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Abstract

The paper develops a simple stochastic new open economy macroeconomic model based on sticky nominal wages. Explicit solution of the wage-setting problem under uncertainty allows one to analyze the effects of the monetary regime on welfare, expected output, and the expected terms of trade. Despite the potential interplay between imperfections due to sticky wages and monopoly, the optimal monetary policy rule has a closed-form solution. To motivate our model, we show that observed correlations between terms of trade and exchange rates are more consistent with our traditional assumptions about nominal rigidities than with a popular alternative based on local-currency pricing.

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1 Introduction

There has been an explosion of academic literature on the "new open economy macroeconomics" in the last several years (see Lane 1999 for an excellent survey).[1] Very recent contributions have sought to understand more deeply the positive macroeconomic effects of uncertainty as well as the normative implications for alternative international monetary regimes. As we showed in Obstfeld and Rogoff (1998), important effects of uncertainty—including effects on economic activity levels—can compound or offset the more obvious welfare effects of variability. These effects are central to accurate regime evaluation, yet they are masked by the linearization techniques commonly used to solve dynamic stochastic models. In this paper we present a simplified sticky-price model with an exact closed-form solution. The model illustrates simply and clearly both the positive effects of uncertainty and the implications for welfare.[2]

The possibilities for modeling nominal rigidities are inherently more numerous in a multicurrency international economy than in the single-money closed economy setting. For example, if output prices are pre-set in nominal terms, in what currencies are they denominated? In an international setting, moreover, it is natural to consider the possibility of segmentation between national markets, with prices for the same product being set in different currencies in different markets. Indeed, much interesting recent work in the new open economy macroeconomics is built on a pricing-to-market paradigm in which prices of imported goods are temporarily rigid in the importing country's currency. In the new models, nominal exchange rate changes tend to have small or negligible short-run effects on international trade flows. This new view contrasts sharply with the traditional Keynesian approach, which assumes that prices are rigid only in exporters' currencies, but not in importers' currencies, so that the exchange rate plays a central role in the international transmission of monetary disturbances.

Before presenting our formal model, we therefore address the empirical issue of which approach is closer to reality. We show that a framework in which imports are invoiced in the importing country's currency implies that unexpected currency depreciations are associated with *improvements* rather than deteriorations of the terms of trade. Section 2 of the paper presents empirical evidence suggesting that this implication is decidedly counterfactual. Instead, the aggregate data suggest a traditional framework in which exporters largely invoice in home currency and nominal exchange rate changes have significant short-run effects on international competitiveness and trade.

The formal model we present in sections 3 through 5 falls clearly in the traditional mold. In principle, the model permits deviations from purchasing power parity to arise either from pricing-to-market or from the presence of nontraded goods, but the only nominal rigidity is that of domestic wages.[3] Because of markup pricing in monopolistic output markets, inflexible wages lead, in equilibrium, to domestic-money price stickiness in the prices of tradable and nontradable products. Exchange rate fluctuations therefore cause sharp changes in both the terms of trade and the real exchange rate. We analyze wage setting in this general equilibrium context, and show how uncertainty impacts the expected level of the real exchange rate, the terms of trade, and relative output and employment levels at home and abroad. As in Obstfeld and Rogoff (1998), the need to set some prices in advance of the resolution of market uncertainties leads to ex ante markup behavior through which uncertainty affects the expected levels of key quantities and prices, possibly with large effects on welfare.

Section 6 explores efficient monetary policy and shows assumptions under which one can solve exactly for efficient policy rules. This section also compares welfare under the optimal monetary regime with welfare under various forms of fixed rates and fixed money growth rules. The welfare rankings are surprisingly elegant and simple. A concluding section summarizes and suggests some directions for future research in this rapidly growing area.

2 Nominal rigidities in new open economy macroeconomic models

In our model, the labor market is the primary locus of nominal rigidity. Changes in international prices and in global demands for national outputs are driven by national labor costs. In contrast to our more traditional setup, much recent literature on international policy transmission assumes that exporters set prices in the currencies of destination markets. In this section we review the implications of different price-setting regimes, present new empirical evidence, and offer an interpretation of the role of exchange rates in the international adjustment process.

2.1 *The expenditure-switching effects of exchange rate changes in traditional Keynesian models*

A central idea in the Keynesian approach to international macroeconomics is the *expenditure-switching effect* of a nominal exchange rate change. A country with a depreciating currency, the argument goes, will experience a fall in the relative price of its exports and a resulting redirection of world expenditure in favor of its products.

Some notation helps to clarify the pricing assumptions that underlie this Keynesian prediction. Let $P_{\rm F}$ be the home-currency price paid for goods imported from abroad, $P^*_{\rm H}$ the foreign-currency price received on home goods exported abroad, and $\mathcal{E}$ the home-currency price of foreign currency (the exchange rate). In standard Keynesian models in the Mundell-Fleming-Dornbusch tradition, domestic imports have prices that are sticky in terms of the *foreign* currency, so that a depreciation of the home currency in the foreign exchange market (a rise in $\mathcal{E}$) causes a proportional rise in $P_{\rm F}$. Correspondingly in those models, because domestic export prices are sticky in domestic currency, a rise in $\mathcal{E}$ (which is an appreciation of the foreign currency) causes a proportional fall in $P^*_{\rm H}$. The relative price of home imports in terms of home exports—the home terms of trade—can be expressed as

$$TOT = \frac{P_{\rm F}}{\mathcal{E} P^*_{\rm H}}. \tag{1}$$

Under the standard pricing assumptions, a rise in $\mathcal{E}$ therefore does not affect the *denominator* in eq. (1), but it raises the *numerator* proportionally. Thus, a domestic currency depreciation automatically raises TOT, the relative price of foreign imports, shifting domestic demand toward domestically produced tradables and away from imports. At the same time, domestic demand also switches from foreign tradables to domestic nontradables. Global demand will shift toward domestic tradables products too, provided trade barriers do not fully insulate foreign economies from cheaper home-country exports.

In this account, the degree of exchange-rate *pass-through* to import prices is 1 and occurs rapidly, so that a home depreciation raises import prices sharply. An extreme example with unitary pass-through occurs when trade is costless and free, so that the law of one price holds. In that case the price a country pays for an import always equals the exchange-rate adjusted price charged abroad, so that $P_{\rm F} = \mathcal{E} P^*_{\rm F}$, $P_{\rm H} = \mathcal{E} P^*_{\rm H}$, and $TOT = \mathcal{E} P^*_{\rm F} / P_{\rm H}$. When traded goods prices are fixed in the currency of the exporting country, exchange rate changes must feed fully and immediately into the foreign prices of those goods to prevent arbitrage opportunities.

It is true that a large body of evidence spanning several decades concludes that the law of one price does not hold closely even for fairly disaggregated commodity categories (see Rogoff 1996). In particular, national markets for manufacturing goods appear more segmented and therefore less susceptible to international arbitrage than in the simplest international macromodels. Border effects are important determinants of international price discrepancies, and nominal currency movements appear to be important in driving measured cross-border departures from the law of one price (Engel and Rogers 1996, 1998). In the short and even medium runs, it is hard to discern any difference in how exchange rate changes affect the relative international prices of nontradable CPI components and supposedly tradable CPI components, at least in bilateral comparisons of industrial countries with the United States (Engel 1999b). As we will discuss shortly, however, we do not believe that this evidence warrants abandoning the conventional Keynesian approach.

As the United States dollar depreciated from its 1985 peak, the apparently sluggish response of the United States current account deficit puzzled many observers and led to a more complex model of international price behavior. According to the *pricing-to-market* approach, a name coined by Krugman (1987), international markets for manufacturing goods are sufficiently segmented that producers can, at least over some time horizon, tailor the prices they charge to the specific local demand conditions prevailing in different national markets. Under pricing-to-market (PTM), a European firm exporting to the U.S. might find it optimal to lower its American price markup in the face of a depreciation of the dollar against the euro. In that case, its U.S. dollar prices would not rise one-for-one with the dollar's nominal exchange rate; the degree of pass-through to its U.S. prices would be less than 1. Instead, the European firm tolerates a fall in the per-unit profits on its U.S. sales (while presumably maximizing its total profits from global sales). Goldberg and Knetter (1997) survey evidence on PTM. They conclude that for the industries that have been studied at the microeconomic level, one-half is the median fraction by which exporters to the United States offset dollar depreciation (within a year) by raising their export prices.

2.2 *The effects of exchange rate changes with local-currency pricing*

Recently the literature on international business cycles has started to incorporate the PTM paradigm. But the approach taken in that literature goes much further than simply assuming that producers have the power to engage in third-degree price discrimination across different national export destinations. An additional assumption is that exporters' prices are quoted in the *buyers'* currencies and are temporarily rigid in terms of those currencies. (Devereux 1997 refers to this pricing convention for exports as *local-currency pricing*, or LCP.) This literature therefore supplements the PTM assumption with an assumption on exporters' *invoicing* practices, a topic the empirical PTM literature has largely left to one side. Under this form of price stickiness for tradables, exchange rate changes lead to proportional short-run deviations from the law of one price: with import prices pre-set in the importer's rather than the exporter's currency, the short-run degree of exchange rate pass-through to import prices is precisely zero.

In innovative theoretical work, Betts and Devereux (1996, 1998) incorporated PTM *cum* LCP (PTM-LCP) into a model based on Obstfeld and Rogoff (1995), thereby showing a tractable way to wean that class of models from assuming the law of one price for tradables. (See also Tille 1998a, 1998b.) Bacchetta and van Wincoop (1998) also assume PTM, as does the Devereux-Engel (1998, 1999) extension of Obstfeld and Rogoff (1998). In important quantitative applications of these models in dynamic general-equilibrium settings, Kollmann (1996), Chari, Kehoe, and McGrattan (1998), Bergin and Feenstra (1999), and Betts and Devereux (2000) illustrate the potential of the approach to replicate some striking international business-cycle regularities, such as the high variability of real and nominal exchange rates and the comovements in international consumption levels.[4]

Recent models incorporating this new incarnation of the PTM approach imply, however, a radical rethinking of the traditional expenditure-switching role of exchange rates. As far as manufactures are concerned, a nominal domestic currency depreciation has no expenditure-switching effect at all in the short run because all prices are temporarily fixed in domestic currency units. When exchange rates change, neither domestic nor foreign consumers perceive any change in the relative price of imports and other domestically available goods. Instead, producers are hypothesized to hold foreign-currency prices constant and allow their foreign markups and profit margins to adjust in proportion to unexpected exchange rate movements. Devereux and Engel (1998, 1999) argue that this behavior can largely insulate an economy from foreign monetary shocks, thereby sharply altering the positive and normative analysis of alternative exchange rate regimes. Engel (1999a) interprets the empirical evidence on international manufacturing prices as showing that among industrial countries, any expenditure-switching role of exchange rates is likely to be quite small in practice.

