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ABSTRACT

Calculations of static I_o-V_o curves, and of the heights of steps induced by rf radiation, have been carried out for current biased Josephson junctions with both a phase-independent quasiparticle conductance G_o and a phase-dependent quasiparticle pair "interference" conductance $G_1 \cos\phi$. Little or no effect of the term in $G_1 \cos\phi$ is found for junctions with zero capacitance. It thus appears that this term can be neglected in many practical applications of weak links. Substantial effects are found for the case of finite junction capacitance. The term in $G_1 \cos\phi$ then enhances the hysteresis of the static I_o-V_o curve by an amount which roughly corresponds to a factor of 2-5 increase in capacitance, and causes the static voltage to increase above the value expected from the shunt conductance alone. The most easily observable effect of the term in $G_1 \cos\phi$ on the rf induced step heights is a pronounced shift in the values of rf current for which maxima occur for low values of normalized rf frequency.

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According to the original work of Josephson[1] as developed by Josephson[2,3] and by Nam,[4] the current in a superconducting tunnel junction is given by

$$I(t) = I_c \sin\phi(t) + [G_o(V) + G_1(V) \cos\phi(t)]V(t). \tag{1}$$

The first term on the right is the usual phase dependent Josephson pair tunneling current. The second term, G_oV is the dissipative quasiparticle tunneling current neglecting coherence.[2] The third term, which is both dissipative and phase dependent, has been described[5,6] as an interference term between the pair and quasiparticle currents when the effects of coherence on the quasiparticle distribution are included.[7] Because the quasiparticle current in a tunnel junction biased well below the gap is small, the interference term has usually been neglected. Recent experiments on the Josephson plasma resonance in Pb-Pb tunnel junctions by Pedersen et al.[6] have shown evidence for the existence of the term in $G_1\cos\phi$ and obtained a value for the ratio $\gamma = G_1/G_o$ of -0.9 ± 0.2. A subsequent calculation by Poulsen[5] from the microscopic theory of tunnel junctions has shown that, for small voltages, the ratio is essentially independent of V, and is equal to −0.93 for the junctions used by Pedersen et al.

It is of interest to explore whether a term in $G_1\cos\phi$ with γ of this order is present in the proper description of weak links other than tunnel junctions. Since the shunt conductance G_o is large in point contacts, Dayem bridges, and proximity effect bridges, the existence of such a term might be of considerable practical importance for device applications. The theory of such weak links is not developed sufficiently to answer this question reliably. In order to obtain an experimental answer, we have calculated such experimentally observable quantities as the static I_o-V_o characteristic and the height of rf induced steps in the presence of voltage-independent G_1 and G_o with a constant negative ratio. In the absence of a shunt capacitance, little or no effect due to the $\gamma\, G_o V \cos\phi$ term is obtained. When nonzero capacitance is introduced, observable effects are predicted. None of the predicted effects, however, are dramatic enough for the existence of the term in $\gamma\, G_o \cos\phi$ to be verified from published data.

We introduce the junction shunt capacitance into Eq. (1) in the usual way,[8,9] letting $I = C\frac{dV}{dt}$ for the capacitor, and replacing V with $\frac{\hbar}{2e}\frac{d\phi}{dt}$. Using convenient dimensionless units[8] we obtain, for the time evolution of the phase in a junction biased with a constant I_o,

$$\beta_c \frac{d^2\phi}{d\tau^2} = i_o - \sin\phi - \frac{d\phi}{d\tau}(1+\gamma \cos\phi), \tag{2}$$

where $i_o \equiv I_o/I_c$, $\tau \equiv 2e\, I_c t/\hbar G_o$, and $\beta_c = 2eI_c\, C/\hbar G_o^{\,2}$. Taking G_o and γ to be constant we have integrated Eq. (2) numerically to find the periodicity of $d\phi/d\tau$ and used this to determine the dependence of the time averaged dc voltage $V_o \equiv \frac{I_c}{G_o}\langle\frac{d\phi}{d\tau}\rangle_\tau$ across the junction on the applied dc current I_o for given values of γ and β_c.

