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EFFICIENT DIVISION OF PROFITS
FROM COMPLEMENTARY INNOVATIONS

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Abstract

Many products—including microprocessors, telecommunications devices, computer software and on-line auction services—make use of multiple technologies, each of which is essential to make or sell the product. The owner of one technology benefits from the existence of complementary technologies. We show that, despite this externality, the structure of payoffs that support efficient R&D investment by duopolists racing to discover a single innovation generalizes to the structure that supports efficient investment for complementary innovations. The paper also examines how alternative intellectual property regimes and legal institutions affect R&D investment in complementary technologies. The results have policy implications for the organization of R&D, the assessment of damages for patent infringement, and allocations of value in patent pools.

Keywords: Patents, innovation, complements, competition policy, oligopoly, licensing.

JEL Classification: L1, L13, L24

I. INTRODUCTION

Many technologies are valuable only when used together, and a firm may be unable lawfully to offer the product enabled by these technologies unless it obtains rights to utilize all of them. The economics literature dealing with complementary intellectual property rights has focused on the potential for double-marginalization to cause inefficient pricing, also called "royalty-stacking".[2] Another question, which is the focus of the present paper, concerns the determination of efficient incentives for research and development to produce complementary intellectual property. How should the value of a product be allocated to owners of intellectual property embodied in the product in order to provide efficient incentives to create the intellectual property in the first place?

The allocation of value can arise in different policy contexts. One key question is the determination of damages for patent infringement when one or a few of many complementary patents are infringed. An example of the tension in the calculation of damages for patent infringement with complementary innovations is the litigation between Eolas Technologies and Microsoft. In 2003 Microsoft was ordered to pay $521 million to Eolas Technologies for infringing one of its patents related to Internet browsing.[3] At the time of its infringement suit, Eolas held only this single patent related to Internet browsing and had no other products. It is questionable whether $521 million for a single patent is a proportional share of the value of all

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[2]   Cournot (1927) first identified the problem of double-marginalization with complements. See also Shapiro (2002), Gilbert (2004), and Lemley and Shapiro (2007).

[3]   Eolas Technologies Incorporated, and The Regents Of The University Of California v. Microsoft Corporation, U.S. District Court For The Northern District Of Illinois, Eastern Division, January 14, 2004. Eolas Technologies was the exclusive licensee of patent number 5,838,906, which was assigned by the inventors to their employer, the University of California. The Court of Appeals for the Federal Circuit vacated the lower court decision for technical reasons. Eolas Technologies and Microsoft subsequently settled the case for an undisclosed amount.

intellectual, physical, and human capital related to Internet browsing. Microsoft alone has more than 10,000 patents, of which at least 500 relate specifically to the Internet.[4] But it is equally questionable whether a proportional share is the correct basis for allocating the value of the Internet.

When a firm owns a single patent that is essential to make or sell a product, that firm arguably has as large a claim on the value the product as does any other patent holder, regardless of the number of patents that the other owner may control. Indeed, when patents are known to be valid and infringed, the patents have no stand-alone value, and a patent holder can obtain injunctive relief to block any activity that infringes its patent, Nash bargaining suggests that the allocation of value corresponds to the number of owners of intellectual property rights rather than to the number of patents that each firm owns. If there are two firms that own the patent rights that cover a product, then under Nash bargaining each firm should claim one-half of the value, even if one firm has only a single patent and the other firm has ten. But does such an allocation of value provide efficient incentives to create intellectual property?

The potential imbalance between the bargaining power of a patent holder and the contribution of the patent to the value of complementary innovations has led Congress to consider legislation that would provide guidance to courts that adjudicate patent infringement damages. The House of Representatives passed HR1908 in 2007, which would require courts to apportion damages for patent infringement when the infringing product has many sources of

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[4]   Based on a January 19, 2009 search of the U.S. Patent and Trademark Office database for patents with claims that include the Internet or browsing.

value other than the infringed intellectual property.[5] The Senate considered similar legislation in 2007 and 2008, although those bills were not enacted into law.[6]

Rules that govern the determination of damages have a primary role in the negotiation of patent terms, as the recourse to a negotiation over royalties is to seek compensation in a court of law (see, *e.g.,* Choi, 2003). It is thus important to ask whether applicable law encourages courts to assess infringement damages that provide efficient incentives for firms to invest in R&D. This is particularly important for information technology, biotech, and other industries, because the scope for disagreements over the appropriate level of damages is very large in these industries in which there are many complementary innovations.

The analysis of the present paper provides an important first step in addressing these issues. After a review of the relevant literature in Section II, we begin in Section III by studying the efficient level of investment in R&D when many discoveries are essential to produce a commercial product. Against this benchmark, we develop a formal analysis of a market in which two firms conduct R&D to develop the essential technologies. To sharpen our focus we examine the polar case of perfect complements, in which each technology is as important as any other. We make several assumptions, which we retain throughout this paper to simplify the analysis and to focus on the central question of efficient incentives for R&D. We assume that each discovery results in a valid patent and none of the essential technologies has a use other than for

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[5]   H.R. 1908, Patent Reform Act of 2007. Available at
        http://www.govtrack.us/congress/bill.xpd?bill=h110-1908, accessed January 13, 2009.

[6]   Senate bill S. 1145: The Patent Reform Act of 2007 included language similar to H.R. 1908 on
        the apportionment of damages. The Senate also considered, but did not pass, patent reform
        legislation in 2008. The proposed 2008 bill did not specifically allow for the apportionment of
        damages but instead emphasized the role of hypothetical negotiation in the determination of a
        reasonable royalty. See S. 3600: The Patent Reform Act of 2008, available at
        http://www.govtrack.us/congress/billtext.xpd?bill=s110-3600, accessed January 13, 2009.

the single product. Hence, no technology is inherently more valuable than another. We also

assume that users derive the same value from the final product that employs the technologies and

licensors charge fixed fees. This assumption implies that there is an available present-value

profit, π, from the final product and allows us to ignore the separate issue of royalty stacking for

complementary innovations.

It is well known that attempts by firms to preempt competitors in a patent race can lead to

socially excessive investment in R&D when the profit from invention is a large fraction of the

social value of the invention, because firms do not internalize the business-stealing effect of

winning the patent race.[7] But the preemption intuition does not obviously extend to competition

with complementary innovations. The reason is that competitors have shared interests in

discoveries. Assuming no intermediate payoffs, there is no profit available to any firm until all

of the essential discoveries have been made.

With complementary innovations, each firm engaged in R&D fails to account for the

positive benefit a discovery has on the value of technologies controlled by other innovators (or

on consumer surplus). This potential externality suggests too little incentive to invest in R&D

for a technology that complements other technologies, just as the double-marginalization

problem with complementary products causes competing firms to choose excessive prices. The

preemption incentive acts in the other direction. Without further analysis, one might expect that

the effects of un-internalized complementarities and business stealing depend on the number of

technologies that must be discovered and have indeterminate effects on the bias in private

investment incentives. Nonetheless, a striking result is that under our assumptions the optimal

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[7]   See, for example, the analyses of patent races surveyed by Reinganum (1989).

reward policy is independent of the number of discoveries that are essential to produce a valuable product.

Section IV considers a duopoly game in which firms invest to discover complementary technologies and derives the payoffs that support efficient investment in R&D. The first case considered corresponds to the states in which only one discovery remains to produce a commercial product. We extend this result to prove that the payoffs that provide efficient incentives for investment in R&D with one remaining discovery also provide efficient incentives for R&D with any number of remaining discoveries. We also show that a payoff regime that allocates the value of invention equally to all patent-holders, along with an appropriately chosen tax, supports efficient investment in R&D for any number of complementary patents.

The reward schemes in Section IV support efficient investment in R&D, but do not necessarily correspond to outcomes in market environments. Nonetheless, the form of the reward function that provides efficient incentives for investment in R&D can provide guidance to Congress and the courts in developing rules for awarding damages in patent and copyright infringement litigation, and to private organizations such as patent pools, which must develop procedures to allocate licensing royalties among pool members.