2.3 *Reservations about the PTM-LCP approach*

Notwithstanding their potential to mimic a selected subset of business-cycle facts, we find recent models built on the PTM-LCP approach highly implausible because their assumptions and predictions appear grossly inconsistent with many other facts.

- A large fraction of measured deviations from the law of one price is the result of nontradable components incorporated in consumer price indexes for supposedly tradable goods. These components include rents, distribution services, advertising, and the like. Thus, it is unclear that the extreme market segmentation and pass-through assumptions of the PTM-LCP approach are necessary to explain the close link between exchange rates and measured deviations from the law of one price.

- The likely time horizon over which trade invoicing induces price stickiness appears too brief to have a large impact on macroeconomic interactions at business-cycle frequencies. (In general currency invoicing applies to contracts of 90 days or less.) Price stickiness induced by wage stickiness seems to us likely to be a more important determinant of persistent macroeconomic fluctuations.

- The direct evidence on currency invoicing is largely inconsistent with the view that exporters set prices predominantly in importers' currencies. The most recent estimates we have seen are those reported for 1992 by ECU Institute (1995), where the conclusion (p. 73) is that: "The national currency remains the principal currency used for the denomination of national exports." According to these numbers, the United States, with 92 percent of exports and 80 percent of imports invoiced in dollars, is an exception in that most of its imports are denominated in home currency. For other countries the percentages of exports and imports, respectively, denominated in home currency are: Japan (40, 17), Germany (77, 56), France (55, 47), United Kingdom (62, 43), Italy (40, 34), Netherlands (43, 39).[5]

- International evidence on markups also seems consistent with a predominance of invoicing in exporters' home currencies. As per our earlier discussion, pass-though to export prices often is found to be less than 1, but seldom 0 (as it would be under LCP). Dynamic evidence also points in this direction. Gagnon and Knetter (1995) present a time-series study of German, Japanese, and United States automobile exports to seven industrial-country destinations. The data frequency is annual. They find that for most destination countries in their sample, PTM is greater in the long run that in the short run—a feature consistent with invoicing in the exporter's currency—whereas only for the United States and Canada is that pattern reversed.

2.4 *Exchange rates and terms of trade in the short run*

The new PTM-LCP models also have a radical implication concerning the manufacturing terms of trade: when a country's currency depreciates, its manufacturing terms of trade *improve*, contrary to the customary presumption. Equation (1) shows this clearly. If P_{F} and P^*_{H} are predetermined, a home currency depreciation (a rise in $\mathcal{E}$) must cause TOT to fall—an *improvement* in the home country's terms of trade. Recall that in contrast, eq. (1) showed that under the more typical assumption of unitary pass-through from exchange rates to import prices, a rise in $\mathcal{E}$ would cause a worsening of the home terms of trade.[6]

The last point suggests one very simple way to evaluate the plausibility of recent PTM-LCP models at the macro level: look at the raw correlations between changes in exchange rates and changes in terms of trade. If manufacturing export prices indeed are predetermined in terms of importers' currencies, then we should see a marked tendency in the data for industrial countries' terms of trade to improve when their currencies depreciate in nominal terms. That is, the correlation between Δe and $\Delta\tau$ should be strongly *negative*.

2.4.1 Comovements of multilateral indexes

For a large sample of industrial countries, table 1 reports the contemporaneous correlation coefficients between quarterly first differences in the IMF's measures of the nominal effective exchange rate and the multilateral terms of trade. The data period is 1982-1998.[7] There are only two negative entries (Netherlands and Portugal) and both are close to zero. On the other hand, there are many sizable positive entries, indicating a strong tendency for the multilateral terms of trade to worsen when the nominal effective exchange rate depreciates. Correlations between the terms of trade and lagged exchange rate changes, which are not reported, are usually positive and are large in a few cases.

Table 1 Correlations of quarterly changes in log exchange rate and terms of trade indexes, 1982-1998

Country	**Contemporaneous correlation coefficient**
Australia	0.51
Austria	0.21
Belgium	0.11
Canada	0.40
Denmark	0.36
Finland	0.22
France	0.16
Germany	0.43
Greece	0.34
Ireland	0.01
Italy	0.27
Japan	0.29
Netherlands	−0.01
New Zealand	0.24
Norway	0.26
Portugal	−0.07
Spain	0.27
Sweden	0.46
Switzerland	0.34
United Kingdom	0.42
United States	0.31

Source: Based on data from IMF, International Financial Statistics.

Even the highest correlations in table 1 are much lower, however, than those typically characterizing the comovements of nominal and real exchange rates. There are two reasons for this. First, the IMF nominal effective exchange rate measure (from line *neu*) covers industrial trading partners only, with weights based on manufacturing trade, while the terms of trade index covers all trading partners and all goods. Second, some of the goods entering the terms of trade index, especially on the import side, are flexible-price commodities. Nonetheless, the virtual absence of negative correlations in table 1 throws doubt on the prevalence of LCP in international trade.

2.4.2 Comovements of exchange rates and bilateral relative competitiveness

A different test looks at the correlation between bilateral nominal exchange rate changes and changes in bilateral relative export prices. Let P_{X} be the home index of nominal export prices (to all destinations) and P_{X}^* the corresponding foreign index (in foreign currency). The ratio $\mathcal{E}P_{\mathrm{X}}^*/P_{\mathrm{X}}$ (where $\mathcal{E}$ is the home-currency price of foreign currency) measures the relative competitiveness of home exports in general relative to foreign exports. In a two-country model, this ratio corresponds exactly to a terms of trade index. But in a more realistic multicountry setting, changes in it, rather than in the bilateral terms of trade, capture more comprehensively the potential expenditure-switching effects of bilateral exchange rate changes. For industrial-country comparisons, an advantage of focusing on $\mathcal{E}P_{\mathrm{X}}^*/P_{\mathrm{X}}$ is a reduction in the extent to which the available price indexes include raw materials alongside manufactured goods.

If export prices are set primarily in the exporter's currency, then P_{X} and P_{X}^* are predetermined and the relative export price ratio $\mathcal{E}P_{\mathrm{X}}^*/P_{\mathrm{X}}$ will vary closely with the bilateral nominal exchange rate. Under LCP, however, foreign exports will be invoiced in a host of partner-country currencies (including that of the home country), and likewise for the home country. Thus the domestic-currency export price indexes will depend not only on the preset local-currency prices but on the corresponding bilateral exchange rates. In that case, the covariance between changes in $\mathcal{E}$ and the relative price of Foreign exports is likely to be much lower than in the traditional case, and quite possibly negative.

Let Home and Foreign be country names and consider an example. Suppose that all shocks that change the Home-Foreign bilateral exchange rate move all other Home exchange rates in the same direction and proportion but do not affect other Foreign exchange rates at all. In that case, a depreciation of Home currency against Foreign's will have no effect on the Foreign currency value of Home's export price index (due to LCP). However, the Foreign currency value of Foreign's export price index will decline to the extent that Foreign's exports to Home (which are priced in Home currency) lose Foreign-currency value. In other words, Foreign will experience a (possibly very small) competitiveness gain against Home when Home's currency depreciates, and $\mathcal{E}$ and the relative price of Foreign exports will tend to covary negatively. This result is, of course, contrary to the conventional presumption.[8]

Table 2 reports the correlations of monthly exchange rate changes and monthly relative export price changes for 15 bilateral pairings of industrial countries. The data period is January 1982-October 1998.[9] The availability of monthly data allows a more detailed look at short-run price and exchange rate behavior than the annual data frequently used in PTM studies do.

Table 2 Correlations of monthly log changes in exchange rates and relative export competitiveness, 1982-1998

	Germany	**Italy**	**Japan**	**U.K.**	**U.S.**
Canada	0.87	0.85	0.74	0.88	0.51
Germany		0.65	0.67	0.74	0.88
Italy			0.73	0.79	0.87
Japan				0.76	0.73
U.K.					0.90

Source: Based on data from IMF, International Financial Statistics.

The salient finding in the table is that the correlations are extremely high, typically 0.70 or higher. The main anomaly is the U.S.-Canada pairing, where the correlation is only 0.51 but where the exchange rate and relative competitiveness are both markedly less variable than in the other cases, possibly the result of exchange rate smoothing intended to offset incipient terms of trade shocks. In all cases the variability in the exchange rate is reasonably close to that in relative competitiveness, though generally not quite as close as in comparisons of movements in real and in nominal exchange rates.[10]

It is conceivable in theory, of course, that high correlations such as those in tables 1 and 2 could arise from anticipated common shocks to prices and exchange rates rather than from unanticipated exchange rate movements interacting with preset prices. The findings constitute a strong challenge to the LCP view of import price determination.[11]

2.5 *Are measured terms of trade irrelevant for international competitiveness?*

The preceding findings also constitute a *prima facie* case that the expenditure-switching effect of exchange rate changes is alive and well among industrial economies, and should be a central feature of open economy models. An argument to the contrary would have to contend that measured terms of trade are somehow irrelevant for the allocation of worldwide demand among countries. Is that view plausible?

Apparent stickiness in terms of domestic currency of the import prices consumers face could result from the pricing practices of domestic importers and distributors, who purchase goods denominated in foreign currency but set retail prices in domestic currency. (See Engel and Rogers 1998 for a model of market segmentation based on domestic distributorships.) In that case, importing firms face international prices but the decisions of the ultimate consumers are based on retail prices and not directly on international terms of trade.[12]

It is still true in this setting, however, that the terms of trade are a powerful determinant of international trade flows in manufacturing. Importing firms typically hold significant inventories of manufactured goods, and their demands for new inventories reflect international prices and longer-term demand forecasts. Importers' demands do *not* simply accommodate day-to-day transactions with final domestic customers.[13] Importers may set prices in domestic currency so as to insure customers against short-term price fluctuations or prevent loss of market share, although often they respond to changes in the cost of new inventory through implicit price changes—altering financing terms and rebates, changing the waiting time for preferred models, and so on.[14] Even in the absence of such nonprice mechanisms for rapidly altering the effective prices charged to final customers, exchange-rate changes would still redirect international expenditure patterns in traditional ways, with domestic owners of distributorships facing the switching incentives (and short-run profit shifts) caused by exchange rate movements.