Static I_o-V_o characteristics corresponding to earlier calculations for $\gamma = 0$[8,9] and also for $\gamma = -0.95$ are shown in Fig. 1 for several values of the capacitance parameter β_c. These static characteristics are independent of γ for zero junction capacitance, as can be shown by analytic integration of Eq. (2) with $\beta_c = 0$.[10] For a given $\beta_c > 0$, and a given bias current, the dc voltage increases as γ goes from 0 to -0.95. Note that for β_c and γ both nonzero, the I_o-V_o characteristic drops below the asymptotic line $I_o = V_o G_o$. This corresponds to a power dissipation greater than I_o^2/G_o over a wide range of current. The voltage at which $I_o = V_o G_o$ scales as $\beta_c^{-1/2}$ for $\beta_c \gtrsim 0.1$. Similar calculations for $\gamma = -0.5$ show smaller deviations, with the crossover moving towards higher currents.[11]

The most pronounced effect of including the $\gamma\, G_o \cos\phi$ conductance is the increase in the hysteresis of the I_o-V_o curve at nonzero β_c. Defining a hysteresis parameter $\alpha \equiv i_{min}/i_o$ in the usual way,[8,9] we plot β_c vs α in Fig. 2 for $\gamma = 0$, -0.5 and -0.95. For purposes of comparison, we have also shifted the curve for $\gamma = 0$ down by an amount corresponding to a reduction in β_c by a factor 0.45 to show that the shape of the curve is most strongly affected in the range of small hysteresis ($\alpha \sim 1$).

That the $\beta_c^{-1/2}$ scaling law is an inherent feature of the model is most easily shown from the definition of β_c;

$$\beta_c \equiv \frac{2e\, I_c}{\hbar C} \left(\frac{C}{G_o}\right)^2 = \omega_p^{\,2}\, \tau_o^{\,2} \qquad (3)$$

where $\omega_p = (2e\, I_c/\hbar C)^{1/2}$ is the zero-phase plasma frequency and $\tau_o = C/G_o$ is the time constant of the junction. When $\omega_p \tau_o \gtrsim 1$ (large β_c) the junction is lightly damped in the sense that the phase evolution is governed by the plasma frequency ω_p and the junction time constant can be neglected; in this regime the modulation of the phase during a cycle (and consequently the deviation of the I_o-V_o

characteristic from the asymptote) is governed solely by γ and the dimensionless ratio $\langle d\phi/dt \rangle / \omega_p = (V_o G_o / I_c)\beta_c^{1/2}$. Since a given feature of the I_o-V_o characteristic, such as crossing the asymptotic conductance, will occur at the same value of $\langle d\phi/dt\rangle/\omega_p$ for all junctions having the same γ and $\beta_c \gtrsim 1$, the voltage V_x at which such a feature occurs will scale as $V_x = \beta_c^{-1/2}\, I_c\, G_o^{-1}[\langle d\phi/dt \rangle / \omega_p]_x = \text{constant} \times \beta_c^{-1/2}$. When the junction is heavily damped, $\beta_c << 1$, the phase response will be governed by the junction time constant τ_o and the scaling law breaks down. Our calculations show that $\beta_c < 0.1$ is necessary before deviations from the $\beta_c^{-1/2}$ law become significant.

The deviation of the static I_o-V_o curves for nonzero γ can be most easily understood by examining Fig. 3, in which we plot a single period of the time evolution of the phase for various values of γ and β_c at a fixed value of $i_o = 1.2$. First consider the case $\gamma = \beta_c = 0$. For ϕ near $\pi/2$, the bias current flows primarily through the superconducting junction, so there is little dissipation in the shunt resistor. When ϕ is near $3\,\pi/2$, the junction current is opposite to the bias current, so the current (and dissipation) in the shunt resistor is large. As the phase spends more time near $\pi/2$ during a cycle than near $3\,\pi/2$, the average dissipation is less than the value I_o^2/G_o for the shunt alone. This corresponds to a time-averaged forward supercurrent, and an average voltage $V_o < I_o/G_o$.

As β_c is increased for $\gamma = 0$, the phase-time characteristics change as shown in Fig. 3(a). The inertial effect of the capacitance smooths out the ac supercurrent, equalizing the time spent near $\pi/2$ and

$3\pi/2$. As $\beta_c \to \infty$ at constant i_o, the voltage approaches the value $V_o = I_o/G_o$ and the time-averaged supercurrent vanishes.

