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Abstract

A crucial test for discriminating between the Bohr-Mottelson model and the Davydov-Filippov model is proposed. The analysis of the beta transition rates from ^{186}Re and ^{188}Re to the K = 2 bands in ^{186}Os and ^{188}Os unfavours the asymmetric rotor model. The beta transition rate to the K = 2 state is evaluated on basis of the microscopic description of the gamma vibration, which accounts for the experimental retardation factor fairly well.

In a previous paper[1)] we discussed the equilibrium shape of deformed nuclei by comparing the Bohr-Mottelson model[2)] with the Davydov-Filippov model[3)]. The essential difference between the two models is, in particular, concerned with the so called gamma-vibrational band of K = 2. In the BM model the K = 2 band is regarded as a gamma-vibrational state of an axially symmetric rotor, whereas in the DF model it is interpreted as a kind of rotational state of an asymmetric rotor generated by a rotation with respect to the near-symmetric axis. The DF model explained many experimental results more quantitatively than the perturbation calculation in the BM model. However, recently Faessler, Greiner, and Sheline[4], using the method of direct diagonalization of the Hamiltonian, have refined the BM model. This yields more realistic results where the perturbation approach is not valid. For instance, the ground state rotational bands of deformed nuclei, which were observed in (α,xn)[5] or (HI,xn) reactions[6], have been explained quantitatively by this theory as well as by the DF model[6].

It is important to note that, as far as the ground band, the beta-vibrational band and the K = 2 band are concerned, the matrix elements relevant to the band mixing, E2 transition rates and energy level spacings are almost the same for the both models despite the essential difference in the Hamiltonian, the E2 transition operator and the wave functions. In other words, the matrix elements and subsequent quantities in the BM model can be transformed into those in the DF model simply by replacing the parameter γ_{00}, the zero-point amplitude of gamma vibration, by the asymmetry parameter γ_0. This means that the dynamical variable γ can be fixed at a constant value equal to the zero-point amplitude γ_{00} as far as such quantities as E2 transition rates and energy level spacings are concerned. Since any difference between their predictions is not inherent in the assumption concerning the equilibrium shape, it is essentially impossible to distinguish between these models on the basis of such quantities. However, it was emphasized in the paper[1] that the both models exhibit quite different features for higher band schemes as a consequence of the essential difference. The comparison of experimental information about the K = 4 band favored the BM model over the DF model. A similar discussion was also made by Faessler, Greiner, and Sheline in a more quantitative way[4].

The present note is concerned with an additional way to distinguish between these models from beta decays. This is more crucial and straightforward than any other test, because it is based on the discrimination of the wavefunction of the K = 2 band itself.

We represent in general the wavefunctions of the ground, the first and the second 2+ states of even nuclei as follows:

$$|0+\rangle = \sqrt{\frac{1}{8\pi^2}}\, D^0_{00}\, |\Psi_0\rangle$$

$$|{}^1 2+\rangle_m = \sqrt{\frac{5}{8\pi^2}}\, \frac{1}{\sqrt{1+\xi^2}}\, [D^2_{m0}|\Psi_0\rangle + \xi \frac{1}{\sqrt{2}}\, \{D^2_{m2}|\Psi_2\rangle + D^2_{m-2}|\Psi_{\tilde{2}}\rangle\}] \quad ,$$

$$|{}^2 2+\rangle_m = \sqrt{\frac{5}{8\pi^2}}\, \frac{1}{\sqrt{1+\xi^2}}\, [\frac{1}{\sqrt{2}}\{D^2_{m2}|\Psi_2\rangle + D^2_{m-2}|\Psi_{\tilde{2}}\rangle\} - \xi\, D^2_{m0}|\Psi_0\rangle] \quad ,$$

where $|\Psi_0\rangle$ and $|\Psi_2\rangle$ are intrinsic wavefunctions associated with the K = 0 and K = 2 bands, respectively, ξ is the band mixing amplitude, approximately given by

$$\xi \approx \frac{\sqrt{2}}{3p\sqrt{p}} \quad , \; p \equiv E({}^2 2+)/E({}^1 2+) \quad ,$$