Another possible response to our empirical findings is to argue that many measured export prices reflect intra-firm transactions and therefore will not necessarily switch international demands. Indeed, a very large share of American exports are exports by multinationals based in the U.S. to their foreign affiliates. In response to currency changes, however, firms with multinational operations will intensify their production and export activities in countries where relative wages decline, thereby shifting demand toward those locations. Given nominal wages worldwide, a dollar depreciation, for example, leads multinationals worldwide to source more intensively from their U.S. operations, increasing the global demand for U.S. exports. Rangan and Lawrence (1993) find that this pattern held during the dollar depreciation of the late 1980s, and that the American terms-of-trade deterioration for the period indeed led multinationals to supply themselves more heavily out of goods produced in the United States.[15]

Based on the evidence we have reviewed, we adopt below a model with nontradables in which sticky wages occupy center stage in the determination of international competitiveness. To highlight the expenditure-switching role of exchange rates, our model abstracts from intermediaries between consumers and traded-goods markets and assumes that exchange-rate changes feed through to import prices in the short run. While our assumptions may not match reality exactly, we believe that they come closer than the existing generation of PTM-LCP models. In particular, they capture the mechanism through which exchange rate changes redistribute aggregate demand internationally in the presence of nominal rigidities.

3 A stochastic sticky-wage model with traded and nontraded goods

Our model is highly stylized but nevertheless includes a number of features essential to understanding the welfare effects of international macroeconomic policy. Most strikingly, the model shows how uncertainty, including monetary uncertainty, affects expected economic activity levels, expected international prices, and welfare. The effects of exchange rate uncertainty are central to any discussion of international monetary regimes. Our approach provides the first tractable framework in which the general-equilibrium effects of uncertainty are modeled exactly, allowing a rigorous welfare analysis of alternative global monetary regimes.

The main elements of the new open economy macroeconomic model below are for the most part quite familiar, even though they take a few pages to lay out in full. These preliminaries are necessary, however, before we can arrive at the more novel aspects of the paper, which really only begin with the wage setting process. The reader will note that we make many simplifying assumptions the sake of generating tractable closed-form solutions. We will try to highlight some places where these assumptions could potentially be relaxed to look at more general issues.

3.1 *Country size and market structure*

The world economy consists of two equally-sized countries, Home and Foreign. Home produces an array of differentiated tradable goods indexed by the interval $[0, 1]$. Foreign's tradables are indexed by the interval $(1, 2]$. In addition, each country produces an array of differentiated nontraded goods indexed by $[0, 1]$.

As in the models of Hau (1999a) and Obstfeld and Rogoff (1996, section 10.4)—and following Blanchard and Kiyotaki's (1987) closed-economy formulation—firms produce differentiated goods out of differentiated labor inputs indexed by $[0, 1]$. Think of each worker as occupying a point in the interval $[0, 1]$ and acting as a monopolistic supplier of a distinctive variety of labor services to both the nontradable and tradable sectors.

The most crucial assumption in our model is that workers set next period's nominal wages (in their domestic currency) in advance of production and consumption. They then supply all the labor that firms subsequently demand in the light of realized economic shocks. (Our main focus will be on money and productivity shocks.) Given the nominal wage constraint, this policy is rational for sufficiently small shocks, because the real marginal consumption value of the wage exceeds the marginal disutility of effort.[16] While wages are preset, prices of all goods are completely flexible and can be changed in response to current market conditions.[17]

Importantly, we will focus throughout on a single period so as to avoid carrying around time subscripts. In general, the mere fact that wage rigidities last for only one period is not enough to justify ignoring intertemporal linkages, since wealth redistributions via the current account can still generate sustained dynamics (Obstfeld and Rogoff 1995). However, as will later become apparent, the wealth channel would not express itself in a multiperiod version of the special setup we are adopting here anyway, so the extension would be very straightforward.

3.2 *Firms*

Let $Y(i)$ denote the output of differentiated good i and $L(i,j)$ the demand for labor input j by producer i. With this notation, the production functions in the Home traded and nontraded sectors are, respectively,

$$Y_{\mathrm{H}}(i) = \left[\int_0^1 L_{\mathrm{H}}(i,j)^{\frac{\phi-1}{\phi}} \mathrm{d}j\right]^{\frac{\phi}{\phi-1}}$$

and

$$Y_{\mathrm{N}}(i) = \left[\int_0^1 L_{\mathrm{N}}(i,j)^{\frac{\phi-1}{\phi}} \mathrm{d}j\right]^{\frac{\phi}{\phi-1}}.$$

There are parallel production functions (with the same substitution elasticity ϕ) for Foreign-produced tradables, denoted $Y_{\mathrm{F}}(i)$ (for $i \in [1,2]$), and for Foreign nontradables.

Let $W(i)$ denote the nominal wage of worker i. Then W, the price index for labor inputs, is defined as the minimal nominal cost of producing a unit of (tradable or nontradable) output:

$$W = \left[\int_0^1 W(i)^{1-\phi} \mathrm{d}i\right]^{\frac{1}{1-\phi}}. \tag{2}$$

Cost minimization implies firm j's demand function for labor of type i,

$$L(i,j) = \left[\frac{W(i)}{W}\right]^{-\phi} Y(j), \tag{3}$$

a constant elasticity of demand function that is extremely familiar from this class of models of monopolistic competition.

3.3 *Individual preferences*

A Home individual of type i maximizes

$$U^i = \log(C^i) + \frac{\chi}{1-\varepsilon}\left(\frac{M^i}{P}\right)^{1-\varepsilon} - \frac{K}{\nu}\left(L^i\right)^{\nu}, \tag{4}$$

where

$$L^i \equiv \int_0^1 \left[L_{\mathrm{H}}(i,j) + L_{\mathrm{N}}(i,j)\right] \mathrm{d}j$$

and $\nu \geq 1$.[18] In (4), K is a random shift in the marginal disutility of work effort that can be interpreted as a (negative) national productivity shock.[19] For any person i the overall real consumption index C is given by

$$C = \frac{C_{\mathrm{T}}^{\gamma} C_{\mathrm{N}}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}},$$

where preferences over Home and Foreign *tradable* products have an Armington form,

$$C_{\mathrm{T}} = 2C_{\mathrm{H}}^{\frac{1}{2}} C_{\mathrm{F}}^{\frac{1}{2}}. \tag{5}$$

(Foreign preferences are identical but Foreign quantities and prices are denoted by asterisks.) The three consumption subindexes are symmetric and are defined by

$$\begin{aligned} C_{\mathrm{H}} &= \left[\int_0^1 C_{\mathrm{T}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\ C_{\mathrm{F}} &= \left[\int_1^2 C_{\mathrm{T}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\ C_{\mathrm{N}} &= \left[\int_0^1 C_{\mathrm{N}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Domestic-currency price indexes for the three preceding consumption baskets, denoted by P_{H}, P_{F}, and P_{N}, are defined by a formula parallel to the one defining the wage index, eq. (2). Price indexes are, however, based on the individual product prices $P(i)$ and all have the consumption substitution elasticity θ in place of the production elasticity ϕ. The domestic-currency price index for overall real consumption C is

$$P = P_{\mathrm{T}}^{\gamma} P_{\mathrm{N}}^{1-\gamma} \tag{6}$$

and the price index for tradable consumption C_{T} is

$$P_{\mathrm{T}} = P_{\mathrm{H}}^{\frac{1}{2}} P_{\mathrm{F}}^{\frac{1}{2}}. \tag{7}$$

The Home commodity demand functions resulting from cost minimization are

$$C_{\mathrm{T}}(h) = \left[\frac{P_{\mathrm{T}}(h)}{P_{\mathrm{H}}}\right]^{-\theta} C_{\mathrm{H}} \text{ (demand for a typical Home tradable product } h\text{)},$$

$$C_{\mathrm{T}}(f) = \left[\frac{P_{\mathrm{T}}(f)}{P_{\mathrm{F}}}\right]^{-\theta} C_{\mathrm{F}} \text{ (demand for a typical Foreign tradable product } f\text{)},$$

$$C_{\mathrm{N}}(h) = \left[\frac{P_{\mathrm{N}}(h)}{P_{\mathrm{N}}}\right]^{-\theta} C_{\mathrm{N}} \text{ (demand for a typical Home nontradable } h\text{)},$$

with parallel demands by Foreigners. Here,

$$C_{\mathrm{H}} = \tfrac{1}{2}\left(\frac{P_{\mathrm{H}}}{P_{\mathrm{T}}}\right)^{-1} C_{\mathrm{T}}, \quad C_{\mathrm{F}} = \tfrac{1}{2}\left(\frac{P_{\mathrm{F}}}{P_{\mathrm{T}}}\right)^{-1} C_{\mathrm{T}},$$

and

$$C_{\mathrm{T}} = \gamma\left(\frac{P_{\mathrm{T}}}{P}\right)^{-1} C, \quad C_{\mathrm{N}} = (1-\gamma)\left(\frac{P_{\mathrm{N}}}{P}\right)^{-1} C.$$

The first-order condition for individual i's nominal money balances, M^i, is familiar:

$$\frac{1}{C^i} = \chi\left(\frac{M^i}{P}\right)^{-\varepsilon}. \tag{8}$$

Because money has value only for the current period, individuals equate the marginal utility from holding it to the full opportunity cost of acquiring it. (Again, extension to the dynamic case is straightforward following Obstfeld and Rogoff 1996, 1998.)

3.4 *Asset markets and budget constraints*

Everyone owns an equal share of all domestic firms—there are complete markets domestically—and of an initial stock of the domestic currency. However, there is no ex ante equity trade between the countries. As will later become apparent, the assumption of no international equity trade is benign. Given (a) the Cobb-Douglas preferences over traded goods [eq. (5)] and (b) the separability of individuals' utility functions, international equity trade would not affect equilibrium allocations.[20]

A typical Home individual i maximizes the expectation of (4) subject to the budget constraint

$$M^i + PC^i = M_0^i + PT + W(i)L^i + \int_0^1 \left[\Pi_{\mathrm{H}}(j) + \Pi_{\mathrm{N}}(j)\right] \mathrm{d}j, \tag{9}$$

where PT denotes per capita nominal transfers from the Home government and the final right-hand side summand aggregates the profits of all domestic firms. Foreign residents face a parallel constraint. Individuals take firm behavior and lump-sum transfers as given. (Later, in our welfare analysis, we will allow for a richer menu of taxes including wage and production subsidies.)

The government is assumed to rebate all lump-sum transfers in the form of money:

$$PT = M_1 - M_0 \tag{10}$$

Money, M_1, will also be a random variable in our analysis below.

4 Optimal wage setting

Having dispensed with preliminaries, we now come to the critical wage-setting relationship that underlies the main results of the paper.

4.1 An intuitive interpretation of the first-order condition for wages

Using the individual's budget constraint (9) to eliminate C^i in expected utility $\mathrm{E}U^i$, one obtains the first-order condition for the optimal preset nominal wage $W(i)$:

$$W(i) = \left(\frac{\phi}{\phi - 1}\right) \frac{\mathrm{E}\left\{K\left(L^i\right)^{\nu}\right\}}{\mathrm{E}\left\{\dfrac{L^i}{PC^i}\right\}}. \tag{11}$$

The above wage-setting equation requires that, at an optimum, the expected marginal revenue (in marginal utility of consumption units) from lowering the wage slightly must equal the expected marginal disutility from the resulting additional hours worked.[21] Absent uncertainty, eq. (11) would simply give the marginal utility of the real wage as a fixed markup $\phi/(\phi - 1)$ over the marginal disutility of labor, as is standard for a monopolist facing a constant elasticity of demand.

