders theorem, to time-reversal invariance). Therefore the eigenvalue problem is split into two separate eigenvalue problems, one in the L = + 1 subspace (solutions of the form (35)) and one in the L = - 1 subspace (solutions of the form (35')). Moreover, the assumption of time-reversal invariance allows one to take all the matrix elements real, so that the matrix elements symmetric with respect to the first diagonal are equal. The solutions are thus of the form

$$K_1^0 = \frac{1}{\sqrt{F}}\left[f\, K_{++}^0 + K_{--}^0\right], \tag{38}$$

$$K_2^0 = \frac{1}{\sqrt{F}}\left[K_{++}^0 - f\, K_{--}^0\right], \tag{38'}$$

$$K_3^0 = \frac{1}{\sqrt{G}}\left[g\, K_{-+}^0 + K_{+-}^0\right], \tag{38''}$$

$$K_4^0 = \frac{1}{\sqrt{G}}\left[K_{-+}^0 - g\, K_{+-}^0\right], \tag{38'''}$$

where f and g are in general complex numbers, and $F = |f|^2 + 1$, $G = |g|^2 + 1$. The solutions K_1^0 and K_2^0 pertain to the eigenvalues L = + 1, K_3^0 and K_4^0 to L = - 1. The states K_1^0 and K_2^0 can decay into two pions, while the states K_3^0 and K_4^0 cannot. The states which at t = 0 represent a K_+^0 and a K_-^0 are respectively

$$K_+^0 \rightarrow \frac{1}{\sqrt{2}}\left[\frac{\sqrt{F}}{f^2+1}\left(fK_1^0 e^{-(\lambda_1 t)/2} + K_2^0 e^{-(\lambda_2 t)/2}\right) + \frac{\sqrt{G}}{g^2+1}\left(K_3^0 e^{-(\lambda_3 t)/2} - gK_4^0 e^{-(\lambda_4 t)/2}\right)\right] e^{-i\mu t}, \tag{39}$$

$$K_-^0 \rightarrow \frac{1}{\sqrt{2}}\left[\frac{\sqrt{F}}{f^2+1}\left(K_1^0 e^{-(\lambda_1 t)/2} - fK_2^0 e^{-(\lambda_2 t)/2}\right) + \frac{\sqrt{G}}{g^2+1}\left(gK_3^0 e^{-(\lambda_3 t)/2} + K_4^0 e^{-(\lambda_4 t)/2}\right)\right] e^{-i\mu t}. \tag{39'}$$

The λ are the complex eigenvalues of the matrix Λ. The resulting decay curves are very complicate, containing exponential terms and oscillating terms. The behavior of the solutions in the limiting case of predominant neutrino interactions is quite different from the behavior for charged K. We have already remarked that, in contrast to the case of charged K, the eigensolutions (38) - (38''') cannot be eigenstates of C_P (this holds in general). We must assume, as a result of the foregoing discussion, that the matrix elements from K_+ are the same, apart from sign, as those K_- for processes involving neutrinos. To be definite let us assume that the amplitudes from K_+ are equal to those from K_-. The eigenvalue problem in the L = +1 subspace is then the same as for the L = -1 subspace in this limit of predominant neutrino interactions. Therefore, we have $\lambda_1 = \lambda_3$, $\lambda_2 = \lambda_4$, and f = g in the above Eq. (40) - (40'''). Since two different lifetimes have already been observed in the K^0 beams we would be led to identify the two different lifetimes with those which are observed. We can show, however, that this solution is not in agreement with experiment, since it would lead to a large fraction of non-2π decays of the short-lived component. The fraction of decay into 2π of the short-lived component can be calculated by considering that K_+^0 and K_-^0 are produced incoherently and noting that the solutions K_1^0 and K_2^0 are orthogonal and normalized to unity. We see from Eqs. (39) and (39') that, in the limiting case considered, at least one-half of the short-lived component does not decay into two pions. This is in apparent disagreement with experiment, considering that no confirmed evidence exists for the 2π decay of the long-lived component--the anomalous decay modes at times comparable to the shorter lifetimes are no more than about 10%. A general argument shows that the distortion of the solutions due to nonneutrino interactions cannot modify this condition, at least as long as one wants to maintain the feature of approximate coincidence of the four lifetimes into two distinct lifetimes into two distinct lifetimes. From Eqs. (39) and (39') we see that the fraction of the short-lived K^0's in the incoherent K_+^0, K_-^0 beam that can decay into two pions is given by

$$\frac{1}{2} \frac{(|f|^2 + 1)^2}{|f^2 + 1|^2} , \qquad (40)$$

and the fraction that cannot decay into two pions is given by

$$\frac{1}{2} \; \frac{(|g|^2 + 1)^2}{|g^2 + 1|^2} . \tag{41}$$

The condition on the Hermitian matrix Γ of being positive definite leads to the inequality

$$\frac{|f - f^*|^2}{|\,|f|^2 + 1\,|^2} < \frac{4\,\mathrm{Re}\,(\lambda_1)\;\mathrm{Re}\,(\lambda_2)}{[\mathrm{Re}\,(\lambda_1 + \lambda_2)]^2 + [\mathrm{Im}(\lambda_1 - \lambda_2)]^2} . \tag{42}$$