When we choose $\gamma = -0.95$, we obtain the phase-time characteristics shown in Fig. 3(b). A careful comparison of Fig. 3(a) with Fig. 3(b) shows that the ratio of the time spent near $\pi/2$ to that spent near $3\pi/2$ is independent of γ for $\beta_c = 0$. As we increase β_c from zero, the decreased conductance near $\phi = 0$ shortens the junction time constant and decreases the time spent near $\phi = \pi/2$ while the increased time constant near $\phi = \pi$ increases the time spent near $\phi = 3\pi/2$. There is a regime where the phase actually spends more time near $3\pi/2$ than near $\pi/2$, giving a time-averaged reverse supercurrent, so that the dissipation and the time-averaged voltage are greater than for the conductance alone; $V_o > I_o/G_o$. As $\beta_c \to \infty$ the capacitor again smooths out the time evolution of the phase so that the I_o-V_o characteristic approaches the asymptotic value, $V_o = I_o/G_o$.

Because of the present interest in ac Josephson effect devices, the possible influence of a nonzero γ on the ac response of various types of weak links is of practical importance. In order to explore this question we include an rf current source $i_{rf} \cos\Omega_{rf}\tau$ in the right-hand side of Eq. (2), where $\Omega_{rf} = \hbar\omega_{rf}G_o/2eI_c$ and $i_{rf} = I_{rf}/I_c$. As a measure of the rf response of the junction we have computed the dependence of the height of the radiation induced steps with n = 0, 1 and 2 on i_{rf}. For $\beta_c = 0$, no effects of nonzero γ were observed for values of reduced frequency $\Omega_{rf} \lesssim 0.16$. The effects of nonzero γ increase with Ω_{rf}, but remain quite small even for Ω_{rf} as large as 1.7.

They include a small shift in the positions of the minima, a lifting of the minima away from zero step height, and a small decrease in the height of the subsidiary maxima. All of these effects would be difficult to observe. Since junctions for applications generally have $\alpha = 1$, step heights were also computed for $\beta_c = 0.15$, the largest value giving no hysteresis. The largest effects then occur for small values of the product $\Omega_{rf}\beta_c = \omega_{rf}\, C/G_o$. Neither $\beta_c = 0.15$, nor $\gamma = -0.95$ alone has an appreciable effect on the step heights, but in combination they produce a marked shift in the positions of the subsidiary maxima, as shown in Fig. 4 for $\Omega_{rf} = 0.16$.

Even though a finite γ increases the harmonic content of the phase-time curve, the heights of integral order steps are not very sensitive to γ. For these steps, the rf period averages over n periods of the phase oscillation. Although we have not explored the γ-dependence of subharmonic steps, these might have a different sensitivity, as several rf cycles occur during one phase period.

We conclude that a phase dependent quasiparticle-pair conductance term $\gamma G_o \cos\phi$ produces potentially observable effects on the static $I_o - V_o$ curves and the radiation induced steps of Josephson junctions shunted by both resistance and capacitance. In no case, however, are the effects large enough to be recognized in published data on weak links, or to require the inclusion of this term in an equivalent circuit used for device design.

It will, in fact, be difficult to establish unambiguously the existance of the $\gamma G_o \cos\phi$ conductance by direct comparison with $I_o - V_o$ curves for any type of junction. As G_o can only strictly be considered constant near zero bias, unshunted tunnel junctions will have a very small G_o value. Externally shunted junctions are obviously unsuitable, while for the several extant types of weak link the need for simultaneous determination of β_c, G_o, and γ for comparison with Fig. 1 makes such a comparison highly unlikely. Even with optimal values of applied rf fields, the shifts in the amplitudes and positions of the rf steps are presently smaller than the accuracy with which the induced rf currents can be estimated. We do, however, note that the position of the first minima of the $n = 0$ step shown in Fig. 4 does not shift, while successive minima occur at progressively lower rf currents. Thus an investigation of the relative positions of the successive minima of the $n = 0$ step as a function of applied rf power appears to be the most promising and sensitive method for investigating the behavior of the $\gamma G_o \cos\phi$ term in weak link devices.

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FIGURE CAPTIONS

Fig. 1. Normalized static I_o-V_o curves for various values of the junction capacitance parameter β_c and pair quasiparticle interference parameter, γ, in the absence of rf fields.

