Reply to comment by Maier and Kocabas on "A closed-form analytical solution for thermal single-well injection-withdrawal tests"

Yoojin Jung and Karsten Pruess

Earth Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

1. Introduction

We appreciate the comment by Maier and Kocabas (2012) on our research article (Jung and Pruess, 2012). In their comment, they raise the following issues: (1) their solutions presented in Kocabas (2010) and Maier and Kocabas (2012) are mathematically simpler and computationally more efficient than our analytical solutions and (2) the insensitivity of thermal breakthrough curve to the flow velocity, which is one of the important conclusions of our study, only holds for the special case where the injection and the withdrawal flow rate is identical. We address each of these comments, along with a few other relatively minor comments and suggestions, below.

2.1. Efficiency of the Analytical Solution

Regarding the computational efficiency of the analytical solutions, we agree that the solutions developed by Kocabas (2010) and Maier and Kocabas (2012) using the iterated Laplace transform have a simpler form than our solutions and therefore need a shorter computation time. For a simple single-well injection-withdrawal (SWIW) test with no quiescent time, the solution presented in Kocabas (2010) or Maier and Kocabas (2012) can be used to reduce computation time, and, as stated in Maier and Kocabas (2012), it would particularly be useful for parameter estimation problems in which repeated calculations are required. However, our analytical solution and the method used for its derivation bring several unique advantages that may have been underemphasized in our research article (Jung and Pruess, 2012) and that are not attained through the simpler solution form.

First of all, the analytical solution for thermal SWIW tests involving injection, quiescent, and withdrawal phases has successfully been developed only because of the unique approach used for its derivation. Because the initial conditions at the beginning of each phase are not solved to find the transform during the derivation process, the derivation of the analytical solution for the withdrawal period becomes identical for both thermal SWIW tests with and without a shut-in period. All that is needed is to derive the solution for the quiescent period and to use the temperature distribution at the end of the quiescent period as the initial condition of the withdrawal period, in the case of thermal SWIW tests including a quiescent period. This implies that our solution is highly flexible to implement various injection schemes, including multiple shut-in periods. Second, our analytical solution not only predicts temperature return curves at the injection/withdrawal location under the given conditions but also provides a comprehensive understanding of heat transfer in fractured rocks. The first and second terms in the final form of the analytical solution (15) shown in Jung and Pruess (2012) account for the integrated effects of the initial temperature within the fracture at the beginning of the withdrawal period and the heat flux at the fracture-matrix interfaces over time and space, respectively. Therefore, the impact of convective heat transfer within the fracture and conductive heat transfer in the adjacent rock matrix on return temperatures can be distinguished.

Moreover, our solution can compute temperature changes at any location in the entire domain of interest, which includes both fracture and matrix. While in most practical cases the injection/withdrawal well is the only location at which temperature changes can be monitored, it would immensely be useful to have a complete analytical solution that is capable of explaining sensitivity (or insensitivity) of thermal SWIW tests based on the sound knowledge of the temporal and spatial temperature distribution during the tests.

2.2. Insensitivity of Return Temperatures to the Flow Velocity

Concerning our statement that the variation of fluid return temperatures with time is independent on the flow velocity, we wish to clarify several points. As pointed out by Maier and Kocabas (2012), the pumping rate directly controls the flow velocity in the fracture, and therefore affects temperature return curves. When the ratio of the injection to the withdrawal flow rate is λ, similar to the approach in Kocabas (2010), the analytical solution (15) in Jung and Pruess (2012) can be readily revised as follows:

$$T_{f3D}(x_D,t_{3D}) = \int_0^{\min(\frac{t_{3D}}{\lambda},t_{Dq}-x_D)} \frac{\sqrt{\theta}\lambda^2\xi}{2} \frac{\exp\left(-\frac{\theta\lambda^2\xi^2}{4(t_{3D}-\lambda\xi)}\right)}{\sqrt{\pi(t_{3D}-\lambda\xi)^3}} \cdot T_{f3D}(x_D+\xi,0)\,\mathrm{d}\xi$$
$$+\int_0^{t_{3D}}\int_0^{\min(\frac{\tau}{\lambda},t_{Dq}-x_D)} \frac{\sqrt{\theta}\lambda^2\xi}{2} \frac{\exp\left(-\frac{\theta\lambda^2\xi^2}{4(\tau-\lambda\xi)}\right)}{\sqrt{\pi(\tau-\lambda\xi)^3}} \cdot \theta\int_0^{\infty} T_{m3D}(x_D+\xi,\eta,0)\cdot\frac{\sqrt{\theta}\eta}{2} \tag{1}$$
$$\times\frac{\exp\left(-\frac{\theta\eta^2}{4(t_{3D}-\tau)}\right)}{\sqrt{\pi(t_{3D}-\tau)^3}}\,\mathrm{d}\eta\,\mathrm{d}\xi\,\mathrm{d}\tau$$