Our analysis provides insight into the effects of payoff schemes observed in practice. For instance, some patent pools split licensing revenues equally among members of the pool (Layne-Farrar and Lerner (2006)). Our findings suggest that this scheme can generate excessive incentives for R&D if the pool chooses a profit-maximizing royalty and if the optimal tax rate

derived in Section IV is strictly positive.[8] Other payoff schemes correspond to the existing

intellectual property rights regime in which a patent owner can obtain an injunction to bar the use

of its patented intellectual property without a license.[9] When the patented technology is

essential, such an injunction blocks the sale of the final product.[10] Under Nash bargaining with

zero disagreement values for each patent if the licensing negotiations fail, firms will split the

value of the product equally as long as each firm has at least one essential patent.

The resulting equal-profit-share-per-innovator regime generally does not generate

efficient incentives for R&D. If each firm holds at least one essential patent, the incentive to

invest to discover another technology is low because of a free-riding problem. The incremental

return to another patent is zero, which generates zero incremental incentive for R&D, while all

patent holders would benefit equally from earlier discovery of the remaining essential

technologies. On the other hand, if a firm currently has no patents, its incentive to discover

another technology can be inefficiently large because, with two rights-holders, successful

discovery yields a reward of one-half of the final product's total value even when the technology

is only one of a very large number needed to produce the final product. Similarly, a firm that has

to date made all of the discoveries has a strong incentive to preempt a second firm from

successfully obtaining a patent because, if the second firm wins a patent, the incumbent's share

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[8] The governing members of some patent pools apparently are more concerned with technology adoption than with maximizing licensing revenues and choose correspondingly low royalties. The U.S. Department of Justice has commented on low royalties in several Business Review Letters. See, e.g., Letter from Joel I. Klein, Assistant Attorney General, U.S. Department of Justice, to Garrard R. Beeney, Sullivan & Cromwell (June 26, 1997).

[9] Lanjouw and Lerner (2001), Schankerman and Scotchmer (2001), and Farrell and Shapiro (2008) study the consequences of injunctive relief for the determination of licensing terms and patent infringement settlements.

[10] The litigation between NTP and RIM, for example, threatened to shut down RIM's widely used Blackberry service. Similarly, recent litigation between Verizon and Vonnage threatened to shut down Vonnage's voice over IP telephone service.

of the total product profits would fall from 100 percent to 50 percent. Due to these effects running in different directions, a reward regime of equal profit shares for each innovator can lead to too little or too much R&D investment.

II. RELATIONSHIP TO THE LITERATURE

Before presenting our analysis, it is useful to put it in context. Several authors have examined industry settings in which firms engage in a sequence of races to obtain the intellectual property rights to technologies that are non-infringing substitutes for one another.[11] In contrast, we are interested in complementary intellectual property in situations where any one intellectual property owner can block the sale of the final good embodying the complementary technologies. The analysis of the present paper is more closely related to studies of cumulative innovation, such as Green and Scotchmer (1995), Matutes et al. (1996), Scotchmer (1996), O'Donoghue (1998), and Denicolò (2000), which allow for infringing, complementary technologies. However, research on cumulative innovation has focused on the interdependence of the rewards for a basic invention, which has standalone value, and the rewards for complementary inventions that follow and build on the basic invention. In the present analysis there is no sense in which one of the inventions is basic and the others follow-on; all of the inventions are required to generate any value.

Like us, Lemley and Shapiro (2007) examine settings in which a single product may infringe several different patents. Their focus is on identifying the effects of injunctions and the judicial determination of "reasonable royalties" on the total licensing fees paid by product suppliers to rights-holders for use of their intellectual property. Our focus, in contrast, is on

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[11]   Examples include Reinganum (1985), Vickers (1986), Katz and Shapiro (1987), Choi (1991), and Doraszelski (2003).

characterizing the optimal division of profits among innovators in order to promote efficient R&D investment.

Our model of efficient investment falls in the general category of optimal contests. Several papers, including Anton and Yao (1992), Taylor (1995), Moldovanu and Sela (2001), and Che and Gale (2003), consider the design of contests to procure a costly innovation.[12] Gans (2001) explores payoff rules that support efficient duopoly investments in a competition to create regulated infrastructure. These papers capture some of the forces of R&D competition addressed in our analysis, but they lack the element of complementary innovation that is crucial to the problem that we study.

Stewart (1983) derives payoffs that support efficient investment for a single discovery assuming that firms choose the rate of investment at each point in time, patterned after Lee and Wilde (1980). Consistent with our model, Stewart (1983) shows that efficient investment can be sustained as a duopoly equilibrium with an appropriate division of the profit between the winner and the loser of the competition. Mortensen (1982) establishes general conditions under which an allocation of private values to competitors supports efficient investment in a class of dynamic games. Although neither Stewart (1983) nor Mortensen (1982) explicitly consider complementary discoveries, their results suggest that our general conclusions do not hinge on the specific assumed formulation of the R&D competition.

Lastly, Dequiedt and Versaevel (2006) analyze the effects of reward policies for a patent pool on incentives for firms to invest in R&D. Their paper is related to ours in that it studies the effects of patent rewards on R&D investment for complementary innovations. There are,

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[12] Others, such as Wright (1983) and Gallini and Scotchmer (2002), compare contests to alternative procurement mechanisms.

however, important differences. They study dynamic issues that can arise in the presence of

patent pools but do not arise in our model. Specifically, in their model, firms race to become one

of the founders of the pool and the threshold size of the pool affects the incentives to invest in

R&D. Dequiedt and Versaevel (2006) assume a particular reward structure and examine its

effects, but they do not characterize the optimal reward scheme for complementary innovations.

III. EFFICIENT INVESTMENT FOR COMPLEMENTARY INNOVATION

We are interested in situations where various technologies are worth more when used

together than when used separately. Formally, we examine the polar case of perfect

complements: there are L technologies that must be used together in order create either private or

social value. In what follows, π is the available present-value profit and w is the total welfare

derived from the final product that utilizes the L technologies.[13] To simplify the analysis, we

assume that there are no other sources of value, such as physical and human capital, other than

the L technologies that are required to produce the product.[14]

Assumption 1: *If all L technologies have been invented, they generate social value w and an*

available private value to the innovators, π, with $w > \pi$. If fewer than L technologies have been

invented, the social and private benefits are zero.

For each of the L technologies, firms choose how many R&D projects to undertake and

pay a lump sum of c for each project chosen, as in Loury (1979) and Dasgupta and Stiglitz

(1980). We assume a standard functional form for innovation:

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13   The gap between π and w could be surplus enjoyed by both consumers and producers of the final
      product if the latter are not themselves the innovators.

14   There is no reason why one technology is "more essential" than another in our model. For an
      illustration of controversy over relative values of otherwise essential patents, compare Goodman
      and Myers (2005) with Martin and De Meyer (2006).