There is a convenient way to rewrite eq. (11) in terms of means and variances. For the sake of clarity, we do so for an arbitrary degree of risk aversion over consumption, ρ, a modification that replaces $1/C^i$ by $\left(C^i\right)^{-\rho}$ in eq. (11) to yield:

$$W = \left(\frac{\phi}{\phi - 1}\right) \frac{\mathrm{E}\left\{K\left(L\right)^{\nu}\right\}}{\mathrm{E}\left\{\dfrac{L}{P}\left(C\right)^{-\rho}\right\}}. \tag{12}$$

We have dropped the i subscripts in this equation, as we will later find that the solution is symmetric across domestic workers anyway. We show below that if the exogenous random variables moving the economy have a jointly lognormal distribution, all endogenous variables are lognormal as well. With lognormally distributed variables, the preceding first-order condition has the equivalent representation:

$$W = \left(\frac{\phi}{\phi - 1}\right) \frac{\mathrm{E}\{K\} (\mathrm{E}\{L\})^{\nu-1}}{(\mathrm{E}\{C\})^{-\rho}\, \mathrm{E}\left\{\frac{1}{P}\right\}} \exp(\xi). \tag{13}$$

The factor ξ, entirely due to uncertainty, is given by

$$\xi \equiv \frac{\nu(\nu-1)}{2}\sigma_l^2 - \frac{\rho(\rho+1)}{2}\sigma_c^2 + \nu\sigma_{\kappa l} + \rho\sigma_{cl} - \rho\sigma_{cp} + \sigma_{lp}. \tag{14}$$

Here, lower-case letters are natural logarithms of upper-case counterparts, for example, $l \equiv \log L$. By holding constant the variable means in eq. (13), we can use eq. (14) to see how mean-preserving uncertainty affects how workers choose to preset their nominal wages.[22]

For example, holding $\mathrm{E}\{L\}$ and all else equal, higher consumption variance raises the expected marginal utility of consumption, inducing lower wages and greater ex ante labor supply. A positive covariance between consumption and labor supply means that workers tend to be supplying the most effort when they need it least for consumption; this makes them set a higher wage and a lower planned level of labor supply. A positive covariance between consumption and the price level makes the nominal wage contract a hedge for consumption risk, since real wages tend to be high when consumption is low; the result is a lower nominal wage.[23] These effects turn out to be critical later in thinking about how uncertainty affects the terms of trade and the overall level of trade and output.[24]

Of course, the covariances in eq. (14) are highly interdependent in equilibrium; the model's solution will express them all in terms of fundamental exogenous sources of uncertainty, productivity and money.

4.2 *Price setting, the real exchange rate, and the terms of trade*

Monopoly firms are free to see prices at whatever levels they choose. Given, however, our assumption that individuals have constant elasticity of demand preferences, revenue maximizing firms will choose prices for goods sold within their own country that are a constant markup over wages

$$P_{\mathrm{N}} = P_{\mathrm{H}} = \left(\tfrac{\theta}{\theta-1}\right) W, \quad P_{\mathrm{N}}^{*} = P_{\mathrm{F}}^{*} = \left(\tfrac{\theta}{\theta-1}\right) W^{*}, \tag{15}$$

where we have dropped i subscripts since the equilibrium is symmetric across firms and workers within a given country. We assume that firms can, if they choose, charge different exchange-rate-adjusted prices abroad. That is, we allow for pricing to market. However, given the constant elasticity of demand preferences, they end up choosing prices

$$P_{\mathrm{H}}^{*} = \frac{1}{\mathcal{E}}\left(\tfrac{\theta}{\theta-1}\right) W \;= \frac{1}{\mathcal{E}} P_{\mathrm{H}}, \quad P_{\mathrm{F}} = \mathcal{E}\left(\tfrac{\theta}{\theta-1}\right) W^{*} = \mathcal{E} P_{\mathrm{F}}^{*} \tag{16}$$

such that the law of one price holds anyway. (If Home and Foreign demand elasticities differed then, obviously, prices would be in constant proportion and a relative version of the law of one price would hold.)

In sum, eqs. (15) and (16) have two striking implications. One is that nominal wage rigidity leads firms to choose to hold their domestic-currency prices constant, even in the face of aggregate demand shifts. Second, even though we allow for pricing to market, domestic firms still choose to change their foreign-currency prices in proportion to changes in the exchange rate.

Of course, we do not view the assumption of constant elasticity of demand as a strict interpretation of reality, but rather as a strategic simplifying assumption. Potentially important channels of transmission remain dormant in our analysis, even though they are not directly shut off by assumption.

Notwithstanding the constancy of relative sectoral producer prices within the countries, the *real exchange rate* is not constant. The real exchange rate is defined as

$$\begin{aligned} \text{Real exchange rate} \;\equiv\; \frac{\mathcal{E}P^{*}}{P} &= \frac{\mathcal{E}P_{\mathrm{T}}^{*\gamma} P_{\mathrm{N}}^{*(1-\gamma)}}{P_{\mathrm{T}}^{\gamma} P_{\mathrm{N}}^{(1-\gamma)}} = \frac{\mathcal{E}\left(P_{\mathrm{H}}^{*\frac{1}{2}} P_{\mathrm{F}}^{*\frac{1}{2}}\right)^{\gamma} P_{\mathrm{N}}^{*(1-\gamma)}}{\left(P_{\mathrm{H}}^{\frac{1}{2}} P_{\mathrm{F}}^{\frac{1}{2}}\right)^{\gamma} P_{\mathrm{N}}^{(1-\gamma)}} \\ &= \left(\frac{\mathcal{E}W^{*}}{W}\right)^{1-\gamma}, \end{aligned} \tag{17}$$

where we have used eqs. (6) and (7). The real exchange rate is determined by the preset relative nominal wages and the nominal exchange rate. Given wages, nominal exchange rate movements induce larger real exchange rate movements the smaller γ, the share of tradables in consumption. A related point has been emphasized by Hau (1999b), who asks whether larger less open economies tend to have more volatile real exchange rates. His empirical results, interpretation of which naturally requires a number of ancillary assumptions, appear to support the hypothesis.

The terms of trade also move with the exchange rate according to

$$TOT \equiv \frac{\mathcal{E}P_{\mathrm{F}}^{*}}{P_{\mathrm{H}}} = \frac{\mathcal{E}W^{*}}{W},$$

for reasons we have already discussed.

4.3 *Market clearing and the current account*

The Home market for nontradables clears when domestic demand equals domestic supply, $C_{\mathrm{N}} = Y_{\mathrm{N}}$ (and similarly for Foreign). As for tradables, equilibrium requires that

$$\begin{aligned} P_{\mathrm{H}}Y_{\mathrm{H}} &= \tfrac{1}{2}P_{\mathrm{T}}C_{\mathrm{T}} + \tfrac{1}{2}\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*}, \\ P_{\mathrm{F}}Y_{\mathrm{F}} &= \tfrac{1}{2}P_{\mathrm{T}}C_{\mathrm{T}} + \tfrac{1}{2}\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*}, \end{aligned}$$

from which the result

$$\frac{P_{\mathrm{H}}}{P_{\mathrm{F}}} = \frac{Y_{\mathrm{F}}}{Y_{\mathrm{H}}}$$

follows. The government budget constraint (10) and market clearing for nontradables imply that $P_{\mathrm{T}}C_{\mathrm{T}} = P_{\mathrm{H}}Y_{\mathrm{H}}$ and $\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*} = P_{\mathrm{T}}C_{\mathrm{T}}^{*} = P_{\mathrm{F}}Y_{\mathrm{F}}$, from which

$$C_{\mathrm{T}} = C_{\mathrm{T}}^{*}$$

follows. As in Corsetti and Pesenti (1998) and Obstfeld and Rogoff (1998), Home and Foreign per capita consumptions of tradables are always equal. [Exact equality follows because the weights in the utility function (5) are the same as country size. More generally, if they were different, consumption levels would be proportional but not equal.] It is this property of the equilibrium allocation that makes international equity markets superfluous, and implies that current accounts would be zero in an intertemporal version of the model. Of course, this again is an extreme case, but useful because we are not focusing here on current account dynamics.

One final special feature of the model is worth noting, as we will make use of it later in our welfare analysis. Because it is only the traded goods component of consumption that has to be equal across countries, the overall consumption indexes C and C^* need not move together. However, *the Home and Foreign overall spending levels are always equal when measured in units of tradables.* Let Z denote Home spending measured in units of tradables,

$$Z \equiv C_{\mathrm{T}} + \left(\frac{P_{\mathrm{N}}}{P_{\mathrm{T}}}\right) C_{\mathrm{N}}.$$

Because of the Cobb-Douglas preferences we have assumed,

$$\frac{P_{\mathrm{N}}}{P_{\mathrm{T}}} = \left(\frac{1-\gamma}{\gamma}\right) \frac{C_{\mathrm{T}}}{C_{\mathrm{N}}},$$

and therefore

$$Z = C_{\mathrm{T}}/\gamma = C_{\mathrm{T}}^*/\gamma = Z^*$$

4.4 *Equilibrium preset wages*

We now make use of the market equilibrium output and pricing conditions to rewrite the wage setting equations in a form that, ultimately, will allow us to obtain solutions in terms of the underlying exogenous variables. By symmetry, eq. (3) implies that in the aggregate $L = Y_{\mathrm{H}} + Y_{\mathrm{N}}$. Thus, the wage first-order condition (11) implies that in equilibrium

$$W = \left(\frac{\phi}{\phi - 1}\right) \frac{\mathrm{E}\left\{K\left(Y_{\mathrm{H}} + Y_{\mathrm{N}}\right)^{\nu}\right\}}{\mathrm{E}\left\{\frac{Y_{\mathrm{H}} + Y_{\mathrm{N}}}{PC}\right\}}$$

for the case of log preferences over consumption. Furthermore, using the national income identity $PC = P_{\mathrm{H}}Y_{\mathrm{H}} + P_{\mathrm{N}}Y_{\mathrm{N}} = P_{\mathrm{T}}Z$ and the price markup eqs. (15), we obtain

$$\left(\frac{W}{W^*}\right)^{\nu/2} = \frac{\phi\theta}{(\phi - 1)(\theta - 1)} \mathrm{E}\left\{K\mathcal{E}^{\nu/2}Z^{\nu}\right\}. \tag{18}$$

From the Foreign analog of condition (11) we derive, symmetrically,

$$\left(\frac{W^*}{W}\right)^{\nu/2} = \frac{\phi\theta}{(\phi - 1)(\theta - 1)} \mathrm{E}\left\{K^*\mathcal{E}^{-\nu/2}Z^{\nu}\right\}.$$

Combining these two equations leads to

$$\left(\frac{W}{W^*}\right)^{\nu} = \frac{\mathrm{E}\left\{K\mathcal{E}^{\nu/2}Z^{\nu}\right\}}{\mathrm{E}\left\{K^*\mathcal{E}^{-\nu/2}Z^{\nu}\right\}}. \tag{19}$$

Equations (18) and (19) govern the simultaneous determination of wages, expected expenditure, and the expected exchange rate. These nonlinear stochastic equations appear quite complex. Fortunately, however, with our assumption of lognormal disturbances, they lead to a rather simple closed-form solution, which we will now derive.