The right-hand side in (42) is less than $4(\tau_S/\tau_L)$, where τ_S is the shorter lifetime and τ_L the longer. According to present data τ_S/τ_L is of the order of 1/100. An inequality similar to (42) holds for g if one substitutes λ_3 and λ_4 for λ_1 and λ_2. If now we use the identy

$$\frac{1}{2} \; \frac{(|f|^2 + 1)^2}{|f^2 + 1|^2} = \frac{1}{2} \; \frac{(|f|^2 + 1)^2}{(|f|^2 + 1)^2 - |f - f^*|^2}$$

and the similar identity for (41), comparing with (42) and with the corresponding inequality for g, we find that both (40) and (41) are approximately equal to 1/2, apart from terms smaller than $4(\tau_s/\tau_L) \sim 4/100$. Another difficulty in reconciling with a solution with essentially two lifetimes would occur if, in the associated production of Λ^0 with K^0 (not eigenstate of c_P in this theory) a high frequency of visible Λ^0 were found in association with the K^0 with shorter lifetime. Recent results at Columbia point in this direction.

II

We now discuss the decay distribution in this theory, in particular the question of up-down asymmetries in hyperon decay. We shall see that a parity-doublet model with parity nonconservation in neutrino interactions and with parity conservation in other interactions leads to up-down asymmetries for hyperon decays, at least as long as there is sufficient parity mixing in the states representing the physical particles. The problem of the polarization of particles with parity-doublet structure has been discussed by Lee and Yang assuming parity conservation.[11] If parity is not conserved the density matrix has a more complicated structure because of the parity mixing due to the weak interactions. Following Lee and Yang, we describe a particle with parity-doublet structure by a 4-component wave function

$$\psi(t) = \begin{bmatrix} \psi_+(t) \\ \psi_-(t) \end{bmatrix},$$

where ψ_+ and ψ_- refer to the even-parity and odd-parity states respectively, and each of them has two components describing spin up and spin down. The density matrix is given by

$$D(t) = \Sigma\psi(t)\,\psi^\dagger(t),$$

and it is written in the form

$$D(t) = \begin{bmatrix} D_+(t) & D_x(t) \\ D_x(t)^\dagger & D_-(t) \end{bmatrix}. \qquad (43)$$

Under space inversion, $D_+(t)$ and $D_-(t)$ are invariant while $D_x(t)$ and $D_x(t)^\dagger$ change the sign. Lee and Yang introduce the pseudovectors $\vec{P}_+(t)$ and $\vec{P}_-(t)$, the vectors $\vec{P}_r(t)$ and $\vec{P}_i(t)$, the scalars $I_+(t)$ and $I_-(t)$, and the psuedoscalars $I_r(t)$ and $I_i(t)$, defined by the relations

[11] T. D. Lee and C. N. Yang, Phys. Rev. 104, 822 (1956).

$$D_+(t) = \vec{P}_+(t) \cdot \vec{\sigma} + I_+(t) , \qquad (44)$$

$$D_-(t) = \vec{P}_-(t) \cdot \vec{\sigma} + I_-(t) , \qquad (44')$$

$$D_\chi(t) = D_r(t) + i D_i(t) = (\vec{P}_r(t) + i \vec{P}_i(t)) \cdot \vec{\sigma} + (I_r(t) + I_i(t)). \qquad (45)$$

The quantities $I_+(t)$ and $I_-(t)$ are the intensities, and $\vec{P}_+(t)/I_+(t)$ and $\vec{P}_-(t)/I_-(t)$ the polarization vectors for the particles with parity + and - respectively. The limitations that follow from the condition that D(t) be positive definite are reported in the Lee-Yang paper. If the value of the density matrix at the instant of production, D(o), is known, its values at times t later can be obtained by the Wigner-Weinkopf treatment of the decay interactions. In contrast to the case of parity conservation, $D_+(t)$ and $D_-(t)$ have a nonexponential time dependence, but they contain two exponential terms and an oscillating exponential term. We find

$$D_+(t) = \frac{1}{|q^2+1|^2} \left[|q|^2 E_1 e^{-\gamma_1 t} + 2 \,\mathrm{Re}\, (qEe^{i\delta t}) e^{-(\gamma_1+\gamma_2)t/2} + E_2 e^{-\gamma_2 t} \right] , \qquad (46)$$