Assumption 2: *Innovation follows an independent Poisson process. If there are n active R&D projects directed to a technology, the probability that the technology will be discovered before time t is* $1 - e^{-nht}$*, with h > 0.*

Our next assumption greatly simplifies the analysis. Together with Assumption 2, the next assumption allows us to focus on strategies whereby a firm chooses its R&D expenditure conditional solely on the number of innovations completed to date and the distribution of the intellectual property rights for those innovations:

Assumption 3: *The L technologies must be invented sequentially and R&D projects are specific to each technology.*[15]

A. Efficient R&D investment

Let $W(K)$ denote the expected continuation social value of R&D assuming that K technologies have been discovered. Then $W(L) = w$, the social value of the product, and for $K \in \{0,1,2,\ldots,L-1\}$

$$W(K) = \max_{n}\left\{\int_{0}^{\infty}\left[nW(K+1)\right]e^{-(r+n)t}dt - nc\right\} = \max_{n}\left[\frac{nW(K+1)}{n+r} - nc\right]. \tag{1}$$

Let $n^{w}(K)$ denote the socially optimal number of R&D projects when K technologies have been discovered. We treat n^{w} as a continuous variable. From (1) it follows that

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[15]   Assumption 3 can be replaced with the assumption that R&D projects are not targeted to a particular technology. Under this alternative assumption, the optimization program described by equation (1) must be modified by the addition of a constraint that the number of R&D projects undertaken must be non-decreasing in the number of technologies that have been discovered to date. As shown below, this constraint is not binding and the solution derived in the text under Assumption 3 is also valid under the alternative assumption.

$$n^w(K) = \max\left\{ r\left[\left(\frac{W(K+1)}{rc}\right)^{1/2} - 1\right], 0\right\}. \tag{2}$$

Suppose $n^w(K) > 0$ for all K. Then iteratively substituting $n^w(K)$ into (1) for

$K = L - 1, L - 2,...,0$ with $W(L) = w$ yields

$$W(K) = w[1 - (L-K)/\alpha]^2 \quad \text{for } K = 0,1,...,L, \tag{3}$$

where

$$\alpha \equiv \left(\frac{hw}{rc}\right)^{1/2}.$$

Intuitively, $\dfrac{w}{r}$ is a measure of the benefit of more rapid innovation and $\dfrac{c}{h}$ is a measure of

the cost. Hence, α can be interpreted as the benefit of faster innovation relative to the cost.

Without loss of generality, we henceforth scale R&D projects so that $h = 1$.

Substituting equation (3) into equation (2) with $K = 0$ shows that $\alpha \geq L$ is necessary and

sufficient for $n^w(K) > 0$ for all technologies. To insure that the problem is nontrivial, we make

Assumption 4: $\alpha \geq L$.

Given Assumption 4,

$$n^w(K) = r\{\alpha - (L-K)\}. \tag{4}$$

As a further benchmark, we note the monopoly level of investment in R&D, which

analogously maximizes

$$\Pi(K) = \max_n\left\{\int_0^\infty [n\Pi(K+1)]e^{-(r+n)t}dt - nc\right\},$$

with $\Pi(L) = \pi$, the available profit from invention. Paralleling the derivation for the social

optimum, the monopoly level of investment in R&D is

$$n^m(K) = \frac{r}{h}\left\{\beta - (L - K)\right\}$$

where $\beta \equiv \left(\dfrac{h\pi}{rc}\right)^{1/2}$.

With $\pi < w$, it follows immediately that $n^\pi(K) < n^w(K)$ if $n^w(K) > 0$. Absent a subsidy, a single firm will under-invest in R&D. In the next section we explore investment in R&D in a duopoly.

IV. OPTIMAL PAYOFFS FOR DUOPOLY R&D INVESTMENT

We use the welfare results as benchmarks to explore R&D investment incentives by a duopoly for complementary technologies. There are two potential innovators, Firms 1 and 2. Both firms are assumed to have the innovation technology characterized by Assumptions 2 and 3, and Assumption 1 continues to apply to the firms' aggregate payoffs. If a firm is the first to discovery a technology, it receives an infinitely lived patent that excludes others from using that technology. Throughout, we assume that all of the patents are valid and would be infringed by the use in question.[16] The state of the market at time t is determined by the number of discoveries by each firm, (k_1, k_2), where we omit a time subscript to economize on notation. Firm-specific present-value payoffs are $\pi_i(k_1, k_2)$ for $i = 1,2$. We make three additional assumptions regarding these payoffs.

Assumption 5: No intermediate payoffs.

Payoffs to technology owners can occur only after all of the L technologies that are necessary to produce the product have been discovered. Although intermediate progress

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[16]   For analysis of the implications of patents with stochastic validity, see Lemley and Shapiro (2005) and Farrell and Shapiro (2008).

payments are feasible in an environment for contract R&D, the assumption that payoffs depend only on final outcomes is reasonable for a market environment in which profits are derived from useful discoveries.

Assumption 6: Payoffs are non-negative and do not depend on the identity of the firm.

If rewards could depend on the identity of the firm, a social planner could designate a firm and offer the social value of invention, w. Alternatively, the social planner could randomly impose a large enough tax on one of the firms to exclude it from the R&D competition while giving the other firm a reward of w. Either regime would provide efficient R&D incentives, but would turn R&D policy into a game of selecting winners instead of providing a level playing field with incentives for discovery.

Assumption 7: Budget balance. All of the available profit is distributed to the firms engaged in R&D when all of the necessary discoveries have been made.

When there is joint production, as in the theory of teams (see *e.g.,* Holmstrom, 1982), efficient payoffs generally do not have the property that total rewards equal the available profit. Complementary investment has similar properties. Because our focus is on rewards that can be implemented in a market context we assume that total payoffs cannot exceed the available profit from invention (*i.e.,* we do not consider the possibility of R&D subsidies). Later, we consider payoffs that allocate less than the entire available profit to the firms engaged in R&D, corresponding to a tax on invention.

We begin by considering payoffs that support efficient R&D investment in a duopoly when there is only one discovery remaining to produce the commercial product.

A. One Discovery Remaining

Let k_i be the number of discoveries made by Firm i, and consider the continuation game

in which $k_1 + k_2 = L - 1$, so that there is one technology remaining to be discovered. This case is similar to a patent race for a single technology, although firms' incentives to conduct R&D in the present stage may depend on how many patents they obtained in earlier stages.

We let $n_i(k_1, k_2)$ be the equilibrium level of R&D activity chosen by Firm i when Firm 1 has k_1 patents and Firm 2 has k_2 patents. We search for final payoffs $\pi_1(k, L - k)$ and $\pi_2(k, L - k)$ for $k = 0, \ldots, L\text{-}1$ such that the Nash equilibrium investment rates sum to the efficient rate of investment in R&D.

To simplify the notation, define $\pi_1(k) \equiv \pi_1(k, L - k)$ and $\pi_2(k) \equiv \pi_2(k, L - k)$. If Firm 1 has k patents with one discovery remaining, its payoff if it discovers the remaining technology is $\pi_1(k + 1)$ and its payoff if firm 2 discovers the remaining technology is $\pi_1(k)$. Firm 1 chooses a constant n_1 to maximize

$$\Pi_1(k, L - 1 - k) = \int_0^\infty \left[n_1 \pi_1(k + 1) + n_2 \pi_1(k) \right] e^{-(r + (n_1 + n_2))t} dt - n_1 c$$

$$= \frac{n_1 \pi_1(k + 1) + n_2 \pi_1(k)}{n_1 + n_2 + r} - n_1 c \quad . \tag{5}$$

Assuming positive investments by both firms, Firm 1's best response to investment by Firm 2 is

$$N_1(k, L - k - 1) = \left\{ \frac{1}{c} \left[r\pi_1(k + 1) + n_2 \left(\pi_1(k + 1) - \pi_1(k) \right) \right] \right\}^{1/2} - r - n_2 . \tag{6a}$$

Similarly, assuming positive investments by both firms, Firm 2's best response to investment by Firm 1 is

$$N_2(k, L - k - 1) = \left\{ \frac{1}{c} \left[r\pi_2(k) + n_1 \left(\pi_2(k) - \pi_2(k + 1) \right) \right] \right\}^{1/2} - r - n_1 . \tag{6b}$$

If the payoffs support efficient investment in R&D, the Nash equilibrium investments $\hat{n}_1$ and $\hat{n}_2$ satisfy

$$\hat{n}_1(k, L-k-1) + \hat{n}_2(k, L-k-1) = n^w(L-1) = r(\alpha-1). \tag{7}$$

Substituting equations (6a) and (6b) in equation (7) and using $\alpha \equiv \left(\dfrac{w}{rc}\right)^{1/2}$ gives

$$\hat{n}_1(k, L-k-1) = r\left[\frac{w - \pi_2(k)}{\pi_2(k) - \pi_2(k+1)}\right] \tag{8a}$$

and

$$\hat{n}_2(k, L-k-1) = r\left[\frac{w - \pi_1(k+1)}{\pi_1(k+1) - \pi_1(k)}\right]. \tag{8b}$$

These are Nash equilibrium investment rates when payoffs are chosen such that the equilibrium rates sum to the socially efficient rate of investment in R&D with one technology remaining to be discovered.