Or more simply, taking advantage of the form of our final solution (15) in Jung and Pruess (2012), only the initial conditions included in the solution (15) need to be rewritten as $T_{f3D}(x_D+\xi/\lambda,0)$ and $T_{m3D}(x_D+\xi/\lambda,\eta,0)$ to reflect the difference in the injection and the withdrawal flow rate. As demonstrated mathematically, the temperature recovery is indeed dependent on the ratio of the flow rate (or the flow velocity) between the injection and the withdrawal period (see that λ is an independent parameter in the revised solution (1) above). However, the velocity itself still only appears in the dimensionless distance x_D, thus it does not appear explicitly in the final solution (1) for the injection/withdrawal well, for which x_D = 0. Therefore, the temperature recovery at the injection/withdrawal well will be identical for thermal SWIW tests with different flow velocities as long as λ is the same for each test.

This finding has significant implications for the application of thermal SWIW tests. One of the important goals sought from stimulation treatments for development of enhanced geothermal systems (EGS) is enhancing the fracture-matrix interface area available for heat transfer to the injected fluid (e.g., increasing the fracture height, which is equal to fracture-matrix interface areas per unit length, and acquiring access to additional fractures) in order to improve the rate of heat extraction from the reservoir rock. For a fixed injection/withdrawal rate, this change will reduce the flow velocity in the fracture, but have no effect on temperature recovery at the injection/withdrawal location. Even if the flow rate during the withdrawal period differs from that during the injection period (e.g., $\lambda \neq 1$), the return temperature profiles for thermal SWIW tests conducted before and after stimulation treatments will be identical as long as the SWIW tests are repeated with the same injection/withdrawal flow rate ratio λ. The insensitivity of thermal SWIW tests to the injection/withdrawal flow velocity renders it impossible to estimate the effectiveness of stimulation treatments for increasing the fracture-matrix interface area, but is advantageous for evaluating the thermal diffusivity of the rock matrix, since the influence of advective heterogeneity can be disregarded.

Another point claimed by Maier and Kocabas (2012) is that the value λ can account for changes in the fracture geometry during the runtime of SWIW tests. This statement may be true if all the properties of the fractured rocks, including the fracture geometry and the thermal diffusivity of the matrix, are known before thermal SWIW tests. Or if a monitoring well located along the flow path and sufficiently close to the injection well is available, additional information on changes in the fracture geometry may be obtained from temperatures measured at the monitoring well. However, this situation is not likely for most applications, and therefore it is probably unrealistic to expect λ to provide information about changes in the fracture geometry occurring during a SWIW test.

Finally, in addition to the comments on the insensitivity of return temperatures to the flow velocity, Maier and Kocabas (2012) assert that thermal breakthrough curves can be better interpreted when the pumping rate is smaller than the injection rate applied, with which we do agree. For simplicity, if we disregard the effect of conductive heat transfer from the rock matrix, it will take $\lambda \cdot t_{Di}$ to completely recover the cold water injected, where t_{Di} is the dimensionless total injection time. That is, when a lower pumping rate is applied (e.g., $\lambda > 1$), time-dependent temperature changes at the injection/withdrawal well during the withdrawal period can be measured for a longer time. Figures 1a and 1b show the temperature return curves for $\lambda = 2$ and $\lambda = 0.5$, respectively, where the parameter values in Table 1 in Jung and Pruess (2012) are used for computation. As the ratio is increased, the differences between the temperature-return profiles for different half fracture apertures *b* become more distinct. However, it should be noted that the difference is not as distinct as that shown in Figure 3 in Maier and Kocabas (2012). This discrepancy is due to the difference of the range of the dimensionless parameter α, which is defined in Eq. (5) in Maier and Kocabas (2012). If calculated using the parameter values for Figure 1, α ranges approximately from 14 to 34000. On the other hand, in Maier and Kocabas (2012), the highest value is 10, and the lowest is 0.01. The range used in Maier and Kocabas (2012) is in fact more appropriate for SWIW tests using solute tracers than thermal SWIW tests, since solute diffusivities are three orders of magnitude smaller than typical thermal diffusivities.