5 A closed-form solution

In this section we assume that the natural logarithms of the exogenous variables $\{\log M, \log M^*, \log K, \log K^*\}$ are jointly normally distributed. Because the equilibrium conditions developed in the last section are all linear in logs, the entire equilibrium distribution of the resulting model's endogenous variables can be written out analytically. The usefulness of an *exact* solution cannot be exaggerated. For one thing. we are able to illustrate in general equilibrium results suggested by our intuitive discussion in the preceding section of the first-order condition for wages. In the next section, armed with an exact solution, we will easily be able to solve an optimal policy problem that might otherwise be quite difficult to characterize.[25]

Our solution algorithm involves two major steps. One is to express the wage setting equations in terms of logs and variances of logs of endogenous variables. The other step is to calculate how endogenous variables respond to exogenous shocks; this in turn allows us to express the variances of all variables in terms of the variances of underlying shocks.

5.1 *Expected relative wages and global spending: Quasi reduced-form solutions*

We now demonstrate a critical insight of our modeling approach: uncertainty has an impact on the expected levels of consumption, output, and the terms of trade through its effect on ex ante wage setting.

We again denote the natural logarithm of any variable X by the corresponding lower-case letter; thus $x \equiv \log X$. To simplify notation we assume that the means and variances of the (log) productivity shocks are equal: $\mathrm{E}\kappa = \mathrm{E}\kappa^*$ and $\sigma_\kappa^2 = \sigma_{\kappa^*}^2$. Asymmetries between the distributions of the productivity shocks may still arise from differing correlations with national monetary shocks. Taking logs of eq. (19), we therefore obtain

$$\mathrm{E}e + w^* - w = -\nu\sigma_{ze} - \tfrac{1}{2}\left(\sigma_{\kappa e} + \sigma_{\kappa^* e}\right) + \left(\sigma_{\kappa^* z} - \sigma_{\kappa z}\right) \equiv \mathrm{E}\tau. \qquad (20)$$

Above, τ denotes the (log) terms of trade TOT; the (log) real exchange rate is $(1-\gamma)\,\tau$. By combining the preceding expression with (the log of) eq. (18), we obtain the expected value of (log) expenditure measured in tradables,

$$\mathrm{E}z = \omega - \frac{\nu}{2}\sigma_z^2 - \frac{\nu}{8}\sigma_e^2 - \frac{1}{2}\left(\sigma_{\kappa z} + \sigma_{\kappa^* z}\right) - \frac{1}{4}\left(\sigma_{\kappa e} - \sigma_{\kappa^* e}\right), \qquad (21)$$

where

$$\omega \equiv \frac{1}{\nu}\left\{\log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \mathrm{E}\kappa - \frac{1}{2}\sigma_\kappa^2\right\}.$$

The preceding relationships reveal an important effect captured in our stochastic utility-maximizing framework, but invisible in crude linear approximations to models such as ours that ignore variance terms. Uncertainty affects the expected *levels* of expenditure and other macroeconomic aggregates, and thereby, the expected terms of trade.[26] Equations (20) for $\mathrm{E}\tau$ and (21) for $\mathrm{E}z$ are still reduced-form rather than structural relationships because the variances they contain have not yet been expressed in terms of the distributions of the exogenous variables.[27] The equations nonetheless offer some preliminary intuition on the mechanisms through which uncertainty affects relative prices and the level of consumption in this model.

Start with eq. (20) for the expected terms of trade. A higher covariance between world expenditure z and the exchange rate e discourages labor effort because it means that the relative demand for home labor is high precisely when overall global demand for labor is high. To reduce the wide ex post fluctuations in labor supply, Home workers demand relatively higher wages. On average, Home products are in scarcer supply and the expected Home terms of trade $\mathrm{E}\tau$ improve. The intuition behind the effects of $\sigma_{\kappa e}$ and $\sigma_{\kappa z}$ is similar.[28]

Now look at eq. (21). Both expenditure and exchange rate variability, measured by σ_z^2 and σ_e^2, reduce ex ante labor supply and expected spending. The intuition for the covariance effects is similar to that underlying their effects on the terms of trade.[29]

5.2 *Ex post spending, the ex post exchange rate, and nominal wage levels*

Solving for the sticky-wage equilibrium levels of ex post expenditure and the ex post exchange rate leads to a solution for absolute nominal wage levels. Take logs of the money Euler eq. (8) and its Foreign counterpart, and assume $\chi = \chi^*$. Then average the two, applying the definitions of the price indexes, the markup equations for prices, and the equality $C = P_{\mathrm{T}}Z/P$. The result is

$$z = \frac{\varepsilon}{2}(m + m^*) - \frac{\varepsilon}{2}(w + w^*) - \log \chi - \varepsilon \log \left(\frac{\theta}{\theta - 1} \right). \tag{22}$$

A similar calculation in differences gives the exchange rate equation

$$e = \frac{\varepsilon \left(m - m^* \right)}{1 - \gamma + \gamma\varepsilon} - \frac{\left(\varepsilon - 1 \right) \left(1 - \gamma \right) \left(w - w^* \right)}{1 - \gamma + \gamma\varepsilon}. \tag{23}$$

A relative Home money supply increase that occurs after nominal wages are set causes an overshooting increase in e if $\varepsilon > 1$, just as in the nontraded goods model of Obstfeld and Rogoff (1995). If fully anticipated, however, the same change causes a precisely equal change in $w - w^*$, and hence in e.

Ex post surprises concerning the real shocks κ and κ^* do not affect the exchange rate or spending. A surprise rise in κ, say, raises the attractiveness of leisure for Home residents ex post, but they are bound by their labor contracts and do not have the option of raising wages and reducing labor supply after the fact. Even though productivity shocks do not directly affect anything except leisure in the sticky-wage equilibrium, it is not necessarily the case that all other endogenous variables are uncorrelated with κ and κ^*. They still can be correlated if, for example, monetary policy aims to offset productivity shocks.

We combine eqs. (20), (22), and (23) and take expectations to solve for the absolute nominal wage levels

$$\begin{aligned} w &= \mathrm{E}m - \log \left(\frac{\theta}{\theta - 1} \right) - \frac{\left(\mathrm{E}z + \log \chi \right)}{\varepsilon} - \frac{(1 - \gamma) + \gamma\varepsilon}{\varepsilon} \left(\frac{\mathrm{E}\tau}{2} \right), \\ w^* &= \mathrm{E}m^* - \log \left(\frac{\theta}{\theta - 1} \right) - \frac{\left(\mathrm{E}z + \log \chi \right)}{\varepsilon} + \frac{(1 - \gamma) + \gamma\varepsilon}{\varepsilon} \left(\frac{\mathrm{E}\tau}{2} \right). \end{aligned}$$

Observe that, for given expected terms of trade, a higher level of expected log world spending [as specified in eq. (21)] induces workers to set lower nominal wages. Log nominal wages are proportional to expected log money supplies.

Because w and w^* are predetermined, eqs. (22) and (23) fully describe the effects on z and e of unanticipated shocks. To solve fully for nominal wages, and hence for the levels of z and e, we still need reduced form solutions for the variances in the model, which in turn also affect the expected terms of trade $\mathrm{E}\tau$.

5.3 *Solutions for variances*

To complete the model's solution we only need to solve for the covariances in eqs. (20) and (21). The following table summarizes the results for the case when monetary policy does not respond to productivity shocks so that $\sigma_{\kappa z}, \sigma_{\kappa^* z}, \sigma_{\kappa e}$, and $\sigma_{\kappa^* e}$ are all zero:

$$\sigma_e^2 : \quad \left(\frac{\varepsilon}{1-\gamma+\gamma\varepsilon}\right)^2 (\sigma_m^2 - 2\sigma_{mm^*} + \sigma_{m^*}^2)$$

$$\sigma_z^2 : \quad \frac{\varepsilon^2}{4}(\sigma_m^2 + 2\sigma_{mm^*} + \sigma_{m^*}^2)$$

$$\sigma_{ze} : \quad \left(\frac{\varepsilon^2}{1-\gamma+\gamma\varepsilon}\right) \frac{(\sigma_m^2 - \sigma_{m^*}^2)}{2}$$

Later, we shall allow for general endogenous monetary policies.

5.4 *Solution for utility*

In studying policy rules, we will look at their welfare implications in the limiting case as $\chi \to 0$ in eq. (4). The welfare implications of the model are most transparent if we again generalize temporarily to allow an arbitrary positive coefficient of risk aversion ρ. Return to the generalized first-order condition for nominal wages in eq. (12). Since $PC = P_{\mathrm{H}}Y_{\mathrm{H}} + P_{\mathrm{N}}Y_{\mathrm{N}} = \left(\frac{\theta}{\theta-1}\right)WL$, eq. (12) implies

$$\mathrm{E}\left\{C^{1-\rho}\right\} = \frac{\phi\theta}{(\phi-1)(\theta-1)}\mathrm{E}\left\{KL^{\nu}\right\}.$$

In analogy to the procedure in Obstfeld and Rogoff (1998), we can use this relationship to rewrite expected utility (4) (for $\chi \to 0$) as

$$\begin{aligned} \mathrm{E}U &= \mathrm{E}\left\{\frac{C^{1-\rho}}{1-\rho} - \frac{K}{\nu}L^{\nu}\right\} \\ &= \mathrm{E}\left\{\frac{C^{1-\rho}}{1-\rho} - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta}C^{1-\rho}\right\} \\ &= \frac{\nu\phi\theta-(\phi-1)(\theta-1)(1-\rho)}{\nu\phi\theta}\mathrm{E}\left\{\frac{C^{1-\rho}}{1-\rho}\right\}. \end{aligned}$$

Expected utility thus is always directly proportional to $\mathrm{E}\left\{C^{1-\rho}/\left(1-\rho\right)\right\}$ (recall that $\nu \geq 1$). This step simplifies the welfare analysis considerably, because only the distribution of the isoelastic function of C, $C^{1-\rho}$, matters.

