Title

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Forrest I. Boley and Peter R. Forman

January 7, 1964

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

March 18, 1964

ERRATUM

TO: All recipients of UCRL-11174

FROM: Technical Information Division

Subject: UCRL-11174, "Steepening of Large-Amplitude Alfvén Waves," Forrest I. Boley and Peter R. Forman, January 7, 1964.

Please change the last sentence on page 2 to read:

The ratio of the slope of the steepened wave front to the corresponding slope of the unsteepened wave at $t = 0$, $z = 0$ is

$$\delta \equiv (\partial b_\theta/\partial t)/(\partial b_\theta/\partial t)_{t=0,\ z=0} . \qquad (4)$$

Steepening of Large-Amplitude Alfvén Waves*

Forrest I. Boley and Peter R. Forman

University of California
Lawrence Radiation Laboratory
Berkeley, California

January 7, 1964

In this letter we report the experimental observation of the steepening of large-amplitude Alfvén waves in a highly ionized plasma. Calculations concerning the expected steepening time for these waves have been previously reported by Montgomery[1] and Parker.[2,3]

The plasma device in which the waves are propagated has been described elsewhere;[4,5] only the salient features are given here. Hydrogen gas at 0.1 Torr is contained in a 14.6-cm-diam, 86.4-cm-long copper cylinder, the ends of which are closed by quartz plates. A coaxial electrode is located in one of these end plates and a copper screen covers the other. Figure 1 shows the manner in which the ionizing voltage and the wave-inducing signal are applied between the center electrode and the copper cylinder. A uniform magnetic field $B_0 = 8.0$ kG is directed axially along the cylinder. The wave is induced 65 μsec after the initial plasma-forming discharge begins and 40 μsec after the discharge has been short-circuited. At the time of wave propagation, the ion density is approximately 3.5×10^{15} cm^{-3}, and the electron temperature about 1.3×10^{4} °K at the radial position of the wave measurements.

For frequencies far below particle resonances, the waves induced by the oscillatory radial electric field are mainly torsional, with the azimuthal-wave-magnetic-field component b_θ dominant. Such waves with amplitudes $|b_\theta| \ll B_0$ propagate undistorted in the direction of B_0 at very nearly the

Alfvén velocity

$$V_A = B_0/(4\pi\rho)^{1/2} . \tag{1}$$

Here ρ is the plasma mass density. The detailed behavior of these small-amplitude waves has been reported.[6,7]

In this experiment, the wave amplitudes $|b_\theta|$ are as large as 0.6 B_0. To a good approximation, waves of such amplitude must satisfy the requirement

$$(B_0^2 + b_\theta^2)^{1/2}/\rho = \text{constant} , \tag{2}$$

because of the plasma compressibility. To satisfy this requirement, the phase velocity $a^2 = dP/d\rho$ becomes

$$a^2 = \frac{B_0^2 + b_\theta^2}{4\pi\rho} . \tag{3}$$

Thus when b_θ^2 is not small compared to B_0^2, the phase velocity depends upon b_θ because of the dependence of the magnetic pressure P upon the total magnetic field.

An initially sinusoidal wave field, $b_\theta = |b_\theta| \exp[i(\omega t - k_0 z)]$ induced at $z = 0$, is increasingly distorted as points of larger b_θ overtake the lower portions of the wave. Steepening of the leading slopes of the waves results. ~~If we write δ as the ratio of the slope of the steepened wave front to the corresponding slope of the unsteepened wave at t = 0, z = 0 give~~s

$$\delta \equiv (\partial b_\theta/\partial t)/(\partial b_\theta/\partial t)_{t=0,\,z=0} . \tag{4}$$

After a propagation time t, δ becomes approximately[8]

$$\delta = \left[1 - \frac{\omega t/2}{1+(B_0/|b_\theta|)^2} \right]^{-1} , \tag{5}$$

when collisions are neglected. The steepening time is the solution of Eq. (5) for which $\delta = \infty$ and is $t_s = 4/\omega$ for $b_\theta = B_0$, in close agreement with the steepening times calculated previously.[1,3] After propagation over a distance z, the ratio δ is given by

$$\delta = \left[1 - \frac{k_0 z/2}{1+(B_0/|b_\theta|)^2} \right]^{-1} . \tag{6}$$

Thus to compare the results of the experiments reported here to the deductions of the collisionless theory, we determine δ as a function of $|b_\theta| / B_0$.

