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AN EXAMINATION OF THE CHEMICAL EQUILIBRIUM RELATIONS USED IN THE FIREBALL MODEL OF RELATIVISTIC HEAVY ION REACTIONS

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ABSTRACT

The fireball model of relativistic heavy-ion collision uses chemical equilibrium relations to predict cross-sections for particle and composite productions. These relations are examined in a canonical ensemble model where chemical equilibrium is not explicitly invoked.

[Fireball; particle production; relativistic heavy ions; chemical equilibrium]

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I. INTRODUCTION

When two heavy ions collide, many types of particles are produced in the reaction. For example, one can measure n, p, d, t, ^{3}He, ^{4}He cross sections. Pion production increases with increasing incident energy.

The fireball model[1] can make predictions about various abundances in such reactions. Assuming that both thermal and chemical equilibria set in before the freeze-out volume is reached,[2-5] one uses the law of chemical kinetics. If $b_1 A_1 + b_2 A_2 \rightleftharpoons b_3 A_3$ where A_i refers to the species and b_i to the number of such species taking part in the reaction, then

$$b_1 \mu_1 + b_2 \mu_2 = b_3 \mu_3 \qquad (1)$$

at equilibrium.[6] Here the μ_i's are the chemical potentials. This enables one to write equations like $\mu_d = \mu_n + \mu_p$; $\mu_{\pi_+} = \mu_p - \mu_n$; $\mu_{\pi_0} = 0$, etc. The number of particles of a given species is given by

$$n_i = e^{\beta\mu_i} Z(i) \qquad (2)$$

where $Z(i)$ is the partition function of one particles of the species i. Combining Eqs. (1) and (2), we get

$$b_1 (\ln n_1 - \ln Z(1)) + b_2 (\ln n_2 - \ln Z(2)) = b_3 (\ln n_3 - \ln Z(3)) \qquad (3)$$

Equations (1) and (2), together with some conserved quantities such as baryon number, charge, etc., allow one to compute abundances of various species.

Equation (2), for example, uses the grand canonical ensemble which is good when many particles are present. However, the production cross-sections of some particles and composites can be quite low. One may get only one particle or less, on the average, per interaction. The validity of Eqs. (1) and (2) is not transparent in such cases. In view of the fact that much effort is being made at the current time to calculate production cross-sections in the fireball model using Eqs. (1) and (2) we felt it worthwhile to examine them more closely. In our approach, chemical equilibrium is not invoked explicitly.

For completeness, we remind the reader of the steps leading to Eq. (2):

$$n_i = \frac{1}{\beta}\frac{\partial}{\partial\mu}\,\ell n\, Z_{gr}(i) \tag{4}$$

$$Z_{gr} = \sum_{n=0}^{\infty} e^{\beta\mu n}\, Z_n(i) \tag{5}$$

$$Z_n(i) = \frac{1}{n!}\left(Z(i)\right)^n \tag{6}$$

Equation (6) is an approximation which is valid in the high temperature and low density limit. Whenever the FD or Bose statistics approach the MB distribution, Eq. (6) is obtained. One can talk of chemical equilibrium only in such limits. Let us elaborate this point further. Consider ^{16}O in its ground state. One can describe it as eight protons and eight neutrons but one can also describe it as four alpha clusters. In order that the two descriptions be different one requires low density so that the protons and neutrons do not get in the way of the alpha particles too often. In our subsequent sections, therefore, we retain Eq. (6).

II. AN ASSEMBLY OF NEUTRONS, PROTONS AND DEUTERONS

Let us first consider a box of volume V at a fixed temperature which has protons, neutrons and deuterons. Let n_1 be the number of protons, n_2 the number of neutrons, and n_3 the number of deuterons. These numbers may change (a deuteron may dissociate into a proton and a neutron or the reverse) but we have the condition

$$\begin{aligned} n_1 + n_3 &= k_1 \\ n_2 + n_3 &= k_2 \end{aligned} \tag{7}$$

where k_1 and k_2 are fixed. In this section when we sum over n_1, n_2 and n_3, we sum preserving Eq. (7).

