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Inference Evaluation in Deductive, Inductive and Analogical Reasoning*

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Abstract

An experiment with a three factorial design is described which tests the impact of 1) the degree of the mapping isomorphism, 2) the differences in the types of reasoning (deduction, induction, and analogy), and 3) the kind of entities changed (objects, attributes, and relations) on the certainty of the inferences made. All the three factors have been found to have significant main effects and a significant interactions between the first factor and all the rest have also been found. Different particular results are discussed. For example, the certainty in the deductive inferences is not significantly different from the one in induction and analogy when there is no one-to-one mapping between the descriptions. Moreover, deduction, induction and analogy have similar behavior in relation to that factor. This is considered as a possible support of the existence of a uniform computational mechanism for evaluation of inferences in all the three kinds of reasoning, a mechanism which is primarily based on the degree of isomorphism.

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Motivation

There is a long tradition in studying human reasoning. Unfortunately, this tradition separates different types of reasoning like deduction, induction (generalization), analogy and typically researchers try to develop separate models of different types of reasoning. The reason for this separation is the claimed differences in their properties. It seems to me, however, that the commonalities in their properties are underestimated and not well known. This paper tries to explore some of these commonalities.

A hypothesis has been made that there could exist a uniform computational mechanism which underlies all three kinds of reasoning (Kokinov, 1988). A specific model, called Associative Memory-Based Reasoning (AMBR), has been proposed. This model includes a number of interacting and parallel running subprocesses like retrieval, mapping, transfer, evaluation and learning. The experimental verification of this hypothesis follows two different and complementary directions: simulation experiments (Kokinov & Nikolov, 1989, Kokinov, 1992) and psychological experiments. As far as the psychological aspects are concerned Kokinov (1990) has demonstrated some common properties of the retrieval processes whereas this paper tries to explore the similarities in the evaluation process.

One widespread opinion is that deductive reasoning produces *absolutely certain* results, while generalization and analogy produce at best *only plausible hypotheses.* This claim is rooted in formal logic, passes through mathematics and is widely imposed on the whole education system.

The present work tries to throw light on the following issues about the evaluation process:

1. Are people indeed absolutely certain in deductive inferences in real-world situations? Do they consider generalization and analogy as unreliable sources of new facts? Are there any common phenomena in the evaluation processes of deduction, generalization and analogy?

2. Is human certainty predefined and based solely on the type of reasoning or does it depend on other factors, e.g. the quality of the established mapping? Is this restricted to analogy or is it a common property of all three kinds of reasoning as AMBR predicts?

3. Are objects, properties and relations processed differently by the evaluation process?

Experiment

Certainty is the reasoner's estimation of the plausibility of an inference. There are at least two sources of evaluation in AMBR (Kokinov, 1992): 1) the reasoner's estimation of the established mapping between a known situation (the base or premises) and the target one and 2) his/her domain knowledge. The goodness of mapping depends on the degree of isomorphism reached, including the number of correspondence pairs found, and on the activation level of the so-called correspondence nodes reflecting the context.

In the present experiment the relation between the degree of isomorphism and human certainty is explored. That is why the evaluation process should be isolated. This is achieved by directly presenting to the subjects what they should know (thus ignoring the retrieval process) together with its graph representation (thus helping them to establish the mapping). Moreover, the domain knowledge as well as the context effects, being sources of evaluation, should be blocked. This condition is fulfilled by presenting problems from a domain completely unfamiliar to the subjects.

In addition to the effect of the *degree of isomorphism* (where we may have the following cases: one-to-one mapping, overspecified new situation, or underspecified new situation) on the subjects' certainty about the inferences made, an attempt is made to examine the effects on the evaluation process of two other factors: the *type of reasoning* (deductive, inductive, or analogical one) and the *type of elements* changed from one situation to another (objects, attributes, or relations).