The wave magnetic fields are observed by use of 2.4-mm diam, 8-turn, calibrated pickup loops placed within 6-mm-o.d. quartz tubes inserted longitudinally into the plasma. The probes are placed at the radial position of maximum b_θ as determined from previous measurements of the radial variation of low-amplitude waves.[4] Probes were placed at $z = 10$ and 30 cm, but were separated by 180 deg in azimuth to minimize possible interference between them. The probes' output voltages and their time integral yield $\partial b_\theta/\partial t$ and b_θ, respectively. Waveforms of both quantities are displayed on oscilloscopes.

The waves are induced by application of a 0.43-Mc damped sinewave derived from the discharge of the 1-μF, 100-kV wave-generating capacitor shown in Fig. 1. Wave fields ranging from 500 to 5000 G are obtained by varying the voltage to which the wave-generating capacitor is charged. The initial slope of the wave-inducing current was constant within about 7% and had no discernible dependence upon the charging voltage. Thus the independence of any observed wave steepening from possible changes in the current waveform is insured. The wave steepening is observed by measurement of $\partial b_\theta/\partial t$ and b_θ at $z = 10$ cm during the first quarter cycle of the oscillation.

Figure 2 shows the observed wave steepening at $z = 10$ cm plotted as a function of $|b_\theta| / B_0$. The bars indicate the rms deviation of the data. The values of b_θ are deduced from those measured at $z = 10$ cm and extrapolated to $z = 0$ according to the measured attenuation of low-amplitude waves between $z = 10$ and 30 cm. Also shown in Fig. 2 is the steepening calculated according to Eq. (6) and normalized to the observed value of $(\partial b_\theta/\partial t) / |b_\theta|$ for the lowest-amplitude waves. The values of ω and V_A used to calculate k_0 in Eq. (6) were deduced directly from the observed waveforms at the two longitudinal positions.

For propagation over the 10-cm distance discussed above, the wave steepening is seen to be indistinguishable from that expected from a collisionless theory. However, additional measurements of the steepening at $z = 20$, 30, and 40 cm show that beyond 10 to 20 cm the steepening does not progress as rapidly as expected from Eq. (6). For $|b_\theta| = 5$ kG, the largest value of $(\partial b_\theta/\partial t) / |b_\theta|$ occurs at 20 cm and is 2.4 times the corresponding low-amplitude value of this ratio, which essentially agrees with a collisionless theory. However, beyond $z = 20$ cm the steepening does not increase for any of the wave fields used in this experiment.

Thus, although over short distances the steepening does not differ significantly from that expected from collisionless theory, dissipation mechanisms that become increasingly important for the higher frequency components likely inhibit the further steepening of the wave in the plasma studied.

We thank Dr. John M. Stone for assistance with the spectroscopic measurements of plasma temperature and density.

FOOTNOTES AND REFERENCES

[*] Work done under the auspices of the U. S. Atomic Energy Commission.

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FIGURE LEGENDS

Fig. 1. Experimental apparatus.

Fig. 2. Dependence of the wave steepening upon $|b_\theta|/B_0$ at $z = 10$ cm. The bars represent the rms deviation of the experimental data. The solid curve is calculated from Eq. (6) and is normalized to the lowest wave-amplitude value of $(\partial b_\theta/\partial t)/|b_\theta|$.