Fig. 2. Plot of the junction capacitance parameter β_c against the junction hysteresis parameter α for various values of the pair-quasiparticle interference conductance $\gamma G_o \cos\phi$. The dashed line is the $\gamma = 0$ curve shifted downward by an amount corresponding to a reduction in β_c by a factor of 0.45.

Fig. 3. Influence of a finite pair-quasiparticle interference conductance $\gamma G_o \cos\phi$ on the time evolution of the junction phase ϕ. (a) With zero interference current ($\gamma = 0$), at various values of β_c. (b) With finite interference current ($\gamma = -0.95$), at the same values of β_c.

Fig. 4. Plot of the height of the rf-induced dc current steps for n = 0, 1, and 2 as a function of the rf current at the reduced frequency $\Omega_{rf} = 0.16$. The locations of the subsidiary maxima are shifted by the combined effects of a small junction capacitance $\beta_c = 0.15$ and a finite quasiparticle-pair interference term $\gamma = -0.95$.

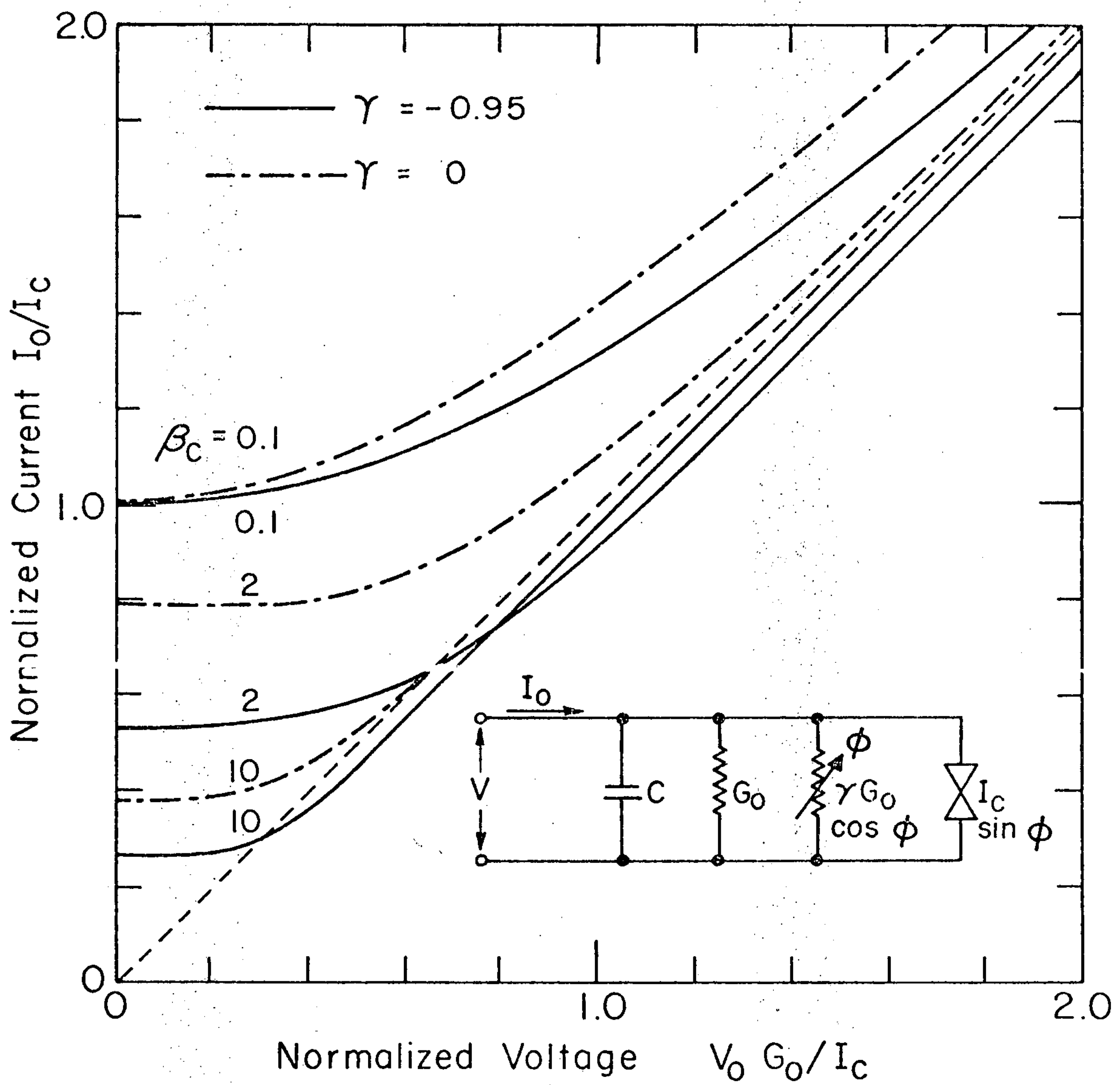

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Fig. 1

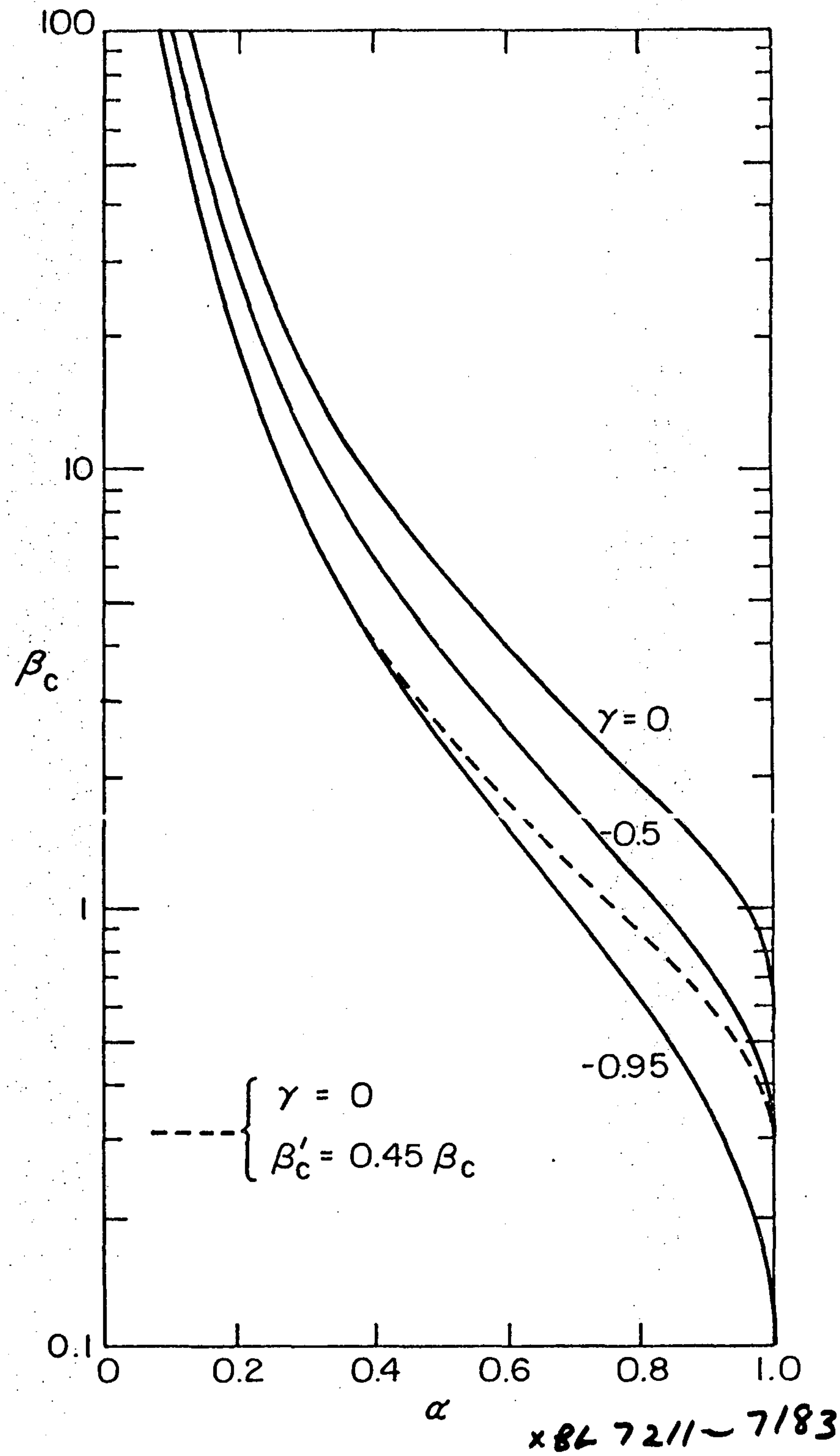
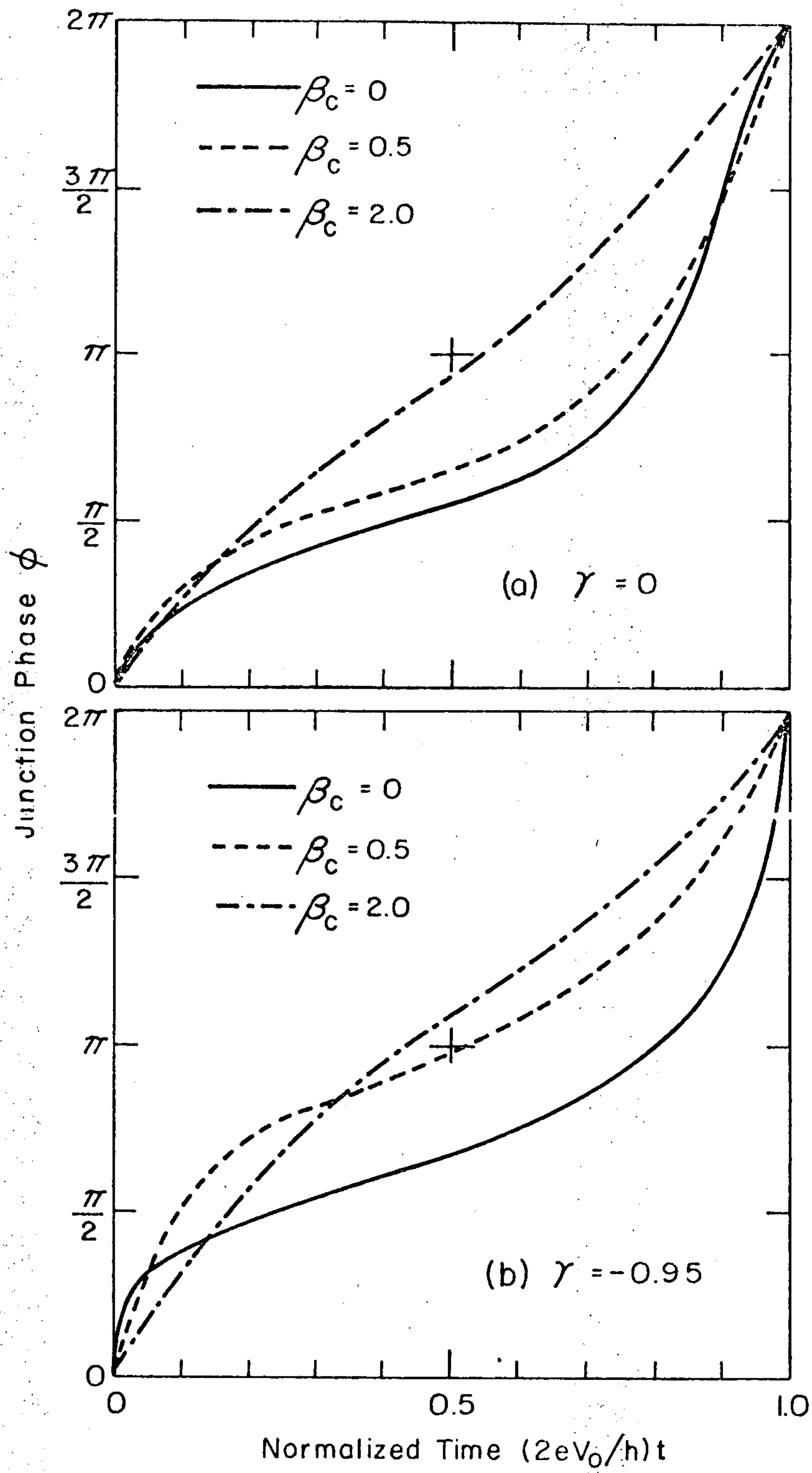

100
10
1
0.1
β_c
0
0.2
0.4
0.6
0.8
1.0
α
γ = 0
-0.5
-0.95
γ = 0
β_c' = 0.45 β_c
XBL 7211-7183