1.   *Efficient payoffs with one discovery remaining and zero taxes.*

We first consider payoffs with no taxation, corresponding to $\pi_1(k) + \pi_2(k) = \pi$. Because we impose budget balance, it is useful to write payoffs in terms of profit shares. Define the payoff shares $s_i(k) \equiv \pi_i(k)/\pi$ and the following parameter, which appears often in what follows:

$$\theta \equiv \frac{2w}{\pi} - 1.$$

The following proposition derives the unique payoffs shares that support efficient investment in R&D with one remaining discovery under Assumptions 1-7.

Proposition 1: *Suppose there is one discovery remaining: $k_1+k_2=L-1$. Given Assumptions 1-7, if $\alpha \geq \theta L$, then the unique final reward scheme that supports efficient R&D is characterized by*

payoff shares $s_i(k) = \pi_i(k)/\pi$:

$$s_1(k) = \frac{1}{2} + (k - \frac{L}{2})\frac{\theta}{\alpha} \tag{9a}$$

and

$$s_2(k) = \frac{1}{2} - (k - \frac{L}{2})\frac{\theta}{\alpha} . \tag{9b}$$

Proof: An efficient reward function must induce an interior solution in which both firms invest

in R&D because the monopoly investment in R&D is less than the efficient level when $\pi < w$.

We prove in the Appendix that payoff shares (9a) and (9b) support a unique interior Nash

equilibrium. At an interior solution, and using budget balance, equations (8a) and (8b) become

$$\hat{n}_1(k, L - k - 1) = r \left[\frac{\frac{w}{\pi} - 1 + s_1(k)}{s_1(k+1) - s_1(k)} \right] \tag{10a}$$

and

$$\hat{n}_2(k, L - k - 1) = r \left[\frac{\frac{w}{\pi} - s_1(k+1)}{s_1(k+1) - s_1(k)} \right] . \tag{10b}$$

Adding the efficiency condition $\hat{n}_1(k, L - k - 1) + \hat{n}_2(k, L - k - 1) = r(\alpha - 1)$ and using $\theta = \frac{2w}{\pi} - 1$,

it follows that the payoff shares that support efficient investment must satisfy

$$s_1(k+1) - s_1(k) = \frac{\theta}{\alpha} > 0 . \tag{11}$$

Iterating equation (11) forward from $k = 0$ gives

$$s_1(k) = s_1(0) + k\frac{\theta}{\alpha} .$$

The budget balance condition $\pi_1(k) + \pi_2(k) = \pi$ requires $s_1(0) = 1 - s_1(L)$, which implies

$s_1(0) = \frac{1}{2}\left(1 - L\frac{\theta}{\alpha}\right)$. Therefore, $s_1(k) = \frac{1}{2} + (k - \frac{L}{2})\frac{\theta}{\alpha}$, and budget balance also requires

$s_2(k) = \frac{1}{2} - (k - \frac{L}{2})\frac{\theta}{\alpha}$. There are no other payoff shares that support efficient R&D investment

levels as an equilibrium with one discovery to go for all $k = 0,\ldots,L$-1 under Assumptions 1-7.

Furthermore $s_1(0) \geq 0$ requires $\alpha \geq \theta L$. **Q.E.D.**

Efficient investment in R&D cannot be sustained as a Nash equilibrium of the duopoly

game under Assumptions 1-7 if $\alpha < \theta L$. We henceforth replace Assumption 4 with the stronger

condition:

Assumption 4′: $\alpha \geq \theta L$.

Note that direct substitution of the optimal payoff shares in the formulas for the

equilibrium duopoly investment rates gives

$$\hat{n}_1(k, L-k-1) = \frac{1}{2}r[\alpha - L + 2k] \tag{12a}$$

and

$$\hat{n}_2(k, L-k-1) = \frac{1}{2}r[\alpha + L - 2(k+1)] \tag{12b}$$

and clearly $\hat{n}_1(k, L-k-1) + \hat{n}_2(k, L-k-1) = n^w(L-1) = r(\alpha - 1)$.

2. *Efficient payoffs with one discovery remaining and a tax.*

The payoffs in Propositions 1 that support efficient investment in R&D have the

uncomfortable property that a firm that makes no discoveries may claim a strictly positive

reward. This could lead to rent seeking by firms that enter the R&D competition merely to claim

a reward or moral hazard by firms in the R&D competition that apply little effort to win the

competition.

In this section we drop Assumption 7 and do not require that all of the profit from a commercial product be allocated to the firms that make the L discoveries necessary to produce the product. In its place, we make the assumption that a firm that makes no discoveries earns zero rewards and we allow the government to impose a tax rate, $0 \le \tau < 1$, on the new product. Hence, the profit available to the inventors equals $(1-\tau)\pi$.[17]

By choosing an appropriate tax rate τ^*, the government can convert a case of $\alpha > \theta L$ to one in which $\alpha = \theta^* L$, where

$$\theta^* \equiv \frac{2w}{\pi(1-\tau^*)} - 1.$$

A straightforward calculation shows that the corresponding tax rate is $\tau^* = \dfrac{\alpha - \theta L}{\alpha + L}$. Given this tax rate, it follows directly from Proposition 1 with $\alpha = \theta L$ that rewarding each firm with a share of after-tax profits equal to its share of discoveries provides efficient investment incentives.

Proposition 2: *Suppose there is one discovery remaining: $k_1 + k_2 = L - 1$. Given Assumptions 1-6 and $\alpha \ge \theta L$, the following payoff shares and tax rate support efficient R&D investment:*

$$\tau^* = \frac{\alpha - \theta L}{\alpha + L} \tag{13}$$

$$s_1(k) = \frac{k}{L}(1-\tau) \tag{14a}$$

$$s_2(k) = \frac{L-k}{L}(1-\tau). \tag{14b}$$

A key question is the structure of payoffs that support efficient investment in R&D as an outcome of the duopoly game when more than one technology remains to be discovered. A

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17   Note that some states have statutes that permit them to appropriate a share of punitive damages awards. See, e.g., Daughety and Reinganum (2000).

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striking result, which we prove in the next section and in the Appendix, is that the payoff shares in Propositions 1 and 2 support efficient investment in R&D for any number of technologies that remain to be discovered.

B. More than One Discovery Remaining

Let $\Pi_i(k_1, k_2)$ denote firm i's expected continuation payoffs at state (k_1, k_2). By definition $\Pi_i(k_1, k_2) = \pi_i(k_1)$ when $k_1 + k_2 = L$. For $k_1 + k_2 \leq L - 1$, we have the following recursion relationship

$$\Pi_i(k_1, k_2) = \int_0^\infty \left[n_1 \Pi_i(k_1 + 1, k_2) + n_2 \Pi_i(k_1, k_2 + 1) \right] e^{-(r + (n_1 + n_2))t} dt - n_i c$$

$$= \frac{n_1 \Pi_i(k_1 + 1, k_2) + n_2 \Pi_i(k_1, k_2 + 1)}{n_1 + n_2 + r} - n_i c \ ,$$

where n_1 and n_2 are the equilibrium investment levels conditional on k_1 and k_2. Here $\Pi_i(k_1 + 1, k_2)$ and $\Pi_i(k_1, k_2 + 1)$ are the continuation values of the game after one more discovery by Firm 1 or Firm 2. Figure 1 illustrates the possible states of the game and the continuation values when $L = 3$.

The following proposition, proved in the Appendix, establishes that the payoff shares given by equations (9a) and (9b) support efficient investment in R&D for any number of technologies under Assumptions 1-7 provided that $\alpha \geq \theta L$.

Proposition 3: *Given Assumptions 1-7 and $\alpha \geq \theta L$, the profit shares in Proposition 1 support efficient duopoly R&D for all $k_1 + k_2 = 0, 1, ..., L - 1$.*

The proof first entails showing that the payoff shares in Proposition 1, which support efficient R&D with one remaining discovery, also support efficient R&D with two remaining

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discoveries. We do this by showing that the game with two remaining discoveries is equivalent to a game with one remaining discovery with the terminal profit and welfare replaced by the expected profit and welfare at the penultimate stage of the game. We then iterate this approach for all remaining discoveries. The Appendix supplies the details.