$$D_-(t) = \frac{1}{|q^2+1|^2} \left[E_1 e^{-\gamma_1 t} - 2 \,\mathrm{Re}(q^* E e^{i\delta t}) e^{-(\gamma_1+\gamma_2)t/2} + |q|^2 E_2 e^{-\gamma_2 t} \right] , \qquad (46')$$

$$D_\chi(t) = \frac{1}{|q^2+1|^2} \left[qE_1 e^{-\gamma_1 t} - |q|^2 E e^{-(\gamma_1+\gamma_2)t/2} e^{i\delta t} + E^\dagger e^{-(\gamma_1+\gamma_2)t/2} e^{-i\delta t} - q^* E_2 e^{-\gamma_2 t} \right] \qquad (46'')$$

with

$$E_1 = |q|^2 D_+(0) + q\, D_\chi(0) + q^*\, D_\chi(0)^\dagger + D_-(0), \qquad (47)$$

$$E_2 = D_+(0) - q^* D_\chi(0) - q\, D_\chi(0)^\dagger + |q|^2 D_-(0), \qquad (47')$$

$$E = q\, D_+(0) - |q|^2\, D_\chi(0) + D_\chi(0)^\dagger - q^*\, D_-(0). \qquad (47'')$$

Equations (46) through (47'') completely specify the density matrix at time t once the two lifetimes, the mass difference, and the parity-mixing parameter q are known. From the density matrix each measurable probability can be derived. In particular, considering the decay of hyperons, the momentum distribution of the decay pion in the hyperon rest system can be written as

$$W(\vec{K}) = \xi + \vec{\eta} \cdot \vec{K}, \qquad (48)$$

where ξ and $\vec{\eta}$ are expressed linearly through the elements of D(t).[10]

For the hyperons emitted in the capture of K^- at rest, $\vec{\eta}$ is parallel to the hyperon momentum, thus giving a forward-backward asymmetry. However, in a production process from particles with kinetic energy, $\vec{\eta}$ has in general a component normal to the plane of production. Therefore both forward-backward asymmetries and up-down asymmetries are expected--even after the states of the other produced particles are summed over. Let us discuss in particular the case in which only the longlived component (of Λ or of Σ) is observed. In this case, from Eqs. (46'') and (47') (to be definite we take particle 2 as the long-lived), we have

$$D_\chi(t) \rightarrow \frac{q^*}{|q^2 + 1|^2}\, e^{-\gamma_2 t}\, [D_+(0) - q^* D_\chi(0) - q D_\chi^\dagger(0) + |q|^2\, D_-(0)]. \qquad (49)$$

The vector $\vec{\eta}$ expressing the average direction of $\vec{K}$ is of the form (constant) $\vec{P}_r(t)$ + (constant) $\vec{P}_i(t)$.[10] The vectors $\vec{P}_r(t)$ and $\vec{P}_i(t)$ can be obtained from Eqs. (48) and (49). From the invariance under rotations and reflections in the production process and from the invariance of the density matrix under $\mathcal{C}_P$ (we assume that the states of the other particles produced are not observed), $\vec{\eta}$ is found to have the form

$$\vec{\eta} = [\vec{P}(0)\ (1 + |q|^2) - 2\, \mathrm{Re}\,(q)\, \vec{P}_r(0)] \times$$

$$[(\text{constant})\, \mathrm{Re}\,(q) + (\text{constant})\, \mathrm{Im}\,(q)], \qquad (50)$$

$$= \vec{P}_-(0)$$

where $\vec{P}_-(0) = \vec{P}_+(0)$ is a pseudovector normal to the production plane and the vector $\vec{P}_r(0)$ lies in the production plane. Equation (50) shows that both up-down asymmetries (except for forward direction) and forward-backward asymmetries occur. The only exceptional case would be $q = 0$, which corresponds to observing only one particle with definite parity.