Thus the problems vary in terms of the relationships between the known and the new situations in three ways:

1. in terms of the degree of *isomorphism* between the descriptions: there is a one-to-one correspondence between the situations (0), in the description of the new situation there is something additional which is unspecified in the known situation and thus remains unpaired (+), in the description of the new situation there is something unspecified and thus something in the known situation remains unpaired (–).

2. in terms of the *inference type*, i.e. the relation type between the two situations: the new situation is a particular case of the known one (deduction - D), the known situation is a particular case of the new one (generalization - G), and there is no pre-specified relation between the situations (analogy - A).

3. in terms of the type of *elements* changed in the second description with respect to the first one: objects (o), attributes (a) and relations (r);

Method

Subjects. Subjects have been 297 volunteers — university students and researchers from different academic disciplines: physics, chemistry, biology, geography, medicine, law, philology. (Mathematicians have been deliberately excluded because a preliminary pilot study has demonstrated that they approach the problems formally and judge always deduction as an absolutely certain inference technique and generalization and analogy as absolutely uncertain ones.)

Procedure. Subjects have been tested individually or in small groups of 5–10 people. They have received a list of 9 target problems preceded by the following written instruction:

> "The present experiment is not a test of your capabilities. It is used to explore some hypotheses about the mechanisms of human thinking. Try to answer the questions although you are unfamiliar with the particular problem domain.
>
> Imagine that you are a researcher in the field of *encelorobes* and you know that all of them consist of *mangovine* and *girofine*, mangovine being *ritalic* and girofine being *tanalic*, and also that mangovine *aceifies* girofine. You also know that encelorobes are *corablic*.
>
> mangovine — ritalic
> | aceifies
> girofine — tanalic
> corablic
>
> During your scientific research you encounter the situations listed below."

For each of them the subjects' certainty in the inferences made has been measured on a 7 point scale. Subjects had as much time as they needed.

Material. Each test problem consists of a situation description and a question. The question is about the subject's certainty of the possible transfer of knowledge (inference) from the known situation described in the instruction to the new one. Situations have been constructed within an artificial nonexisting problem domain in order to avoid subjects' preliminary knowledge about the truth of the inferences and to measure the certainty of the inferences as a result of "pure" reasoning. The concepts used in the situation descriptions are fake but sound scientific (at least in Bulgarian) — something like biochemistry — so that the subjects are not aware of the artificiality of the situations, i.e. they think that correct answers to the questions exist.

Because there is a lot of unfamiliar terminology in the test problems, in addition to the textual description of the situations a simple graph representation is presented.

Several examples of test problems (descriptions of target situations and questions) are presented below in their English translation.

1. You discover a new representative of the class of the *encelorobes* which consists of *didorine* and *caronine*. In it, didorine is *ritalic*, caronine is *tanalic* and the didorine *aceifies* the caronine. It is known that didorine and caronine are kinds of *mangovine* and *girofine* respectively. Are the encelorobes of this type also *corablic?* (The combination of factor levels in this example is [objects, deduction, 0]).

   didorine — ritalic
   | aceifies
   caronine — tanalic

2. Exploring the *stericorobes*, you find out that they consist of mangovine and girofine, the mangovine being ritalic, the girofine being tanalic, the mangovine *privilates* the girofine and the girofine

girovates the mangovine. Are the stericorobes *corablic* as well? (The combination of factor levels in this example is [relations, analogy, +]).

mangovine — ritalic
privilates || girovates
girofine — tanalic

3. It is known that the encelorobes are representatives of the class of the *robes*. Robes consist of mangovine and girofine and the mangovine aceifies the girofine. Are the robes corablic as well? (The combination of factor levels in this example is [attributes, generalization, –]).

mangovine
| aceifies
girofine

Design. The experimental design is a 3×3×3 factorial one, with factors *Elements* (objects, relations, attributes), *Inference Type* (deduction, generalization, analogy) and *Isomorphism* (0,+,–). The dependent variable is the subjects' certainty in the possible inference measured on a 7 point scale.