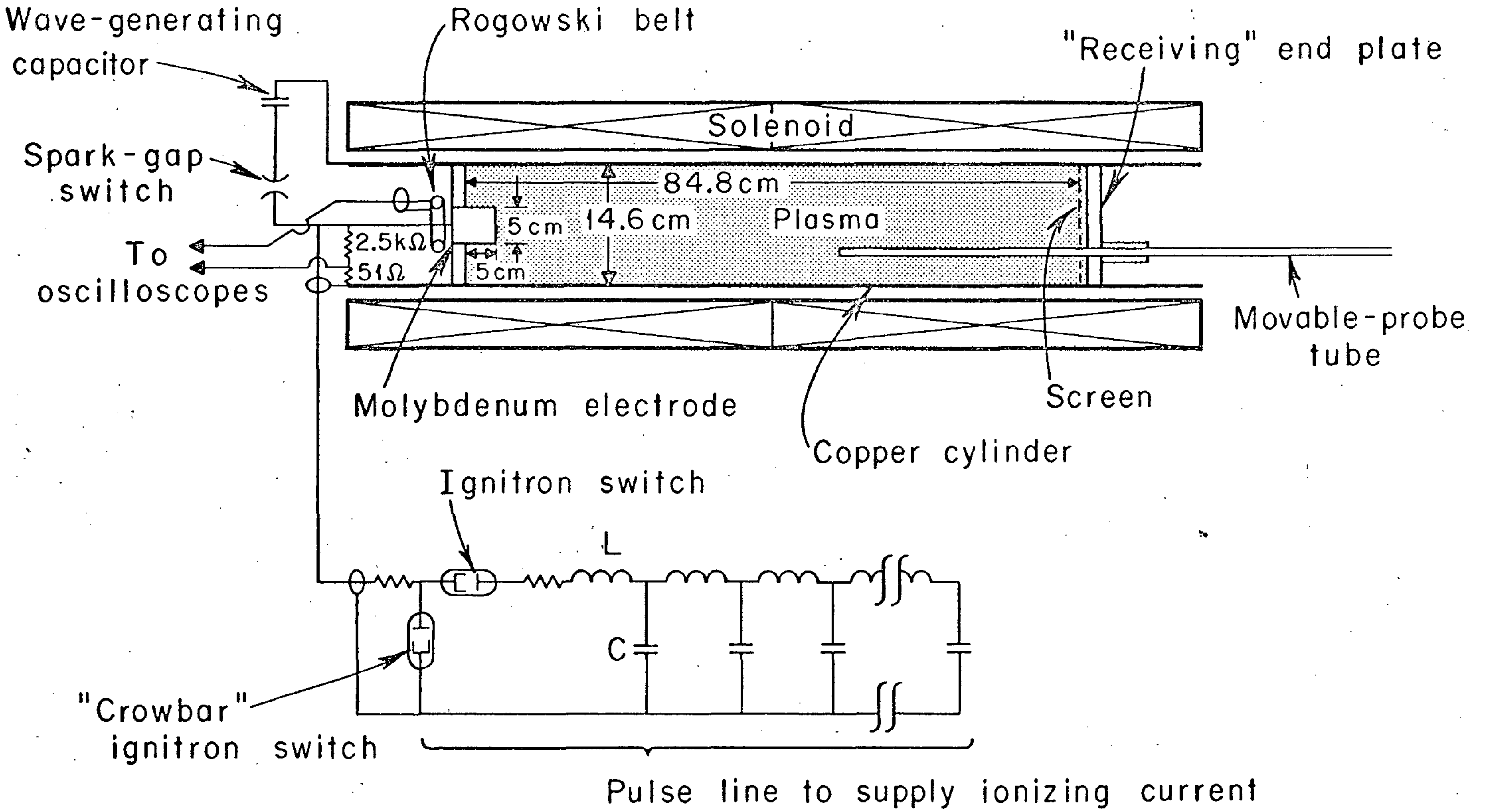
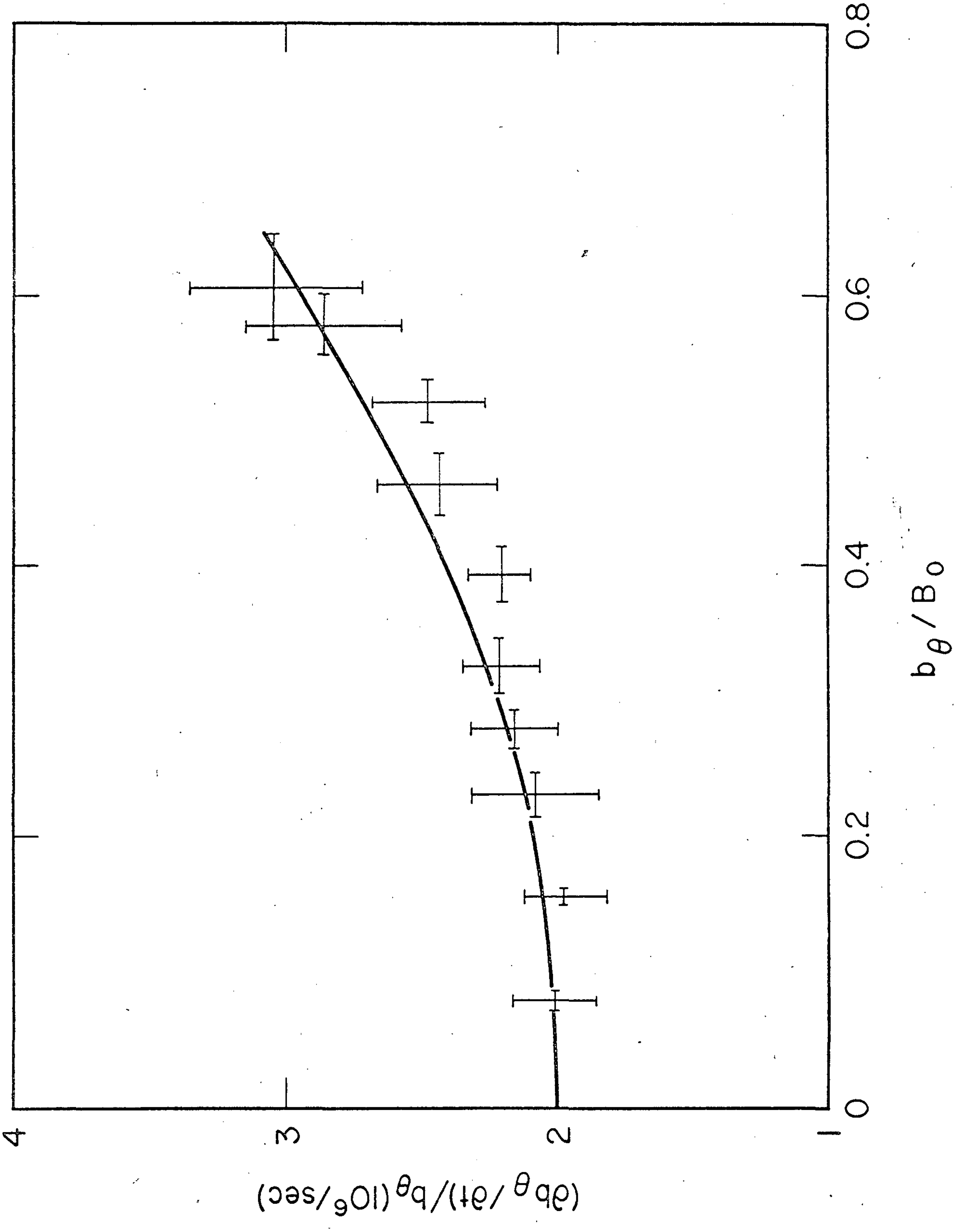

Wave-generating capacitor
Rogowski belt
"Receiving" end plate
Solenoid
Spark-gap switch
84.8cm
5 cm
14.6 cm
Plasma
2.5kΩ
51Ω
5cm
To oscilloscopes
Movable-probe tube
Molybdenum electrode
Screen
Copper cylinder
Ignitron switch
L
C
"Crowbar" ignitron switch
Pulse line to supply ionizing current
MU-31949A

Fig. 1



$(\partial b_\theta/\partial t)/b_\theta\ (10^6/\mathrm{sec})$
1
2
3
4
b_θ/B_0
0
0.2
0.4
0.6
0.8
MU-32989

Fig. 2.