III

If K_+ and K_- have all quantum numbers equal and differ only in the parity assignment, the minimum symmetry required for accounting of the equality of their masses would be the invariance under parity conjugation.[7] Our discussion so far applies to the parity-conjugation model. One can note, however, that in the Gell-Mann's Theory I_3 conservation is equivalent to S conservation and therefore, as long as the I_3 assignment is maintained, one can very well reverse the S value for each particle (and accordingly modify the relation between Q and I_3) without introducing any forbidden reaction (single production, etc.). This freedom allows for a duplication of the number of strange particles--by introducing for each strange particle a particle with opposite S and the same I_3. In a recent paper Pais points out that by taking S pseudoscalar one can use this freedom for generating parity doublets.[12] Because P and S anticommute, the parity operator reverses the strangeness (however, it leaves I_3 unchanged), therefore invariance of the theory under P is sufficient for obtaining the equality of the masses. However, again owing to the anticommutation between P and S, the particles with definite strangeness have no definite parity. This theory is in some sense more economical than the parity-conjugation theory; in fact, no c_P invariance is needed. No forward-backward asymmetries are expected in this theory. Most of the discussion carried out so far for parity-conjugation invariance applies in a similar form to Pais theory. The K^+ beam at time t after the production is a superposition of K^+_+ and K^+_-, or (changing the representation) of K^+ with $S = +1$ and of K^+ with $S = -1$. These latter, however, cannot still generate hyperons because of I_3 conservation. The only possibility allowed is again that with two very different lifetimes. Moreover, the long-lived particles must not generate short-lived particles in interaction, and the only way of obtaining such a situation is assuming that they are close to eigenstates of the strangeness, in which case the short-lived are also, corresponding to the opposite eigenvalue. This again means $q = +1$, and the physical realization would again be of the same kind as discussed for the parity-conjugation model.

[12] A. Pais. A Suggestion Concerning the Reflection Properties of Hyperons and K-Mesons (preprint).

The following differences must be noted however. First, in the Pais theory also the Ξ particles are doublets. Therefore there would be short-lived and long-lived Ξ. Short-lived are produced with short-lived, and long-lived with long-lived. Moreover, the Λ^0 emitted from the decay of the long-lived Ξ^- must again be long-lived if $\Delta S = \pm 1$ holds. Therefore the difficulties concerning Ξ-particles would be in part removed by acceptance of the Pais formulation of parity-doublet theory. Second, since $L = C P$ and S commute in the Pais theory (they anticommute in the ordinary theory) it would not be impossible for the K^0 with definite L to have also definite S -- however, we cannot give any argument showing that this occurs; it would in fact be quite accidental. Moreover, the limit of strong neutrino interactions would essentially lead to the same difficulties as for the Lee-Yang parity-doublet theory. In particular, the present scheme for associated production of $\Lambda^0 + K^0$ with one observed Λ^0 lifetime and two observed K^0 lifetimes occurring with about the same relative frequencies would not in general be fitted in this model. The arguments discussed in Section II concerning the conditions imposed for the consistency of the solution would, moreover, still apply in exactly the same form.

IV

We have examined in this discussion what modifications in the original τ-θ problem are brought about by the recent discovery of parity nonconservation in neutrino interactions, and whether such modifications could permit avoidance of the conclusion that parity must be violated also in other weak interactions not involving neutrinos. We have, however, restricted our discussion to the usual description of neutrino interactions such that the observed neutrino processes occur as direct processes at first order in the weak neutrino coupling. Under this restriction our conclusion is negative. In fact, the only possible solutions would require at least two K mesons, and hence almost unavoidable general parity-doublet structures. Such models would appear in any case more complicated than models with parity nonconservation in weak nonneutrino interactions also. A detailed discussion of the possible cases shows that one solution could be constructed that can simulate the present experimental evidence strongly indicating single particles and single lifetimes. According to this solution there would be short-lived and long-lived particles, which are described by states that very approximately are eigenstates of the parity conjugation operator in the Lee-Yang parity-doublet theory, or eigenstates of the strangeness in the parity-doublet theory recently suggested by Pais. Short-lived particles would be produced in association with short-lived, and long-lived with long-lived. Moreover, long-lived particles would for most experiments originate only long-lived particles after nuclear interaction. This solution can be constructed consistently if a (rather unphysical) predominance of the neutrino interactions is assumed with respect to the parity-conserving weak interactions, and the neutrino interactions are further assumed to satisfy some symmetry requirement with respect to exchange of the two parity states in the parity doublet. However, it is shown that the consistency of the solution requires very unphysical conditions on the decay interactions (large differences in the magnitudes of various matrix elements, strong energy dependences or different behavior of various classes of neutrino interactions). It is shown that the decay of the short-lived particles would practically always occur into neutrino modes. A particular difficulty occurs in connection with the neutral K mesons, which do not fit the scheme properly. In the Lee-Yang form of parity doublets, the presence of the nonobserved forward-backward asymmetries and the nondoublet structure of the Ξ particle would constitute further difficulties. The model would lead to up-down asymmetries even if the interaction directly responsible for the observed hyperon decay conserved parity.

In conclusion, we feel that the unavoidable complication of the scheme, the unphysical conditions on the weak interactions required for its consistency, and some of the specific difficulties encountered--in particular the difficulty found in connection with neutral K mesons--already constitute a disproof of this model. Unfortunately the mere qualitative discovery of up-down asymmetries in hyperon decays would not directly exclude this model. Since a definite quantitative prediction of the model is that short-lived particles that decay rapidly into leptons must be produced with the same cross section as for production of the observed strange particles, any evidence against such processes would constitute a direct experimental disproof.

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