Fig. 2



XBL-7211-7184

Fig. 3

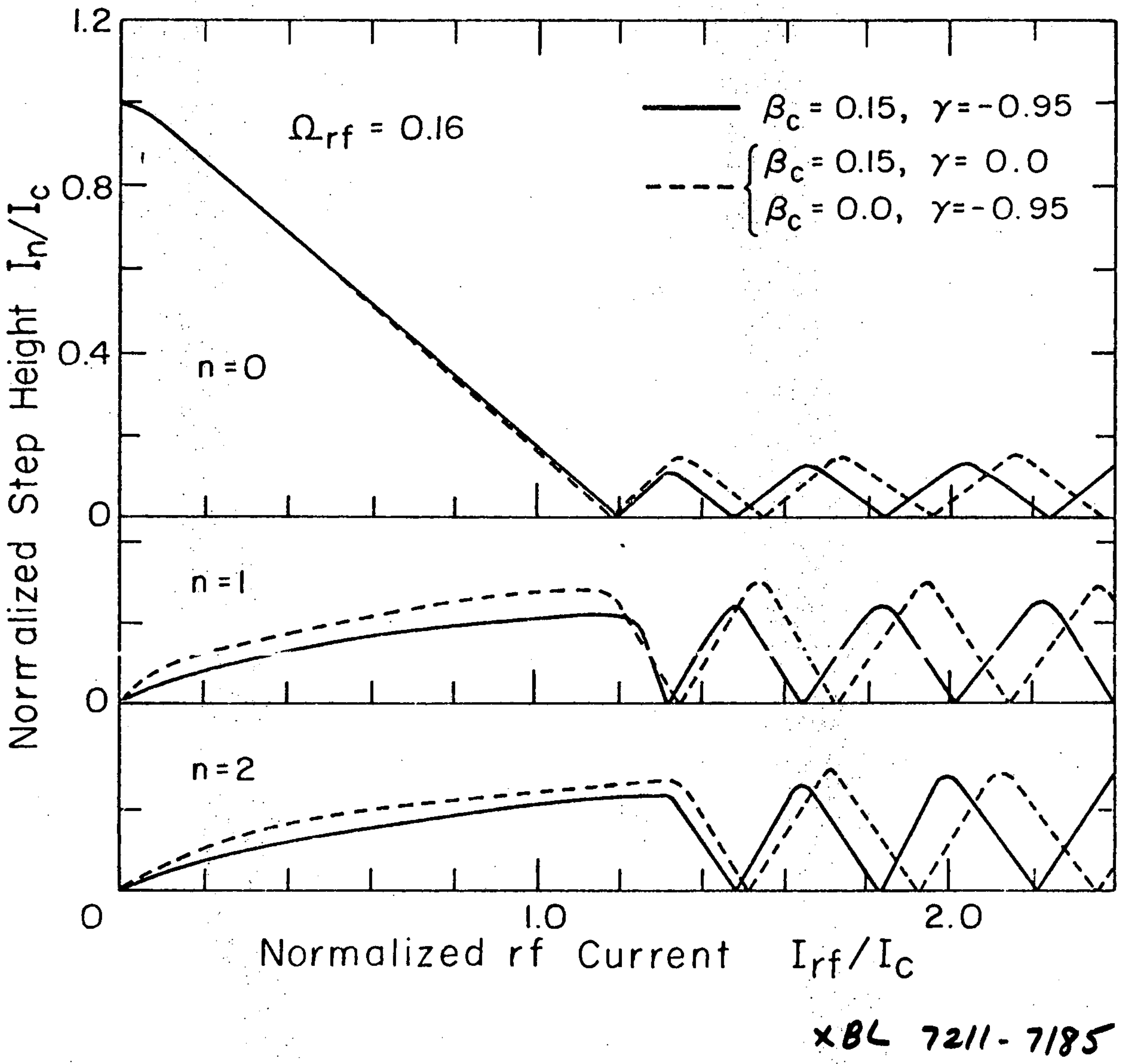

1.2
0.8
0.4
0
Normalized Step Height I_n/I_c
Ω_{rf} = 0.16
β_c = 0.15, γ = -0.95
β_c = 0.15, γ = 0.0
β_c = 0.0, γ = -0.95
n = 0
n = 1
n = 2
0
1.0
2.0
Normalized rf Current I_{rf}/I_c
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Fig. 4

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