More specifically, conditional on $k_1 + k_2 = L - 1$ and socially optimal R&D investment in the last stage of the game, the equilibrium expected welfare and total profit with one remaining discovery are $W(L\text{-}1)$ and $\Pi(L\text{-}1)$. These values assume $k_1 + k_2 = L - 1$, but are independent of the particular values of k_1 and k_2. The Appendix shows that the following payoff shares, applied to the total profit $\Pi(L-1)$, support efficient investment in R&D when $k_1 + k_2 = L - 2$:

$$s_1'(k, L - k - 1) = \frac{1}{2} + (k - \frac{L-1}{2})\frac{\theta'}{\alpha'}$$

and

$$s_2'(k, L - k) = \frac{1}{2} - (k - \frac{L-1}{2})\frac{\theta'}{\alpha'},$$

where

$$\alpha' = \left(\frac{hW(L-1)}{rc}\right)^{\frac{1}{2}},$$

and

$$\theta' = 2\frac{W(L-1)}{\Pi(L-1)} - 1.$$

In other words, the duopoly R&D game with two remaining discoveries can be transformed into a game with one remaining discovery that terminates with the total profit $\Pi(L-1)$ and social welfare $W(L\text{-}1)$. These values are the expected profit and welfare assuming efficient R&D with one remaining discovery. The payoff shares in Proposition 1 support efficient R&D investment for this game, with the level of investment equal to the socially efficient level corresponding to two remaining discoveries. Thus the terminal payoff shares in Proposition 1, which support efficient R&D with one remaining discovery, also support efficient

R&D with two remaining discoveries. This logic can be repeated for any number of remaining discoveries.

Proposition 3 is a central result. It is not at all obvious why the rewards that support efficient investment as the outcome of the duopoly competition with one remaining discovery also should support efficient investment with any number of technologies remaining to be discovered. Yet, under the assumptions in the model, this is indeed the case.

A reward scheme that allocates profit equally to each discovery, along with an optimal tax, also supports efficient investment as the outcome of the duopoly competition with any number of remaining discoveries. This follows immediately because, from Proposition 2, the combination of an optimal tax and equal patent shares results in the same expected payoffs in the penultimate stage of the game as do the optimal payoff shares in Proposition 1. The recursion proof then applies as well to the payoff shares and optimal tax in Proposition 2. We state this result formally.

Proposition 4: *Given Assumptions 1-6 and $\alpha \geq \theta L$, an optimal tax coupled with equal profit shares per innovation support the first-best outcome as an equilibrium of the duopoly game. Specifically, the profit shares in Proposition 2 support efficient duopoly R&D for all* $k_1 + k_2 = 0, 1, ..., L - 1$.

V. DO MARKETS PROVIDE EFFICIENT AWARDS?

Actual patent rewards in market environments can differ substantially from the reward schemes that support efficient investment. For example, we are unaware of the existence of any taxes explicitly intended to curb excess R&D incentives, and we doubt that such taxes are likely to be implemented in practice. Moreover, in contrast to optimal schemes—which feature equal profit shares per innovation—bargaining with the threat of injunction can lead to the same share

of profits for each innovating firm if all patents are essential and firms behave as Nash bargainers with equal reservation values for their patents. This section explores the efficiency implications of alternative reward schemes that offer equal profit shares for each innovation and equal profit shares for each innovator in the absence of corrective taxes.

A. Equal Profit Shares per Innovation

Under a regime of equal profit shares per innovation, firm i receives $\dfrac{k_i \pi}{L}$ when it holds k_i patents and $k_1 + k_2 = L$. A regime of equal profit shares per innovation provides efficient R&D investment incentives when $\alpha = \theta L$ (Proposition 2). When $\alpha > \theta L$, the incremental private return to another innovation exceeds the incremental return with optimal payoff shares and firms have excessive R&D incentives. In the Appendix, we prove

Proposition 5: *Suppose $k_1 + k_2 = L - 1$. If $\alpha > \theta L$, then the firms' aggregate equilibrium investment rate under the equal profit shares per innovation regime exceeds the socially optimal level.*

Numerical simulations suggest that the equal-profit-shares-per-innovation regime also provides excessive R&D incentives when $k_1 + k_2 < L - 1$ and $\alpha > \theta L$. Figure 2 compares equilibrium investment rates under the equal-profit-shares-per-innovation regime to efficient R&D investment rates at different states of the game when $L = 3$. The solid lines in Figure 2 show efficient levels of investment in R&D when there are three, two, or one technology remaining to be discovered. The socially efficient investment rates are independent of the profit from the product holding *w, r, c,* and *h* fixed. The profit levels π in Figure 2 range from the lowest value for which a duopoly equilibrium can support efficient investment (corresponding to

$\theta = \dfrac{\alpha}{L}$, where $\theta = \dfrac{2w}{\pi} - 1$) to the social value, w, corresponding to $\theta = 1$. At the lowest value of

π, equal profit shares per innovation equal the rewards that support efficient investment in R&D.

For higher levels of π, equal profit shares per innovation generate excessive incentives for

investment in R&D.

B. Equal profit shares per innovator

Under a regime of equal profit shares per innovator, each intellectual property owner

receives $\dfrac{\pi}{n}$ when there are n holders of L patents. This regime can be viewed as a reduced form

for a setting in which a patent holder can obtain injunctive relief to block any activity that

infringes its patent, each patent has no value except when used with the $L - 1$ other patents, and

bargaining by the rights holders satisfies the Nash bargaining axioms. When the threat of an

injunction is strong, a firm holding one patent has as much bargaining power as a firm holding

ten. In a duopoly under a regime of equal profit shares per innovation owner, $\pi_i(k) = \dfrac{\pi}{2}$ for all

$k \in \{1, 2, ..., L - 1\}$, $\pi_1(L) = \pi_2(0) = \pi$, and $\pi_1(0) = \pi_2(L) = 0$.

Incentives to invest in R&D are weak under a regime of equal profit shares per innovator

if each firm has at least one patent. Each firm has a payoff of $\dfrac{\pi}{n}$ regardless of the number of

discoveries provided that the total number of technologies required to produce a commercial

product have been discovered. There is no value to preempting a discovery by another firm in

this case as it does not affect the division of profit. The only R&D incentive is the value of

reaching the requisite number of discoveries at an earlier date. But the private payoff from

accelerating commercialization is only $\dfrac{\pi}{n}$, which is less than the social value of the product.

Hence firms have too little incentive to invest in R&D when each firm has at least one patent.

It does not follow that R&D incentives are necessarily too weak in a regime of equal profit shares per innovator. Consider a duopoly in which Firm 1 has $k_1 \le L - 1$ discoveries as of a particular date and Firm 2 has no discoveries. Firm 2 can increase its payoff from zero to $\dfrac{\pi}{2}$ if it makes at least one discovery before Firm 1 makes all of the remaining discoveries, and Firm 1 can obtain a payoff of π instead of $\dfrac{\pi}{2}$ if it succeeds in making all of the remaining discoveries.

Thus, there can be a strong preemption incentive in a regime of equal profit shares per innovator when one of the firms has made all of the discoveries prior to reaching the total number of discoveries required for a commercial product.

Starting at an initial state in which neither firm has made a discovery, there are some parameter values for which R&D incentives are excessive in a regime of equal profit shares per innovator. But for other parameter values R&D incentives are weak relative to the socially optimal level. R&D incentives in the equal profit shares per innovator regime are clearly too weak if profits are a small fraction of the social value of the product. In contrast, a regime of equal profit shares per innovation plus an optimal tax can generate efficient R&D incentives with large social spillovers provided that $\alpha \ge \theta L$.