and $|\Psi_{\tilde{2}}\rangle$ stands for the time reversed state of $|\Psi_2\rangle$. In the BM model, $|\Psi_0\rangle$ and $|\Psi_2\rangle$ refer to the $n_\gamma = 0$ and 1 vibrational modes, respectively. In this case, it is expected that the beta transition from the K = 1 parent state to the K = 2 band may be fairly retarded compared with the beta transition to the ground band, because the former transition involves a change in n_γ ($\Delta n_\gamma = 1$). Similar phenomena are well known for spherical nuclei[7], where the beta transition from a parent nucleus of 1+ spin to the first 2+ state is retarded. On the other hand, in the framework of the DF model the $K = 2$ band is of the same intrinsic state as the ground state band, that is $|\Psi_2\rangle = |\Psi_0\rangle$, so that the relative decay rates depend simply on the geometrical factors (the Clebsch-Gordan coefficients)[8]. Previously analyses of beta transitions were attempted in this way by Davydov[9] and by Sakai[10].

Now we deal with the beta transitions from 1- states of odd-odd nuclei, in which case the beta-decay operator G_β with $\lambda = 1$, $\Delta K = 1$, yes, is involved. There are two such examples, namely, ^{186}Re and ^{188}Re, which lie in the region where the Dayvdov-Filippov model is known to be successful. The experimental data[11] are summarized in table 1, and the relevant decay scheme is illustrated in fig. 1.

The general form of the parent state is

$$|1-\rangle_m = \sqrt{\frac{3}{8\pi^2}} \; \frac{1}{\sqrt{1+\eta^2}} \left[\frac{1}{\sqrt{2}} \{D^1_{m1}|\Phi_1\rangle - D^1_{m-1}|\Phi_{\tilde{1}}\rangle\} + \eta \, D^1_{m0} \; \frac{|\Phi_0\rangle + |\Phi_{\tilde{0}}\rangle}{\sqrt{2}} \right]$$

where $|\Phi_1\rangle$ and $|\Phi_0\rangle$ are intrinsic wavefunctions associated with $K = 1$ and $K = 0$, respectively, and η is the mixing amplitude of the $K = 0$ component. It will be shown below that η is very small, as expected for the deformed region.

The ratio of the ft values for decays to the ground and the first 2+ states becomes

$$\frac{ft(1- \to {}^{1}2+)}{ft(1- \to 0+)} = \left[\frac{\eta'\langle 1100|00\rangle + \langle 111-1|00\rangle}{\eta'\langle 1100|20\rangle + \langle 111-1|20\rangle} \right]^2 = 2.0 \times \left(\frac{1-\eta'}{1+2\eta'}\right)^2 ,$$

where

$$\eta' \equiv \frac{\langle \Psi_0 \| G_\beta \| \Phi_0 \rangle}{\langle \Psi_0 \| G_\beta \| \Phi_1 \rangle} \, \eta$$

is the effective mixing ratio contributing to these transitions. In the above expression the effect of the admixture of the K = 2 state is neglected.

Since the experimental values are close to 2.0, we obtain $\eta' \approx 0$. (See column 3 of table 2.) There is another solution, $\eta' \approx -2$, which infers the presence of a very large admixture of a K = 0- state. The Nilsson configuration of $\{5/2+[402]_p, 3/2-[512]_n\}_{K=1-}$ is given to both ^{186}Re and ^{188}Re from the consideration of neighboring odd nuclei. A possible configuration with K = 0- may be $\{3/2+[402]_p, 3/2-[512]_n\}_{K=0-}$, which has a Coriolis matrix element connecting this with the primary state. However, $\eta' \approx -2$ is larger than expected from theoretical consideration, unless the energy difference between these two configurations is extremely small. As a matter of fact, the first three levels of ^{188}Re, 1- at 0 keV, 2- at 63.6 keV, and 3- at 156.0 keV, which were observed by Burson et al.[12] and by Takahashi et al.[13], clearly constitute a quite regular rotational band of K = 1-. The magnetic moments of ^{186}Re and ^{188}Re which were recently determined by Armstrong and Marrus[14], also support the $\{5/2+[402]_p, 3/2-[512]_n\}_{K=1-}$ configuration. Therefore we will prefer $\eta' = 0$ in the following discussion.