Six different samples of 9 problems out of the full set of 27 problems have been prepared to randomize the possible order effects. Each sample included problems corresponding to each of the levels of *Inference Type* and *Isomorphism* and to some of the levels of *Elements*. Subjects were randomly assigned to one of these 6 versions. As a result each problem has been presented to about 100 people.

Results

As a result of the experiment 2676 observations have been obtained. A 3 × 3 × 3 ANOVA has been conducted in order to explore the results. In short, the main effects of all three factors are significant.

Mean plausibility ratings vary as a function of the degree of *isomorphism*, $F(2, 2649) = 36.50, p < .001$. Mean plausibility is 5.38 in one-to-one mapping situations, 4.72 when the target situations is overspecified, and 4.87 when the target situation is underspecified.

Mean plausibility ratings vary also as a function of the *inference type*, $F(2, 2649) = 35.18, p < .001$. Mean plausibility is 5.31 in deduction, 5.03 in generalization, and 4.62 in analogy.

Mean plausibility ratings vary also as a function of the *element* changed, $F(2, 2649) = 8.69, p < .001$. Mean plausibility is 4.82 when an object is changed, 5.16 when an attribute changed, and 4.98 when a relation changed.

There are also significant interactions between *Isomorphism* and *Element*, $F(4, 2649) = 27.66$, $p < .001$ (Figure 1a), between *Isomorphism* and *Inference Type*, $F(4, 2649) = 2.75, p < .05$ (Figure 1b), as well as between all the three factors, $F(8, 2649) = 4.19$, $p < .001$.

Discussion

A number of conclusions about main effects, interactions and simple effects can be drawn from the analysis.

1) In contrast with AMBR's predictions *Inference Type* does have a significant effect on the certainty evaluation. It remains, however, to be explored what are the exact causes of this phenomenon: whether this is due to differences in the built- in mechanisms performing the separate types of reasoning, to everyday experience in commonsense reasoning, or to reasoning patterns implanted by the education system.[1]

Let us consider the interaction between *Inference Type* and *Isomorphism* (Figure 1b). Although deduction differs significantly from analogy and generalization and dominates them in the case of a one-to-one mapping (0),

[1] or possibly to the material design (e.g. the base and target have more explicitly paired elements in the cases of deduction and generalization than in the case of analogy, so again the level of isomorphism can be the cause of the decreasing subjects' certainty in analogical inference).