In the log utility of consumption ($\rho = 1$) case, we can invoke lognormality to write a Home resident's expected utility as

$$\mathrm{E}U = \mathrm{E}c - \frac{(\phi-1)\left(\theta-1\right)}{\nu\phi\theta} = \log \mathrm{E}C - \tfrac{1}{2}\sigma_c^2 - \frac{(\phi-1)\left(\theta-1\right)}{\nu\phi\theta},$$

as one can verify directly. Macroeconomic variability affects welfare directly, as well as through its effect on expected real consumption.[30] Using the definition of z and eqs. (20) and (21), we can alternatively express expected utility for $\rho = 1$ as

$$\begin{aligned} \mathrm{E}U &= \mathrm{E}z + \left(\frac{1-\gamma}{2}\right)\mathrm{E}\tau - \frac{(\phi-1)\left(\theta-1\right)}{\nu\phi\theta} \\ &= \frac{1}{\nu}\left\{\log\left[\frac{(\phi-1)\left(\theta-1\right)}{\phi\theta}\right] - \frac{(\phi-1)\left(\theta-1\right)}{\phi\theta} - \mathrm{E}\kappa\right\} + \Omega, \quad (24) \end{aligned}$$

where the additive term

$$\Omega \equiv -\frac{\nu}{2}\sigma_z^2 - \frac{\nu}{8}\sigma_e^2 - \frac{1}{2\nu}\sigma_\kappa^2 - \frac{(1-\gamma)\nu}{2}\sigma_{ze} - \frac{(2-\gamma)}{2}(\sigma_{\kappa z} + \frac{1}{2}\sigma_{\kappa e}) - \frac{\gamma}{2}(\sigma_{\kappa^* z} - \frac{1}{2}\sigma_{\kappa^* e})$$

involves only variances and covariances. In this log case, expected utility depends positively and exclusively on expected log expenditure measured in tradables and the expected log terms of trade.[31] (More precisely, the expected log real exchange rate: given total spending measured in tradables, z, real consumption C is higher when nontradables are cheaper, i.e., when τ is higher.) Foreign expected utility is given by $\mathrm{E}U^* = \mathrm{E}U - (1-\gamma)\mathrm{E}\tau$, which can be expressed in terms of a corresponding Foreign variance term Ω^*. For later purposes, it is important to observe that in eq. (24), the monopoly distortion term $\frac{(\phi-1)(\theta-1)}{\phi\theta}$ enters additively into utility but is absent from Ω. That is, the parameters θ and ϕ do not enter into Ω (given the joint distribution of shocks) even after one substitutes in all of the variances, written in terms of underlying exogenous shocks.[32]

6 Efficient monetary policies and exchange rate regimes

As we noted in Obstfeld and Rogoff (1998), a major advantage of an explicitly optimization-based approach to open-economy macroeconomics is the capacity to conduct rigorous welfare analysis of monetary policies and alternative monetary regimes. In this section we present applications of our model in both areas. As a first application, we show that in our model constrained-efficient monetary policy rules will replicate the flexible price equilibrium and feature a procyclical response to productivity shocks. The second application is to assess relative welfare under several alternative arrangements for managing exchange rates and global monetary growth.

6.1 *Efficient monetary policy rules*

We now assume that national monetary authorities can observe the productivity shocks K and K^* after wages are set and then set money supplies in response. Under the assumption of precommitment, how do efficient monetary policy rules look?

Let us first clarify the sense in which we will understand "efficient" market allocations here. Since we are looking at precommitment to policy *rules*, there is no scope for monetary surprises to offset monopolistic distortions. Instead, we wish to characterize policy rules that are *constrained-efficient*, in the sense of maximizing an average of Home and Foreign utilities subject to the optimal wage setting behavior of workers and price setting behavior of firms that we analyzed in section 4. When we refer to an "efficient" allocation below, we thus mean a market allocation that cannot be altered without making one country worse off, given the constraint imposed by the optimizing price setting behaviors of the monopolistic actors.

6.1.1 Flexible-price equilibrium

To characterize efficient monetary policy, we first recall that, absent an endogenous monetary response, shocks to K and K^* have no effect on any variable except the utility from leisure (because labor is demand determined). That would not be the case under flexible wages, of course. Let tildes denote flexible-wage equilibrium values. Then it is easy to show that the equations

$$\begin{aligned}\tilde{L} &= \tilde{L}_{\mathrm{H}} + \tilde{L}_{\mathrm{N}} = \left[\frac{(\phi-1)(\theta-1)}{K\phi\theta}\right]^{1/\nu}, \\ \tilde{L}^* &= \tilde{L}^*_{\mathrm{F}} + \tilde{L}^*_{\mathrm{N}} = \left[\frac{(\phi-1)(\theta-1)}{K^*\phi\theta}\right]^{1/\nu},\end{aligned}$$

determine total equilibrium labor effort.[33] Clearly, under flexible wages,

$$\frac{\mathrm{d}l}{\mathrm{d}\kappa} = \frac{\mathrm{d}l^*}{\mathrm{d}\kappa^*} = -\frac{1}{\nu} < 0.$$

Workers wish to supply more labor after a positive productivity shock (fall in K or K^*), and under wage flexibility they can do so Note also that even under flexible wages, $\tilde{L}$ and $\tilde{L}^*$ are below the levels $K^{-1/\nu}$ and $K^{*-1/\nu}$ that would be dictated by a planner with equal country weights (see Obstfeld and Rogoff, 1996, ch. 10). For use in a moment, one can easily calculate expected welfare under flexible wages as

$$\mathrm{E}\tilde{U} = \frac{1}{\nu}\left\{\log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \frac{(\phi-1)(\theta-1)}{\phi\theta} - \mathrm{E}\kappa\right\} = \mathrm{E}\tilde{U}^*, \qquad (25)$$

where we have imposed $\mathrm{E}\kappa = \mathrm{E}\kappa^*$.[34] The variance terms contained in Ω enter eq. (24) but not eq. (25) because in the former case, wages are set in advance with an eye to uncertainty. Observe also that even though $\mathrm{E}\tilde{U} = \mathrm{E}\tilde{U}^*$ when $\mathrm{E}\kappa = \mathrm{E}\kappa^*$, $\mathrm{E}U$ need not equal $\mathrm{E}U^*$ (that is, Ω and Ω^* can differ) because of asymmetries in the joint distribution of economic shocks.

6.1.2 Dual distortions and the efficiency of the flexible-price equilibrium

Is it efficient to have monetary policy rules aim to mimic the flexible-wage equilibrium, as in 1970s-style rational expectations monetary models? In general, the answer is not trivial, since wage stickiness is not the only distortion here. In addition, monopoly pervades labor and goods markets, and interactions between the sticky-wage and monopoly distortions can greatly complicate policy analysis in general. Here, however, that is not the case, as we now show.[35]

Proposition 1 *A global monetary policy that gives the same real allocation as under flexible wages is efficient.*

Proof. As is well known, the monopoly distortions to labor supply and output can be eliminated completely here by a giving workers a (proportional) wage subsidy of $1/(\phi-1)$ and paying firms a (proportional) production subsidy of $1/(\theta-1)$. Introducing these subsidies (in both countries) minimizes the monopoly distortion term

$$f(\theta,\phi;\nu) \equiv \frac{1}{\nu}\log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta}$$

in eq. (24) and its Foreign analog, but has no effect on the additive covariance terms Ω and Ω^* for given monetary policy rules. (The latter claim is easy to prove by the same logic showing that θ and ϕ do not enter Ω.) Likewise, it is clear that introducing the optimal subsidies globally under *flexible* prices will affect expected utility only by minimizing the term $f(\theta,\phi;\nu)$ in eq. (25).

Assume that optimal subsidies are in place and consider Home and Foreign monetary policy rules that replicate the flexible-wage allocation ex post, despite wage stickiness. The resulting allocation clearly is efficient, that is, it is impossible to make one country better off (in terms of expected utility) without hurting the other. Now suppose that, in the absence of optimal subsidies, we could find monetary policy rules such that (say) $\mathrm{E}U > \mathrm{E}\tilde{U}$ but $\mathrm{E}U^* \geq \mathrm{E}\tilde{U}^*$. Because the effects of the distortions affect sticky-wage and flex-wage utility identically and additively through the term $f(\theta,\phi;\nu)$, we would then conclude that the same policies, when applied in the presence of optimal subsidies, yield an allocation that Pareto-dominates the flexible-wage allocation with optimal subsidies. But we have already seen that this is impossible. This shows that even in the absence of optimal subsidies, monetary policies that replicate the flexible-wage equilibrium produce an efficient allocation. ■

The only remaining question is whether monetary policy can indeed induce the flexible-price allocation ex post. In the special case assumed above, where the κ shock moves $\tilde{L}_{\mathrm{H}}$ and $\tilde{L}_{\mathrm{N}}$ uniformly, as does monetary policy, the answer plainly is yes. By suitably adjusting Home and Foreign monetary policies, the economy can achieve a (constrained) efficient allocation. Loosely speaking, there are two targets and two instruments in this case. Policy rules replicating the flexible-price equilibrium[36] can be shown to have the form

$$m = \mathrm{E}m + \frac{1}{2\nu\varepsilon}\left\{\gamma(\varepsilon - 1)(\kappa^* - \mathrm{E}\kappa^*) - [2 + \gamma(\varepsilon - 1)](\kappa - \mathrm{E}\kappa)\right\},$$

$$m^* = \mathrm{E}m^* + \frac{1}{2\nu\varepsilon}\left\{\gamma(\varepsilon - 1)(\kappa - \mathrm{E}\kappa) - [2 + \gamma(\varepsilon - 1)](\kappa^* - \mathrm{E}\kappa^*)\right\}.$$

Surprisingly, although we have only proved that the preceding policy rules are efficient, they are also *optimal* from either country's individual perspective. That result and its far-reaching implications for policy design are discussed in Obstfeld and Rogoff (1999).