References

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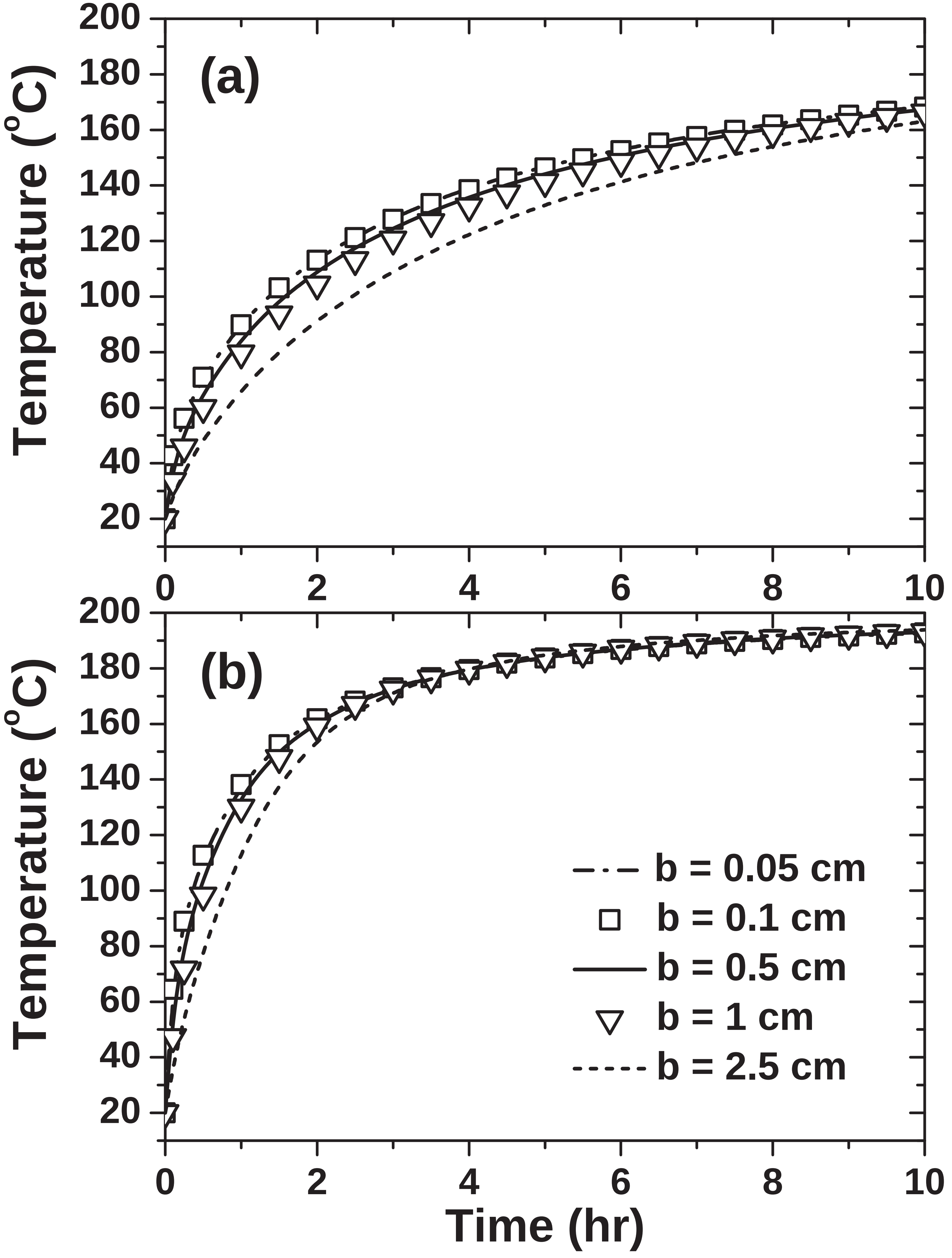
Figure 1. Temperature return profiles (a) for the ratio of the injection to the withdrawal flow rate $\lambda = 2$ and (b) for $\lambda = 0.5$. b is the half fracture aperture.



(a)
(b)
Temperature (°C)
Time (hr)
b = 0.05 cm
b = 0.1 cm
b = 0.5 cm
b = 1 cm
b = 2.5 cm