Even if social spillovers are small, a regime of equal profit shares per innovator is likely to provide insufficient duopoly incentives for R&D if L is large. If L is large, it is likely that most R&D investment will occur in states for which both firms have at least one patent, and in these states the incentives for R&D are weak relative to the socially optimal level. Starting from

an initial state in which neither firm has a patent, the probability of transitioning to a state in which both firms have at least one discovery is positive if both firms invest in R&D and therefore is likely if L is sufficiently large. Furthermore, if a firm with no discoveries stops investing in R&D, then the remaining firm is a monopolist and will invest less than the socially efficient level. It follows that the expected level of R&D is less than the socially optimal level if L is large because either there is a high probability that eventually both firms will have a patent, in which case they will under-invest in R&D, or only one firm will invest in R&D, in which case the firm, as a monopolist, will under-invest in R&D.

Thus, for many economic situations, in particular the economically important case in which a product has large social spillovers, a regime of equal profit shares per innovator, which corresponds to a market environment with injunctive relief for patent infringement, provides insufficient incentives to invest in research and development for complementary innovations. Moreover, the addition of a properly tailored tax to a regime of equal profit shares per innovation can provide socially efficient incentives to invest in R&D.

A regime of equal profit shares per innovator has an additional disadvantage in that it provides incentives for patentees with two or more patents to out-license their patents to independent firms or assign them to separate subsidiaries in order to increase payoffs. For example, suppose $L = 4$, Firm 1 has three patents and Firm 2 has one patent. With this pattern of ownership, each firm receives half of the profit from the commercial product in a regime of equal profit shares per innovator. But Firm 1 could assign its patents to three separate divisions

or license two of them to separate companies. In this way it can increase the total reward for its patents to three-quarters of the profit from the commercial product.[18]

VI. CONCLUSION

When technologies are valuable only when used together, firms undertaking R&D to create these technologies are in a complementary relationship. This complementary relationship can give rise to a free-riding problem because technology owners would like others to incur costly investments to bring a product to fruition. However, the firms are also competitors to the extent that rewards are positively related to the amount of intellectual property that they create, which gives rise to business-stealing effects. We investigated innovation reward schemes that balance these forces to support efficient R&D investment by a duopoly. We found the striking result that, under some assumptions, the reward policy that supports efficient R&D investment for a single technology also supports efficient investment when a product requires the discovery of many complementary technologies.

When $\alpha < \theta L$, efficient investment in R&D cannot be sustained as a Nash equilibrium of the duopoly game absent subsidies. When $\alpha > \theta L$, efficient investment in R&D can be sustained as a Nash equilibrium. The efficient reward policy has the property that the gain from another discovery is less than its share of all of the technologies required to produce a commercial product. That is, if L technologies are essential to produce the product that generates total profit π, then another discovery earns less than π/L. This outcome can be achieved by a reward to the loser of the R&D competition, which squeezes the margin for success, or by taxing the profit and

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[18]   Arguably, Lucent-Alcatel pursued such a strategy. Following Lucent's merger with Alcatel, it assigned patents to a separate organization, the Multimedia Patent Trust, which pursued infringement litigation for its patents.

distributing the remainder to the firms in the R&D competition in proportion to their share of discoveries.

These results are what one expects in a competition for a single discovery, for which a winner-take-all reward can provide too much incentive for investment in R&D in the presence of business stealing effects as long as social spillovers from the discovery are not too large. But it is not intuitive that these results should so directly extend to the case of complementary innovations, where each firm has an incentive to free-ride on discoveries made by others. Nonetheless, we find that the reward structure that supports efficient duopoly R&D with only one remaining discovery also provides efficient incentives for investment in R&D when there are many discoveries remaining that are essential to produce a useful product.

In particular, we find that taxing profit at an appropriate rate and distributing the rest to firms in proportion to their number of discoveries can support efficient investment in R&D for complementary technologies. In principle, this rule could be implemented in an actual policy setting, although not surprisingly the optimal tax depends on various cost and technology parameters, as well as on the number of essential technologies and the extent of social spillovers.

We also investigated the efficiency consequences of alternative reward schemes. A payoff regime that distributes profits in proportion to the number of innovators rather than the number of discoveries corresponds to the outcome of a Nash bargaining game in which firms can obtain injunctive relief against infringement and have no alternative uses for their discoveries. Our analysis suggests that such a payoff regime likely generates too little incentive to invest in R&D when the number of essential technologies is sufficiently large.

Recent legislation introduced in the House and the Senate (but not passed by both chambers) would apportion patent infringement damages to reflect the contribution made by the

patent to the product's value. We find support for such a rule in our analysis, which concludes that a payoff regime that combines equal profit shares per patent with an optimal tax supports efficient R&D investment in the duopoly game when $\alpha \geq \theta L$.[19]

We have derived these results under the assumption that the set of essential technologies is exogenously given. Under some reward schemes (*e.g.,* equal profit shares per innovation) firms could have incentives to engage in strategic behavior to increase the number of patents covering a given technology. Similarly, under a regime of equal profit shares per innovator, a firm holding multiple patents would have incentives to sell all but one of those patents to other firms or assign them to independent subsidiaries in order to increase its share of total profits. Lastly, firms can have incentives to obtain intellectual property rights to what they claim are technologies necessary to offer a product, even if they are not. The regime of equal profit shares per innovator can be particularly vulnerable to this type of behavior because a single intellectual property right can be sufficient to claim a large share of the value of the final product. An interesting issue for future research is to determine how considerations of private strategic behavior shape the socially optimal sharing rules.

We close by observing that our analysis is also of some relevance to patent pools, which offer bundles of intellectual property under package licenses. A bundled license can mitigate the problem of royalty stacking and lower the transactions costs involved in assembling the intellectual property rights necessary to make or sell a product. A patent pool offering such a license has to provide internal governance rules that address the allocation of licensing revenues among its members. Typically, patent pools are formed after key innovations have already been

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[19]   We recognize that such a rule may not be appropriate if some patents are more valuable than others, even if all are "essential" to make or use a particular product. Some patents may be more valuable because they are more likely to be valid or because they are essential for more products.

developed. Hence, the pool's principal concerns in developing its allocation rules are the effects

of those rules on incentives for patent owners to join and remain in the pool. However, prior to

innovation's having occurred (and prior to the formation of the patent pool), potential inventors

will form predictions of how patent pools are likely to divide rewards among their members.[20]

Intellectual property policy can affect private patent pool governance decisions and, thus,

influence the private allocations schemes that potential innovators anticipate when choosing their

R&D investment levels. This is another interesting area for further research.

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[20]   Some patent pools allocate licensing revenues through an internal negotiation over patent values,
and others have lower royalties for patentees that join the pool after its initial formation.
Voluntary patent pools raise interesting issues of commitment and membership that do not arise
in the analysis of legally-imposed sharing regimes. See, e.g., Layne-Farrar and Lerner (2006) and
Lerner, Strojwas and Tirole (2007).

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APPENDIX

A.1   Proof that payoff shares in Proposition 1 support a unique Nash equilibrium if

$\alpha \geq \theta L$:

The best-response functions (6a) and (6b) can be written as

$$N_1(n_2) = \left\{ \frac{\pi}{c} \left[rs_1(k+1) + n_2 \left(s_1(k+1) - s_1(k) \right) \right] \right\}^{1/2} - r - n_2 \qquad (A.1a)$$

and

$$N_2(n_1) = \left\{ \frac{\pi}{c} \left[r(1 - \tau - s_1(k)) + n_1 \left(s_1(k+1) - s_1(k) \right) \right] \right\}^{1/2} - r - n_1. \qquad (A.1b)$$

By direct calculation, there is at least one Nash equilibrium corresponding to the payoff shares in

Proposition 1. The proof that there is at most one follows from the four following facts:

1. The best-response functions (A.1a) and (A.1b) and the optimal payoff shares imply that

    any interior equilibrium must satisfy

    $$n_2 = n_1 + r(L - 1 - 2k) \equiv \varsigma(n_1).$$

2. The optimal payoff shares and (A.1b) imply that any interior equilibrium must satisfy

    $$n_2 = \left\{ \left(\frac{\pi(\theta + 1)}{c(\alpha + \theta)} \right) \left[r(L - k) + n_1 \right] \right\}^{1/2} - r - n_1 \equiv \varphi(n_1)$$

3. If $\alpha \geq \theta L$, then $\varphi(0) > r(L - 1 - 2k) = \varsigma(0)$.

4. $\varphi''(n_1) < 0 = \varsigma''(n_1)$ for all n_1, as can be shown by direct calculation.