Notice that only when $\gamma = 0$ (no goods traded) or $\varepsilon = 1$ (no over/undershooting) is it optimal for a country's monetary policy to respond exclusively to the domestic productivity shock. For $\varepsilon > 1$, for example, domestic m, in response to a Foreign κ^* shock, must offset the effect of m^* on world spending z but reinforce the effect on the exchange rate e.[37]

Of course, an optimal policy will not, in general, entail fixed exchange rates unless the Home and Foreign productivity shocks are perfectly correlated. Optimal monetary policies are, in a sense, *procyclical.* For example, a fall in K, which is a positive "productivity" shock that would elicit greater labor supply and output under flexible wages, optimally induces an expansionary Home monetary response when wages are set in advance. The same shock simultaneously elicits a contractionary *Foreign* monetary response when $\varepsilon > 1$, but the net *global* monetary response (the response of $m + m^*$) is always positive. The implied global monetary rule takes the procyclical form:

$$m + m^* = \mathrm{E}m + \mathrm{E}m^* - \frac{1}{\nu\varepsilon}\left[(\kappa - \mathrm{E}\kappa) + (\kappa^* - \mathrm{E}\kappa^*)\right].$$

In a more general case, where there can be separate productivity shocks to the Home (Foreign) traded and nontraded goods sectors, our results still go through because prices are flexible in this model. In a model where relative goods prices cannot adjust immediately, however, the problem becomes more complex.[38]

6.2 *Welfare under alternative monetary regimes*

We have already solved for the optimal monetary policy and shown that, in general, it involves allowing the exchange rate to fluctuate in response to cross-country differences in productivity shocks. Given that our framework omits certain strategic and political factors that might be important in practice (for example, we have assumed away any inflationary bias in monetary policy, see Rogoff 1985), it is reasonable to ask just how much better is the optimal regime than some popular alternatives, such as fixed rates (or a common currency).

Here we analyze how three alternative monetary regimes perform in mitigating the effects of uncertainty in productivity. The first regime is an *optimal* float ($Optimal\ float$). The second regime is world monetarism *à la* McKinnon ($World\ monetarism$), under which two countries fix the exchange rate while also fixing an exchange rate-weighted average of the two national money supplies. The third regime is an *optimal* fixed rate regime ($Optimal\ fix$).

Utility in the optimal floating regime is given by eq. (25), which we rewrite as

$$\mathrm{E}U^{Optimal\ float} = \frac{1}{\nu}\left\{\log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \frac{(\phi-1)(\theta-1)}{\phi\theta} - \mathrm{E}\kappa\right\}.$$

When the only shocks are to κ and κ^*, it is also easy to express expected utility for the case of world monetarism (which here simply calls for the monetary authorities in both countries to fix M and M^*, since productivity shocks do not directly affect the exchange rate). Making use of eq. (24), and noting that e and z are constant in this case [recall eqs. (22) and (23)], one arrives at

$$\mathrm{E}U^{World\ monetarism} = \mathrm{E}U^{Optimal\ float} - \frac{1}{2\nu}\sigma_{\kappa}^{2}.$$

Under the optimal fixed exchange rate regime, the global monetary authorities instead use monetary policy to offset global shocks but constrain relative money supply movements to ensure that the exchange rate remains fixed. We restrict our attention to a policy that equally weights utility in Home and Foreign so that (given the symmetry across the two countries), the optimal policy must be symmetric. This implies that $\mathrm{E}\tau\ (=\tau) = 0$. We also simplify our derivation by observing that, given the model's is loglinearity, the optimal monetary policy rule must be loglinear as well. It is easiest to express the rule implicitly in the form of an aggregate expenditure rule:

$$z = -\alpha\left(\kappa + \kappa^{*}\right).$$

Given that the exchange rate is constant, one can then solve for Ω ($= \Omega^{*}$) in eq. (24) as

$$-\frac{1}{2\nu}\sigma_{\kappa}^{2} - \frac{\nu}{2}\sigma_{z}^{2} - \sigma_{\kappa z} = -\left(\frac{1}{2\nu} + \nu\alpha^{2} - \alpha\right)\sigma_{\kappa}^{2},$$

where we have assumed that κ and κ^{*} are uncorrelated. (The simplification is not important since global productivity shocks do not create any wedge between the optimal fixed and the optimal floating rate regimes.). Maximizing the above expression with respect to α yields

$$\alpha = \frac{1}{2\nu},$$

so the optimal rule is

$$z = -\frac{\left(\kappa + \kappa^{*}\right)}{2\nu}$$

and therefore

$$\mathrm{E}U^{Optimal\ fix} = \mathrm{E}U^{Optimal\ float} - \frac{1}{4\nu}\sigma_{\kappa}^{2} > \mathrm{E}U^{World\ monetarism}.$$

Clearly, the critical parameters are the variance of the productivity shocks and the elasticity of utility with respect to effort. These comparisons, of course, assume that all the regimes offset gratuitous financial-market shocks.

By developing a simple log-linear model, we have been able to get some interesting insights into optimal monetary policy.[39] Of course, our analysis of welfare issues has only scratched the surface of the area, leaving open a broad range of issues on strategy and international monetary policy, and on optimal policy rules in more complex settings. Nonetheless, we believe that our approach is instructive in suggesting how to make progress in these other cases.

7 Conclusion

In this paper we have developed a remarkably simple and tractable stochastic model following the approach of the "new open economy macroeconomics."

The model that can be used to answer a variety of theoretical and policy questions, including questions about welfare under alternative monetary regimes. Thanks to the model's log-linearity, we are able not only to derive exact closed form solutions for levels and variances of all the endogenous variables in the model, but we are also able to derive exact welfare results, despite the complication that there are two sources of market imperfection, monopoly and wage stickiness. We show that a constrained optimal global monetary policy is procyclical with respect to productivity shocks, and demonstrate how to calculate the welfare costs of keeping the exchange rate fixed in response to asymmetric shocks. We also consider the stabilization cost of instituting a regime of global monetarism in which the monetary authorities forgo offsetting global shocks. Because the welfare results are so simple and tractable, the model is potentially quite useful for analyzing issues of international macroeconomic policy coordination in a stochastic setting.[40]

We have also provided empirical evidence supporting a major building block of our model, the assumption that nominal exchange rate changes play a key role in the short run in shifting world demand between countries. That assumption, which also plays a central role in the traditional monetary model of Mundell, Fleming, and Dornbusch, implies that there is substantial pass-through of exchange-rate changes to the foreign-currency prices of domestic exports (and vice versa). In our model both the domestic-currency and foreign-currency prices of home goods are equally flexible in principle. But domestic wages are rigid, so that (optimal) markup pricing turns out to involve rigidity in the domestic-currency prices, but not the foreign-currency prices, of home goods. Because our model allows for pricing to market, there still can be systematic international price differences due to demand elasticities that differ at home and abroad. Our model explains some of the apparent failure of the law of one price by the fact that many "traded" goods contain a very large nontradable component by the time they reach final consumers. We also argue that a substantial component of rigidity in retail prices for imported goods may originate in the pricing policies of domestic importing firms that intermediate between foreign exporters and domestic consumers, though we do not explicitly model such interactions, leaving that interesting task to future research.

We do not necessarily view our model as "better" than the plethora of interesting new open economy macroeconomic models built on the assumption that all prices, of both imports and exports, are equally rigid in domestic-currency terms. These interesting new models have been used to study a host of important issues, including, for example, the purchasing power parity puzzle, the linkage between macroeconomic volatility and international trade, and the welfare effects of international monetary transmission. Our approach, however, turns out to be more tractable for some questions, and the empirical findings we present suggest that, in any event, its underlying pricing assumptions are not worse than those of the recently popular PTM-LCP alternative. It is clear, however, that much interesting theoretical and empirical work remains to be done.

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Footnotes

1. For more updated references, see Brian Doyle's new open economy macroeconomics homepage at http://www.princeton.edu/~bmdoyle/open.html.

2. In important independent work, Bacchetta and van Wincoop (1998) show how a framework related to the one in Obstfeld and Rogoff (1998) can elucidate the effects of exchange rate uncertainty on international trade. Using the mode of analysis suggested in Obstfeld and Rogoff (1998), Devereux and Engel (1998, 1999) analyze the welfare implications of alternative exchange rate regimes under uncertainty. We mention briefly some relevant precursor literature. Rankin (1998) develops a very interesting analysis of a small open economy with complete asset markets and competitive production, in which monopolistic labor suppliers preset money wages. While (like us) he examines the positive effects of monetary uncertainty, he does not systematically explore the welfare effects of policies. An earlier complete-markets model with nominal rigidities is that of Svensson and van Wijnbergen (1989). Svensson and van Wijnbergen (1989), building on Svensson's (1986) closed-economy model, provide an early discussion of price-setting in advance by maximizing firms facing uncertainty. The appendix to Svensson (1986) briefly discusses the welfare impact of an infinitesimal degree of money-supply variability, but such higher-moment effects are not the main focus of his paper.

3. This emphasis on nominal wage rigidities accords with the extensive empirical evidence, starting with the work of MacDougall (1951, 1952), that relative unit labor costs are a prime determinant of international demand for a country's exports. Bernanke and Carey (1996) argue that nominal wage stickiness is an essential ingredient in understanding the Great Depression. Akerlof, Dickens, and Perry (1996) discuss evidence on rigidities in nominal United States wages, with emphasis on downward rigidity.

4. Kollmann's is one of the few studies that incorporates nominal wage along with nominal price rigidities. Lane's (1999) survey provides more detailed discussion of these and other contributions.

5. ECU Institute (1995) concludes, however, that the practice of invoicing exports in the importer's currency shows an increasing trend over time, except for the United States.

6. Several recent PTM-LCP models assume that only some tradable goods are invoiced in the buyer's currency. In these models the negative correlation between the exchange rate and the overall terms of trade is attenuated, but the expenditure switching effect of the exchange rate is, correspondingly, strengthened.

7. Results for 1973-1998 look very similar. Some of the terms of trade indexes used in the table are based on export and import unit values, others use direct price indexes. For some countries there are gaps in the data.

8. A second example is the case in which Home and Foreign have no trade with each other, but have identical trade patterns with third countries. In that case relative export prices are invariant with respect to changes in the Home-Foreign bilateral exchange rate or any other exchange rates, since prices are set in destination currencies and identical trade weights prevent exchange rate shifts from altering the two countries' export-price indexes differentially. As a result, the correlation between the exchange rate and relative export price is zero.

9. Results for 1973-1998 look very similar. Price data are export unit values.

10. Obstfeld (1998) examines the case of France and Germany in detail, and shows that the variability of relative international competitiveness is systematically higher in floating exchange rate periods, when the variability of the nominal exchange rate is higher. In most cases in table 2, the variabilities of the exchange rate and relative competitiveness are on the order of 2 to 3 percent per month. In the Canadian case, both are on the order of 1 percent per month.

11. Recent PTM-LCP models have other counterfactual implications that we do not take up here. For example, they imply that if import and export *quantities* are sluggish in the short run, exchange rate movements will have a "reverse J-curve" effect that improves the trade balance simply by raising the value of exports relative to imports.

12. Micro studies of PTM generally have been based on export rather than final-customer prices. Thus, it is likely that there is even less pass-through to final consumers than the incomplete pass-through that seems to characterize export prices.

13. Goldberg and Knetter (1997, p. 1267) suggest that inventory demands may lie behind the high variability of measured export volumes compared to that of prices.