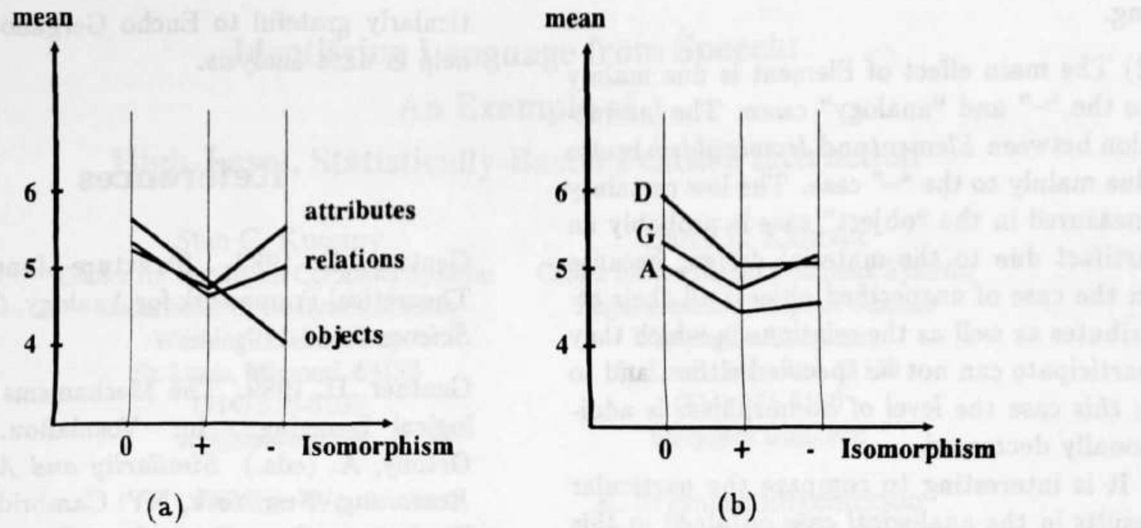

Figure 1: Interactions between: (a) Isomorphism and Elements; (b) Isomorphism and Inference Type

in the other two cases (which are more typical real-world situations) and especially in the case of underspecification of the new situation (-) the results are generally indistinguishable from one another and even sometimes analogy or generalization dominate deduction.

In order to explore the causes of this interaction, let us consider the simple effect of decreasing the certainty level of deductive inference in the non-isomorphic situations. Note that from the point of view of formal logic, if it is known that an object belongs to a class, no more information is needed in order to draw a deductive conclusion with absolutely certainty. As the experiment demonstrates, however, this is simply not true with human reasoning. Moreover, it is clear that each representative of a class has a number of elements (attributes, relations, components) that are specific for it, so there are no logical reasons for questioning the deductive inference solely on the ground that the situation is overspecified (+). According to common sense, the certainty should be at least the same as in the isomorphic case.[2] However, as Figure 1b shows, there is a significant decrease in the subjects' certainty in this situation.

[2]Compare with the "mirror" case, i.e. generalization in the underspecified situation (-), where the subjects' certainty increases.

The high certainty of deduction may be an effect of education. The examples that are most instructive in this respect are given by mathematics, and there only pure isomorphic situations are presented. When, however, subjects are involved in a complex reasoning process in realistic non-isomorphic situations, the reasoning pattern imposed by education may not be triggered and only the built-in mechanisms of evaluation based on isomorphism are activated.

So one possible explanation of the interaction is that the significant differences between deduction, generalization, and analogy in the "0" case are imposed by the learned reasoning patterns, which do not function in the non-isomorphic cases.

Finally, deduction, generalization and analogy have similar behavior in relation to *Isomorphism* (Figure 1b). Moreover, this is true for each of the *Elements* levels: in the case of an object change, the worst case for all inference types is an unspecified object in the new situation, whereas in the case of an attribute or relation change the worst case is the presence of additional attributes or relations. In general, such similarity in behavior is considered to indicate the existence of a uniform computational mechanism for evaluation of inferences in all the three kinds of reasoning.

2) The main effect of Element is due mainly to the "–" and "analogy" cases. The interaction between *Element* and *Isomorphism* is also due mainly to the "–" case. The low certainty measured in the "object" case is probably an artifact due to the material design, because in the case of unspecified objects all their attributes as well as the relations in which they participate can not be specified either, and so in this case the level of isomorphism is additionally decreased.

It is interesting to compare the particular results in the analogical case obtained in this experiment with those of the experiments of Gentner demonstrating the superiority of relations over attributes in the mapping and evaluation process in analogy (Gentner, 1983, 1989, Goldstone et al., 1991). In the case of a one-to-one mapping as well as in the case of overspecification no such priority is demonstrated in the present experiment. In the case of underspecification, however, relations do have a significant priority.

3) Finally, *Isomorphism* has the greatest main effect and in correspondence with AMBR's predictions, the one-to-one mapping gives more certainty than many-to-one or one-to-many mappings. Moreover, *Isomorphism* influences both other factors which should be related to the functioning of a mechanism at the very basic level. This supports the AMBR's hypothesis that the primary mechanism for evaluating inferences is based on the goodness of mapping evaluation.

The results from this complex experiment are very rich and controversial, so I do not consider them as conclusive but rather as a starting point for further experimentation.

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