    By facts (1) and (2), any interior equilibrium must satisfy $\varphi(n_1) = \varsigma(n_1)$. If the two

functions intersect at least once, then by fact (3), we know that $\varphi(n_1)$ first intersects $\varsigma(n_1)$ from

above. By fact (4), there can be no other intersection. Hence, there is at most one interior

equilibrium.

We next show that an equilibrium in which only one firm invests does not exist.

Suppose $n_1 > 0$. Firm 2 will invest a strictly positive level if

$$\frac{\partial}{\partial n_2}\left\{\pi\left[\frac{n_1 s_2(k+1) + n_2 s_2(k)}{n_1 + n_2 + r}\right] - n_2 c\right\} > 0 \text{ at } n_2 = 0.$$

This condition requires that

$$\frac{\pi}{rc}\left[s_2(k) - \frac{n_1}{n_1 + r}s_2(k+1)\right] > \frac{n_1 + r}{r} \tag{A.2}$$

Given optimal pay-off shares from Proposition 1, the left-hand side of (A.2) is strictly greater

than $\frac{\pi}{rc}\left(\frac{1-\tau}{L}\right) = 2\frac{\alpha^2}{\alpha + L}$, while the right-hand side is less than α, because a monopoly invests at

less than the efficient level (equal to $r(\alpha - 1)$). Thus, if Firm 1 invests, Firm 2 also will invest at

a strictly positive level if

$$2\frac{\alpha^2}{\alpha + L} > \alpha,$$

which is always satisfied if $\alpha \geq \theta L$. The same holds for Firm 1 if Firm 2 invests. Hence, the

unique equilibrium investment levels are the efficient levels identified in Proposition 1. **Q.E.D.**

A.2   Proof of Proposition 3:

The proof proceeds by induction. Given the payoff shares in Proposition 1 that support efficient

investment when $k_1 + k_2 = L - 1$, the equilibrium expected welfare with one remaining discovery

is $W(L - 1) = w\left(\frac{\alpha - 1}{\alpha}\right)^2$ and the equilibrium expected total profit is $\Pi(L - 1)$. Define the

transformed payoff shares:

$$s_1'(k, L - 1 - k) = \frac{1}{2} + (k - \frac{L-1}{2})\frac{\theta'}{\alpha'} \tag{A.3a}$$

$$s_2'(k,L-1-k) = \frac{1}{2} - (k - \frac{L-1}{2})\frac{\theta'}{\alpha'}, \tag{A.3b}$$

where

$$\alpha' = \left(\frac{hW(L-1)}{rc}\right)^{\frac{1}{2}},$$

and

$$\theta' = 2\frac{W(L-1)}{\Pi(L-1)} - 1.$$

We show that the Firm 1's expected profit with one remaining discovery, $\Pi_1(k,L-1-k)$, is equal to $s_1'(k,L-1-k)\Pi(L-1)$. Therefore the duopoly game with two remaining discoveries is equivalent to the duopoly game with one remaining discovery in which the final payoffs are $\Pi(L-1)$ and $W(L-1)$, and Proposition 1 establishes that R&D investment is efficient in this game. The logic extends to any number of remaining investments.

Given $k_1 = k$ and $k_2 = L - k - 1$, Firm 1's expected profit is

$$\Pi_1(k,L-1-k) = \pi\left[\frac{n_1(k,L-1-k)s_1(k+1,L-1-k) + n_2(k,L-1-k)s_1(k,L-k)}{n_1(k,L-1-k) + n_2(k,L-1-k) + r}\right] - n_1(k,L-1-k)c$$

The payoff shares in Proposition 1 imply that $n_1(k,L-k-1) + n_2(k,L-k-1) = n^w(L-1)$.

Using this result and the expressions for the payoff shares, the sum of the firms' payoffs with one discovery remaining is

$$\Pi(L-1) = \pi\left[\frac{n^w(L-1)}{n^w(L-1) + r}\right] - n^w(L-1)c = \frac{\alpha-1}{\alpha}(\pi - \alpha rc). \tag{A.4}$$

Furthermore, direct calculation of equation (A.4), noting that $n_1(k,L-1-k) = \frac{1}{2}r(\alpha - L + 2k)$

and $n_2(k,L-1-k) = \frac{1}{2}r(\alpha + L - 2(k+1))$ yields

$$\Pi_1(k,L-k-1) = \pi\left[\frac{1}{2}\left(\frac{\alpha-1}{\alpha}\right) + \left(k - \frac{L-1}{2}\right)\frac{\theta}{\alpha}\right] - \alpha rc\left[\frac{1}{2}\left(\frac{\alpha-1}{\alpha}\right) + \left(k - \frac{L-1}{2}\right)\frac{1}{\alpha}\right] \tag{A.5}$$

A laborious calculation shows that equation (A.5) is equivalent to

$\Pi_1(k, L-1-k) = \Pi(L-1)s_1'(k, L-1-k)$ with $s_1'(k, L-1-k)$ given by equation (A.3a). A similar calculation shows that $\Pi_2(k, L-1-k) = \Pi(L-1)s_2'(k, L-1-k)$.

The proof assumes that $s_1'(k, L-1-k) \geq 0$, which requires that $\alpha' \geq \theta'(L-1)$. This condition must hold at each stage for the payoff shares in equations (A.3a,b) to support efficient investment in R&D. Direct calculation using the expressions for α', θ', $\Pi(L-1)$ and $W(L-1)$ shows that $\alpha' \geq \theta'(L-1)$ if $\alpha \geq \theta L$. By induction, this result implies that $\alpha \geq \theta L$ is a sufficient condition for the payoff shares to be non-negative for any values of k_1 and k_2. **Q.E.D**

A.3 Proof of Proposition 5:

Label the best response functions under the equal profit shares per innovation regime with the superscript "p." From equations (6a) and (6b) with $\pi_1(k, L-k) = \pi \dfrac{k}{L}$ and $\pi_2(k, L-k) = \pi \dfrac{L-k}{L}$, we have

$$N_1^p(n_2; k, L-1-k) = \left\{ \frac{\pi}{cL}\left[r(k+1) + n_2\right]\right\}^{1/2} - r - n_2, \qquad (A.6a)$$

and

$$N_2^p(n_1; k, L-1-k) = \left\{ \frac{\pi}{cL}\left[r(L-k) + n_1\right]\right\}^{1/2} - r - n_1. \qquad (A.6b)$$

These two equations imply that, in equilibrium, $r(k+1) + n_2 = r(L-k) + n_1$, or

$$n_2 = r(L-1-2k) + n_1. \qquad (A.7)$$

One can simultaneously solve equations (A.6a,b) and (A.7) to find the equilibrium investment levels:

$$2n_1 + r(L-2k) - \left\{ \frac{\pi}{cL}\left[r(L-k) + n_1\right]\right\}^{1/2} = 0. \qquad (A.8)$$

Equation (A.8) is equivalent to

$$4n_1^2 + \left[4z - y\right]n_1 + \left[z^2 - y(z + rk)\right] = 0$$

where $z \equiv r(L - 2k)$ and $y \equiv \dfrac{\pi}{cL}$.