The ratio of the ft values for the first and second 2+ states is

$$\frac{ft(1- \to {}^{2}2+)}{ft(1- \to {}^{1}2+)} = \left[\frac{\sqrt{2}\langle 111\text{-}1|20\rangle + \xi\,\Gamma\langle 1111|22\rangle}{-\sqrt{2}\,\xi\,\langle 111\text{-}1|20\rangle + \Gamma\langle 1111|22\rangle}\right]^2$$

$$= \left(\frac{0.577 + \xi\,\Gamma}{-0.577\,\xi + \Gamma}\right)^2 ,$$

where
$$\Gamma \equiv \frac{\langle \Psi_2 \| G_\beta \| \Phi_1 \rangle}{\langle \Psi_0 \| G_\beta \| \Phi_1 \rangle} .$$

The quantity Γ means the intrinsic retardation amplitude of the K = 2 band compared with that of the ground-state band.

In fig. 2 is shown the relation between the ft value ratio and the energy ratio p with parameter Γ. As mentioned before, the DF model requires $\Gamma = 1$. Apparently, the experimental values reveal great deviation from the DF prediction. The values of Γ that fit the experimental data are presented in column 4 of table 2.

In the following we will attempt a rough estimate of Γ on the basis of the microscopic description of the gamma-vibrational state. We express the intrinsic wavefunctions using the Bogoliubov-Valatin transformation as follows:

$$|\Phi_1\rangle = \alpha'_{f_p}{}^{+}\alpha'_{f_n}{}^{+}|\Psi'_0(i)\rangle$$

$$|\Psi_0\rangle = |\Psi_0(f)\rangle$$

$$|\Psi_2\rangle = Q_2^{+}|\Psi_0(f)\rangle = \sum_{i,j} f_{ij}\alpha_i^{+}\alpha_j^{+}|\Psi_0(f)\rangle ,$$

where f_{ij}, partial amplitude of a two-quasi particle component (i,j) in the gamma-vibrational state, is calculated by solving the dispersion equations in the random phase approximation. Then Γ is approximately given by

$$\Gamma \approx 2\,\frac{\Sigma\langle j_p|G_\beta|f_n\rangle f^{*}_{f_p j_p} U_{j_p}(f)U_{f_n}(f) + \Sigma\langle\tilde{f}_p|G_\beta|\tilde{j}_n\rangle f_{f_n j_n} V_{f_p}(f)V_{j_n}(f)}{\langle\tilde{f}_p|G_\beta|f_n\rangle V_{f_p}(f)U_{f_n}(f)} .$$

There are three beta decay operators responsible for these transitions: $\int\vec{\sigma}\times\vec{r}$, $\int\vec{r}$, and $\int\vec{\alpha}$. If we use the relation

$$\int\vec{\alpha} = iW_0\int\vec{r}$$

according to Bogdan[15], we obtain

$$\Gamma^2 \sim \frac{1}{10}$$

for the $^{186}Re \rightarrow {}^{186}Os$ decay. Although this value is still too large and dependent on many assumptions involved in the calculation, this estimate shows that the retardation factor is accounted for qualitatively by the microscopic description of the gamma vibration.

As a conclusion, we can say that the asymmetric-rotor model fails in explaining the beta decays to the K = 2 state. In other words, the experiments show that the K = 2 state should be intrinsically different from the ground state, which contradicts with the basic idea of the asymmetric-rotor model. On the other hand, our tentative calculation based on the microscopic description of the gamma-vibrational state accounted for the retardation factor Γ fairly well.

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References and Footnotes

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Table 1
Experimental data on the beta transitions from odd-odd 1- nuclei in the deformed region taken from ref. 11)

Parent nucleus	Daughter nucleus	Level energy (keV)		log ft		
		$^{1}2+$	$^{2}2+$	0+	$^{1}2+$	$^{2}2+$
$^{186}_{75}Re_{111}$	$^{186}_{76}Os_{110}$	137	768	7.7	8.0	9.0
$^{188}_{75}Re_{113}$	$^{188}_{76}Os_{112}$	155	633	8.1	8.5	9.4

Table 2
Results of the analyses

Parent nucleus	Daughter nucleus	η'	Γ
^{186}Re	^{186}Os	-0.03	0.21 or -0.16
^{188}Re	^{188}Os	-0.08	0.31 or -0.14

Figure Captions

Fig. 1. The partial decay scheme of ^{186}Re and ^{188}Re.