14. Of course, the potential for nonprice features of transactions to influence resource allocation has figured importantly in discussions of wage as well as price stickiness.

15. Devereux and Engel (1999) develop a model with internationalized production. In that model, however, expenditure-switching effects are excluded by assumption because all domestically-consumed goods must be produced entirely out of domestic labor.

16. Obstfeld and Rogoff (1996) and Corsetti and Pesenti (1998) discuss the voluntary participation constraint in more detail.

17. Note the contrast between our setup and that of Bacchetta and van Wincoop (1998) and Devereux and Engel (1998) who assume perfect nominal wage flexibility but sticky output prices. The same assumptions are implicitly adopted in Obstfeld and Rogoff (1998). Neither extreme set of assumptions is right, but we view those of the present paper as closer to reality. The opposite view is argued by Kimball (1995).

18. The utility component $\log(C)$ could be replaced by $C^{1-\rho}/(1-\rho)$, $\rho \neq 1$, where ρ is the coefficient of risk aversion, equal here to the inverse of the intertemporal substitution elasticity. Setting $\rho = 1$ simplifies the formulas below; we discuss parenthetically the loss of generality due to that simplifying assumption.

19. Think of the variable L as denoting efficiency labor rather than actual hours worked, H, with $H = K^{1/\nu}L$. In this interpretation, technology is labor-augmenting, for example, technological advance (a lower K) allows a plumber to fix more sinks in a given amount of time. It is not difficult to modify the model to encompass differential productivity shocks between traded and nontraded goods production, but this extension is not essential to our present purpose.

20. In the models of Devereux and Engel (1998) and Betts and Devereux (2000), which feature segmented product markets but complete markets in

contingent nominal payments, nominal consumption levels move in a synchronized fashion. We prefer to avoid the assumption of complete markets in contingent nominal payments, because we cannot think of a coherent account of why workers should be able to sign such contracts with other consumers but not with firms.

21. Note that the effect of a lower wage change on the marginal disutility of labor is

$$-K\left(L^i\right)^{\nu-1}\frac{\mathrm{d}L^i}{\mathrm{d}W(i)}=-K\left(L^i\right)^{\nu-1}\left[\phi\frac{L^i}{W(i)}\right],$$

where the equality follows from the constant wage elasticity ϕ of labor demand.

22. The covariance σ_{XY} of lognormally distributed random variables X and Y is related to the covariance σ_{xy} of their natural logs by

$$\sigma_{XY}=\mathrm{E}\{X\}\,\mathrm{E}\{Y\}\left[\exp\left(\sigma_{xy}\right)-1\right].$$

Thus, holding (the necessarily positive) level means constant, an increase in the covariance between logs implies an increase in the covariance between levels (and conversely). (Setting $X=Y$, the same assertion holds for variances.) In addition, σ_{XY} and σ_{xy} have the same sign.

23. A positive covariance between κ and l means that labor supply will tend to be unexpectedly irksome when it is also unexpectedly high; this pattern raises the expected marginal disutility of effort and so, the nominal wage. A high covariance between labor supply and the price level induces workers to raise the nominal wages they demand, since ex post real wages will tend to be low when ex post labor supply is high.

24. The channel through which uncertainty operates here is closely analogous to, but distinct from, the one proposed by Obstfeld and Rogoff (1998) and Bacchetta and van Wincoop (1998), in which uncertainty affects the preset prices and expected output levels of producers.

25. Our "exact" solutions do require that labor be demand-determined even for large shocks, for which the voluntary participation constraint may be violated. However, the approximation can be made arbitrarily precise by making the variances of the shocks sufficiently small.

26. Even if EZ is held constant, a rise in σ_z^2 raises Ez because the exponential function is convex. But the assumption of lognormally distributed variables allows us to infer from eq. (21) that

$$\log \mathrm{E}Z = \omega - \frac{(\nu - 1)}{2}\sigma_z^2 - \frac{\nu}{8}\sigma_e^2 - \frac{1}{2}\left(\sigma_{\kappa z} + \sigma_{\kappa^* z}\right) - \frac{1}{4}\left(\sigma_{\kappa e} - \sigma_{\kappa^* e}\right).$$

Thus when $\nu > 1$, a rise in σ_z^2 must lower EZ, other things equal. We can similarly derive from eq. (20) the quasi reduced-form expressions for $\log \mathrm{E}\{TOT\}$ and $\log \mathrm{E}\{1/TOT\}$. The present simplified model, unlike that of Bacchetta and van Wincoop (1998), does not imply a relationship between exchange-rate volatility and international trade. However, easy modifications of the model—for example, making the supply of nontradables relatively insensitive to labor input—would introduce the potential for such effects. The mechanisms at work would differ from the one featured by Bacchetta and van Wincoop (1998), who focus upon the possibility of utility functions nonseparable in consumption and leisure.

27. For example, even though it will later turn out that temporary productivity shocks have no effect on the exchange rate or labor supply in the sticky-wage equilibrium, $\sigma_{\kappa z}$ and $\sigma_{\kappa e}$ may still be nonzero if monetary policy responds endogenously to disturbances. It is thus important that these correlations be retained until we have had a chance to delve more deeply into the nature of optimal monetary policy.

28. For example, a positive $\sigma_{\kappa e}$ implies that the "pain of effort" parameter κ is high when the Home currency is weak (e is high) and therefore relative demand for Home products is high. The effect of $\sigma_{\kappa^* e}$ is explained similarly, after noting that $\sigma_{\kappa^* e} = -\sigma_{\kappa^*(-e)}$.

29. For $\rho > 1$ and sufficiently large, an increase in σ_z^2 can actually be associated with an *increase* in Ez in the relationship corresponding to eq. (21), for reasons explained in Obstfeld and Rogoff (1998). The possibility follows from an analysis based on eq. (12).

30. For an arbitrary $\rho > 0$,

$$\begin{aligned} \mathrm{E}\left\{\frac{C^{1-\rho}}{1-\rho}\right\} &= \frac{\left(\mathrm{E}\{C\}\right)^{1-\rho}}{1-\rho}\exp\left[-\frac{(1-\rho)\rho}{2}\sigma_c^2\right] \\ &= \frac{\left(\mathrm{E}\{C\}\right)^{1-\rho}}{1-\rho}\left(1+\frac{\sigma_C^2}{\mathrm{E}\{C\}^2}\right)^{-(1-\rho)\rho/2}. \end{aligned}$$

This equation shows more generally that expected utility is increasing in expected consumption and decreasing in the variance of consumption.

31. To derive the preceding expressions from the general case by letting $\rho \to 1$, one must be careful to write the utility function as

$$U = \frac{C^{1-\rho} - 1}{1-\rho} - \frac{K}{\nu} L^{\nu}.$$

To see why welfare is increasing with ϕ and θ in eq. (24), observe that the expression $\log x - x$ is increasing in x for $x < 1$.

32. To be precise, this statement implicitly assumes that θ and ϕ do not enter arbitrarily into the monetary reaction function, though it is easy to show that such a policy would be suboptimal.

33. Under flexible wages, the Home worker's Euler condition (for example) would be

$$\frac{1}{PC} W = \frac{\phi}{\phi - 1} K L^{\nu-1}.$$

Since $PC = P_{\text{H}} Y_{\text{H}} + P_{\text{N}} Y_{\text{N}} = \left(\frac{\theta}{\theta-1}\right) W (Y_{\text{H}} + Y_{\text{N}}) \equiv \left(\frac{\theta}{\theta-1}\right) WY$ and $L = Y$, we have

$$\frac{1}{Y} = \frac{\phi\theta}{(\phi - 1)(\theta - 1)} K Y^{\nu-1},$$

or

$$\tilde{Y} = \tilde{L} = \left[\frac{(\phi - 1)(\theta - 1)}{K\phi\theta}\right]^{1/\nu}.$$

34. Note that

$$\begin{aligned} \mathrm{E}\tilde{U} &= \mathrm{E}\left\{\log(\tilde{C}) - \frac{K}{\nu}\tilde{L}^{\nu}\right\} \\ &= \mathrm{E}\left\{\log \tilde{Y} - \frac{K}{\nu}\tilde{Y}^{\nu} + \frac{\gamma}{2}\log\left(\frac{\tilde{W}}{\tilde{\mathcal{E}}\tilde{W}^*}\right)\right\}, \end{aligned}$$

where the last is derived making use of the markup pricing equations and the relationship $PC = \left(\frac{\theta}{\theta-1}\right) WY$. From the Home and Foreign first-order conditions for wage setting, using $PC = P_T Z$, we derive

$$\frac{\tilde{W}}{\tilde{\mathcal{E}}\tilde{W}^*} = \frac{K}{K^*}\left(\frac{\tilde{Y}}{\tilde{Y}^*}\right)^{\nu-1}.$$

Substituting this into the expression for $E\tilde{U}$ above and making use of the expressions from the preceding footnote, we obtain eq. (25) of the text.

35. The following proposition generalizes to the case $\rho \neq 1$. Ireland (1996) provides the same characterization of optimal monetary policy in an infinite-horizon closed-economy model with log consumer preferences over consumption. Our proof below clarifies some of the intuition behind the finding, and the sense in which it is special to the case of fixed markups (which monetary policy cannot affect) and homothetic consumption preferences. Erceg, Henderson, and Levin (1999) investigate optimal monetary policy in a numerical closed-economy model; however they assume that fiscal instruments adjust continuously to eliminate monopolistic distortions.

36. To derive the following policy rules, equate the log flexible-price aggregate output levels $\tilde{y}$ and $\tilde{y}^*$ (see footnote 33) to the log sticky-price output levels, given by:

$$y = -\tfrac{1}{2}(w - w^* - e) + z, \;\; y^* = \tfrac{1}{2}(w - w^* - e) + z.$$

The policy rules then follow from expressing z and e in terms of m and m^*.

37. There is an indeterminacy in the preceding policy rules. Here, policy will fully accommodate the expectations Em and Em^*, whatever they are. That is, the (fully anticipated) trend inflation rate is not determined, and one must add conditions on inflation expectations (such as $Em = \log M_0$) to get determinacy. Ireland (1996), who works with a cash-in-advance model of money demand in which some goods can be purchased on credit, contains a similar indeterminacy but over a narrower range of policies. In his setup, optimal monetary policy must result in a nominal interest rate of zero. A similar "optimal quantity of money" result could be derived here were we to reintroduce the monetary component of utility.

38. Benigno (1999) looks at the problem of how a national monetary policy should respond to different regional shocks.

39. We remind the reader of the qualification that our linearization is actually only approximate, since it does not take into account the voluntary participation constraints that can arise for large shocks—when the real wage no longer exceeds the marginal cost of working.

40. Corsetti and Pesenti (1998) apply their elegantly simple nonstochastic new open economy macroeconomic model to analyze policy coordination in a deterministic setting.