At a given temperature the probability that we obtain a state which has n_1 protons in eigenstate A, n_2 neutrons in eigenstate B, and n_3 deuterons in eigenstate C is

$$P^{ABC}_{n_1 n_2 n_3} = \frac{e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} e^{-\beta E_C(n_3)}}{\sum\limits_{\substack{ABC \\ n_1 n_2 n_3}} e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} e^{-\beta E_C(n_3)}} \tag{8}$$

The probability of obtaining n_1 protons, n_2 neutrons, and n_3 deuterons is

$$P_{n_1 n_2 n_3} = \frac{\sum\limits_{ABC} e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} e^{-\beta E_C(n_3)}}{\sum\limits_{\substack{ABC \\ n_1 n_2 n_3}} e^{-\beta E_A(n_1)} e^{-\beta E_B(n_2)} e^{-\beta E_C(n_3)}} \tag{9}$$

The quantity $\sum\limits_{A} e^{-\beta E_A(n_1)}$ is, however, the canonical partition function for n protons. We now use Eq. (6) and obtain

$$P_{n_1 n_2 n_3} = \frac{\frac{1}{n_1!}(Z(1))^{n_1} \frac{1}{n_2!}(Z(2))^{n_2} \frac{1}{n_3!}(Z(3))^{n_3}}{\sum \frac{1}{n_1!}(Z(1))^{n_1} \frac{1}{n_2!}(Z(2))^{n_2} \frac{1}{n_3!}(Z(3))^{n_3}} \tag{10}$$

For deuteron cross-sections, the only quantity needed is

$$\bar{n}_3 = \sum n_3 \, P_{n_1 n_2 n_3} \tag{11}$$

Consider Eq. (11) in two limits: $\bar{n}_3 << 1$, $\bar{n}_1 >> 1$, $\bar{n}_2 >> 1$ and $\bar{n}_3 >> 1$, $\bar{n}_1 >> 1$, $\bar{n}_2 >> 1$. In the first case,

$$\bar{n}_3 \approx \frac{\frac{1}{(k_1-1)!}(Z(1))^{k_1-1} \frac{1}{(k_2-1)!}(Z(2))^{k_2-1} \; Z(3)}{\frac{1}{k_1!}(Z(1))^{k_1} \frac{1}{k_2!}(Z(2))^{k_2}}$$

$$= k_1 k_2 \frac{Z(3)}{Z(1)Z(2)} \approx \bar{n}_1 \bar{n}_2 \frac{Z(3)}{Z(1)Z(2)}$$

which is the same as in Eq. (3). In the limit $\bar{n}_3 >> 1$, since all the numbers involved are big, we maximize $P_{n_1 n_2 n_3}$ to find the most probable distribution and assert that the most probable distribution will be the actual distribution seen as the distribution is sharply peaked. Thus we consider

$$\delta\left\{n_1 \ln Z(1) - n_1! + n_2 \ln Z(2) - n_2! + n_3 \ln Z(3) - n_3!\right\} = 0$$

Using $dn_1 = dn_2 = -dn_3$ and $\frac{\partial}{\partial n} n! \approx \ln n$, we obtain Eq. (3) again.

Since Eq. (3) holds in both the limits, it may be reasonable in between also, although it is difficult to see this analytically. Numerical estimates of errors will be given later.

III. PRODUCTION OF PIONS

There are some cases where no conservation laws such as Eq. (7) exist. An example is the production of π_0's.

$$\bar{n}_{\pi_0} = \frac{\sum_{n=0}^{\infty} n \frac{1}{n!} Z^n}{\sum_{n=0}^{\infty} \frac{1}{n!} Z^n} = Z \tag{12}$$

which is exactly the same as the grand canonical result with $\mu_{\pi_0} = 0$. For π_+ and π_- however, conservation laws similar to Eq. (7) exist.

To estimate errors we considered $k_1 = 25$, $k_2 = 35$ [Eq. (7)]. This corresponds roughly to the optimum value[1] of the impact parameter 'b' for the case of ^{20}Ne on U. We considered a model of n,p and d and also a model of n, p, π_+, π_- and π_0. For the values of Z(i)'s appropriate for heavy ion reactions, we never found deviations greater than 5% from the grand canonical predictions.

There is more information contained in Eq. (10) than what is contained in cross-section measurement [Eq. (11)]. Equation (10) can be used to calculate multiplicity distribuitons. As an example, for π_0's since there is no conservation law, the probability for producing n π_0's at a given impact parameter is

$$P_n = \frac{1}{n!} Z^n \Big| \sum_{m=0}^{\infty} \frac{1}{m!} Z^m$$

For Eq. (12), $\bar{n} = Z$ and thus P_n reduced to the Poisson distribution, $P_n = \frac{1}{n!} (\bar{n})^n e^{-\bar{n}}$. The argument here is more general than that of Ref. 7. Because of conservation laws, such simple arguments do not apply to π_+ or π_-, although the more complicated expressions for these may produce values close to a Poisson distribution.

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