The roots of this quadratic equation are

$$n_1 = \frac{-B \pm (B^2 - 4AC)^{1/2}}{2A} \quad,$$

where $A = 4$, $B = 4z - y$, and $C = z^2 - y(z + rk)$. By assumption, $\beta \equiv \left(\dfrac{\pi}{rc}\right)^{1/2} > L$, which

implies that $y > rL$. It is evident that $z \le rL$. Hence, C is negative. Because $C < 0 < A$, only

the larger root is admissible, and

$$n_1^P = -\tfrac{1}{2}z + \tfrac{1}{8}\left(y + \sqrt{y^2 + 8yrL}\right). \tag{A.9}$$

Under the efficient-investment regime (denoted by *),

$$N_2^*(n_1; k, L - 1 - k) = \left\{\left(\frac{r\alpha\theta}{\theta + 1}\right)\left[r(L + \frac{\alpha}{\theta} - 2k) + 2n_1\right]\right\}^{1/2} - r - n_1 \ .$$

It is readily shown that (A.7) must be satisfied in this regime as well. Hence, the equilibrium

investment levels under the efficient-investment regime can be found by solving

$$2n_1 + r(L - 2k) - \left\{\left(\frac{r\alpha\theta}{\theta + 1}\right)\left[r(L + \frac{\alpha}{\theta} - 2k) + 2n_1\right]\right\}^{1/2} = 0 \ ,$$

which is equivalent to

$$4n_1^2 + \left[4z - 2x\right]n_1 + \left[z^2 - x(z + r\frac{\alpha}{\theta})\right] = 0 \ ,$$

where $z \equiv r(L - 2k)$ and $x \equiv r\dfrac{\alpha\theta}{\theta + 1}$.

The roots of this quadratic equation are

$$n_1 = \frac{-\hat{B} \pm (\hat{B}^2 - 4\hat{A}\hat{C})^{1/2}}{2\hat{A}} \quad ,$$

where $\hat{A} = 4$, $\hat{B} = 4z - 2x$, and $\hat{C} = z^2 - x(z + r\frac{\alpha}{\theta})$. Observe that $\hat{C} < 0$ if $x > z$. For $x \le z$, $\hat{C}$

is increasing in z. Thus, taking $k = 0$, $\hat{C}$ is bounded from above by $\hat{C} \le (rL)^2 - xr(L + \frac{\alpha}{\theta})$.

Recall $\alpha > \theta L$. Hence,

$$\hat{C} \le (rL)^2 - 2xrL = r^2 L\left(L - 2\frac{\alpha\theta}{\theta+1}\right) < r^2 L^2 \left(1 - 2\frac{\theta^2}{\theta+1}\right) < 0.$$

Because $\hat{C} < 0 < \hat{A}$, only the larger root is admissible, and

$$n_1{}^* = -\tfrac{1}{2}z + \tfrac{1}{8}\left(2x + \sqrt{(2x)^2 + \frac{16xr\alpha}{\theta}}\right) \quad . \tag{A.10}$$

By equations (A.9) and (A.10),

$$n_1^p - n_1^* = y + \sqrt{y^2 + 8yrL} - 2x - \sqrt{(2x)^2 + \frac{16xr\alpha}{\theta}} \quad .$$

Observe that

$$2x = r\frac{2\alpha\theta}{\theta+1} \quad ,$$

$$y \equiv \frac{\pi}{cL} = r\frac{2}{\theta+1}\frac{\alpha^2}{L} = 2x(\frac{\alpha}{\theta L}) \equiv 2x\lambda \quad ,$$

where $\lambda \ge 1$, and $\frac{16xr\alpha}{\theta} = 8yrL$. Therefore, $n_1^p - n_1^* > 0$. The fact that (A.7) holds for both

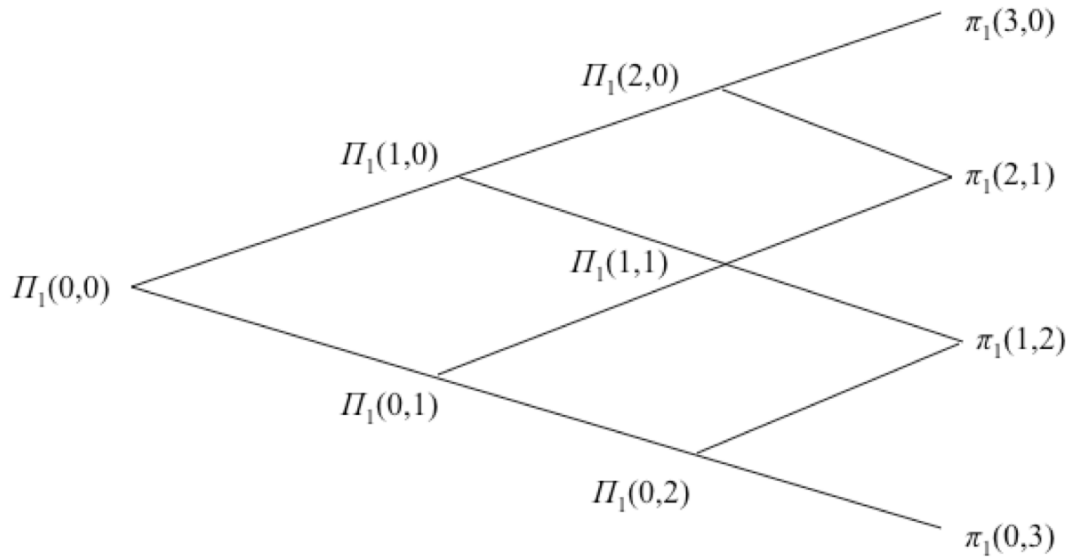regimes implies that $n_2^p - n_2^* > 0$ as well. **Q.E.D.**

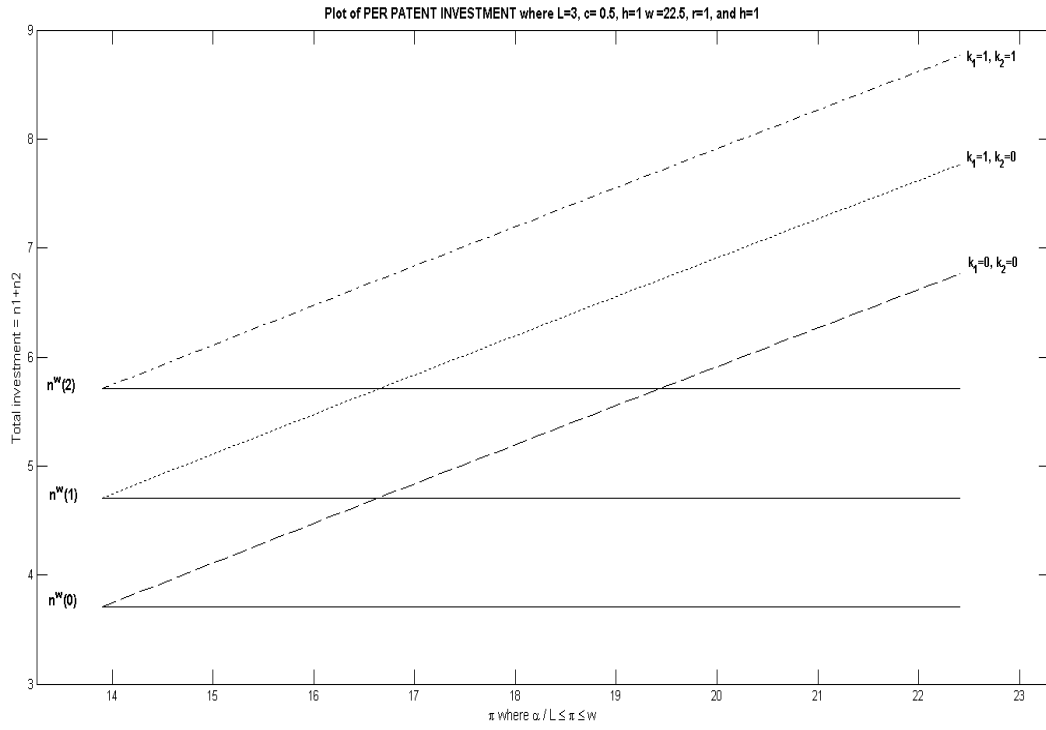**Figure 1. The states corresponding to $L = 3$.**

Figure 2. Investment rates with per-patent payoffs corresponding to different discovery states when $L = 3$.