Fig. 2. The ratio $ft(1 \rightarrow {}^{2}2)/ft(1 \rightarrow {}^{1}2)$ is illustrated versus the ratio $E({}^{2}2+)/E({}^{1}2+)$ for various values of Γ. The experimental values are also presented. The $\Gamma = 1$ curve corresponds to the prediction of the asymmetric rotor model.

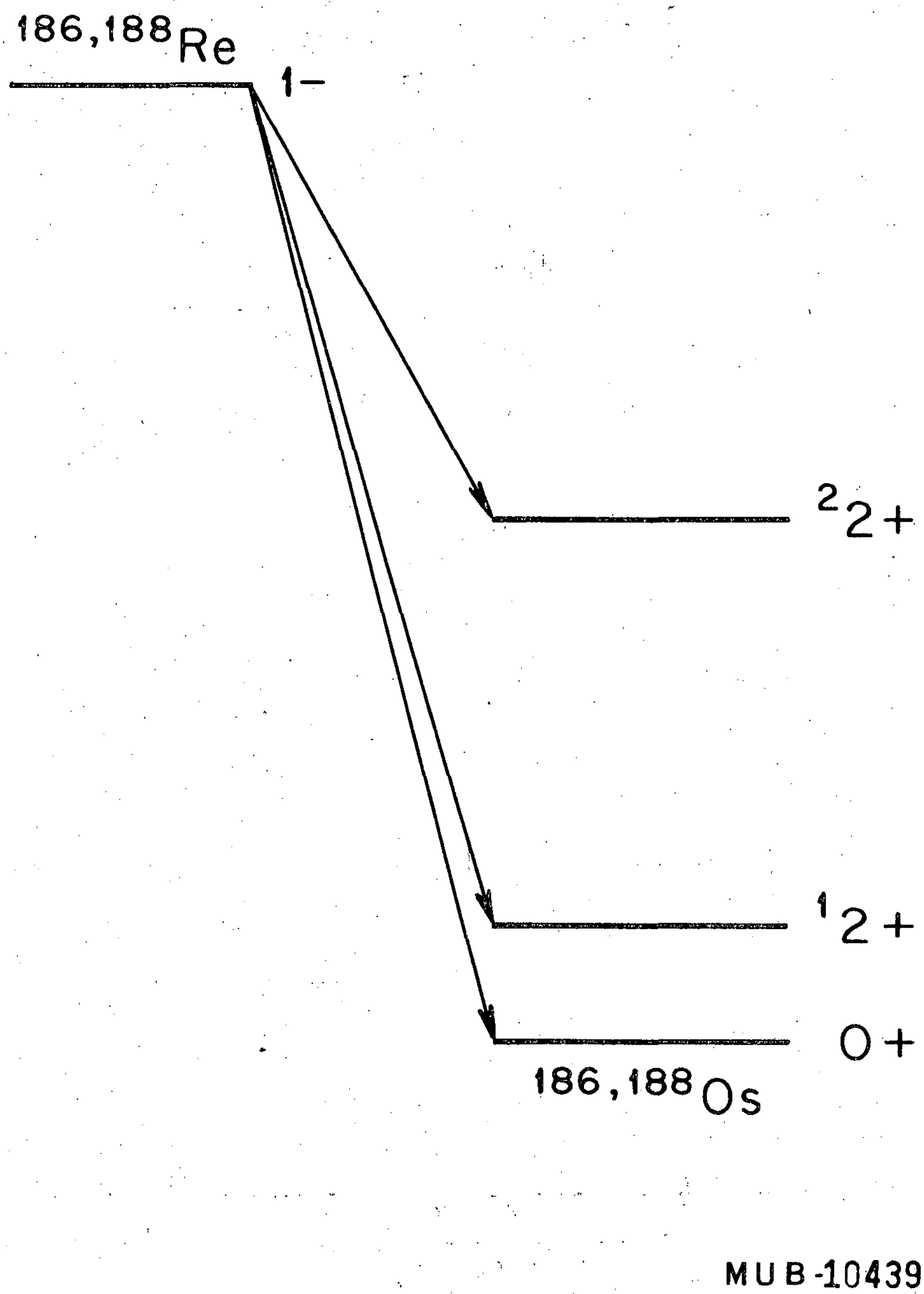

Fig. 1

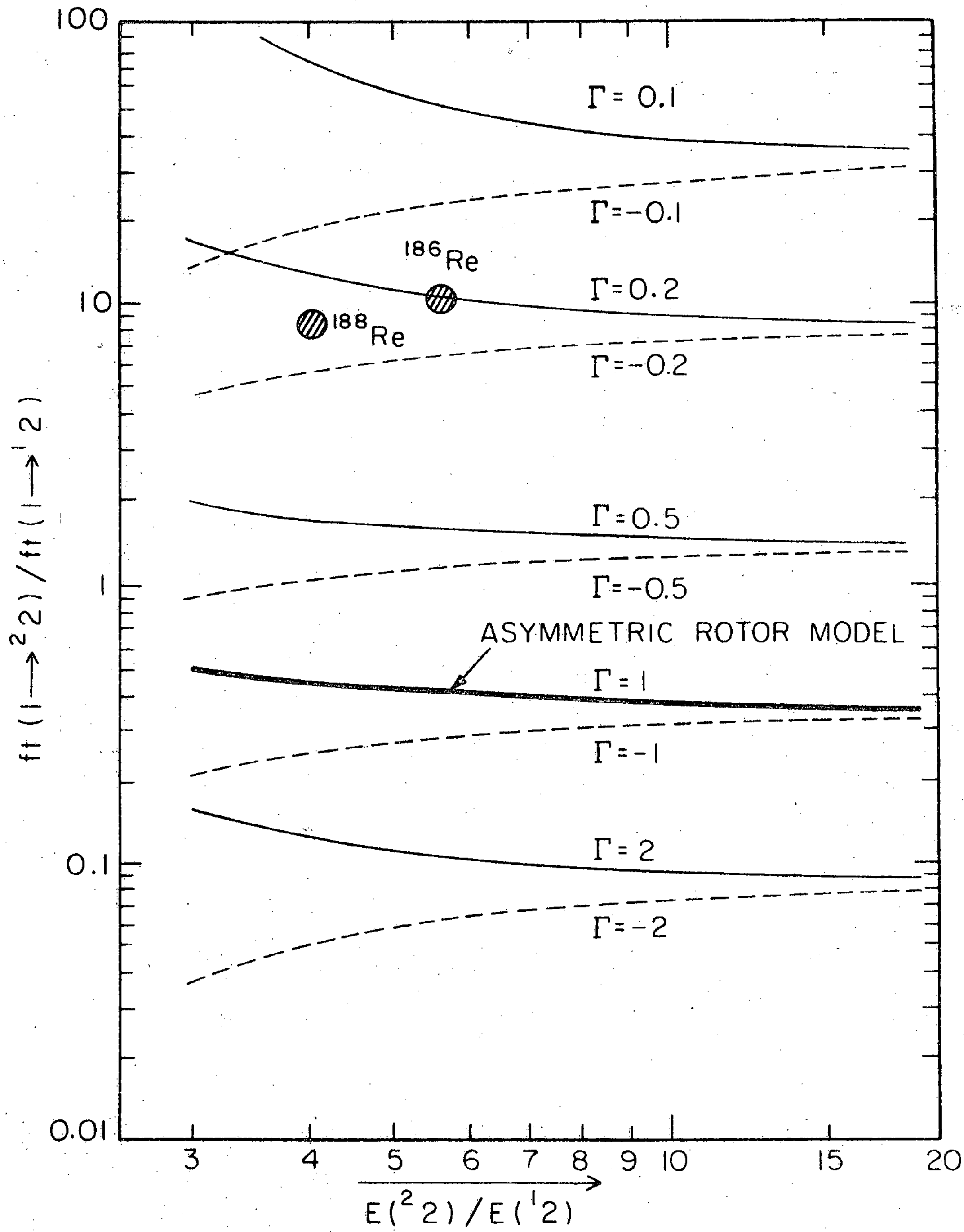

Fig. 2

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